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Nonlinear dynamics of a parametric analytical model for beam-cable-beam structures

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Abstract

A parametric model is proposed to analytically describe the nonlinear dynamics of the structural system composed by two vertical cantilever beams connected by a suspended sagged cable. Focus is made on the geometric nonlinearities that characterize the boundary interactions between the linear beams and the nonlinear Irvine cable. The closed form solution of the linear eigenproblem governing the undamped small-amplitude vibrations enables – first – the clear distinction between global modes, dominated by the beam dynamics, and local modes, dominated by the cable vibrations, and – second – the parametric assessment of some parameter combinations corresponding to integer frequency ratios (1:1, 2:1) between global and local modes. Such internal resonances open the way to different phenomena of linear and nonlinear interactions, which can sustain the transfer of mechanical energy between the interacting modes and, consequently, the onset of high amplitude local oscillations. After the reduction to a single mode basis, the qualitative and quantitative relevance of the system nonlinearities is analyzed. In particular, the effects of the global, local, hybrid nature of the modal shapes on the softening/hardening behavior of the frequency response are investigated.

Keywords: Nonlinear dynamics; Cable structures; Internal resonance; Parametric excitation.

1. Introduction

The structural employment of long-span and highly flexible cables with load-bearing or stabilizing functions has a well-established tradition in many engineering applications. Since the pioneer studies, dating back to the seventies or early eighties of the past century [1-6], the extreme slenderness and low-damping of structural cables has attracted the researchers' interest, mainly focused on the prediction and mitigation of different nonlinear phenomena [7-12].

More recently, a certain attention has been devoted to the formulation of refined but synthetic cable-beam models, in order to analyse the rich variety of linear and nonlinear interaction mechanisms, which can be responsible of high-amplitude cable vibrations, in consequence of mechanical energy transfers from the cable-supported system [13-18].

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The present paper illustrates an original nonlinear model governing the nonlinear dynamics of a structural system composed by two free standing structures, modelled as cantilever beams, coupled by a suspended cable. The linearized system is known to possess a dense natural spectrum, in which global and local modes strongly interact to each other [19, 20]. Moving from this standpoint, the nonlinear free undamped oscillations of the cable-beam system are analysed, by means of a one-degree-of-freedom model. In particular, the influence of the modal localization and hybridization phenomena on the quadratic and cubic nonlinearities is discussed, in the significant range of the parameter space.

2. Nonlinear dynamic model

The structural model realized by two vertical unshearable inextensible cantilever beams, connected at the free ends by a suspended parabolic cable, is considered (Figure 1a). The length L_b of the beams and the horizontal chord L_c of the cable are spanned by the rectilinear abscissae x_{b1}, x_{b2}, x_c respectively. The varied configuration of the system is described by the dynamic variables $v_{b1}(x_{b1},t), v_{b2}(x_{b2},t), v_c(x_c,t)$ for the transversal beams and cable displacements, and the variable $u_c(x_c,t)$, for the longitudinal cable displacement (Figure 1b). Denoting Ω_1 the unknown fundamental frequency of the system, the dependent and independent variables can be expressed in the nondimensional form

$$\tau = \Omega_1 t, \quad \tilde{x}_c = \frac{x_c}{L_c}, \quad \tilde{x}_{b1} = \frac{x_{b1}}{L_b}, \quad \tilde{x}_{b2} = \frac{x_{b2}}{L_b}, \quad \tilde{v}_c = \frac{v_c}{L_c}, \quad \tilde{u}_c = \frac{u_c}{L_c}, \quad \tilde{v}_{b1} = \frac{v_{b1}}{L_b}, \quad \tilde{v}_{b2} = \frac{v_{b2}}{L_b}, \quad \omega = \frac{\Omega}{\Omega_1} \tag{1}$$

where ω stands for the nondimensional frequency. Denoting m_b, m_c the mass densities, H and d the pretension and midspan sag of the cable, EA_c its axial rigidity, EI_b the flexural rigidity of the beam, the mechanical parameters are

$$\alpha = \frac{L_b}{L_c}, \quad \rho = \frac{m_c}{m_b}, \quad \chi = \frac{EI_b}{L_b^2 EA_c}, \quad \mu = \frac{EA_c}{H}, \quad \nu = \frac{d}{L_c} \tag{2}$$

where α defines the height-to-width system proportion, ρ stands for the cable-beam mass ratio, χ accounts for the ratio between the beam (flexural) and cable (axial) stiffness. Furthermore, μ describes the elastic (axial) to transversal (geometric) stiffness of the cable and ν expresses the cable shallowness.

Adopting the classic quasi-static condensation of the longitudinal cable motion, as admissible in the low-frequency oscillation range, the nonlinear equations of motion governing the free damped dynamics of the system read

$$\beta_b^4 \ddot{v}_{b1} + 2\varsigma_b \omega \dot{v}_{b1} + \omega^2 v_{b1}^{IV} = 0 \qquad \beta_b^4 \ddot{v}_{b2} + 2\varsigma_b \omega \dot{v}_{b2} + \omega^2 v_{b2}^{IV} = 0 \tag{3}$$

$$\beta_c^2 \ddot{v}_c + 2\varsigma_c \omega \dot{v}_c - \omega^2 v_c'' + \omega^2 \mu e(8\nu - v_c'') = 0 \tag{4}$$

where the tilde has been omitted, while dot and prime stand for differentiation with respect to the nondimensional time and abscissa, respectively. The coefficients ς_b, ς_c introduce viscous damping terms accounting for the dissipation.

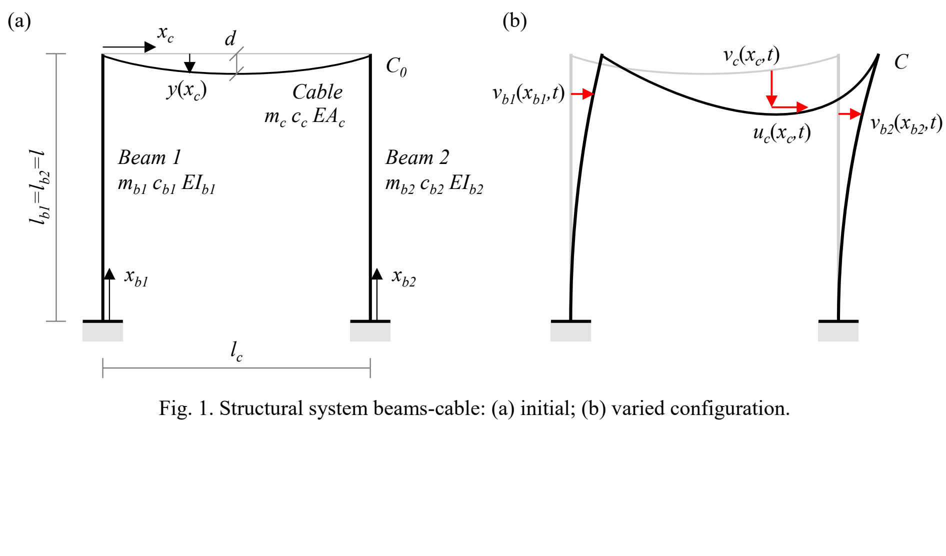

Fig. 1. Structural system beams-cable: (a) initial; (b) varied configuration.

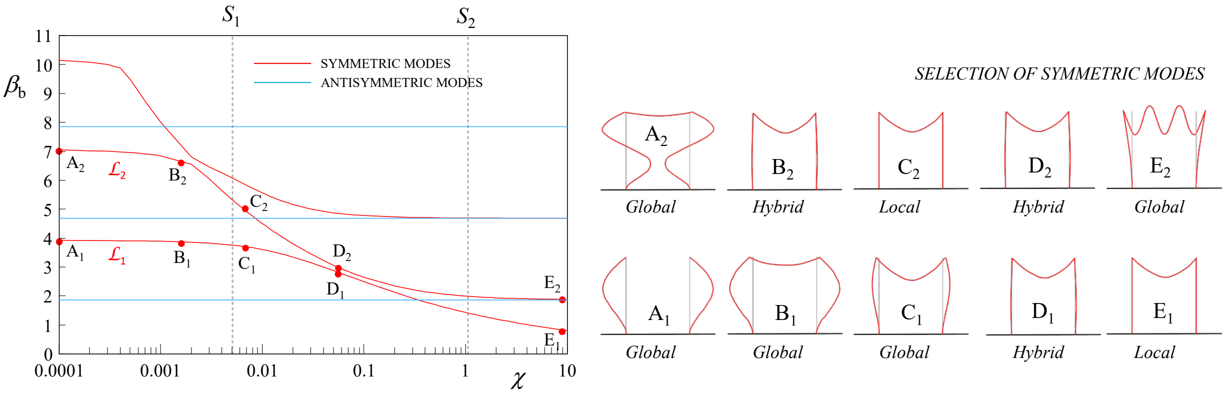

Fig. 2. Eigensolution (wavenumber and eigenfunctions) versus the varying χ-parameter (α=0.4622, ρ=0.00023, μ=2213, ν=0.00125, see [19]).

The geometric (kinematic) nonlinearities affecting the system dynamics are embodied by the quadratic and cubic terms in the cable equation, owing to the time-dependent elongation $e(\tau)$ rising up from the static condensation

$$e(\tau)=\alpha_b v_{b2}(1,\tau)-\alpha_b v_{b1}(1,\tau)+\int_0^1\left(8\nu\, v_c(x_c,\tau)+\tfrac{1}{2}v_c'(x_c,\tau)^2\right)dx_c \tag{5}$$

It is worth noting that further quadratic and cubic nonlinearities affect the mechanical boundary conditions, since the cable elongation participates (through the dynamic tension $\mu e(\tau)$) in the transversal equilibrium at the beam tips.

2.1. Closed-form linear eigensolution

The solution of the integral-differential eigenproblem governing the linear undamped dynamics consists of the exact frequencies ω and the related modal vectors $\boldsymbol{\varphi}=(\varphi_c(x_c),\varphi_{b1}(x_{b1}),\varphi_{b2}(x_{b2}))$, in which the cable $\varphi_c(x_c)$ and beam eigenfunctions $\varphi_{b1}(x_{b1}),\varphi_{b2}(x_{b2})$ are determined in closed form, depending on the nondimensional wavenumbers

$$\beta_c^2=\omega^2\frac{m_c L_c^2}{H}\Omega_1^2,\qquad \beta_b^4=\omega^2\frac{m_b L_b^4}{EI_b}\Omega_1^2,\qquad \beta_c=\frac{\beta_b^2}{\alpha}\sqrt{\mu\rho\chi} \tag{6}$$

The analysis of the eigensolution discloses a rich scenario of modal forms, depending on the system parameters [19]. Indeed *global modes*, dominated by the beams dynamics with quasi-static participation of the cable, coexist with *local modes*, dominated by the transversal cable dynamics [21]. *Antisymmetric* global (AG) modes do not entail any cable stretching. Consequently, only local modes (L) and *symmetric* global (SG) modes actually depend on the beam-cable stiffness ratio and undergo marked variations of the frequency ω (or wavenumber β_b , through the univocal relation (6)) and modal shape in the χ -range (Figure 2). Internal 1:1 resonances or nearly-resonances may occur when the χ -dependent frequencies of local and global modes approach each other, giving rise to *crossing* points (not-interacting AG-L modes) or *veering* zones (linearly interacting SG-L modes). The frequency veering determines the *hybridization* of the SG-L modes, which exchange their modal shapes in a rapid but continuous way across the veering zone.

Superharmonic and subharmonic internal resonance conditions between a global symmetric and a local modes can be detected for the systems S_1 and S_2, whose χ -values give 2:1 or 1:2 ratios of the L and SG frequencies (Table 1).

Table 1. Superharmonic and subharmonic internal resonance conditions between a global symmetric and a local modes.

System	χ	β_b (L)	β_b (SG)	β_c (L)	β_c (SG)	ω (L)	ω (SG)	Internal resonance
S_1	0.0051	5.32294	3.76237	3.12629	1.56188	8.05847	4.02599	Subharmonic (2:1)
S_2	1.055	1.41145	1.99547	3.16154	6.31914	0.56661	1.13250	Superharmonic (1:2)

3. Nonlinear one-degree-of-freedom model

A one-degree-of-freedom nonlinear model can be formulated by expressing the displacement vector $\mathbf{u} = (v_c, v_{b1}, v_{b2})$ as $\mathbf{u} = q(\tau)\ \boldsymbol{\varphi}(x_c, x_{b1}, x_{b2})$, where q is the unknown amplitude of a particular mode $\boldsymbol{\varphi}(x_c, x_{b1}, x_{b2})$ of interest. Therefore, the free nonlinear dynamics of the one-degree-of-freedom model is governed by the equation

$$m\ddot{q} + \xi\dot{q} + kq + c_2 q^2 + c_3 q^3 = 0 \tag{7}$$

where m is unitary and $k = \omega^2$ in the following, since the mode can be properly normalized with respect to the system masses. The nonlinear coefficients c_2 and c_3 introduce the quadratic and cubic nonlinearities, respectively.

Since linear models have been adopted for the cantilever beams, it is worth remarking that the nonlinear coefficients almost entirely depend on the cable mechanical properties. Nonetheless, the modal coupling at the beam-cable joints can be verified to influence the c_2-parameter in a not-negligible way. Consequently, some attention can be paid to analyzing how the nonlinear coefficients vary in the parameter space, with focus on the χ-range in which the beam-cable interactions let symmetric modes undergo the localization and hybridization processes.

Figure 3 illustrates the variation of the nonlinear coefficients versus the χ-parameter for the two symmetric modes corresponding to the wavenumber loci $\mathcal{L}_1$ (*low-frequency* mode M_1) and $\mathcal{L}_2$ (*high-frequency* mode M_2) in Figure 2. The actual importance of the nonlinearities in the hybridization regions is confirmed by the large values attained by the two coefficients c_2 and c_3 for the χ-values corresponding to hybrid modes (e.g. mode D_1, but also mode B_2 in Figure 2). Furthermore, by virtue of the modal (linear) coupling, hybrid modes tend to exalt the nonlinearities affecting the boundary conditions at the beam-cable joints. Remarkably, this specific *hybrid contribution* offered by the cable-beam boundary interaction can become dominant in the c_2-parameter, at least for shallow cables (low ν-values). Qualitatively, this remark reflects the unique maximum value attained by the c_2 and c_3 curves, which corresponds to the highest hybridization factor [21] between the *low-frequency* and *high-frequency* modes M_1 and M_2.

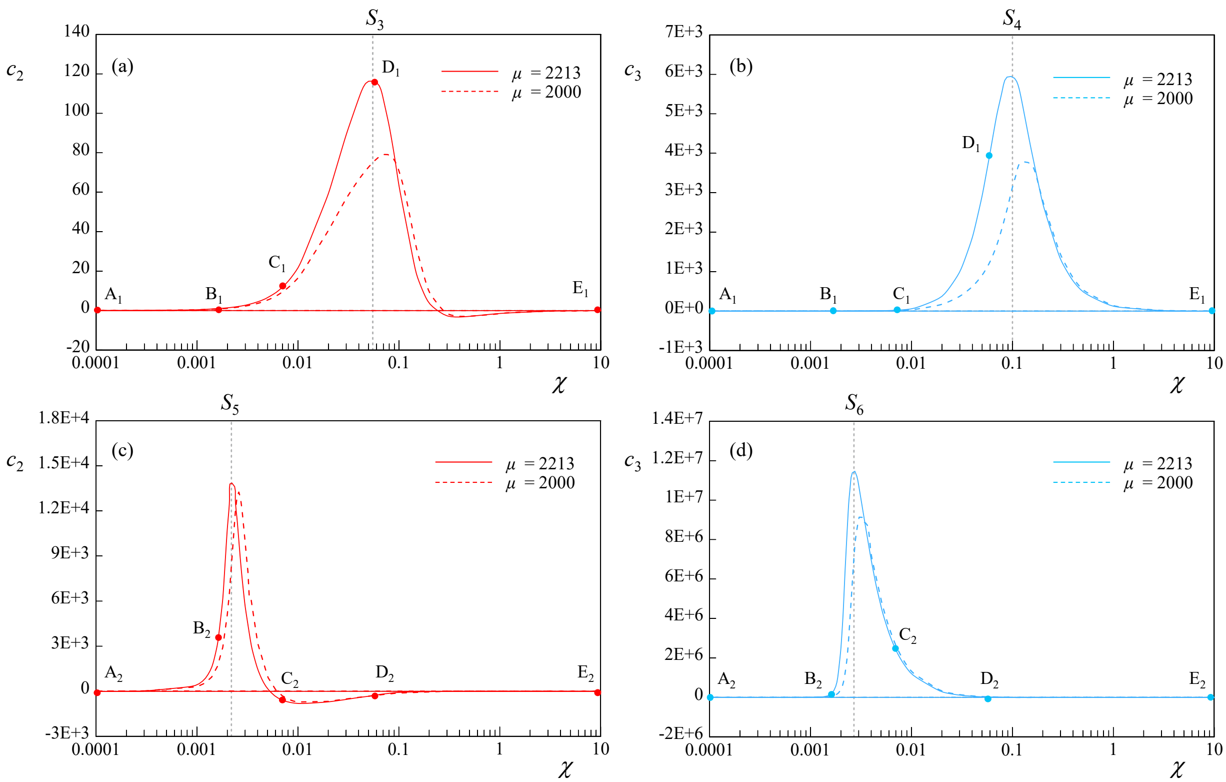

Fig. 3. Nonlinear coefficients versus the varying χ-parameter for: (a),(b) low-frequency mode M_1, (c),(d) high-frequency mode M_2.

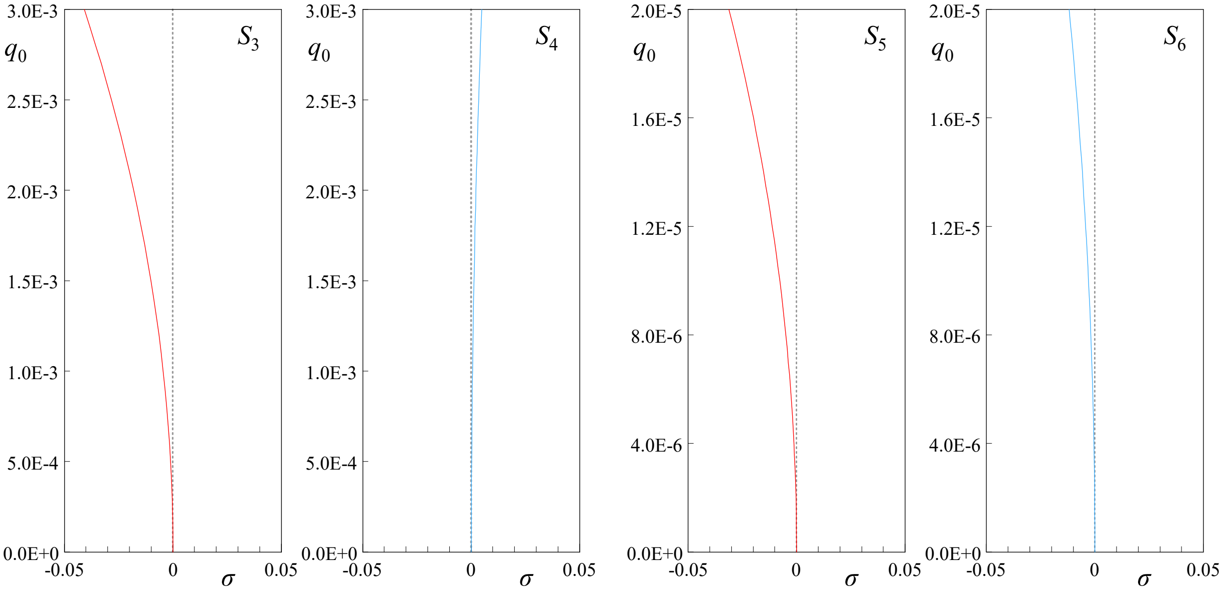

Fig. 4. Backbone curves (frequency versus amplitude) for the highly nonlinear systems S_3-S_6.

3.1. Free vibrations

The nonlinear behaviour of the one-degree-of-freedom model can be readily assessed by describing how the stationary response amplitude depends on the frequency of a harmonic external excitation. The response peak can also be regarded as the amplitude-dependent *frequency* $\varpi(a) = \omega + \sigma(a)$ of the nonlinear system freely vibrating with a certain (small) oscillation amplitude a. The essential relation $\sigma(a)$ has been determined by means of the standard Multiple Scale Method, up to the second approximation order for the solution of the eq. (7). The frequency-amplitude (namely *backbone*) curves are illustrated in Figure 4, where q_0 stands for the lowest order approximation of the oscillation amplitude a. Results are presented for the highly-nonlinear systems S_3-S_5, corresponding to the maxima of the c_2 and c_3 curves for the modes M_1 and M_2, whose dynamic characteristics are collected in Table 2.

The curves show how the quadratic nonlinearities, responsible for the softening behavior of the S_3,S_5,S_6 systems, tend to prevail on the cubic nonlinearities. A moderate hardening behavior can be achieved only by the S_4 system, which corresponds to the c_3-maximum for the *low-frequency* mode M_1. Considering that the hardening effect can be fully attributed to the cable dynamics (as long as the c_3-parameter is independent of the beam eigenfunctions), this observation confirms that the modal hybridization (strongly contributing to the peak value of the c_2-parameter) can strongly characterize the nonlinear dynamics of the beam-cable-beam system. The occurrence of dynamic bifurcations in the forced dynamic response, as well as the possible occurrence of autoparametric excitations in two-degrees-of-freedom models (based on a pair of internally resonant global-local modes), are under current investigations.

Table 2. Dynamic characteristics and nonlinear behavior of the highly-nonlinear systems in Figure 3

Mode	System	χ	β_b	β_c	ω	c_2	c_3	Nonlinear behavior
M_1	S_3	0.05	2.87857	2.86273	2.35670	max	-	softening
M_1	S_4	0.1	2.49635	3.04475	1.77239	-	max	hardening
M_2	S_5	0.0022	6.46657	3.03039	11.89315	max	-	softening
M_2	S_6	0.0027	6.20261	3.08867	10.94203	-	max	softening

4. Conclusions

A parametric model has been presented to describe the geometrically nonlinear dynamics of the structural system composed by two cantilever beams connected by a parabolic cable. Leveraging the closed form solution of the linear eigenproblem governing the small undamped free oscillations, a nonlinear one-degree-of-freedom model has been formulated. The importance of the quadratic and cubic nonlinearities has been parametrically analysed, with specific discussion concerning the global, local or hybrid nature of the modal basis. The modal hybridization has been found to exalt the nonlinear characteristics of the system, with particular reference to its inherent quadratic nonlinearities, responsible also for the softening behaviour of the backbone curve characterizing the frequency response function.

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