On the Positive Relation between the Wage Share and Labor Productivity Growth with Endogenous Size and Direction of Technical Change

Luca Zamparelli*†

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Abstract

This paper combines induced innovation and endogenous growth to investigate two issues: the relation between the wage share and labor productivity growth and the potential influence of the saving rate on the steady state wage share. We assume that myopic competitive firms choose the size and direction of technical change to maximize the growth rate of profits. First, we find technological conditions sufficient to ensure that labor productivity growth is a positive function of the wage share. Second, we show that the steady state wage share depends on the saving rate if, and only if, R&D investment affects the marginal rate of transformation between labor and capital productivity growth. Both results have important policy implications as they clarify under what conditions any factor affecting the wage share or the saving rate will have an impact on labor productivity growth or steady-state income distribution.

Keywords: Factor income shares, Induced innovation, R&D, Growth models

JEL Classification: D24, E25, D33, O30, O41

*Department of Social Sciences and Economics, Sapienza University of Rome. P.le Aldo Moro 5, Rome Italy 00185. Email: luca.zamparelli@uniroma1.it

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1 Introduction

Since the early stages of political economy, income distribution has been central to the discussion of economic growth and technical change. British Classical economists thought that capital accumulation would be financed by profits, so they looked at the profit rate as the ultimate regulator of output and capital growth. They also recognized the importance of profits in eliciting innovations, as competing capitalists would seek to introduce cost-reducing production techniques to earn above-average profit rates. Similarly, Schumpeter (1911[2008], 1942) argued that technical change is the source of temporary monopolistic profits, and that their existence is essential to provide the necessary incentives for innovation. The Schumpeterian insights have become the foundation of the endogenous growth literature that developed during the 1990s (see Segerstrom et al., 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). By introducing the monopolistic competition framework into general equilibrium models, this literature established a positive causal relation between the size of monopolistic profits that accrue to innovators and the amount of resources invested to produce technical change (R&D investment).

While all these lines of thinking emphasize the importance of profits in fostering productivity growth, the notion that high real wages or real wage growth may spur labor productivity growth is also well established in both economic theory and economic history. The Habakkuk hypothesis (Habakkuk, 1962) maintains that during the 19th century the pace of labor-saving technical change was faster in the United States than in Britain because of its scarcer and more expensive labor supply. Allen (2009) singles out the high price of labor relative to energy costs as one of the fundamental forces that triggered the British industrial revolution. More recently, Fontanari and Palumbo (2023) argue that stagnating real wages may have contributed to the slowdown of US productivity in recent decades.

From a theoretical standpoint, this connection is rooted in the incentive to introduce labor-saving innovations by competitive, profit-maximizing, firms facing high labor costs. This has been formally developed and investigated within various analytical frameworks. The theory of induced technical change traces back to Hicks's conjecture that "a change in the relative prices of the factors of production is itself a spur to invention .... directed to economizing the use of a factor which has become relatively expensive" (Hicks 1932, p.124). Later, Kennedy (1964) and von Weizsacker (1962) independently proved this result. They assumed the existence of an innovation possibility frontier (IPF) that describes the trade-off between freely available capital- and labor-augmenting innovations as shown in Figure 1. Competitive firms choose a point on the IPF, that is the *direction* of technical change, to maximize the rate of unit cost reduction, or equivalently, the growth rate of the profit rate, given the levels and prices of employed labor and capital. The firms' optimal choice produces a relation between the direction, or bias, of technical change and functional income distribution: labor- (capital-) productivity growth becomes a positive function of the wage (profit) share.

At the macroeconomic level, the mechanism of induced innovation, also known as the induced innovation hypothesis (Funk, 2002), has been implemented in both neoclassical (Drandakis and Phelps, 1965; von Weizsacker 1966) and Classical (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2005) growth models with exogenous labor supply. An important implication of these models concerns long-run income distribution. In the steady state, the wage share only depends on the shape of the innovation possibility frontier; it is 'exogenous' in the sense that changes in the economy's saving preferences do not affect it. In particular, the curvature of the IPF at the point where capital

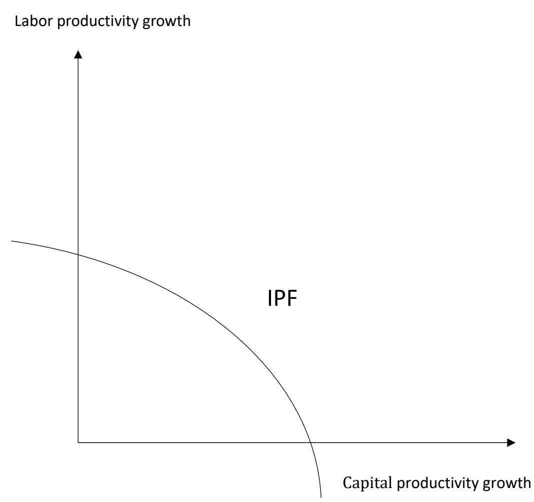

Figure 1: Innovation Possibility Frontier

productivity growth is zero uniquely determines the long-run level of the wage share.

The same positive relation between wage share and labor productivity growth is found in a recent literature, which has introduced endogenous, costly, technical change in Classical growth models. In these contributions (Foley et al. 2019, Ch.9; Tavani and Zamparelli, 2021), competitive firms choose the intensity, or *size*, of technical change rather than its direction. In fact, capital productivity is fixed, and firms can only augment labor productivity. Specifically, they need to decide how to allocate resources between the alternative uses of physical capital accumulation and labor-saving R&D investment. In this context, a higher wage share makes R&D investment relatively more profitable, so that firms divert funds from physical capital to R&D investments, thus raising labor productivity growth. Contrary to the induced innovation theory, the saving rate affects long-run income distribution in Classical growth models with endogenous technical change intensity and exogenous labor supply. In this framework, the wage share is not constrained by the slope of the IPF when capital productivity growth is zero and will adjust to balance the warranted and natural growth rate, which are both affected in different ways by the saving rate.

This paper offers a synthesis of induced and endogenous technical change to investigate both the relation between the wage share and labor productivity growth and the long-run determinants of the wage share. We assume that the set of capital- and labor- saving innovations is not exogenously given to firms but depends on the intensity of their of R&D investment. For any given level of R&D investment, a distinct IPF exists; higher levels of R&D increase the size of the innovation set by pushing the IPF outward. In line with the induced innovation tradition, we assume that firms maximize the instantaneous rate of growth of profits subject to the innovation technology set. In maximizing their objective function, they simultaneously choose the allocation of funds between capital accumulation and R&D investment, which determines the size of technical change, and whether to direct technological progress more toward capital- or labor- saving innovations, the direction of technical change. This integration is relevant because the emerging relation between the wage share and labor productivity growth is not necessarily positive, contrary to both the literatures we have reviewed, and because it opens up the possibility for the saving rate to affect the steady-state wage share within the induced innovation framework.

Specifically, we make the following two contributions. First, we find a technological sufficient condition for a positive relation between the wage share and labor productivity growth. This occurs when the productivity of R&D investment is independent of the choice of direction of technical change, implying that the wage share separately affects the optimal direction and size of technical change. We prove this result for any factor elasticity of substitution smaller than one. Second, we embed our microeconomic analysis into Classical and neoclassical growth models with exogenous labor supply. We show that the saving rate affects the long-run distribution of income if, and only if, R&D investments change the marginal rate of transformation between labor and capital productivity growth. This happens when the elasticities of labor and capital productivity growth to R&D investment are different. With no obvious reasons to believe that returns to R&D expenditure differ along the capital or labor productivity dimensions, this result can be seen as a generalization of the original conclusion of induced innovation literature that the long run labor share is merely a function of the slope of the IPF and thus independent of the saving rate.

Both results have relevant policy implications. On the one hand, any change in labor market institutions that affects the wage share may have an indirect effect on labor productivity growth. On the other hand, fiscal policy reforms that influence an economy's saving propensity could have long-run distributional consequences.

At this stage, it is useful to anticipate the intuition of our results. With respect to the first one, we should notice that the overall effect of the wage share on labor productivity growth depends on how it affects both the direction of technical change and the amount of R&D investment a firm makes. A rise in the wage share always makes it more convenient to direct technical change toward labor productivity growth to save the more expensive factor of production. The wage share effect on R&D investment, however, is not as straightforward. On the one hand, it is always true that a higher wage share makes increasing labor productivity more convenient than raising the physical capital stock, thus creating an incentive for higher R&D investment. However, when the problem of choosing the size and direction of technical change does not decompose, the benefit of investing in R&D depends on the direction of technical change. This introduces an additional indirect influence by the wage share on the incentive to invest in R&D that may go in the opposite direction and offset the positive direct effect.

Comparing our steady state analysis with the findings of the original induced innovation literature and of the recent Classical growth models with endogenous technical change sheds light on the rationale of our second result. When technical change is exogenous and costless, the steady state wage share is independent of the saving rate as it is determined by the slope of the IPF, or the marginal rate of transformation between labor and capital productivity growth. When technical change is endogenous, both the size and the shape of the innovation set depend on R&D investment. The saving rate influences long-run income distribution if its effect on R&D investments also changes the marginal rate of transformation between labor and capital saving innovations, which still determines the equilibrium wage share. Therefore, the effect of the saving rate purely depends on its potential impact on technology. This mechanism is quite different from the way the saving rate affects income distribution in Classical growth models with endogenous technical change, where changes in the wage share depend on the way shocks to the saving rate impact labor market tightness.

At the onset of the induced innovation literature, a few contributions have investigated the simultaneous choice of direction and intensity of technical change. Kamien and Schwartz (1969) explored the problem from the microeconomic point of view of a competitive firm. Nordhaus (1967) solved the infinite horizon problem of a benevolent planner who maximizes the discounted value of consumption per capita. von Weizsacker (1966) analyzed a competitive two-sector economy. Their analysis, however, establishes the standard effect of the wage share on the direction of technical change without exploring its overall effect on labor productivity growth (see Kamien and Schwartz 1969, p. 676, eq. 36).

More recently, the joint determination of intensity and direction of technical change has also been analyzed by Acemoglu (2002, 2007, 2010) within the endogenous growth framework based on monopolistic competition developed in the 1990s. He focuses more on the relation between relative factor scarcity, rather than relative factor shares, and factor productivity growth. He shows that the factors elasticity of substitution is crucial in determining the sign of this relation. When the elasticity is lower (higher) than one, a scarcer labor supply will favor labor (capital) augmenting innovations. Our contribution shows that in a competitive framework, additional restrictions besides factor complementarity (i.e., an elasticity of substitution lower than one) are necessary to ensure a positive relation between labor productivity growth and the wage share.

This paper is also related to early 2000s literature that combined endogenous growth and perfect competition. Like in Hellwig and Irmen (2001), Bester and

Petrakis (2003), and Irmen (2005) firms are willing to pay for innovations to earn temporary rents that are eliminated as soon as the new technology becomes public knowledge. Out of these contributions, only Bester and Petrakis (2003) have analyzed the link between income distribution and innovations. They assume that capital productivity is fixed, and R&D investment can only improve labor productivity growth. They find the latter is a positive function of the unit labor cost, but only in a partial equilibrium framework.

Finally, Zamparelli (2015) has introduced the endogenous direction and intensity of technical change in a Classical growth model with exogenous labor supply. On the one hand, he does not find an explicit relation between labor productivity growth and the wage share; on the other, even though he finds that the saving rate affects the wage share, he does not discuss the technological assumptions necessary for this result.

The rest of the paper is organized as follows. Section 2 presents the microeconomic problem of the firm and derives the relation between the wage share and labor productivity growth when capital and labor are perfect complements. Section 3 investigates the steady state connection between the saving rate and the wage share within a Classical growth framework. Section 4 generalizes the analysis of the previous two sections to the neoclassical case, where factors of production are substitutable. Section 5 concludes.

2 The Model

2.1 Households and firms

The economy is populated by a fixed number (normalized to one) of identical households, who are endowed with one unit of homogeneous labor (L) and own a certain share of the capital stock (K). Households supply labor inelastically and, if employed, earn the real wage rate w; they also earn profit income on the capital they own. They save a constant fraction (s) of their total income. Since there are no financial markets, aggregate savings are directly employed to either accumulate capital stock or improve the technology of the representative firm.[1]

2.2 Technology

The final good Y is the numeraire and can be used both for consumption and investment in physical capital or R&D. It is produced by using labor and capital in fixed proportions.[2] There is no depreciation. Letting A and B denote, respectively, labor and capital productivity, the production function is

$$Y = \min\{AL, BK\}. \tag{1}$$

The modeling of technological change includes insights from both the induced innovation literature and the endogenous growth theory. As anticipated in the Introduction, the former represented the evolution of technology through an IPF, which states an inverse relation between the freely available maximum growth rates of labor and capital productivity. The frontier is decreasing and strictly concave in order to capture the increasing complexity in the trade-off between labor-augmenting and capital-augmenting innovations. On the other hand, the endogenous growth literature (see for example Aghion, 2010) posited that technical change is a costly activity that requires investment in physical or human resources. If we let g_x be the growth rate of variable x, we can define an innovation possibility set as

[1]The assumption of a representative firm may appear restrictive, but it is equivalent to assuming a fixed number of firms, each of which has access to the same technology and to the same fraction of aggregate savings.

[2]Section 4 generalizes the production technology to any positive factor elasticity of substitution.

$$P(g_A, g_B, b) \leq 0, \tag{2}$$

where $b \equiv R/Y$ and R is the amount of final good invested in R&D. P represents an innovation technology that uses one input, b, to produce two outputs, labor and capital productivity growth. Efficiency requires firms to choose points on the set boundary, that is points where $P(g_A, g_B, b) = 0$, otherwise they could increase productivity growth at no cost. For a given level of b, say $\bar{b}$, $P(g_A, g_B, \bar{b}) = 0$ implicitly defines a transformation curve between g_A and g_B, that is the highest achievable level of labor productivity growth for any level of capital productivity growth. In fact, the P set generates a family of innovation possibility frontiers, each indexed by a different level of R&D investment. As shown in Figure 2, where $b_1 > b_0$, higher investments push the frontier up and to the right.

This representation of technology is flexible enough to encompass both exogenous and endogenous growth. Exogenous growth assumes that technical change is available without cost or investment: $P(g_A, g_B, 0) = 0$, with either g_A, g_B or both strictly positive. When growth is endogenous, innovation is costly, so no investment yields zero productivity growth $P(0, 0, 0) = 0$. Finally, notice that normalization of R&D investment by total output is imposed to rule out explosive growth. This is a standard result in endogenous growth models when R&D inputs consist of an accumulable factor such as physical output, and it is typically justified with the increasing complexity of discovering new ideas.

In order to make the firm's optimization problem tractable, we generalize the innovation set proposed by Kamien and Schwartz (1969). They assumed that for a given level of R&D spending, the growth rates of productivity growth are related through

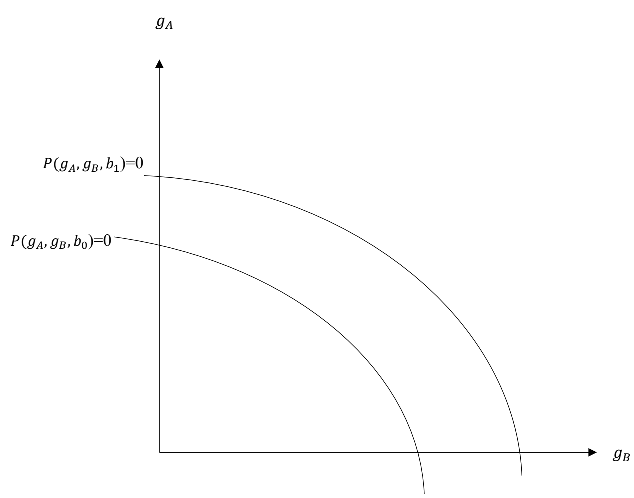

Figure 2: Innovation Possibility Set

$$g_A = f(g_B) = f(\beta), \tag{3}$$

where β is defined by (3) and $\beta, f(\beta) \geq 0$ while $f', f'' < 0$. $f(\beta)$ represents the specific innovation IPF associated to a given level of R&D investment. Different levels of R&D investment push the frontier inward or outward. Unlike Kamien and Schwartz (1969), we do not restrict these shifts to radial homothetic contractions and expansions. Accordingly, we posit

$$g_A = H(f(\beta), b) \tag{4}$$

$$g_B = F(\beta, b), \tag{5}$$

where H and F are twice differentiable and, on the one hand, $H'_b, F'_b > 0$ and $H''_{b,b}, F''_{b,b} < 0$ convey the idea that productivity growth is an increasing and concave function of R&D investment. On the other hand, $F'_\beta, H'_f > 0$ imply that factor productivity growth increases when the direction of technical change is biased in their respective direction. We also add $F''_{\beta,\beta} = H''_{f,f} = 0$ so that the direction of technical change raises linearly each productivity growth rate.

Improvements in technology allow innovators to earn instantaneous rents. The new knowledge becomes freely available to all producers immediately after rents are obtained. This assumption represents the classical notion of competition where firms introduce innovations to earn a temporary advantage over their competitors that disappears as soon as rival firms can imitate the new technology. A similar framework is present in Hellwig and Irmen (2001), Bester and Petrakis (2003), and Irmen (2005), who have introduced endogenous technical change into neoclassical perfectly competitive growth models.

2.3 Income distribution, saving allocation and optimal productivity growth

The owners of the representative firm have no incentive to operate the firm with spare capacity or to hire unproductive labor; therefore, $AL = BK$, so the number of employed workers in the economy is $L = BK/A$. We denote the wage share as $\omega \equiv wL/Y = w/A$, equal to the unit labor cost. Accordingly, total profits are

$$\Pi = Y - wL = Y(1-\omega) = BK(1-\omega). \tag{6}$$

Savings are spent to accumulate physical capital or invested in R&D. From the standpoint of profit-maximization, the two types of investment pose a trade-off. They both increase total profits. While capital accumulation increases the size of a firm, innovations raise its profits per unit of capital by reducing unit costs.[3] Letting μ be the share of savings invested in R&D, the R&D investment share of output is:

$$b = R/Y = \mu sY/Y = \mu s. \tag{7}$$

Physical capital accumulation, on the other end, obeys:

$$g_K = (1-\mu)sY/K = (1-\mu)sB. \tag{8}$$

In order to define the objective function of the representative firm, we extend the original proposal by Kennedy (1964). He assumed that firms take input lev-

[3]Our model, in which firms simultaneously choose investments in physical capital and in R&D, is similar to the endogenous growth literature from the early 2000s; in this framework, competitive firms maximize short-run profits by investing in increasing productive capacity and labor productivity growth (see Hellwig and Irmen (2001) and Bester and Petrakis (2003) in particular). The Schumpeterian endogenous growth literature based on monopolistic competition, on the other hand, has a different structure. In most models, R&D investment is decided by competitive firms, whereas the optimal amount of resources invested to produce intermediate inputs (the measure of circulating capital) is chosen under monopolistic conditions (see for example Aghion and Howitt (2010, Chs. 4 and 5)).

els and prices as given and choose the direction of technical change to maximize the instantaneous rate of growth of the profit rate. This myopic behavior is justified because the temporary rents from innovating dissolve instantaneously as the new technology becomes public knowledge. In our setting, the representative firm still acts myopically, but besides the direction of technical change it also chooses the allocation of savings between R&D investment and capital accumulation to maximize the rate of growth of profits.[4] Differentiation of total profits with respect to time yields $\dot{\Pi} = \dot{B}K(1-\omega)+\dot{K}B(1-\omega)+BK\omega(g_A-g_w)$, where the time derivative of variable x is denoted by $\dot{x}$. The corresponding rate of growth of profits is

$$g_\Pi = g_B + g_K + (g_A - g_w)\omega/(1-\omega). \tag{9}$$

Substituting from Equations (4), (5), (7) and (8), the firm's problem is to choose β and μ to maximize $g_\Pi = F(\beta, s\mu)+s(1-\mu)B+H(f(\beta), s\mu)\omega/(1-\omega)-g_w\omega/(1-\omega)$. Note that real wage growth does not affect the firms' optimal plan since they take it as given when making their maximizing choices. As we show in the next section, the dynamic behavior of wages depends on the economy's aggregate behavior. If we denote the optimal level of a choice variable by $*$, the first order conditions with respect to β and μ are

$$-f'(\beta^*)\frac{H'_f(f(\beta^*), s\mu^*)}{F'_\beta(\beta^*, s\mu^*)} = \frac{1-\omega}{\omega} \tag{10}$$

$$F'_\mu(\beta^*, s\mu^*) + \frac{\omega}{1-\omega}H'_\mu(f(\beta^*), s\mu^*) = B. \tag{11}$$

We are interested in understanding under what conditions this system necessarily produces a positive effect of the wage share on labor productivity growth.

[4]Note that the rate of growth of the profit *rate* and the rate of growth of profits coincide when the level of capital stock is given, as originally assumed by Kennedy (1964).

Notice that

$$\frac{dg_A^*}{d\omega} = H_f'(f(\beta^*), s\mu^*)f'(\beta^*)\frac{d\beta^*}{d\omega} + sH_\mu'(f(\beta^*), s\mu^*)\frac{d\mu^*}{d\omega}.$$

Since $f' < 0$ and $H_f', H_\mu' > 0$, a sufficient condition for $\frac{dg_A^*}{d\omega} > 0$ is that $\frac{d\beta^*}{d\omega} < 0$ and $\frac{d\mu^*}{d\omega} > 0$. This condition is ensured by some technological restrictions summarized in the following

Proposition 1. *When the production technology is Leontief, if* $F_{\mu,\beta}''(\beta, s\mu) = F_{\beta,\mu}''(\beta, s\mu)=H_{\mu,\beta}''(f(\beta), s\mu) = H_{\beta,\mu}''(f(\beta), s\mu) = 0$ *then* $\frac{dg_A^*}{d\omega} > 0$.

Proof. See the Appendix. □

Let us focus on the innovation technology assumptions of Proposition 1. When the second-order mixed partial derivatives of a function of two variables are null, the function is additively separable in its two arguments. Both capital and labor productivity growth can be expressed as the sum of two distinct functions of the direction of technical change and R&D investment. To better grasp the intuition of this functional structure, we investigate a specific example that satisfies these assumptions. Let us posit $g_A = f(\beta) + (s\mu)^\alpha$, and $g_B = \beta + (s\mu)^\alpha$, with $\alpha \in (0, 1)$. The first order conditions with respect to β and μ are

$$-f'(\beta^*) = \frac{1-\omega}{\omega}, \tag{12}$$

and (after some minor manipulations)

$$\mu^* = \frac{1}{s}\left(\frac{\alpha}{B}\frac{1}{1-\omega}\right)^{\frac{1}{1-\alpha}}. \tag{13}$$

Equations (12) and (13) show that the choice of direction and intensity of technical change decomposes into two parts. Equation (12) demands the equality between the slope of the IPF and the relative unit factor cost; this is the exact condition that produced the positive relation between the wage share and labor productivity growth under the original induced innovation hypothesis. In fact, total differentiation of (12) yields $d\beta^*/d\omega = 1/\left(f''(\beta^*)\,(\omega)^2\right) < 0$: for a given amount of R&D investments (the position of the IPF), a rise in the wage share directs technical change away from capital productivity growth and in favor of labor productivity growth. Equation (13), on the other hand, shows that R&D investments are a positive function of the wage share because raising productivity growth becomes relatively more profitable than capital accumulation when unit labor costs increase. We can use the optimal values for β and μ to solve for the equilibrium labor productivity growth as

$$g_A^* = f(\beta^*) + \left(\frac{\alpha}{B}\frac{1}{1-\omega}\right)^{\frac{\alpha}{1-\alpha}},$$

which shows that an increase in the wage share unequivocally raises labor productivity growth given $f'(\beta^*)d\beta^*/d\omega > 0$. To understand why this is the case, we must look at the objective function g_Π. Profits growth is a weighted sum of capital accumulation and capital and labor productivity growth, but only the weight attached to labor productivity growth is an increasing function of the labor share $(\omega/(1-\omega))$. The firm makes two choices: it allocates funds between capital accumulation and R&D investment; and, given the amount of R&D investment, it decides whether to improve capital or labor productivity growth. When these two choices are independent, as in our example, a higher wage share makes labor productivity more profitable than capital productivity growth and, on the other hand, it makes investing in R&D more rewarding than accumulating capital. The two effects of the labor share on the direction and size of technical change move in the same direction to contribute to labor-saving technical change. This occurs because the wage share affects the two choices only through the weight of labor productivity growth in the objective function.

We now use a second example to show that violating the assumptions in Proposition 1 may result in a nonpositive relation between the wage share and labor productivity growth. Consider the alternative specification $g_A = f(\beta)\,(s\mu)^{\alpha}$, and $g_B = \beta\,(s\mu)^{\alpha}$. The two productivity growth functions are not additively separable and $H''_{\mu,\beta} = H''_{\beta,\mu} = f'(\beta)\alpha s^{\alpha}\mu^{\alpha-1} < 0$ and $F''_{\mu,\beta} = F''_{\beta,\mu} = \alpha s^{\alpha}\mu^{\alpha-1} > 0$. We can find the first order conditions with respect to β and μ as

$$-f'(\beta^*) = \frac{1-\omega}{\omega}, \tag{14}$$

and

$$\mu^* = \frac{1}{s}\left(\frac{\alpha}{B}\left(\beta^* + \frac{\omega}{1-\omega}f(\beta^*)\right)\right)^{\frac{1}{1-\alpha}}. \tag{15}$$

Equation (14) is identical to Equation (12) and finds the optimal direction of technical change β^* as a negative function of the wage share. Conversely, (15) shows that μ^* does not solely depend on the wage share, as it is also affected by the optimal choice of technical change β^*. This has important consequences on the relation between labor productivity growth and the wage share. If we substitute from the two first order conditions into $g_A = f(\beta)\,(s\mu)^{\alpha}$ we find:

$$g_A^* = f(\beta^*(\omega))\left(\frac{\alpha}{B}\left(\beta^*(\omega) + \frac{\omega}{1-\omega}f(\beta^*(\omega))\right)\right)^{\frac{\alpha}{1-\alpha}}, \tag{16}$$

where we emphasized the dependence of β^* on the wage share. On the one hand, a rise in the wage share produces a bias in technical change that unequivocally raises labor productivity growth: $f'(\beta^*)d\beta^*/d\omega > 0$. The effect on the size of R&D investment μ^*, on the other hand, is ambiguous. Let us go back to the objective function g_Π to see why. At the margin, μ^* equalizes the return from investing in labor and capital productivity growth to the return from increasing the capital stock. A higher wage share has two conflicting effects on the returns from investing in R&D. It increases both the weight attached to labor productivity growth in the objective function $(\omega/(1-\omega))$ and labor productivity growth itself through $f(\beta^*(\omega))$, but it lowers capital productivity growth through $\beta^*(\omega)$, thus making the overall effect on aggregate productivity growth uncertain. If the fall in capital productivity growth is strong enough, investing in capital accumulation may become more profitable than R&D investment, which would produce a decline in μ^*. Accordingly, a negative relation between the wage share and labor productivity growth may emerge when the optimal choice of the size of technical change depends on the optimal direction of technical change.

The assumptions in Proposition 1 rule out this possibility. The second-order mixed partial derivatives can be expressed as $\frac{d}{d\beta}\left(\frac{dg_i^*}{d\mu}\right)$ and $\frac{d}{d\mu}\left(\frac{dg_i^*}{d\beta}\right)$, where $i = A, B$. Our assumptions require $\frac{d}{d\beta}\left(\frac{dg_i^*}{d\mu}\right) = \frac{d}{d\mu}\left(\frac{dg_i^*}{d\beta}\right) = 0$, which means that the marginal productivity of R&D investment, or the return from R&D, is independent of the direction of technical change. When this happens the two choice variables become sole functions of the wage share. Each of the two first order conditions (10) and (11) individually determines the effect of a shock to the wage share on, respectively the direction and the size of technical change, which ensures $\frac{d\beta^*}{d\omega} < 0$ and $\frac{d\mu^*}{d\omega} > 0$ and a rise in labor productivity growth for the reasons illustrated in our first example.

2.3.1 Discussion

We have established that labor productivity growth is necessarily a positive function of the wage share when factor productivity growth rates are additively separable functions of the direction and size of technical change. Let us now dig deeper into the economic plausibility of this result. If we go back to the early stages of the development of the induced innovation theory, we can gain some insights from Nordhaus's (1973) radical rejection of the theory. The absence of path dependence in innovation technology was one of his main concerns; he found problematic that the evolution of labor and capital productivity would not affect the relative difficulty of introducing factor augmenting innovations: "..the rate of capital-augmenting technological change is everywhere independent of the level of labor augmentation. Thus as technological change accumulates, there is no effect on the trade-off between labor and capital augmenting technological change." (p. 215) Following his logic, we would expect that if technical change is pursued along a constant direction, a certain β, the relative productivity growth of capital and labor would remain constant. We can explore this hypothesis with respect to our two examples. When productivity growth rates are additively separable functions we have $g_A/g_B = (f(\beta) + (s\mu)^{\alpha}) / (\beta + (s\mu)^{\alpha})$, which is a function of R&D expenditure unless $f(\beta) = \beta$. In our second example, on the contrary, $g_A/g_B = f(\beta)/\beta$ independent of R&D investment along any direction of technical change.

From this standpoint, it appears that the innovation technology capable of ensuring a positive relation between the wage share and labor productivity growth is not the most plausible. Still, we should keep in mind that it only provides a sufficient condition so that even when violated, a positive shock to the wage share may produce a rise in labor productivity growth.

3 Income distribution implications

As anticipated in the Introduction, the induced innovation hypothesis has been embedded in both neoclassical and Classical growth models with exogenous labor supply. An important result common to both frameworks is that long-run income distribution depends solely on technology, and specifically on the curvature of the IPF; this implies that the saving rate does not affect the steady state wage share. In this section, we show how the generalization of innovation technology to simultaneously encompass both the choice of direction and the size of technical change affects the role played by the saving rate in the steady state equilibrium of Classical growth models. We develop our result in the framework offered by Shah and Desai (1981), since they have been the first to introduce the induced innovation hypothesis into a Classical growth model. They did so by adding a costless, freely available, IPF to Goodwin's (1967) growth cycle model. The aggregate economy is described by three differential equations, and the output-capital ratio, the labor share and the employment rate are the three state variables (see also van der Ploeg, 1987; Foley, 2003; Julius, 2005). Since firms do not perform R&D investment, labor productivity growth only depends on capital productivity growth, say $g_A = j(g_B)$, while all savings are invested in physical capital accumulation so that $g_K = sB$. Notice also that when exogenous labor supply (N) is normalized to one the employment rate coincides with total employment L. In our notation, the dynamical system is:

$$-j'(g_B^*) = \frac{1-\omega}{\omega}$$

$$g_L = g_B^* + sB - j(g_B^*)$$

$$g_\omega = g_w - j(g_B^*) = m(L) - j(g_B^*),$$

where $g_w = m(L)$ is a real wage Phillips curve describing the positive effect of labor market tightness on real wage growth. Steady states require that capital productivity growth be turned off, so that $g_B^* = 0$ determines the long run wage share. If we denote steady state values by ss we can find ω_{ss} as solution to

$-j'(0) = \frac{1-\omega_{ss}}{\omega_{ss}}$, that is $\omega_{ss} = 1/(1-j'(0))$. The steady state wage share is determined by the slope of the IPF where capital productivity growth is zero, irrespective of the saving rate.

Le us now explore how the dynamical system changes when innovations are costly and require investment, that is when we adopt the innovation technology $g_A = H(f(\beta), s\mu)$ and $g_B = F(\beta, \mu)$. In particular, in order to obtain analytical conclusions, let us slightly modify our second example and assume $g_A = f(\beta)\,(s\mu)^{\gamma}$ and $g_B = \beta\,(s\mu)^{\alpha}$, with $\gamma \in (0,1)$. Notice that when $\gamma = \alpha$ we are back to our second example. If, on the contrary, $\gamma \neq \alpha$, R&D investments affect labor and capital productivity growth asymmetrically and the expansion of the innovation possibility frontiers is non-homothetic. In this case, choosing β and μ to maximize $g_\Pi = \beta\,(s\mu)^{\alpha} + s(1-\mu)B + f(\beta)\,(s\mu)^{\gamma}\,\omega/(1-\omega)$ yields the following first order conditions

$$-f'(\beta^*)\,(s\mu^*)^{\gamma-\alpha} = \frac{1-\omega}{\omega} \tag{17}$$

$$\beta^*\alpha\,(s\mu^*)^{\alpha-1} + f(\beta^*)\gamma\,(s\mu^*)^{\gamma-1}\,\omega/(1-\omega) = B. \tag{18}$$

As a first point, notice that the left-hand side of (17) is the marginal rate of transformation between labor and capital productivity growth. We can calculate it by plugging $\beta = g_B/\,(s\mu)^{\alpha}$ into $g_A = f(\beta)\,(s\mu)^{\gamma}$ and finding $-\frac{dg_A}{dg_B} = -f'(\beta)(s\mu)^{\gamma-\alpha}$. Firms choose the optimal direction of technical change by equalizing the slope of the IPF to the relative unit factor cost. The difference with the original induced innovation theory is that the slope of the IPF depends in principle on the size of R&D investment. Next, focus on the system of Equations (17) and (18). It implicitly finds β^*, μ^* as functions of the saving rate and the two state variables wage share and output-capital ratio, say

$\beta^* = \beta(s, \omega, B)$ and $\mu^* = \mu(s, \omega, B)$. We can use them to define a differential equation for the output-capital ratio as $g_B = \beta(s, \omega, B)\,(s\mu(s, \omega, B))^{\alpha}$. The rest of the dynamical system is

$$g_L = \beta^*\,(s\mu^*)^{\alpha} + s(1-\mu^*)B - f(\beta^*)\,(s\mu^*)^{\gamma}$$

$$g_\omega = m(L) - f(\beta^*)\,(s\mu^*)^{\gamma}\,.$$

We know that in the steady-state capital productivity growth is turned off, that is $\beta^* = 0$. First note that if $\gamma = \alpha$, Equation (17) finds the steady state wage share independently of the saving rate as $\omega_{ss} = 1/(1 - f'(0))$. This is the original result of the induced innovation hypothesis, where long-run income distribution depends only on the slope of the IPF when capital productivity growth is zero. However, this is not the case when $\gamma \neq \alpha$. Equation (17) yields $\mu_{ss} = (-f'(0))^{\frac{1}{\alpha-\gamma}} \left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{\alpha-\gamma}}/s$, while from Equation (18) we find $\mu_{ss} = \left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{1-\gamma}}/s$. We can use both equations jointly to obtain an isocline in the (ω_{ss}, B_{ss}) space: $B_{ss} = \gamma\frac{f(0)}{-f'(0)^{\frac{1-\gamma}{\alpha-\gamma}}}\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1-\alpha}{\alpha-\gamma}}$. If we turn to the law of motion of the employment rate and we set the steady state condition $g_L = 0$ while using $\mu_{ss} = \left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{1-\gamma}}/s$, we obtain an additional isocline in the (ω_{ss}, B_{ss}) plane: $B_{ss} = f(0)\gamma^{\gamma}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\gamma}\left(\gamma\frac{\omega_{ss}}{1-\omega_{ss}} + 1\right)^{1-\gamma}/s^{1-\gamma}$, as shown in the Appendix.

The two isoclines jointly determine the long-run values of the wage share and the capital-output ratio. Since the saving rate enters the second isocline through both capital accumulation and the size of R&D investment, it also affects the steady state wage share. In the Appendix we show that the two isoclines can be used to find ω_{ss} as solution to

$$s = \left(-f'(0)\right)^{\frac{1}{\alpha-\gamma}} \left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}} \left(\frac{\omega_{ss}}{1-\omega_{ss}} + 1/\gamma\right).$$

We can now state

Proposition 2. *A rise in the saving rate has a positive, null or negative effect on the steady state wage share depending on whether* $\alpha \gtreqless \gamma$.

Proof. See the Appendix. □

We can interpret this result by looking at Equation (17) evaluated at the steady state:

$$-f'(0)\left(s\mu_{ss}\right)^{\gamma-\alpha} = \frac{1-\omega_{ss}}{\omega_{ss}}.$$

It states that the steady state wage share is determined by the marginal rate of transformation between labor and capital productivity growth when capital productivity growth is zero. We have already seen how this implies that the steady state wage share is independent of the saving rate when $\alpha = \gamma$. This occurs because under this condition R&D investments improve labor and capital productivity growth at the same rate, which ensures the homothetic expansion and contraction of the IPF family. Along any ray coming out of the origin, the marginal rate of transformation is constant. It is independent of the amount of R&D investment performed and, in turn, of the saving rate. In particular, this is also true when $\beta = 0$, which is the steady state condition.

In contrast, the marginal rate of transformation depends on the amount of R&D investment and the saving rate when $\alpha \neq \gamma$. If $\gamma > \alpha$, the marginal rate of transformation is an increasing function of expenditure in R&D. Higher R&D investments make the slope of the IPF steeper and this will require a lower equilibrium wage share. Symmetrically, if $\gamma < \alpha$, the slope of the IPF becomes flatter with more expenditure in R&D, which makes the equilibrium wage share rise in response.

The possibility that the saving rate affect long-run income distribution thus depends on its potential influence on the shape of innovation possibility set. As such, this mechanism is quite different from the way the saving rate affects the wage share in Classical growth models with endogenous technical change, where distributional changes are the results of higher capital accumulation and labor demand relative to the exogenous labor supply. On the contrary, our result appears more in line with the conclusions of the induced innovation literature where the equilibrium wage share is a mere function of the curvature of the IPF. In fact, even when the size of technical change is endogenous, the steady state wage share still only depends on the marginal rate of transformation between labor and capital productivity growth. The saving rate becomes relevant only if R&D investments have different returns on labor and capital productivity growth, that is when $\alpha \neq \gamma$. Assessing this possibility may be an empirical issue and future research on the topic may help verify it. In the meantime, the more plausible and intuitive assumption is that R&D efforts are equally productive along both the labor and the capital directions. From this standpoint, our result appears more like a generalization than a confutation of the original induced innovation hypothesis.

4 The neoclassical version of the model

As we mentioned, the induced innovation hypothesis has been embedded in both neoclassical and Classical growth models with exogenous labor supply. Thus far, we have worked within the Classical growth framework. One distinctive feature of this approach consists in the non-substitutability among factors of production, which implies no full employment, except by a fluke. As a robustness check to our results, we implement our analysis in a neoclassical growth model, which assumes that factors of production are substitutes, they are paid according to their marginal products and are fully employed at all times. We show below that a positive elasticity of substitution affects our Proposition 1, but plays no role in Proposition 2, since the latter is founded on the innovation rather than the production technology.

Let us assume that factors of production can be combined according to a CES production function

$$Y = F[K, L] = [(BK)^{\rho} + (AL)^{\rho}]^{\frac{1}{\rho}}, \tag{19}$$

where $\rho \in (-\infty, 1]$ determines the elasticity of substitution between capital and labor (σ), with $\sigma = 1/(1-\rho)$. We have thus far assumed $\rho \to -\infty$, that is no factor substitution. Our first step is to calculate the objective function g_{π}. Remember that $\Pi = Y(1-\omega) = [(BK)^{\rho} + (AL)^{\rho}]^{\frac{1}{\rho}}(1-\omega)$. If we define $[(BK)^{\rho} + (AL)^{\rho}]^{\frac{1}{\rho}} \equiv \Lambda^{\frac{1}{\rho}}$, then $\dot{\Pi} = (1-\omega)\Lambda^{\frac{1}{\rho}-1}\{(BK)^{\rho}(g_B + g_K) + (AL)^{\rho}(g_A + g_L)\} + \Lambda^{\frac{1}{\rho}}(g_A - g_w)\omega/(1-\omega)$, or $\dot{\Pi} = (1-\omega)\Lambda^{\frac{1}{\rho}}\left\{\frac{(BK)^{\rho}(g_B+g_K)+(AL)^{\rho}(g_A+g_L)}{\Lambda} + (g_A - g_w)\omega/(1-\omega)\right\}$. Hence $g_{\pi} = \dot{\Pi}/\Pi = \frac{(BK)^{\rho}(g_B+g_K)+(AL)^{\rho}(g_A+g_L)}{(BK)^{\rho}+(AL)^{\rho}} + (g_A - g_w)\omega/(1-\omega)$. If we divide both sides of the first addend by $(AL)^{\rho}$ and set $k \equiv BK/AL$, we have $g_{\pi} = \frac{k^{\rho}(g_B+g_K)+(g_A+g_L)}{k^{\rho}+1} + (g_A - g_w)\omega/(1-\omega)$. Since factors of production are paid their marginal products, then $1-\omega = F_K' K/Y$, which, after a few minor manipulations, yields $1-\omega = \frac{k^{\rho}}{k^{\rho}+1}$. Hence, $\omega = \frac{1}{k^{\rho}+1}$ and $g_{\pi} = (1-\omega)(g_B+g_K)+\omega g_L+g_A\omega(2-\omega)/(1-\omega)-g_w\omega/(1-\omega)$. Next note that in the neoclassical model the (constant) labor force is continuously fully employed so that $g_L = g_N = 0$, and $g_{\pi} = (1-\omega)(g_B+g_K)+g_A\omega(2-\omega)/(1-\omega)-g_w\omega/(1-\omega)$. We know from (4) and (5) that $g_A = H(f(\beta), s\mu)$ and $g_B = F(\beta, s\mu)$. We can calculate $g_K = (1-\mu)sY/K = (1-\mu)s\,[(BK)^{\rho} + (AL)^{\rho}]^{\frac{1}{\rho}}/K =$

$$(1-\mu)s\left[B^\rho + (B/k)^\rho\right]^{\frac{1}{\rho}} = (1-\mu)sB\left[1/(1-\omega)\right]^{\frac{1}{\rho}}.$$

4.1 Labor augmenting technical change and the wage share

Before proceeding, we shall note that when the factor elasticity of substitution is positive, we cannot interpret the parameter A as an exact measure of labor productivity. When output is produced according to (19) we have $Y/L = [(BK/L)^\rho + A^\rho]^{\frac{1}{\rho}} \neq A$. Accordingly, we refer to g_A as labor-augmenting technical change rather than labor productivity growth. Analogously, g_B indicates capital-augmenting technical change rather than capital productivity growth.

We are now in a position to choose β and μ to maximize $g_\pi = (1-\omega)(F(\beta, s\mu) + (1-\mu)sB(1-\omega)^{1-\frac{1}{\rho}} + H(f(\beta), s\mu)\omega(2-\omega)/(1-\omega) - g_w\omega/(1-\omega)$. The first order conditions are

$$-f'(\beta^*)\frac{H'_f(f(\beta^*), s\mu^*)}{F'_\beta(\beta^*, s\mu^*)} = \frac{(1-\omega)^2}{\omega(2-\omega)} \tag{20}$$

$$F'_\mu(\beta^*, s\mu^*)(1-\omega)^{1/\rho} + \frac{\omega(2-\omega)}{(1-\omega)^{2-1/\rho}}H'_\mu(f(\beta^*), s\mu^*) = B. \tag{21}$$

Equation (20) closely resembles Equation (10). We show in the Appendix that, under the assumptions of Proposition 1, it ensures that β is a negative function of ω, that is, a rise in the wage share directs technical change toward labor-augmenting technical change. On the other hand, Equation (21) differs from Equation (11) in that the elasticity of substitution appears in the new version of the first order condition. Accordingly, a positive effect of the wage share on R&D investment ($\frac{d\mu^*}{d\omega} > 0$) requires some additional technological restrictions. We can state

Proposition 3. *When the elasticity of substitution between factors of production is positive, if* $F''_{\mu,\beta}(\beta^*, s\mu^*) = F''_{\beta,\mu}(\beta^*, s\mu^*) = H''_{\mu,\beta}(f(\beta^*), s\mu^*) = H''_{\beta,\mu}(f(\beta^*), s\mu^*) = 0$ *and* $\sigma < 1$, *then* $\frac{dg_A^*}{d\omega} > 0$.

Proof. See the Appendix. $\square$

In contrast to Proposition 1, the sufficient condition for a positive effect from the wage share on labor augmenting innovations now depends on the elasticity of substitution. We can appreciate the reason by looking at the objective function g_π. In this case, the growth rate of profits is a weighted sum of capital accumulation, capital- and labor- augmenting technical change. When $\sigma > 1$, the weight attached to capital accumulation is a positive function of the wage share. A rise in the wage share provides an incentive to invest in physical capital, which may result in declines in R&D investment and labor productivity growth. On the other hand, when $\sigma < 1$ the weight of capital accumulation in profit growth is a negative function of the wage share so that an increase in the wage share reduces the return from investing in physical capital relative to R&D. No such mechanism is at play in Section 2, where the contribution of capital accumulation to profit growth is independent of the wage share.

4.2 Steady state income distribution

We now turn to the second contribution of our paper, and explore the influence of the saving rate on steady-state income distribution when the factor elasticity of substitution is positive. Since in a neoclassical model the labor force is continuously fully employed, there is no dynamic equation for the employment rate. The steady state equilibrium stabilizes the stock of capital-augmenting technologies and the wage share (B_{ss} and ω_{ss}) at levels that ensure zero capital-augmenting technical change and equality between the warranted and natural growth rates. The dynamical system consists of two equations in the state variables B and k.

As in Section 3 we assume $g_A = f(\beta)\,(s\mu)^\gamma$ and $g_B = \beta\,(s\mu)^\alpha$. Choosing β

and μ to maximize $g_{\Pi} = (1-\omega)\beta\left(s\mu\right)^{\alpha} + (1-\omega)^{1-1/\rho}s(1-\mu)B + f(\beta)\left(s\mu\right)^{\gamma}\omega(2-\omega)/(1-\omega) - g_w\omega/(1-\omega)$ yields the following system of first order conditions

$$-f'(\beta^*)\left(s\mu^*\right)^{\gamma-\alpha} = \frac{(1-\omega)^2}{2-\omega} \tag{22}$$

$$(1-\omega)\beta^*\alpha\left(s\mu^*\right)^{\alpha-1} + f(\beta^*)\gamma\left(s\mu^*\right)^{\gamma-1}\omega(2-\omega)/(1-\omega) = B(1-\omega)^{1-1/\rho}. \tag{23}$$

This implicitly finds β^*, μ^* as functions of the saving rate, the wage share and B, say $\beta^* = \beta(s, \omega, B)$ and $\mu^* = \mu(s, \omega, B)$. We can use them to define a differential equation for capital-augmenting technical change as $g_B = \beta^*\left(s\mu^*\right)^{\alpha}$. The second dynamic equation describes the evolution of the factor ratio k as $g_k = \beta^*\left(s\mu^*\right)^{\alpha} + (1-\mu^*)sB\left[1/(1-\omega)\right]^{\frac{1}{\rho}} - f(\beta^*)\left(s\mu^*\right)^{\gamma}$. We know that in the steady state capital-augmenting technical change is turned off, that is $\beta^* = 0$. First, notice that if $\gamma = \alpha$, Equation (22) finds the steady state wage share independently of the saving rate as the solution to $-f'(0) = \frac{(1-\omega_{ss})^2}{2-\omega_{ss}}$. Just as in original induced innovation literature, income distribution depends only on the slope of the IF when capital-augmenting technical change is zero. Things change when $\gamma \neq \alpha$. Equation (22) yields $\mu_{ss} = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^2}\right)^{\frac{1}{\alpha-\gamma}}/s$, while from Equation (23) we find $\mu_{ss} = \left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}(2-\omega_{ss})}{\left(1-\omega_{ss}\right)^{2-\frac{1}{\rho}}}\right)^{\frac{1}{1-\gamma}}/s$. We can combine the two equations to obtain an incline in the (ω_{ss}, B_{ss}) space: $B_{ss} = \gamma\frac{f(0)}{-f'(0)^{\frac{1-\gamma}{\alpha-\gamma}}}\frac{\left(1-\omega_{ss}\right)^{\frac{2(1-\alpha)}{\alpha-\gamma}+\frac{1}{\rho}}}{\left(\omega_{ss}(2-\omega_{ss})\right)^{\frac{(1-\alpha)}{\alpha-\gamma}}}$. If we turn to the law of motion of k and we set the steady state condition $g_k = 0$ while using $\mu_{ss} = \left(\frac{\omega_{ss}(2-\omega_{ss})}{\left(1-\omega_{ss}\right)^{2-\frac{1}{\rho}}}\right)^{\frac{1}{1-\gamma}}/s$, we obtain an additional isocline in the (ω_{ss}, B_{ss}) plane: $B_{ss} = f(0)\gamma^{\gamma}\frac{\left(\omega_{ss}(2-\omega_{ss})\right)^{\gamma}}{\left(1-\omega_{ss}\right)^{2\gamma-\frac{1}{\rho}}}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^2}+1\right)^{1-\gamma}/s^{1-\gamma}$, as shown in the Appendix.

We can use the two inclines to determine the long-run values of the wage share and the stock of capital-augmenting technologies. In the Appendix we show that ω_{ss} solves

$$s = (-f'(0))^{\frac{1}{\alpha-\gamma}} \left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2-\frac{1}{\rho}}} \right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}} \left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}} + 1/\gamma \right).$$

We can now state

Proposition 4. *A rise in the saving rate has a positive, null or negative effect on the steady state wage share depending on whether* $\alpha \gtreqless \gamma$.

Proof. See the Appendix. □

Proposition 4 is identical to Proposition 2, as is its interpretation. This happens because the steady-state results depend solely on innovation technology, while production technology plays no role. The saving rate affects long-run income distribution if, and only if, the marginal rate of transformation between labor and capital productivity growth depends on R&D investment. This additional generalization appears to confirm the original induced innovation hypothesis.

5 Conclusions

Most advanced economies have recently experienced a slowdown in productivity growth (Dipped, 2021). The notion that declining, or low, real wages may be contributing to this trend is becoming increasingly popular in the public debate: "Faced with reduced labour costs, employers have lesser incentives to substitute capital for labour, especially in labour intensive sectors, which hinders diffusion of artificial intelligence and other technologies." (ILO, 2018). More generally, several commentators have suggested that rising income inequality is likely a relevant factor in explaining the present sluggish level of economic activity known as 'secular stagnation'. This relation may operate through both demand side factors, such as a higher average propensity to save (see for example Summers, 2014; Storm, 2017; and Kiefer et al., 2020), and supply side elements, like limited incentives to innovate due to low labor costs (Petach and Tavani, 2020).

The simultaneous rise in income inequality and productivity slowdown is also at the center of our paper. We have reviewed different strands of economic literature that, by focusing either on the direction or on the size on innovation, have provided strong micro-foundations for a positive relation between the wage share and labor productivity growth. We have found technological restrictions that ensure this relation holds even when firms simultaneously choose both the direction and the size of innovation: when factors of production are complements, a rise in the wage share necessarily increases labor productivity growth if the productivity of R&D investment is independent of the choice of direction of technical change. This condition implies that the wage share affects separately the optimal direction and size of technical change. Furthermore, we have shown that the saving rate may have an effect on the steady state wage share, but only if R&D investments change the marginal rate of transformation between labor and capital productivity growth. Since this requires the counter-intuitive condition that R&D returns be different along the labor and capital dimensions, our result appears in line with the original induced innovation literature conclusion that the long-run labor share is a mere function of innovation technology.

6 References

Acemoglu, D. (2002), Directed Technical Change, *Review of Economic Studies*, 69 (4): 781-810.

Acemoglu, D. (2007), Equilibrium Bias of Technology, *Econometrica*, 75 (5): 1371-1410.

Acemoglu, D. (2007), When Does Labor Scarcity Encourage Innovation? *Journal of Political Economy*, 118(6): 1037-1078.

Aghion, P. and Howitt, P. (1992), A Model of Growth Through Creative Destruction. *Econometrica*, 60(2): 323-351.

Aghion, P., and Howitt, P. (2010), *The Economics of Growth.* Cambridge, MA: MIT Press.

Allen, Robert C. (2009), *The British Industrial Revolution in Global Perspective.* New York: Cambridge Univ. Press.

Bester, H. and Petrakis, E. (2003), Wages and Productivity Growth in a Competitive Industry. *Journal of Economic Theory*, 109 (1): 52-69.

Dieppe, A. ed. (2021), *Global Productivity: Trends, Drivers, and Policies.* Washington, DC: World Bank.

Drandakis, E. M., and Phelps, E. S. (1965), A model of induced invention, growth and distribution. *Economic Journal*, 76 (304): 823–40.

Foley, D. K., Michl, T. R., and Tavani, D., (2019), *Growth and Distribution, Second Edition.* Cambridge, MA: Harvard University Press.

Foley, D. K. (2003), Endogenous technical change with externalities in a classical growth model. *Journal of Economic Behavior and Organization* , 52 (2): 167-189.

Fontanari, C. and Palumbo, A. (2023), Permanent scars: The effects of wages on productivity. *Metroeconomica*, 74(2): 351-389.

Goodwin, R. (1967), A Growth Cycle. In: C. H. Feinstein, ed. *Socialism,* *Capitalism, and Economic Growth.* Cambridge: Cambridge University Press.

Grossman, G. and Helpman, E. (1991), *Innovation and Growth in the Global Economy.* Cambridge, MA: MIT Press.

Habakkuk, H. J. (1962), *American and British Technology in the Nineteenth Century.* London: Cambridge Univ. Press.

Hellwig, M. and Irmen, A. (2001), Endogenous Technical Change in a Competitive Economy. *Journal of Economic Theory*, 101 (1): 1-39.

Hicks, J.R. (1932[1960]), *The Theory of Wages.* London: Macmillan.

ILO (2018), Statement by Guy Ryder, ILO Director-General, to the International Monetary and Financial Committee, https://www.ilo.org/global/about-the-ilo/newsroom/statements-and-speeches/WCMS_626195/lang--en/index.htm.

Irmen, A. (2005), Extensive and Intensive Growth in a Neoclassical Framework. *Journal of Economic Dynamics and Control* 29 (8): 1427-1448.

Julius, A. J. (2005), Steady state growth and distribution with an endogenous direction of technical change. *Metroeconomica*, 56 (1): 101-125.

Kamien, M. I. and Schwartz, N. L. (1969), Induced Factor Augmenting Technical Progress from a Microeconomic Viewpo. *Econometrica*, 37(4): 668-684.

Kennedy, C. (1964), Induced bias in innovation and the theory of distribution. *Economic Journal*, 74 (295): 541-547.

Kiefer, D., Mendieta-Munoz, I., Rada, C. and Von Arnim, R. (2020), Secular stagnation and income distribution dynamics. *Review of Radical Political Economics*, 52(2): 189-207.

Nordhaus, W. D. (1967), The Optimal Rate and Direction of Technical Change, in Shell, K., ed. *Essays on the Theory of Optimal Economic Growth.* Cambridge, MA: MIT Press.

Nordhaus, W. D. (1973), Some Skeptical Thoughts on the Theory of Induced Innovation. *The Quarterly Journal of Economics* 87 (2): 208-219.

Petach, L. and Tavani, D. (2020), Income shares, secular stagnation and the long-run distribution of wealth. *Metroeconomica,* 71 (1): 235-55.

Schumpeter, J. A. (1911[2008]), *The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest and the Business Cycle*, New Brunswick (U.S.A) and London (U.K.): Transaction Publishers.

Schumpeter, J. (1942), *Capitalism, Socialism and Democracy.* New York: Harper and Roe Publishers.

Segerstrom, P. S., Anant, T. and Dinopoulos, E. (1990), A Schumpeterian Model of the Product Life Cycle. *American Economic Review*, 80(5): 1077-1092.

Shah, A., and Desai, M. (1981), Growth cycles with induced technical change. *The Economic Journal*, 91 (364): 1006-1010.

Storm, S. (2017), The New Normal: Demand, Secular Stagnation, and the Vanishing Middle Class. *International Journal of Political Economy*, 46(4): 169-210.

Summers, L. (2014), U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. *Business Economics*, 49(2): 65-73.

Tavani, D. and Zamparelli, L. (2021), Labor-Augmenting Technical Change and the Wage Share: New Microeconomic Foundations, *Structural Change and Economic Dynamics*, 2021, 56: 27-34.

van der Ploeg, F. (1987), Growth Cycles, Induced Techincal Change, and Perpetual Conflict over the Distribution of Income. *Journal of Macroeconomics*, 9 (1): 1-12.

von Weizsacker, C.C. (1962[2019]), A New Technical Progress Function. *German Economic Review*, 11 (3): 248-265.

von Weizsacker , C.C. (1966), Tentative notes on a two sector model with induced technical progress. *Review of Economic Studies*, 33 (3): 245–51.

7 Appendix

7.1 Proof of Proposition 1

If we totally differentiate the system of first order conditions (10) and (11) with respect to β^*, μ^* and ω, after rearranging and dropping the arguments of the function for parsimony, we find

$$\left(f'' \frac{H'_f}{F'_\beta} + \frac{f'}{(F'_\beta)^2}\left(H''_{f,f} f' F'_\beta - F''_{\beta,\beta} H'_f\right)\right) \frac{d\beta^*}{d\omega} + s \frac{f'}{(F'_\beta)^2}\left(H''_{f,\mu} F'_\beta - F''_{\beta,\mu} H'_f\right) \frac{d\mu^*}{d\omega} = \frac{1}{\omega^2}$$

$$\left(F''_{\mu,\beta} + \frac{\omega}{1-\omega} H''_{\mu,\beta}\right) \frac{d\beta^*}{d\omega} + s\left(F''_{\mu,\mu} + \frac{\omega}{1-\omega} H''_{\mu,\mu}, s\mu^*)\right) \frac{d\mu^*}{d\omega} = -\frac{H'_\mu}{(1-\omega)^2}.$$

Let us focus on the role played by the second-order mixed partial derivatives. From Young's theorem we know $F''_{\mu,\beta} = F''_{\beta,\mu}$ and $H''_{\mu,\beta} = H''_{\beta,\mu}$ and from the chain rule $H''_{\beta,\mu} = H''_{f,\mu} f'$. When all the mixed partial derivatives are null, the system simplifies to

$$\frac{d\beta^*}{d\omega} = \frac{1}{\omega^2 f'' \frac{H'_f}{F'_\beta}} < 0$$

$$\frac{d\mu^*}{d\omega} = -\frac{H'_\mu}{s(1-\omega)^2} / \left(F''_{\mu,\mu} + \frac{\omega}{1-\omega} H''_{\mu,\mu}\right) > 0.$$

This shows that the two conditions sufficient for the positive effect of the wage share on labor productivity growth are satisfied, and $\frac{dg^*_A}{d\omega} > 0$ follows necessarily.

7.2 Steady state solution and proof of Proposition 2

Let us start with $g_L = 0$ and $\beta_{ss} = 0$. We have $s(1-\mu_{ss})B_{ss} = f(0)\left(s\mu_{ss}\right)^{\gamma}$. Plugging $\mu_{ss} = \left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{1-\gamma}}/s$ into the previous equation and rearranging yields $sB_{ss} = B_{ss}\left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1}{1-\gamma}} + f(0)\left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{\gamma}{1-\gamma}}$,which we can solve for B_{ss} to find $B_{ss} = f(0)\gamma^{\gamma}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\gamma}\left(\gamma\frac{\omega_{ss}}{1-\omega_{ss}}+1\right)^{1-\gamma}/s^{1-\gamma}$.

Next, use the two isoclines in the (B_{ss}, ω_{ss}) space to find: $\gamma\frac{f(0)}{-f'(0)^{\frac{1-\gamma}{\alpha-\gamma}}}\left(\frac{1-\omega_{ss}}{\omega_{ss}}\right)^{\frac{1-\alpha}{\alpha-\gamma}} = f(0)\gamma^{\gamma}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\gamma}\left(\gamma\frac{\omega_{ss}}{1-\omega_{ss}}+1\right)^{1-\gamma}/s^{1-\gamma}$. Simplifying, elevating to the power of $1/(1-\gamma)$ and rearranging yields:

$$s = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2-\frac{1}{\rho}}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}\left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1/\gamma\right).$$

We can totally differentiate the previous equation w.r.t. ω_{ss} and s to find:

$$ds = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left\{\frac{1+\gamma-\alpha}{\alpha-\gamma}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}\frac{1}{(1-\omega_{ss})^{2}}\left(\frac{\omega_{ss}}{1-\omega_{ss}}+1/\gamma\right)+\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}\frac{1}{(1-\omega_{ss})^{2}}\right\}d\omega_{ss}.$$

Hence $ds = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}\frac{1}{(1-\omega_{ss})^{2}}\left\{\frac{1+\gamma-\alpha}{\alpha-\gamma}\left(\frac{\omega_{ss}}{1-\omega_{ss}}+1/\gamma\right)+\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)\right\}d\omega_{ss}$, $ds = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}\frac{1}{(1-\omega_{ss})^{2}}\left\{\frac{1}{\alpha-\gamma}\frac{\omega_{ss}}{1-\omega_{ss}}+\frac{1+\gamma-\alpha}{\alpha-\gamma}\frac{1}{\gamma}\right\}d\omega_{ss}$, and finally $ds = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}}{1-\omega_{ss}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}\frac{1}{(1-\omega_{ss})^{2}}\frac{1}{\alpha-\gamma}\left\{\frac{\omega_{ss}}{1-\omega_{ss}}+\frac{1+\gamma-\alpha}{\gamma}\right\}d\omega_{ss}$. Since all factors multiplying $d\omega_{ss}$ are positive save for $(\alpha-\gamma)$, we can conclude that $sign\frac{d\omega_{ss}}{ds} = sign\left(\alpha-\gamma\right)$.

7.3 Proof of Proposition 3

If we totally differentiate the system of first order conditions (20) and (21) with respect to β^*, μ^* and ω, after rearranging and dropping the arguments of the function for parsimony, we find

$$\left(f''\frac{H'_f}{F'_\beta}+\frac{f'}{(F'_\beta)^2}\left(H''_{f,f}f'F'_\beta-F''_{\beta,\beta}H'_f\right)\right)\frac{d\beta^*}{d\omega}+s\frac{f'}{(F'_\beta)^2}\left(H''_{f,\mu}F'_\beta-F''_{\beta,\mu}H'_f\right)\frac{d\mu^*}{d\omega}=\frac{2}{(1-\omega)\omega^2}$$

$$\left(F''_{\mu,\beta}(1-\omega)^{1/\rho}+\frac{\omega(2-\omega)}{(1-\omega)^{2-1/\rho}}H''_{\mu,f}f'\right)\frac{d\beta^*}{d\omega}+s\left(F''_{\mu,\mu}(1-\omega)^{1/\rho}+\frac{\omega(2-\omega)}{(1-\omega)^{2-1/\rho}}H''_{\mu,\mu}\right)\frac{d\mu^*}{d\omega}=$$

$$=F'_\mu\frac{1}{\rho}(1-\omega)^{1/\rho-1}-H'_\mu\frac{(2-\omega)^2+\omega(2-\omega)(2-1/\rho)}{(1-\omega)^{3-1/\rho}}.$$

Under our assumptions $F''_{\mu,\beta}=F''_{\beta,\mu}=H''_{\mu,\beta}=H''_{\beta,\mu}=0$ and from the chain rule $H''_{\beta,\mu}=H''_{f,\mu}f'$, the system simplifies to

$$\frac{d\beta^*}{d\omega}=\frac{2}{\omega^2(1-\omega)f''\frac{H'_f}{F'_\beta}}<0$$

$$\frac{d\mu^*}{d\omega}=\frac{F'_\mu\frac{1}{\rho}(1-\omega)^{1/\rho-1}-H'_\mu\frac{2(1-\omega)^2+\omega(2-\omega)(2-1/\rho)}{(1-\omega)^{3-1/\rho}}}{s\left(F''_{\mu,\mu}(1-\omega)^{1-1/\rho}+\frac{\omega(2-\omega)}{(1-\omega)^{1-1/\rho}}H''_{\mu,\mu}\right)}.$$

Let us consider the second equation. The denominator of $\frac{d\mu^*}{d\omega}$ is negative since $F''_{\mu,\mu},H''_{\mu,\mu}<0$. Therefore $\frac{d\mu^*}{d\omega}>0$ requires $F'_\mu\frac{1}{\rho}(1-\omega)^{1/\rho-1}<H'_\mu\frac{2(1-\omega)^2+\omega(2-\omega)(2-1/\rho)}{(1-\omega)^{3-1/\rho}}$. This condition now depends on the elasticity of substitution parameter ρ. Remembering $\sigma=1/(1-\rho)$, we have that $\sigma<1$ for $\rho<0$. If we cross-multiply the previous inequality by ρ when $\rho<0$ we must invert the inequality sign of our condition. Therefore $\frac{d\mu^*}{d\omega}>0$ requires $F'_\mu(1-\omega)^{1/\rho-1}>\rho H'_\mu\frac{2(1-\omega)^2+\omega(2-\omega)(2-1/\rho)}{(1-\omega)^{3-1/\rho}}$, if $\rho<0$. We can divide both sides by $(1-\omega)^{1/\rho-1}$ and rearrange to find $F'_\mu/H'_\mu>\rho\frac{2(1-\omega)^2+\omega(2-\omega)(2-1/\rho)}{(1-\omega)^2}$. The right-hand side is always negative since $\rho<0$ and $(2-1/\rho)>2$. The left-hand side is always positive under our assumptions, so that $\frac{d\mu^*}{d\omega}>0$. This shows that the two conditions sufficient for the positive effect of the wage share on labor productivity growth are satisfied, and $\frac{dg_A^*}{d\omega}>0$ follows necessarily when $\sigma<1$.

7.4 Proof of Proposition 4

Let us start with $\beta_{ss} = 0$. We have $s(1-\mu_{ss})B_{ss}/(1-\omega)^{1/\rho} = f(0)\,(s\mu_{ss})^{\gamma}$. Plugging $\mu_{ss} = \left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2-\frac{1}{\rho}}}\right)^{\frac{1}{1-\gamma}}/s$ into the previous equation and rearranging yields $sB_{ss} = B_{ss}\left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2-\frac{1}{\rho}}}\right)^{\frac{1}{1-\gamma}} + f(0)\left(\frac{f(0)\gamma}{B_{ss}}\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2-\frac{1}{\rho}}}\right)^{\frac{\gamma}{1-\gamma}}(1-\omega)^{1/\rho}$, which we can solve for B_{ss} to find $B_{ss} = f(0)\gamma^{\gamma}\frac{(\omega_{ss}(2-\omega_{ss}))^{\gamma}}{(1-\omega_{ss})^{2\gamma-\frac{1}{\rho}}}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right)^{1-\gamma}/s^{1-\gamma}$.

Next, use the two isoclines in the (B_{ss},ω_{ss}) space to find: $\gamma\frac{f(0)}{-f'(0)^{\frac{1-\gamma}{\alpha-\gamma}}}\frac{(1-\omega_{ss})^{\frac{2(1-\alpha)}{\alpha-\gamma}+\frac{1}{\rho}}}{(\omega_{ss}(2-\omega_{ss}))^{\frac{(1-\alpha)}{\alpha-\gamma}}} = f(0)\gamma^{\gamma}\frac{(\omega_{ss}(2-\omega_{ss}))^{\gamma}}{(1-\omega_{ss})^{2\gamma-\frac{1}{\rho}}}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right)^{1-\gamma}/s^{1-\gamma}$. Simplifying, elevating to the power of $1/(1-\gamma)$ and rearranging yields:

$$s = (-f'(0))^{\frac{1}{\alpha-\gamma}}\left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right).$$

Define $\left(\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}\right)^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right) \equiv (F_1(\omega_{ss}))^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}F_2(\omega_{ss}) \equiv G(\omega_{ss})$. Total differentiation of $s = (-f'(0))^{\frac{1}{\alpha-\gamma}}G(\omega_{ss})$ w.r.t. ω_{ss} and s shows that $sign\frac{d\omega_{ss}}{ds} = sign\left[G'(\omega_{ss})\right]$. Let us calculate $G'(\omega_{ss}) = \frac{1+\gamma-\alpha}{\alpha-\gamma}(F_1(\omega_{ss}))^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}F_1'(\omega_{ss})F_2(\omega_{ss}) + (F_1(\omega_{ss}))^{\frac{1+\gamma-\alpha}{\alpha-\gamma}}F_2(\omega_{ss})$. Notice that, after few minor manipulations, $F_1'(\omega_{ss}) = 2/(1-\omega_{ss})^3$ and $F_2'(\omega_{ss}) = 2\gamma/(1-\omega_{ss})^3$. Therefore $G'(\omega_{ss}) = \left[2/(1-\omega_{ss})^3\right](F_1(\omega_{ss}))^{\frac{1+\gamma-\alpha}{\alpha-\gamma}-1}\left\{\frac{1+\gamma-\alpha}{\alpha-\gamma}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right)+\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}\right\}$. When $\alpha > \gamma$ all factors defining $G'(\omega_{ss})$ are positive so that $G'(\omega_{ss}) > 0$. When $\alpha < \gamma$ the sign of $G'(\omega_{ss})$ will equal the sign of the expression in curly brackets. Let us discuss $\frac{1+\gamma-\alpha}{\alpha-\gamma}\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right)+\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}} < 0$. Notice that $\alpha-\gamma < 0$ so that we must invert the inequality sign when we cross multiply by it: $(1+\gamma-\alpha)\left(\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}+1\right)+\gamma\frac{\omega_{ss}(2-\omega_{ss})}{(1-\omega_{ss})^{2}}(\alpha-\gamma) > 0$. Then $(1+\gamma-\alpha)\,\gamma\omega_{ss}\,(2-\omega_{ss})+(1+\gamma-\alpha)\,(1-\omega_{ss})^{2}-\gamma\omega_{ss}\,(2-\omega_{ss})\,(\alpha-\gamma) > 0$, and finally $\gamma\omega_{ss}\,(2-\omega_{ss})+(1+\gamma-\alpha)\,(1-\omega_{ss})^{2} > 0$, which is always true. Hence $G'(\omega_{ss}) < 0$, when $\alpha < \gamma$.

8 Highlights

- Labor productivity growth depends on the size and direction of technical change.
- We examine the impact of the wage share on labor productivity growth.
- The distribution/productivity growth relation depends on the innovation technology.
- We study the influence of the saving rate on the steady-state wage share.
- The saving rate is unlikely to affect long-run functional income distribution.