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Hypotheses testing in mixed–frequency volatility models: a bootstrap approach

Test d'ipotesi nei modelli di volatilità a frequenza mista: un approccio bootstrap

Vincenzo Candila, Lea Petrella

Abstract It is widely recognized that standard likelihood–based inference suffers from the presence of nuisance parameters. This problem is particularly relevant in the context of Mixing–Data Sampling (MIDAS) models, when volatility forecasting is the research topic and where often covariates' data are sampled at a different (usually lower) frequency than the asset returns. In this framework, testing the significance of the MIDAS terms brings together the presence of nuisance parameters that under the null hypothesis are not identifiable. This circumstance interferes with the asymptotic distribution of the common statistical tests employed in this framework. In particular, the asymptotic distribution is no more a χ^2 distribution. The present paper proposes a bootstrap likelihood ratio (BLR) test to overcome this problem, simulating the likelihood ratio test distribution. Using a Monte Carlo experiment, the proposed BLR test presents quite good performances in terms of the test's size and power.
Abstract *E' ampiamente riconosciuto che gli approcci inferenziali basati sulla massima verosimiglianza soffrano della presenza di nuisance parameters. Questo problema è particolarmente rilevante nel contesto di modelli Mixing–Data Sampling (MIDAS), usati nell'ambito delle previsioni di volatilità. In questo framework, testare la significatività dei termini MIDAS comporta la gestione dei nuisance parameters che, sotto l'ipotesi nulla, sono non identificabili. Questa circostanza interferisce con la distribuzione asintotica dei test statistici comunemente usati in questo ambito. In particolare, la distribuzione asintotica non risulta più essere una* χ^2*. Il presente lavoro propone un bootstrap likelihood ratio (BLR) test per superare questo problema, simulando la distribuzione del likelihood ratio test. Attraverso una simulazione Monte Carlo, il test BLR proposto presenta ottime performance, in termini di size e potenza.*

Key words: Likelihood ratio test, MIDAS, nuisance parameter, bootstrap.

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1 Introduction

The financial econometrics literature has paid particular attention to the estimation of asset returns volatility during the last four decades. In this framework, empirical evidences suggest that the volatility has a slow–moving feature around which the conditional second moments of returns oscillate. Starting from this characteristic, a new type of volatility models, based on the decomposition of volatility into two components, namely a short and a long–run component, has been proposed (for more details, see the review of Amado et al., 2019). At the same time, it is quite common in financial data analysis that observations came at a different frequency (usually lower) than the returns' ones. The Mixing–Data Sampling (MIDAS) methods proposed by Ghysels et al. (2007) are designed to solve this problem. When the MIDAS techniques are applied within the GARCH framework, the long–run component of the models can depend on variables observed at different frequencies than daily (see, for example, Engle et al. (2013) and Conrad and Kleen (2020)). Recently, the MIDAS methods have also been applied in the quantile regression framework to forecast the Value–at–Risk (Candila et al., 2020).

Unfortunately, as stated in Ghysels et al. (2007), testing the null hypothesis of no influence of the MIDAS component can be problematic since the weights associated with each realization of the low–frequency variable, seen as nuisance parameters, are not identifiable. This circumstance has a fundamental impact on the asymptotic distribution of the commonly used tests, like the Wald or the Likelihood Ratio (LR) tests (see Hansen (1996) and Andrews (2001) for a complete survey on this topic).

In the context MIDAS variables within a volatility model, our paper aims at investigating the profitability of using a bootstrap LR (BLR) test where the distribution of the test is obtained using a suitable bootstrap procedure. Resorting to the bootstrap to derive the LR test distribution is not new at all: see, for instance, the contributions of Di Sanzo (2009) and Busetti and Di Sanzo (2012). But this is the first time the BLR test is used within the volatility models employing MIDAS components.

In terms of results, the size and power of the proposed BLR are calculated through an extensive Monte Carlo experiment in a GARCH model framework. Comparing the results with the standard LR test, the BLR appears to have an empirical size closer to the nominal one and quite good empirical power.

The rest of the paper is as follows: Section 2 illustrates the models and the proposed BLR test, while Section 3 presents the Monte Carlo experiment.

2 Bootstrap Likelihood Ratio test

Let $r_{i,t}$ be the log–return of an asset representing the first log–difference of the closing prices for the day i in the period (week or month) t. Then, let us consider the formalization of the GARCH–MIDAS model proposed by Engle et al. (2013):

$$r_{i,t} = \sigma_{i,t}\varepsilon_{i,t} = \sqrt{\tau_t \times g_{i,t}}\varepsilon_{i,t}, \quad \text{with} \quad i = 1,\cdots,N_t \quad \text{and} \quad t = 1,\cdots,T, \tag{1}$$

where, $\sigma_{i,t}$ representing the conditional standard deviation at day i and period t, consists of two (multiplicative) components: τ_t and $g_{i,t}$. In particular, τ_t is defined as the long–run component of the volatility at period t and $g_{i,t}$ the short–run term at day i for period t. Moreover, $N = \sum_{t=1}^{T} N_t$ is the total number of days considered with N_t being the number of days in the period t. In Eq. (1), $\varepsilon_{i,t}$ is the *iid* innovation term, with $E\left(\varepsilon_{i,t}\right) = 0$ and $E\left(\varepsilon_{i,t}^2\right) = 1$, and with a finite fourth moment.

Following the common dynamics specifications of the short– and the long–run components proposed in the GARCH–MIDAS literature, we consider for $g_{i,t}$ the unit–mean reverting GJR–GARCH(1,1) process given by:

$$g_{i,t} = (1-\alpha-\gamma/2-\beta) + \left(\alpha + \gamma \cdot \mathbb{1}_{\left(r_{i-1,t}<0\right)}\right)\frac{\left(r_{i-1,t}\right)^2}{\tau_t} + \beta g_{i-1,t}, \tag{2}$$

where $\mathbb{1}_{(.)}$ is an indicator function and $\alpha > 0$; $\beta \geq 0$; $\gamma \geq 0$; $\alpha + \beta + \gamma/2 < 1$.

The component τ_t is:

$$\tau_t = \exp\left(m + \theta \sum_{k=1}^{K} \delta_k(\omega) MV_{t-k}\right), \tag{3}$$

where $m \in R$, $\theta \in R$ represents the response to the one–sided filter of the past K realizations of the MIDAS terms i.e. the low–frequency variable MV_t through the weighting function $\delta_k(\omega)$. The most common used $\delta_k(\omega)$ in this context is the Beta function:

$$\delta_k(\omega) = \frac{(k/K)^{\omega_1-1}(1-k/K)^{\omega_2-1}}{\sum_{j=1}^{K}(j/K)^{\omega_1-1}(1-j/K)^{\omega_2-1}}. \tag{4}$$

Under this configuration, the parameter space is then $\Theta = \{\alpha,\gamma,\beta,m,\theta,\omega_1,\omega_2\}$. Given K and a distributional assumption for $\varepsilon_{i,t}$ in (1) it is possible to calculate the maximum likelihood (ML) estimator for Θ.

In order to test the significance of the MIDAS component in (3), the following null hypothesis is considered:

$$H_0 : \theta = 0. \tag{5}$$

Typically, one can evaluate such a null using the a Wald or a LR test. We focus on this latter case. Let $\widehat{\Theta}_0$ be the ML estimate of Θ under the null $\theta = 0$, that is the "restricted" model. The correspondent log–likelihood at $\widehat{\Theta}_0$ is denoted by $\ell\left(\widehat{\Theta}_0\right)$. Let $\widehat{\Theta}$ be the ML estimate of Θ under the alternative $\theta \neq 0$ i.e. in the "unrestricted" model. The corresponding log–likelihood at $\widehat{\Theta}$ is denoted by $\ell\left(\widehat{\Theta}\right)$. The LR test is:

$$LR = 2\left[\ell(\widehat{\Theta}) - \ell(\widehat{\Theta}_0)\right]. \tag{6}$$

Assuming a significance level α, test statistic in (6) should reject H_0 when

$$LR > CV_\alpha, \tag{7}$$

where CV_α is the $(1-\alpha)th$ quantile of the LR distribution under the null. Under some regularity conditions, it can be shown that the LR test follows asymptotically a Chi–square (χ^2) distribution. In our context, since under the null hypothesis in (5) the parameters ω_1 and ω_2 in (4) are not identified, the distribution of LR in (6) is no more a χ^2 distribution. For this reason, here we propose a bootstrap procedure to simulate the distribution of LR test (6) under the null (5). The proposed BLR procedure is as follows:

1. Estimate the unrestricted and restricted models. Compute the LR statistic as in Eq. (6).
2. Let $\widehat{\sigma}_{i,t}$ be the estimated volatility obtained from the restricted model. Compute the standardized residuals $\widehat{\varepsilon}_{i,t}$ under the null, for $i = 1, \cdots, N_t$ and $t = 1, \cdots, T$, that is:
$$\widehat{\varepsilon}_{i,t} = \frac{r_{i,t}}{\widehat{\sigma}_{i,t}}.$$
Let $\widehat{\varepsilon}^*_{i,t}$, be the bootstrap residual, obtained from resampling with replacement from the standardized residual series $\widehat{\varepsilon}_{i,t}$.
3. Compute the bootstrap replicates of $r_{i,t}$, denoted by $r^*_{i,t}$, through:
$$r^*_{i,t} = \widehat{\sigma}^*_{i,t}\widehat{\varepsilon}^*_{i,t},, \quad \text{for} \quad i = 1, \cdots, N_t \quad \text{and} \quad t = 1, \cdots, T,$$
where $\widehat{\sigma}^*_{i,t}$ is the bootstrap volatility:
$$\widehat{\sigma}^*_{i,t} = \sqrt{\widehat{\tau}^*_t \times \widehat{g}^*_{i,t}},$$
with the long–run term under the null identified as $\widehat{\tau}^*_t = \exp(\widehat{m})$ and the short–run term as
$$\widehat{g}^*_{i,t} = \left(1 - \widehat{\alpha} - \widehat{\gamma}/2 - \widehat{\beta}\right) + \left(\widehat{\alpha} + \widehat{\gamma} \cdot \mathbb{1}_{\left(r^*_{i-1,t} < 0\right)}\right) \frac{\left(r^*_{i-1,t}\right)^2}{\widehat{\tau}^*_t} + \widehat{\beta}\widehat{g}^*_{i-1,t},$$
where $\widehat{\alpha}, \widehat{\gamma}, \widehat{\beta}$ and $\widehat{m}$ are the ML estimates of the restricted model. In order to obtain (recursively) the bootstrap realizations of $r^*_{i,t}$ for the N days, start the procedure with $\widehat{\sigma}^*_{1,t} = \widehat{\sigma}_{1,t}$. Finally, estimate the restricted and unrestricted models on the series $r^*_{i,t}$. Hence, calculate the LR on the bootstrap returns $r^*_{i,t}$, denoted by LR^*.
4. Repeat the previous step B times, obtaining $\left(LR^{*(1)}, \cdots, LR^{*(B)}\right)$, which is the bootstrap distribution of LR.
5. The estimate of (the bootstrap) CV_α, based on $\left(LR^{*(1)}, \cdots, LR^{*(B)}\right)$ and labelled as $\widehat{CV}_\alpha$, is obtained as the $1-\alpha$ quantile of the bootstrap distribution of LR.

   Finally, the null in (5) is rejected through the BLR test if $LR > \widehat{CV}_\alpha$.

3 Monte Carlo Experiment

In this section, we consider a Monte Carlo experiment to learn about the profitability of using the BLR test when testing MIDAS components. For this goal, we generate R samples of data from the following data generating process (DGP):

$$r_{i,t} = \sqrt{\tau_t \times g_{i,t}}\varepsilon_{i,t}, \quad \text{with} \quad i = 1,\cdots,N_t, \quad \text{and} \quad t = 1,\cdots,T, \tag{8}$$

where:

$$\varepsilon_{i,t} \sim iid\ t_{(7)}, \tag{9}$$

$$\tau_t = \exp\left(m_0 + \theta_0 \sum_{k=1}^{K} \delta_k(\omega) MV_{t-k}\right), \tag{10}$$

$$g_{i,t} = (1-\alpha_0-\gamma_0/2-\beta_0) + \left(\alpha_0 + \gamma_0 \cdot \mathbb{1}_{(r_{i-1,t}<0)}\right)\frac{(r_{i-1,t})^2}{\tau_t} + \beta_0 g_{i-1,t}. \tag{11}$$

In Eq. (9), the error term $\varepsilon_{i,t}$ follows a standardized Student's t distribution with 7 degrees of freedom which allows for fat tails of real financial asset returns. We assume that the simulated stationary variable MV_t follows an AR(1): $MV_t = \varphi MV_{t-1} + e_t$, with $\varphi = 0.7$.

Using the R package *rumidas* (Candila, 2021), the DGP in (9) is simulated $R = 250$ times, according to two sample sizes: $N = \{500, 1000\}$. The true values of the parameters are:

$$\{\alpha_0 = 0.01, \gamma_0 = 0.1, \beta_0 = 0.9, m_0 = -1, \omega_{2,0} = 1.1\}.$$

The parameter of interest θ has instead the following values: $\theta_0 = \{0, 0.5, 1\}$.

The results of our experiment are illustrated in Table 1, where the estimated probabilities of rejecting the null across the R replicates are reported. More in detail, Panel A shows the empirical sizes for the BLR and the LR tests that is the occurrence of null rejection when the null is true. The results of the LR test are evaluated using the χ^2 distribution. Independently of the significance level adopted (0.01, 0.05, and 0.1) and of the sample length, the empirical size of the BLR appears much more in line with the actual size. When the null is false, as in Panels B and C, both the tests appear to have reasonable powers. These results support the use of the proposed BLR test instead of the LR one when mixed frequency models are employed.

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Table 1 BLR and LR empirical sizes and powers

Signif. level	0.01	0.05	0.10	0.01	0.05	0.10
Panel A: $\theta = 0$		$N = 500$			$N = 1000$	
BLR	0.012	0.056	0.136	0.016	0.040	0.108
LR	0.060	0.132	0.200	0.060	0.160	0.220
Panel B: $\theta = 0.5$		$N = 500$			$N = 1000$	
BLR	0.980	0.980	1.000	0.980	1.000	1.000
LR	1.000	1.000	1.000	1.000	1.000	1.000
Panel C: $\theta = 1$		$N = 500$			$N = 1000$	
BLR	0.980	1.000	1.000	1.000	1.000	1.000
LR	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Panel A reports the empirical sizes of the BLR and LR tests, for the null in (5), that is the number of times (across the $R = 250$ replications) that the null is rejected (given that the null is true). The other panels report the empirical powers, that is the number of times (across the $R = 250$ replications) that the null is rejected (given that the null is false).

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