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Introduction to Methods for Nonlinear Optimization

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Preface

Optimization (or mathematical programming) concerns the study of "decision problems", where the aim is to determine the minimum or the maximum points of a real function (the objective) in a prefixed set (the feasible set). Many real problems arising in the fields of economics, engineering, chemistry, physics, statistics, computer science, management science, operations research and mathematics itself can be formulated as optimization problems.

This book is an introduction to the important area of nonlinear optimization, known also as nonlinear programming, that studies optimization problems where some of the problem functions can be nonlinear. In particular, we restrict our study to continuous nonlinear optimization problems, defined on finite dimensional real spaces, under the assumption that the problem functions are smooth. Even with these restrictions, nonlinear programming has many relevant applications in the solution of nonlinear systems of equations and inequalities, in the construction and the analysis of mathematical models, in the solution of (almost all) engineering problems, in the computation of the control laws of discrete-time dynamical systems. At present, nonlinear optimization techniques represent also a basic ingredient of *machine learning* methods, for the computation of the parameters of the learning system on the basis of the available data.

The book contains three blocks of chapters concerning:

– Basic theory and optimality conditions (Chaps. 1–7)
– Unconstrained and constrained algorithms (Chaps. 8–23)
– Advanced topics (Chaps. 24–26).

The first two blocks consist of short chapters, and each of these chapters may correspond, in the authors' experience, to the notes for one or two lessons (with each lesson of approximately 2 hours). Some more space is given to the matter of the third block. In each chapter, we add a few exercises and a terminal section on notes and references. As the main emphasis of the book is on solution methods, we believe that the best instructive exercise could be the realization of a computer code, related to the methods illustrated in the book, under the guidance of the teacher.

The basic concepts and results of linear algebra, of calculus on R^n and of convex analysis, are reported in Chaps. 27, 28, and 29. In particular, in Chap. 29 on convex analysis, we give also the proof of all the results used in the book, since, in many cases, this chapter must be included in a course. Then, the only prerequisite is the content of basic courses on linear algebra and calculus.

Most of the basic theoretical results reported in the text are formally proved in order to allow the students, potential readers of the book, to acquire a sound background in optimization. As regards “classical” optimization subjects, we attempted to give the simplest proofs available in the literature or adapted from the best known sources. Particular attention is devoted to the study of global convergence properties of the algorithms, since the definition of globalization techniques represents one of the major contributions of optimization to computational methods.

The book contains also more specialized topics, which are receiving an increasing interest in the last years, like those concerning derivative-free methods, nonmonotone methods, decomposition algorithms, which have been, and currently are, research topics of the authors, and are not usually reported in introductory textbooks.

The material given in the book can be used for constructing different courses, both at an (advanced) undergraduate level and at a graduate level.

A short course (say of 30 hours) can be based on elements of convex analysis (Chap. 29), basic definitions and existence results (Chap. 2), optimality conditions (at least Chaps. 3, 4, and 5). This material can be combined, for instance, with a basic course on linear programming. Note, however, that linear programming is not a prerequisite and that the basic theory of linear programming can also be derived, as special case, from the optimality conditions of nonlinear programming.

A course of 60 hours for advanced undergraduate students could be based, for instance, on the same material indicated above, with the addition of of a selection of unconstrained algorithms that includes at least Chaps. 8, 9, 10, and 11.

Similarly, a course of 60 hours for graduated students could be focused on a selection of classes of methods presented in the text, both for unconstrained and constrained optimization problems. The choice of the arguments will depend on the type of master degree and on the background and scientific interests of the potential readers of the book.

Some important topics like, for instance, sparse optimization, incremental and online algorithms, complexity analysis, non-smooth optimization, global optimization, cone programming, semidefinite programming have been leaved out for space limitations. We believe, however, that the study of the material reported in the book can be a useful prerequisite for further extensions. The basic techniques introduced here can also be useful in the study of other sectors of decision sciences like variational inequalities, multi objective optimization and game theory.

Finally we remark that most of the material on unconstrained methods is extracted from the book (in Italian):

L. Grippo and M. Sciandrone, *Metodi di ottimizzazione non vincolata*, Springer-Verlag Italia, Milano, 2011, which contains also some of the proofs omitted here.

Rome, Italy
January 2023

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