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Giovanni C. Porzio

Carla Rampichini

Chiara Bocci

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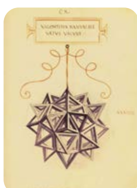

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Contributed Papers

UNCONDITIONAL M-QUANTILE REGRESSION

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ABSTRACT: In this paper we develop the unconditional M-quantile regression for modeling unconditional M-quantiles in the presence of covariates. Extending the paper by Firpo *et al.* (2009), we assess the impact of small changes in the explanatory variables on the M-quantile of the unconditional distribution of the dependent variable by running a mean regression of the recentered influence function of the unconditional M-quantile on the covariates. The proposed methodology is applied on the Survey of Household Income and Wealth (SHIW) 2016 conducted by the Bank of Italy.

KEYWORDS: Influence function, M-estimation, RIF regression, Robust method

1 Introduction

Quantile Regression (QR), as proposed by Koenker & Bassett Jr (1978), has proven to be a powerful tool to explore conditional distributions in many empirical applications. However, if one is interested in how the whole unconditional distribution of the dependent variable responds to changes in the covariates, using the well-known QR would yield misleading inferences (see Firpo *et al.* 2009 and Borah & Basu 2013). Motivated by this interest, Firpo *et al.* (2009) proposed the Unconditional Quantile Regression (UQR) approach for modeling unconditional quantiles of a dependent variable as a function of the explanatory variables. This method builds upon the concept of Recentered Influence Function (RIF) which originates from a widely used tool in robust statistics, namely the Influence Function (IF) discussed in Hampel *et al.* (2011). The RIF of a distributional statistic ν is obtained by adding back the statistic to the IF and it can be thought of as the contribution of an individual observation on ν. In the regression framework where covariates are available, Firpo *et al.* (2009) proposed to replace the dependent variable with the RIF to model the unconditional quantiles of the response and evaluate the effect of changes in the law of the covariates on unconditional quantiles. When the interest of the research is concentrated on the entire distribution of a response variable, in addition to the classical QR, a possible alternative is represented by the M-quantile regression (MQR) approach proposed by Breckling & Chambers (1988). This method provides a "quantile-like" generalization of the mean regression based on influence functions, combining in a common framework the robustness and efficiency properties of quantiles and expectiles (Newey & Powell 1987), respectively.

In this article, we extend the UQR of Firpo *et al.* (2009) to the M-quantile regression framework. We develop the Unconditional M-quantile Regression (UMQR) to model the M-quantiles of the unconditional distribution of the response variable. In order to analyze how the entire unconditional distribution of the outcome is affected by changes in the distribution of explanatory variables, we regress the RIF of the unconditional M-quantile on the covariates and denote such effect as Unconditional M-Quantile Partial Effect (UMQPE).

2 Methodology

Let Y denote a scalar random variable with absolutely continuous distribution function F_Y. The M-quantile of order $\tau \in (0,1)$ of Y is defined as the solution, $\theta_\tau \in \mathbb{R}$, of the following estimating equation:

$$\int \psi_\tau(y-\theta_\tau)dF_Y(y) = 0, \tag{1}$$

where $\psi_\tau(u) = \mid \tau - \mathbf{1}_{(u<0)} \mid \psi(u/\sigma_\tau)$, with ψ being the first derivative of a convex loss function ρ and σ_τ is a suitable scale parameter. In this work, we consider the well-known Huber influence function (Huber (1964)):

$$\psi(u) = u\mathbf{1}_{(|u|\leq c)} + c\,\mathrm{sign}(u)\mathbf{1}_{(|u|>c)}, \tag{2}$$

where c denotes a tuning constant bounded away from zero that can be used to trade robustness for efficiency in the model fit. In particular, M-quantiles nicely include quantiles when $c \to 0$, $\psi(u) = \mathrm{sign}(u)$, and expectiles when $c \to \infty$, $\psi(u) = u$.

To build the UMQR model, it follows from Firpo *et al.* (2009) and Hampel *et al.* (2011) that the RIF of the M-quantile θ_τ is defined as:

$$RIF(y;\theta_\tau) = \theta_\tau + IF(y;\theta_\tau) = \theta_\tau + \frac{\psi_\tau(y-\theta_\tau)}{\int \psi'_\tau(y-\theta_\tau)dF_Y(y)}, \tag{3}$$

where $IF(y;\theta_\tau)$ is the IF of θ_τ and $\psi'(u) = \mathbf{1}_{(|u|<c)}$ is the derivative of ψ in (2). In a regression framework when covariates $\mathbf{X} \subset \mathbb{R}^k$ are available, from (3) we define the UMQR model as follows:

$$\mathbb{E}[RIF(Y;\theta_\tau) \mid \mathbf{X} = \mathbf{x}] = \theta_\tau + \mathbb{E}\Big[\frac{\psi_\tau(y-\theta_\tau)}{\int \psi'_\tau(y-\theta_\tau)dF_Y(y)}\Big|\mathbf{X} = \mathbf{x}\Big]. \tag{4}$$

Our objective is to identify how small changes in the distribution of $\mathbf{X}$ affect the M-quantile of the unconditional distribution of Y. From (4) and Firpo *et al.* (2009), the unconditional effect of the τ-th M-quantile, that we denote Unconditional M-quantile Partial Effect, α_τ, is formally defined as:

$$\alpha_\tau = \int \frac{d\mathbb{E}[RIF(Y;\theta_\tau) \mid \mathbf{X} = \mathbf{x}]}{d\mathbf{x}} dF_{\mathbf{X}}(\mathbf{x}) = \frac{1}{s_\tau} \int \frac{d\mathbb{E}[\psi_\tau(Y-\theta_\tau) \mid \mathbf{X} = \mathbf{x}]}{d\mathbf{x}} dF_{\mathbf{X}}(\mathbf{x}), \tag{5}$$

where $F_{\mathbf{X}}$ is the distribution function of $\mathbf{X}$ and $s_\tau = \int \psi'_\tau(y-\theta_\tau)dF_Y(y)$. As suggested by Firpo *et al.* (2009), we can estimate α_τ in (5) via a mean regression of the $RIF(Y;\theta_\tau)$ as dependent variable onto $\mathbf{X}$ by using a two-step procedure. Specifically, an estimate $\widehat{\theta}_\tau$ of θ_τ is obtained by solving (1) via Iterative Reweighted Least Squares, substitute $\widehat{\theta}_\tau$ in (3) and then regress the $RIF(Y;\widehat{\theta}_\tau)$ on $\mathbf{X}$.

3 Application

We investigate the effect of economic and socio-demographic characteristics on italian households' log-consumption using data from the SHIW 2016. We fit the UMQR at different points of the unconditional distribution of the response and compare the results with standard conditional M-quantile regressions. The tuning constant c in (2) has been set to 1.345 and 100. In the second case, we obtain the Unconditional Expectile Regression (UER). The results in Table 1 highlight that the impact of income, gender, age and education is very different on the conditional and unconditional distributions of consumption, especially in the tails. This demonstrates the ability of the UMQR to extend mean regression for estimating the effect of covariates, not only at the center, but also at different parts of the unconditional distribution of interest.

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Variable	MQR			UMQR			ER			UER		
τ	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
Log-Income	**0.570**	**0.595**	**0.442**	**0.447**	**0.391**	**0.429**	**0.483**	**0.413**	**0.263**	**0.450**	**0.413**	**0.436**
	(0.011)	(0.007)	(0.010)	(0.038)	(0.032)	(0.038)	(0.011)	(0.008)	(0.011)	(0.038)	(0.033)	(0.038)
Gender	−0.019	−0.011	**−0.043**	−0.011	**−0.024**	**−0.038**	−0.023	**−0.026**	**−0.046**	−0.010	**−0.026**	−0.035
	(0.016)	(0.009)	(0.014)	(0.018)	(0.012)	(0.018)	(0.016)	(0.011)	(0.016)	(0.017)	(0.012)	(0.018)
Age	−0.002	0.001	0.004	**−0.013**	**0.006**	**0.013**	−0.001	0.004	**0.008**	**−0.011**	0.004	**0.011**
	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)
Marital status												
never married	**−0.062**	**−0.084**	**−0.164**	**−0.094**	**−0.141**	**−0.187**	**−0.095**	**−0.138**	**−0.201**	**−0.101**	**−0.138**	**−0.176**
	(0.020)	(0.012)	(0.018)	(0.025)	(0.017)	(0.022)	(0.020)	(0.014)	(0.020)	(0.024)	(0.017)	(0.022)
separated	**−0.066**	**−0.056**	**−0.127**	**−0.102**	**−0.151**	**−0.155**	**−0.111**	**−0.137**	**−0.207**	**−0.105**	**−0.137**	**−0.141**
	(0.025)	(0.015)	(0.022)	(0.034)	(0.024)	(0.030)	(0.025)	(0.017)	(0.026)	(0.033)	(0.024)	(0.030)
widowed	−0.040	**−0.063**	**−0.119**	**−0.116**	**−0.136**	**−0.111**	**−0.074**	**−0.123**	**−0.193**	**−0.110**	**−0.123**	**−0.107**
	(0.022)	(0.013)	(0.020)	(0.029)	(0.019)	(0.025)	(0.022)	(0.015)	(0.022)	(0.028)	(0.019)	(0.025)
Education level												
elementary school	**0.175**	**0.120**	**0.151**	**0.488**	**0.125**	−0.037	**0.188**	**0.161**	**0.187**	**0.446**	**0.161**	−0.000
	(0.039)	(0.023)	(0.035)	(0.069)	(0.024)	(0.022)	(0.039)	(0.027)	(0.040)	(0.066)	(0.027)	(0.022)
middle school	**0.240**	**0.203**	**0.316**	**0.645**	**0.269**	**0.060**	**0.281**	**0.294**	**0.398**	**0.590**	**0.294**	**0.094**
	(0.041)	(0.024)	(0.037)	(0.070)	(0.028)	(0.029)	(0.041)	(0.028)	(0.042)	(0.067)	(0.030)	(0.028)
high school	**0.248**	**0.235**	**0.383**	**0.652**	**0.355**	**0.147**	**0.313**	**0.363**	**0.500**	**0.598**	**0.363**	**0.168**
	(0.042)	(0.025)	(0.038)	(0.072)	(0.033)	(0.037)	(0.042)	(0.029)	(0.043)	(0.069)	(0.034)	(0.036)
university	**0.298**	**0.297**	**0.521**	**0.631**	**0.440**	**0.506**	**0.391**	**0.484**	**0.705**	**0.608**	**0.484**	**0.515**
	(0.045)	(0.027)	(0.040)	(0.076)	(0.040)	(0.053)	(0.045)	(0.031)	(0.046)	(0.073)	(0.042)	(0.052)
Employment status												
self-employed	**−0.087**	0.010	**0.083**	**−0.060**	0.021	**0.121**	**−0.058**	0.023	**0.081**	**−0.046**	0.023	**0.107**
	(0.024)	(0.014)	(0.022)	(0.021)	(0.019)	(0.038)	(0.024)	(0.017)	(0.025)	(0.020)	(0.018)	(0.037)
not-employed	0.008	**0.027**	0.035	−0.046	**0.037**	0.037	−0.002	0.014	0.017	**−0.052**	0.014	0.031
	(0.021)	(0.013)	(0.019)	(0.025)	(0.016)	(0.025)	(0.021)	(0.015)	(0.022)	(0.024)	(0.015)	(0.024)

Table 1. *M-quantile and Expectile regression results at* $\tau = (0.1, 0.5, 0.9)$. *Parameter estimates are displayed in boldface when significant at the 5% level.*

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