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Quantifying rate dependence of hysteretic systems

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Abstract

Hysteresis nonlinearities are formally defined as deterministic, rate-independent operators for a great variety of systems. Rate independence frequently occurs in problems in which the time scales of interest are much longer than the intrinsic time scales of the system. In this paper we propose a measure of rate dependence and numerically evaluate the corresponding metric for two rate-dependent systems, namely, a linearly viscous damper and a class of shape memory materials exhibiting thermomechanical behavior [1]. The rate-independent extended Bouc-Wen model of hysteresis [2] is used to validate the robustness of our rate-independence criterion. On the other hand, the shown rate dependence in shape memory materials working in nonisothermal conditions is associated with the ensuing thermomechanical coupling.

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Keywords: Hysteresis, rate independence, Homeomorphisms, Shape memory materials

1. Introduction

Hysteresis nonlinearities are typically defined as deterministic, rate- independent operators. Rate independence means that the material responses are invariant with respect to invertible transformations of the time scale [3]. Rate independence frequently occurs in problems in which the time scales of interest are much longer than the intrinsic time scales of the system. A good survey of rate-independent systems is given by Mielke [4].

Rate dependence in the mechanical response of engineering materials and systems can arise depending on the considered ranges of the loading rates. In this paper we introduce a measure of rate dependence, which is an appropriate norm of the difference between the original output of the system and that of the system subject to a time-transformed input. We evaluate this metric for various models, including the purely viscous damper, the extended Bouc-Wen model of hysteresis [2,5], and a Shape Memory Alloy (SMA) material model that accounts for thermomechanical coupling between the mechanical and thermal states of the material [1,6,7].

The rate-dependence measure is evaluated for simple one-parameter families of homeomorphic transformations of the time scale. The proposed metric can also be extended to account for random homeomorphisms. Such extension relies on a body of works dealing with probability measures on spaces of homeomorphisms [8] or methods for generating random probability measures [9–11].

The present work aims to explore the effectiveness and robustness of the rate-independence measure by employing simple one-parameter homeomorphisms as a first step towards a full study over the space of random homeomorphic time transformations that make use of suitable probability measures.

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2. Hysteresis operators and simple homeomorphisms

A hysteresis operator $\mathcal{H}$ maps input functions $u(t)$ into output functions $x(t)$, where t denotes the time independent variable, i.e.

$$x(t) = \mathcal{H}[u](t), \qquad t \in [0, T]. \tag{1}$$

The rate-independence property of $\mathcal{H}$ can be expressed as

$$\mathcal{H}[u(\varphi(t))] = x(\varphi(t)) \tag{2}$$

for all continuously increasing functions $\varphi : [0, T] \to [0, T]$. We propose to measure rate dependence according to

$$\varepsilon = \left\| \mathcal{H}[u(\varphi(t))] - x(\varphi(t)) \right\|, \tag{3}$$

where the norm $\|\cdot\|$ may be an appropriate average over a class of functions φ. In this study we use the L_2 norm.

Here a one-parameter family of simple homeomorphisms on $[t_i, t_{i+1}]$ is employed to define the time transformation as

$$t_{new} = t_{i+1} - \frac{t_{i+1} - t_i}{1 + \frac{1-a}{a}\frac{t-t_i}{t_{i+1}-t}}, \qquad a \in [0, 1]. \tag{4}$$

The value $a = 1/2$ corresponds to the identity transformation. Figure 1 shows this family of functions for various values of a. Note that the time window is fixed since the time transformations have the purpose of changing continuously the rate of the input function (i.e., the transformations above the identity transformation entail faster signals while those below lead to slower signals). This overcomes the cumbersome computations associated with harmonic inputs exhibiting frequencies varying in a wide bandwidth.

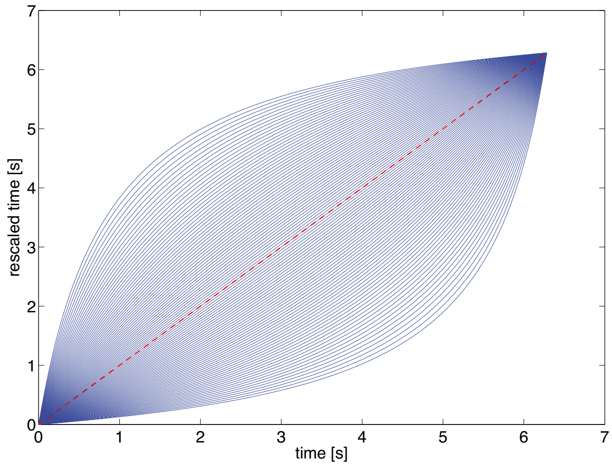

Fig. 1: Simple homeomorphisms for $a \in [0.1, 0.9]$.

3. The simplest rate-dependent system: the purely viscous damper

To set the stage for the computations, we start considering the following first order ordinary differential equation:

$$\dot{x}(t) = f(x(t), u(t)), \tag{5}$$

whose solution is denoted by $\alpha_t[u(t); x_0]$ with $u(t)$ representing the input. Here, the dependence of the solution on the initial condition x_0 was made explicit. Applying the time transformation $\phi(t)$ to the input yields the differential equation

$$\dot{x}(t) = f(x(t), u(\phi(t))) \tag{6}$$

with solution $\beta_t\left[u\left(\phi\left(t\right)\right);x_0\right]$. The system is rate-independent if

$$\beta_t\left[u\left(\phi\left(t\right)\right);x_0\right] = \alpha_{\phi(t)}\left[u\left(t\right);x_0\right] \tag{7}$$

for all continuously increasing functions $\varphi : [0,T] \to [0,T]$. Thus our proposal is to compute the rate dependence measure as

$$\varepsilon = \left\|\beta_t\left[u\left(\phi\left(t\right)\right);x_0\right] - \alpha_{\phi(t)}\left[u\left(t\right);x_0\right]\right\|. \tag{8}$$

A linearly viscous damper subject to an input force is the simplest example of rate-dependent system. Let the input be described by the ramp $u(t) = vt$ where v is the time rate of change of the load. Thus the balance of forces requires $c\dot{x} = vt$ where c is the damper viscosity. By nondimensionalizing time so that the governing equation becomes $\dot{x} = t$, the solution in the original time scale can be represented as $\alpha_t = x_0 + t^2/2$. On the other hand, by introducing the rescaled input $u(\phi(t))$, the solution of $\dot{x} = \phi(t)$ is expressed as

$$\beta_t = x_0 + \int_0^t \phi\left(\tau\right)d\tau,$$

while the original solution with the rescaled time is $\alpha_{\phi(t)} = x_0 + \frac{1}{2}\phi\left(t\right)^2$.

The rate dependence of the purely viscous damper can thus be expressed as

$$\varepsilon = \left\|x_0 + \int_0^t \phi\left(\tau\right)d\tau - \left(x_0 + \tfrac{1}{2}\phi\left(t\right)^2\right)\right\| = \left\|\int_0^t \phi\left(\tau\right)d\tau - \tfrac{1}{2}\phi\left(t\right)^2\right\|. \tag{9}$$

This metric is shown in Figure 2 which gives the variation of ε with the parameter a regulating the family of simple homeomorphisms. As expected, for the identity transformation corresponding to $a = 1/2$, ε vanishes while away from it the nontrivial values of ε clearly indicate the expected rate dependence.

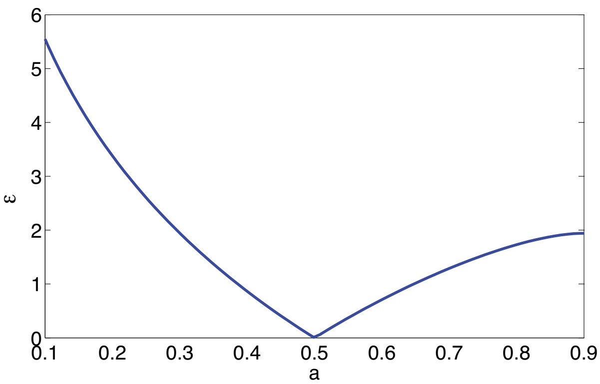

Fig. 2: Rate-dependence measure ε for the purely viscous damper as a function of a.

4. The extended Bouc-Wen model of pinched hysteresis

The Bouc-Wen model of hysteresis [12,13] defines a rate-independent relationship between two variables (the input and the output such as the input displacement x and the output force f). To account for pinching around the origin (as observed in the hysteresis loops of mixed steel-SMA wire ropes subject to flexural cycles), an extended model was proposed in [2] according to

$$f = k_e x + z, \quad \dot{z} = \left[k_d h - (\gamma + \beta\,\mathrm{sgn}(\dot{x}z)\,|z|^n\right]\dot{x} \tag{10}$$

where $(k_e, k_d, \beta, \gamma, n)$ are model parameters and z indicates the purely hysteretic part of the restoring force. Pinching of the hysteresis loops is obtained through the function $h(x) = 1 - \xi e^{-\frac{x^2}{x_c}}$ with $\xi \in [0, 1)$ regulating the pinching severity

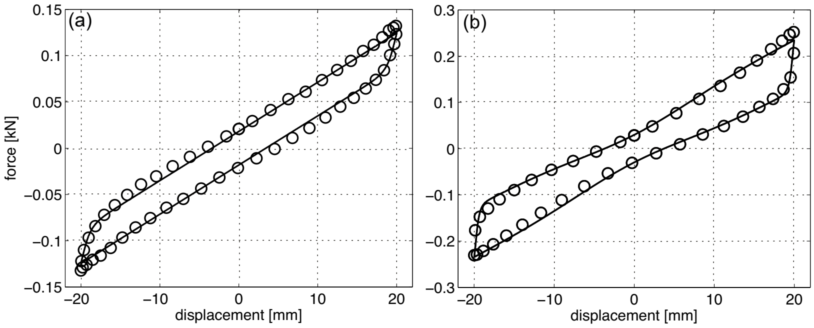

Fig. 3: Experimental force-displacement cycles (denoted by circles) and numerical identifications (denoted by solid lines) of an assembly of (a) steel and (b) SMA wire ropes subject to flexural cyclic loads.

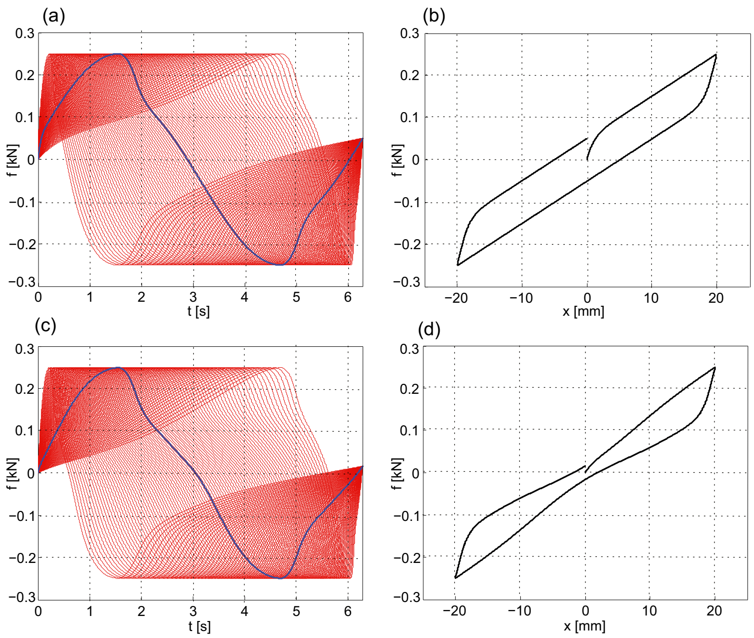

Fig. 4: Restoring force outputs for the baseline displacement input (blue line) and the rescaled displacement inputs (red lines) as function of time in (a), (c) and outputs in (b), (d) where (a), (b) and (c), (d) are obtained employing the classic Bouc-Wen model (b) and the extended model with pinching (d), respectively.

and $x_c > 0$ defining the pinching extension in terms of displacement amplitude. The stiffness at the origin of the cycles is $k_e + k_d$ while after a threshold displacement it achieves the constant value of k_e. The parameters (β, γ, n) regulate the shape of the hysteresis loops. The extended Bouc-Wen model given by Eq. 10 was employed to reproduce the bending hysteresis cycles of steel and SMA ropes, respectively, shown in Fig. 3 [2,5]. One end of the ropes was clamped while the other end was displaced in the direction transversal to the ropes and the delivered restoring force was measured by a load cell. Here we take x as the input displacement just as in [14].

Both the classic Bouc-Wen model and the pinched version were employed as benchmark models to assess the numerical robustness of the rate-independence criterion. Figure 4 shows the baseline input displacement signals, the time-transformed input, and the associated force-displacement cycles. The input signals are obtained from the linear ramp using Eq. 4. As expected the restoring forces obtained for the baseline and transformed input signals coincide in agreement with the rate independence of the models testified by the small values of ε (i.e., order of 10^{-3}).

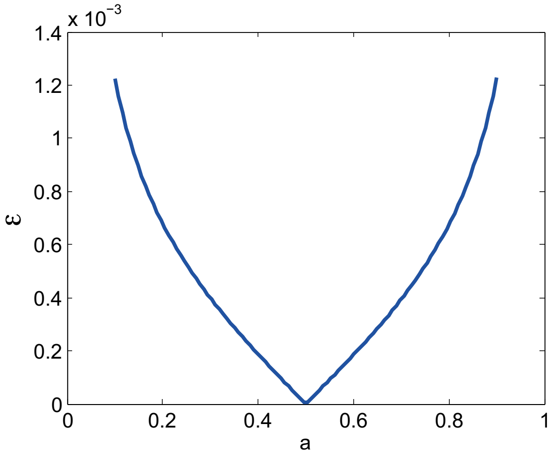

Fig. 5: Rate-independence measure ε for the Bouc-Wen model of hysteresis. ε is of the order of 10^{-3}.

5. Shape memory materials

Shape Memory Alloys (SMA) are metallic alloys that exhibit functional properties such as pseudoelasticity and shape memory effects. Their typical macroscopic behavior is the result of the occurrence, at the microscopic scale, of phase transformations between the two solid phases, Austenite and Martensite. Phase transformations can be activated both by thermal and mechanical loads and the corresponding latent heat induces a significant coupling between thermal and mechanical states.

A thermomechanical constitutive formulation is employed according to Bernardini and Pence [1,7]. Two evolution equations for the Martensite fraction ξ and temperature T are given by

$$\dot{\xi} = -\frac{1}{Z_1 + Z_3 Z_4}\left(Z_2\dot{\epsilon} + Z_3\frac{r}{c}\right), \quad \dot{T} = -\frac{1}{Z_1 + Z_3 Z_4}\left(Z_4 Z_2\dot{\epsilon} - Z_1\frac{r}{c}\right), \tag{11}$$

where ϵ is the elongation, c is the specific heat and r the convective rate of heat exchange with the environment. The functions Z_i can be expressed in the form:

$$Z_1 := \frac{\partial\Pi}{\partial\xi} - \frac{\partial\Lambda}{\partial\xi}, \quad Z_2 := \frac{\partial\Pi}{\partial\epsilon} - \frac{\partial\Lambda}{\partial\epsilon}, \quad Z_3 := \frac{\partial\Pi}{\partial T}, \quad Z_4 := \left(\Lambda - T\frac{\partial\eta}{\partial\xi}\right), \tag{12}$$

where Π is the driving force of the phase transformations and Λ is the dissipative threshold level that the driving force must attain and sustain in order to promote the occurrence of phase transformations. The two quantities Π and Λ can be derived, as discussed in [1,7], from a free energy and a dissipation function in order to ensure a full consistency with the second law of thermodynamics. The dissipation function corresponding to the model is a convex and positively homogeneous function of the phase fraction rates. Hence, as shown in previous works, for fixed T, the first equation in Eq. 11 yields a rate-independent hysteresis operator.

Once the phase fraction ξ is computed as a solution of the evolution equations, the stress output can then be computed as $\sigma = E(\xi)(\varepsilon - \xi M)$, where E is the effective elastic stiffness and M is the upper bound for the macrosopic transformation strain, typical of the active Martensite variant.

Although, for fixed temperature, the model is rate-independent, due to the inherent thermomechanical coupling, Eqs. 11 properly describe the rate-dependent behavior which is typically observed in experiments.

The proposed test has been applied to a typical nonisothermal SMA response. Specifically, the same parameterized family of simple homeomorphisms (with $a \in [0.1, 0.9]$) considered in previous examples has been used to check the rate dependence of the response shown in Fig. 6 which is characterized by a full hysteresis loop with complete phase transformations (the two linear branches correspond to the material completely transformed either in pure Austenite or pure Martensite). Figure 7 shows the time histories of input and output quantities under rescaled time. The temperature

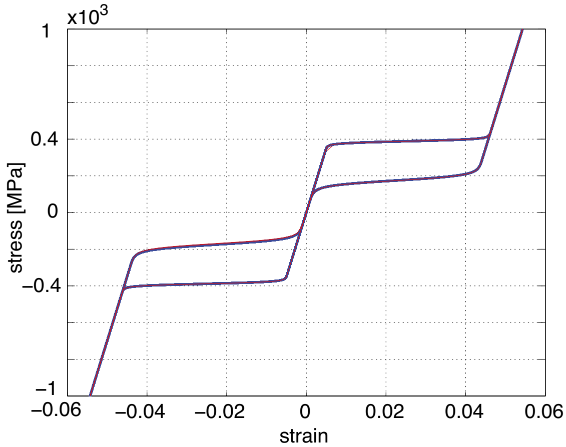

Fig. 6: Stress-strain curves for a typical SMA material.

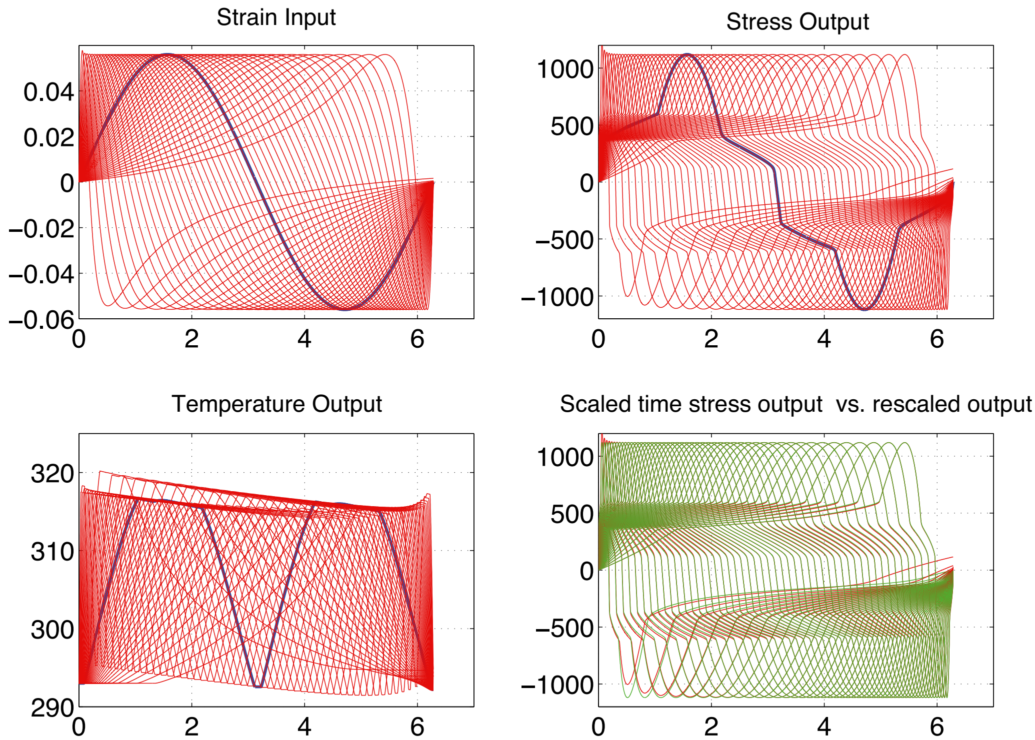

Fig. 7: Baseline strain input and rescaled strain inputs, corresponding stress and temperature outputs, and superposition of the stress output to the rescaled input (red line) and rescaled stress (green line). The noticeable difference between the red and green lines shows the rate dependence typical of nonisothermal response regimes in SMA materials.

outputs clearly show that the thermomechanical coupling is active and significant so that a rate-dependent behavior is to be expected. The rate-dependence norm ε versus a, as depicted in Figure 8, shows a behavior qualitatively similar to that of the purely viscous damper (Fig. 2). This behavior can be considered as a fingerprint of rate dependence upon the action of simple homeomorphisms, namely, nonzero values, sharp zero minimum at $a = \frac{1}{2}$ which corresponds to

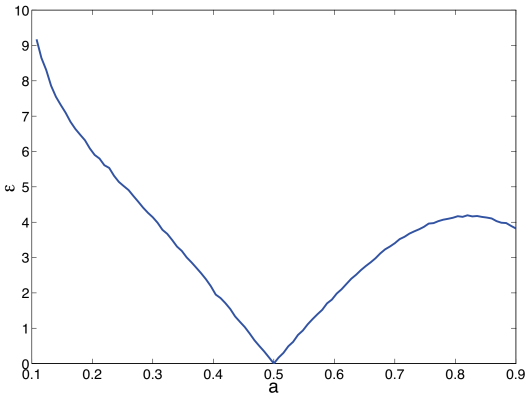

Fig. 8: Variation of the rate independence measure ϵ with a for a shape memory material in non- isothermal conditions.

the identity. The proposed test is thus capable of properly detecting the rate dependence of the nonisothermal SMA model.

6. Conclusions

A measure of rate dependence in materials and systems behaviors is proposed as a norm of the difference between the output of the rescaled input and the rescaled output according to the time transformation. The rate dependence of the purely viscous damper and that of a shape memory alloy (SMA) model in nonisothermal conditions is clearly manifested through the nontrivial values of the rate-dependence measure. In SMAs, the rate dependence is a direct consequence of the thermomechanical coupling. As a benchmark problem, the Bouc-Wen model of hysteresis with or without pinching was employed to test the inherent rate independence.

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