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Contents

Introduction

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The fascinating and challenging idea that some scale invariance principle can lead many natural as well as artificial phenomena such as the pulmonary vessels or the financial markets, is not new and from long time has attracted the interest of theoretical and applied mathematicians. Scaling, that is the property of those phenomena which preserve to some extent the same (geometric or probabilistic) structure regardless their granularity in space or time, is a peculiar feature of fractals, and characterizes many stochastic processes: the (fractional) Brownian motion, the Bessel process, the α-stable Lévy motion, or the generalized Hermite processes, which include, along with some of the examples aforementioned, special cases as the Rosenblatt process and other non-Gaussian stochastic processes. The interest towards fractional calculus and models dates back to the celebrated contributions of great mathematicians such as Cauchy, Leibniz, Liouville, Abel, Caputo, Riesz and many others. Since then a very extensive literature piled up on fractional calculus and its applications in many different areas spanning from physics to data analysis, from biology to medicine, to finance, demonstrating the great versatility of fractional models.

The increasing relevance and the interest that this topic is earning in finance is ascribable not only to the intriguing mathematical properties that fractional and multifractional models inherently display. There is something more: the potential that these models show to describe financial markets in all their complexity, overcoming several simplistic assumptions whose validity and effectiveness is made more and more questionable by the evolution of financial engineering and by its weight on market functioning. In fact, what can be referred to as standard theory of financial asset prices was developed and systematized in the early 1970s, by conceptualizing what seemed reasonable about agents' behaviour: rationality and risk aversion, free information available simultaneously to all interventions, liquid and divisible assets, and other minor technical requirements. The idea that market prices were to completely and instantly incorporate all available information turned into the Efficient Market Hypothesis, one of the cornerstones on which the asset pricing theory is based still today. Nonetheless, with the exponential growth of financial engineering, things articulated more and

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more. As early as 1978, Jensen acknowledged that as econometric sophistication increased, more and more inconsistencies in the paradigm arose that could no longer be ignored[1].

Since then, despite the widespread expectation that financialization and globalization would have made markets more efficient, actually these processes could be at least partly responsible for the weakening of the realism of the original assumptions. An example is given by the controversial results about the impact of derivatives trading on the underlying bonds or stocks markets: on the one hand, evidence exists that derivatives enlarge information on the underlying market, by reducing the asymmetric information among agents, the transaction costs and the market risks; on the other hand, it is still controversial whether derivatives reduce or amplify volatility. At the same time, it is unclear whether - while protecting individual positions - they generate systemic risks and, hence, financial shocks.

To make a long story short, the discordance of the current model with many stylized facts along with a more subtle awareness of both the connections that define and rule financial markets and the behavioural biases that affect agents as human beings, suggest a substantial review of the standard paradigm. In this framework, fractional and multifractional models represent a natural generalization of processes which look now too narrow to capture the whole complexity of markets. The frontier on this topic is challenging and wide, and covers several directions: it ranges from the deep understanding of the very foundations of fractional calculus, to the generalizations built on a time-changing granularity, from the improvement of the computational tools, to the analysis of the scaling properties of the fractional distributions. Some of these topics are discussed in the articles of this special issue, which combines both theoretical and applied contributions.

The first article, entitled "*Wavelet series representation for multifractional multistable Riemann-Liouville process*", is a joint work by Antoine Ayache and Julien Hamonier. They generalize a previous result and construct a wavelet-type random series representation for a random field, which generates a multifractional multistable Riemann-Liouville process. The almost surely convergence of the representation in some spaces of continuous function is proved and an estimate is deduced for its almost sure rate of convergence in the same spaces. As a by-product of their work, the Authors find that the representation provides also an efficient method to simulate the paths of the multistable process.

[1] Verbatim: "[...] *As better data become available (e.g., daily stock price data) and as our econometric sophistication increases, we are beginning to find inconsistencies that our cruder data and techniques missed in the past. It is evidence which we will not be able to ignore.*", Jensen, M.C., (1978) Some anomalous evidence regarding market efficiency. Journal of Financial Economics, 6(2-3):95–101.

The second contribution, "*A study of Chinese market efficiency Shanghai versus Shenzhen: Evidence based on multifractional models*", by Pierre R. Bertrand, Marie-Eliette Dury and Bing Xiao, is an in-depth analysis of the emerging Chinese equity market in the period 2006-2019. The usage of multifractional statistics to estimate the Hurst exponent of the time series leads to highlight a regime switch in the level of market efficiency after the reform in October 2011, intended to push the China's stocks pricing mechanism toward a more market-oriented approach.

"*The volume-volatility relationship: a fractal analysis for a stock index*" is the title of the third contribution, by Massimiliano Frezza. Defining the volatility in terms of the (time-changing) pointwise Hölder regularity exponent of the price process, the Author investigates both the contemporaneous and the causal relationship of this measure with the trading volume and finds evidence of contemporaneous correlation for the Japanese market. Using a vector autoregressive analysis, the Author also tests the dynamic relationship and argues a bidirectional causality.

Multifractionality in the foreigh exchange rates is investigated in the fourth contribution, "*Fractal analysis of the multifractality of foreign exchange rates*", authored by Matthieu Garcin. The Author proposes a method to extract realized Hurst exponents from log-prices and uses this information to estimate different models of dynamics. Assuming a Multifractional Process with Random Exponents, Garcin finds that data justify using a fractional model for the dynamic of the Hurst exponent itself. The result is a nested fractality model, whose key parameter is the Hurst exponent of the Hurst exponents of the log-price series. The analysis is applied to the log-prices of several FX rates between 2006 and 2016, sampled every minute.

The fifth and concluding contribution is "*The Origins of Randomness: Granularity, Information and Speed of Convergence*", by Charles S. Tapiero, Pierre Vallois and Sergio Bianchi. The article investigates how granularity of the data sampled over parametrized fractional time intervals affects the notion of randomness. This happens because of the information embedded in the granularity itself: larger and smaller granular intervals, as well as limit (and parametrized) intervals, necessarily affect data measurements and the consequences of fractionalization on some stochastic processes widely used in finance is discussed.

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