16th Cologne-Twente Workshop on Graphs and Combinatorial Optimization

CNAM
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June 18-20, 2018

Proceedings of the Workshop

General Chair: Leo Liberti
Editors: Emiliano Traversi, Fabio Furini, Leo Liberti

CTW 2018 *(CNAM, Paris, France, 18-20 June)*

This conference is supported by a hell of an organizing committee. Special thanks go to Amélie Lambert (local arrangements), Lucas Létocart (website), Fabio Furini (email), Emiliano Traversi (proceedings). All complaints should go to Leo Liberti (sigh).

Schedule

	Mon 18 June						**Tue 19 June**						**Wed 20 June**					
	Room PP		*Room Z*		*Room Y*		*Room PP*		*Room Z*		*Room Y*		*Room PP*		*Room Z*		*Room Y*	
09:00-09:30	Registration (hall) and opening (PP)																	
09:30-10:00	*Complexity*	Anapolska	*Math. Progr. I*	Iommazzo	*Networks I*	Oustry	*Games II*	Yang	*Transportation I*	Bauguion	*Energy I*	Schwenk	*Graphs IV (Cordone)*	Kumbargoudra	*Math. Progr. III*	Aoudia	*Energy II*	Vanier
10:00-10:30		Pradhan		Lee		Gunnec		Furini		Righini		Thomopulos		Tian		Traversi		Mencarelli
10:30-11:00	coffee (hall)						coffee (hall)						coffee (hall)					
11:00-11:30	**Seiller (plenary, PP)**						**Wiegele (plenary, PP)**						*Graphs V*	Zheng	*Transportation II*	Pisacane	*Games III*	Boehnlein
1130-12:00														Behmaram		Bruglieri		Pacifici
12:00-14:00	lunch (on your own)						lunch (on your own)						Closing (PP)					
14:00-14:30	*Graphs I*	Nguyen	*Algorithms I*	Hommelsheim	*Graph Embeddings*	Silva	*Clustering*	Edelmann	*Math. Progr. II*	Francois	*Scheduling*	Pan						
14:30-15:00		Gomes da Silva		Vernet		Serocold		Gentile		Casazza		Schaudt						
15:00-15:30		Hossain		Vandomme		Lavor		Cordone		Marinelli		Nicosia						
15:30-16:00	coffee (hall)						coffee (hall)											
16:00-16:30	*Graphs II*	Obreja	*Comb. Opt. (Schrader)*	Vretta	*Games I*	Lozovanu	*Graphs III*	Gishboliner	*Algorithms II*	Klootwijk	*Networks II*	Ghanem						
16:30-17:00		Wolfler		Apke		Kern		Hu		Verma		Baste						
17:00-17:30								Del-Vecchio		Weller		Danisch						
19:00-21:00							Cocktail (salle des textiles)											

Did you know that CNAM hosts a Sciences Museum? This is one of the most crucial places in the novel "Foucault's Pendulum" by Umberto Eco (possibly my favorite writer). Many years ago I had applied to an assistant professorship at CNAM. I did not get the position, but during the interview I could not refrain from declaring that one of my strong motivations to apply was working in a place celebrated in a novel I loved. The hiring committee burst out laughing, and maybe that's why I wasn't offered the position. In any case you should go and visit the museum (same building, different entrance). Do not miss the part of the museum which hosts Foucault's pendulum, which hangs from the dome of the church of St. Martin-des-Champs (literally: St. Martin-in-the-Fields, which describes a sister church in London, equally central, but of a different confession I think).

The seminar rooms are the *Paul Painlevé* (PP), the *Robert Faure* (Z) and the *Jean-Baptiste Say* (Y) amphitheatres, located in Access 1, lower ground floor. Opening, plenary and closing sessions will take place in the PP amphitheatre. The cocktail event on Tuesday evening will take place in the *salle des textiles* room, located in Access 3, 1st floor. Coffee pauses will take place in the hall before the three amphitheatres. http://cedric.cnam.fr/~courtiep/planCnam/plan_Cnam_3e_arrondissement.html

Session chairs. The last speaker of the session will chair the session, with two exceptions for PhD-only sessions: *Combinatorial Optimization* (Mon 18, Room PP, 16-17) chaired by R. Schrader, and *Graphs III* (Wed 20, Room PP, 9:30-10:30) chaired by R. Cordone. Session chairs must remind speakers to load up slides on laptops, and keep the sessions on time. Session chairs are encouraged to be cruel and despotic as regards times allotted, since there are parallel sessions. If a speaker will not get your hints, standing is often not enough: just cut him/her short and invite the next speaker (as the last speaker in the session, you have every incentive to do so, but please don't be the chair who overruns his own time slot). Conversely, if a speaker ends before the time is up, you should encourage some questions/discussion/debate: e.g. invite questions from the audience and leave a pause long enough to be slightly awkward, then possibly someone will ask a question just to fill in the horrible silence, and then other questions may follow. If no-one asks, you can start off the debate by asking a session yourself. In any case, keep all slots to exactly 30 minutes (parallel sessions regime).

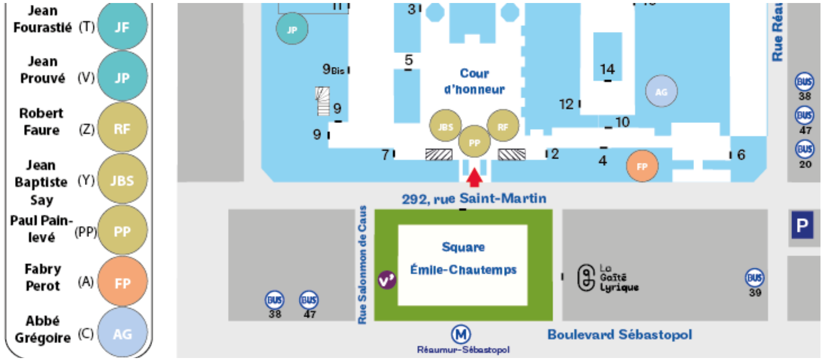
CTW 2018 *(CNAM, Paris, France, 18-20 June)*

Invited speakers

Thomas Seiller, Univ. Paris-Nord, *Mon 18, PP, 11-12*
From Proofs to Programs, Graphs and Dynamics. Geometric perspectives on computational complexity.

Angelika Wiegele, Alpen-Adria Univ. Klagenfurt, *Tue 19, PP, 11-12*
Modeling and Solving Combinatorial Optimization Problems using Semidefinite Programming

Enter CNAM by the entrance labelled "1". The amphitheatres are underground, underneath the entrance court (see picture below). The "Salle des textiles" (where the cocktail event takes place) is labeled by "3", on the first floor.



The proceedings of this workshop are distributed in a PDF file which is available for download at www.lix.polytechnique.fr/~liberti/ctw18-proceedings.pdf.

A special issue of Discrete Applied Mathematics will be dedicated to the topics of the CTW18. Watch out for calls for papers to this issue during summer/autumn/winter 2018.

Speaker	**Title**	***Session***
Anapolska	Minimum Color-Degree Perfect b-Matchings	*Complexity*
Aoudia	Star forest polytope on complete graph	*Math. Progr. III*
Apke	A Characterization of Interval Orders with Semiorder Dimension Two	*Comb. Opt.*
Baste	Temporal matching in link stream: kernel and approximation	*Networks II*
Bauguion	Multimodal transportation plan adjustment with passengers behaviour constraints	*Transportation I*
Behmaram	On matching and distance property of m-barrele Fullerene	*Graphs V*
Boehnlein	Make or Buy: Revenue Maximization in Stackelberg Scheduling	*Games III*
Bruglieri	The Electric Vehicle Relocation Problem in Carsharing Systems with Collaborative Operators	*Transportation II*
Casazza	Dual bounds for a Maximum Lifespan Tree Problem	*Math. Progr. II*
Cordone	Some polynomial special cases for the Minimum Gap Graph Partitioning Problem	*Clustering*
Danisch	A Modular Overlapping Community Detection Algorithm: Investigating the “From Local to Global” Approach	*Networks II*
Del-Vecchio	A new centrality measure: spectral closeness.	*Graphs III*
Edelmann	Graph partitioning using matrix differential equations	*Clustering*
François	Mixed Integer Linear Programming Approach for a Distance-Constrained Elementary Path Problem	*Math. Progr. II*
Furini	Attacking the Clique Number of a Graph	*Games II*
Gentile	An algorithm for computing lower bounds for the Microaggregation problem	*Clustering*
Ghanem	How to exploit structural properties of dynamic networks to detect nodes with high temporal closeness	*Networks II*
Gishboliner	A Generalized Turan Problem and its Applications	*Graphs III*
Gomes Da Silva	Equitable total chromatic number of two classes of complete r-partite p-balanced graphs	*Graphs I*
Gunnec	Influence Maximization in Social Networks under Deterministic Linear Threshold Model	*Networks I*
Hommelsheim	Robust Matching Augmentation	*Algorithms I*
Hossain	Multicoloring of Pattern Graphs for Sparse Matrix Determination	*Graphs I*
Hu	On the spectra of general random mixed graphs	*Graphs III*
Iommazzo	A methodology for addressing the Algorithm Configuration problem on mathematical programming solvers	*Math. Progr. I*
Kern	The asymptotic price of anarchy for k-uniform congestion games	*Games I*
Klootwijk	Probabilistic Analysis of Optimization Problems on Generalized Random Shortest Path Metrics	*Algorithms II*
Kumbargoudra	Total k-rainbow Domatic Number	*Graphs IV*
Lavor	New advances on the branch-and-prune algorithm for the discretizable molecular distance geometry problem	*Graph Embeddings*
Lee	Gomory by column generation	*Math. Progr. I*
Lozovanu	Nash Equilibria in Mixed Stationary Strategies for m-Player Cyclic Games on Networks	*Games I*
Marinelli	A star-based reformulation for the maximum quasi-clique problem	*Math. Progr. II*
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Nguyen	On some tractable constraints on paths in graphs and in proofs	*Graphs I*
Nicosia	Single machine scheduling with bounded job rearrangements	*Scheduling*
Obreja	Extremal Graphs with respect to the Modified First Zagreb Connection Index	*Graphs II*
Oustry	Optimal Deployment of Wireless Networks	*Networks I*
Pacifici	Two Stackelberg Knapsack games	*Games IV*
Pan	A hybrid heuristic for multi-activity tour scheduling	*Scheduling*
Pisacane	Solving the Green Vehicle Routing Problem with Capacitated Alternative Fuel Stations	*Transportation II*
Pradhan	Algorithmic aspects of neighborhood total domination in graphs	*Complexity*
Righini	Dynamic programming for the Electric Vehicle Orienteering problem with multiple technologies	*Transportation I*
S. Schaudt	Parallel machine scheduling with unit time distinct due windows	*Scheduling*
Schwenk	A Green Energy Grid Coupling Problem (GEGCP)	*Energy I*
Serocold	Rigidity of 1-coordinated frameworks in 2 dimensions	*Graph Embeddings*
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Thomopulos	A Constrained Shortest Path formulation for the Two-Reservoir Hydro Unit Commitment Problem	*Energy I*
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Vandomme	Fully leafed induced subtrees (extended abstract)	*Algorithms I*
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Zheng	Implicit heavy subgraph conditions for hamiltonicity of almost distance-hereditary graphs	*Graphs V*

Preface

This volume collects the abstracts of invited plenary and accepted contributed talks presented at the 16th Cologne-Twente Workshop (CTW) on graphs and combinatorial optimization, which took place at the Conservatoire National d'Arts et Métiers (CNAM) in Paris, 18-20 June 2018. Only those accepted abstracts for which the authors gave an explicit consensus of appearance are collected in this volume. The copyright of each single abstract rests with its authors. This volume is posted online at `http://www.lix.polytechnique.fr/~liberti/ctw18-proceedings.pdf`. Following tradition, a special issue of Discrete Applied Mathematics (DAM) dedicated to this workshop and its main topics of interest will be edited.

The CTW workshop series has been initiated by Ulrich Faigle, around the time he moved from Twente University to the University of Köln. After many CTW editions in Twente and Köln, it was decided that CTWs were mature enough to move about: in 2004 the CTW was organized in Villa Vigoni (Como, Italy) by F. Maffioli (Politecnico di Milano) and myself. Since then, the CTW visited Italy again many times and in many places (and more visits are planned), France and Turkey. The first edition of CTW in France occurred when I chaired the 8th edition of CTW in 2009 in Paris (at CNAM). In this second French edition of CTW, which I am again chairing, I aimed at more or less the same organization style as in 2009: the wonderful CNAM venue, which affords beautiful buildings, a wonderful science museum, a central Paris location close to lots of small, quaint and (relatively) cheap restaurants where you can while lunch breaks away; a cocktail on the second day; but other than that, an orga nization which is as simple as possible. For the first time, we shall not distribute paper copies of these proceedings. Instead, we shall distribute a single sheet of paper with the timetable and the list of talk titles with presenting authors (`http://www.lix.polytechnique.fr/~liberti/ctw18-program.pdf`).

The scientific program of this CTW edition (codenamed CTW18) includes two plenary talks (by Dr. Thomas Seiller and Prof. Angelika Wiegele), and 57 contributed (accepted) talks. The 57 accepted talks were selected from an initial set of 69: counter to computer science habits, this is not a "selective workshop". Having been initially set up by discrete applied mathematicians, it still follows the mathematical tradition whereby the main purpose of workshops is to present and discuss (possibly preliminary) results, rather than publish proceedings articles which are fully accomplished and have an archival nature. CTWs are not selective, and hence, in today's academic publish-or-perish worldview, not as attractive as they used to be. Are they still necessary? Among the initial motivations for CTWs we find a special attention to young (nonpermanent) researchers: MSc and PhD students as well as postdoctoral fellows. Another initial motivation was to provide a venue where preliminar y work could be presented and discussed. In this sense, this edition is perfectly in line with these two motivations (which I personally find very valid). At CTW18, 31 out of 57 contributed talks will be given by MSc, PhD or Postdocs. Half of the registered participants are MSc, PhD or Postdocs. While some talks relate to accomplished works, many have a preliminary/ongoing nature.

The governance of the CTW workshop series is assured by a "steering committee" which also acts as "programme committee", in the sense that it screens contributed abstracts and rejects those which are scientifically objectionable, written extremely poorly, or off topic. New members of the steering committee are sometimes chosen from CTW organizers. Currently, this committee counts 19 researchers from Germany, Italy, Turkey and France. Organizing committees are newly formed for each CTW edition. This year we have Fabio Furini (Paris-Dauphine), Amélie Lambert (CNAM), Lucas Létocart (Paris-Nord), Ivana Ljubic (ESSEC, Paris), Emiliano Traversi (Paris-Nord), Roberto Wolfler Calvo (Paris-Nord) and myself (CNRS & Ecole Polytechnique).

Not every CTW edition features invited plenaries, but this one does. Two young and brilliant researchers were invited: Thomas Seiller and Angelika Wiegele. Thomas is a CNRS researcher affiliated to the Computer Science Dept. (LIPN) at Paris-Nord. His research focuses on a certain unusual semantics for linear logic which holds some promise as a tool for separating complexity classes. Although this topic is far from the usual CTW crowd, I believe it is important enough that this community should know about it. Thomas was asked to give a "tutorial" on this line of research. Angelika, an associate professor at the Mathematics Dept. of Alpen-Adria University in Klagenfurt, Austria, is a well-known member of the mathematical programming community. She specializes in semidefinite programming applied to combinatorial optimization problems. She is one of those rare researchers who pursue the whole "pipeline" of a scientific result in mathematical programming, from theorems through algorithms to software (see e.g. `doi.org/10.1007/s10107-008-0235-8` to `biqmac.uni-klu.ac.at` and `biqbin.fis.unm.si`).

I very much hope you will all enjoy this 2018 edition of CTW.

Leo Liberti
CTW18 General Chair
CNRS LIX, Ecole Polytechnique

Organization

The CTW18 venue is the Conservatoire National d'Arts et Métiers (CNAM) in Paris (lecture halls PP, Y and Z) located in the third arrondissement of Paris (France).

The CNAM has several sites, and the rooms of CTW18 are located in the main site, 292 rue Saint-Martin, 75003 Paris.

The seminar rooms are the *Robert Faure* (Z), *Paul Painlevé* (PP) and *Jean-Baptiste Say* (Y) amphitheatres, located in Access 1, lower ground floor. The cocktail event on Tuesday evening will take place in the *salle des textiles* room, located in Access 3, 1st floor.

Scientific Committee:

- Ali Fuat Alkaya (U Marmara)
- Alberto Ceselli (U Milano)
- Roberto Cordone (U Milano)
- Ekrem Duman (U Ozyegin)
- Ulrich Faigle (U Koeln)
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- Fabio Furini (U Paris Dauphine)
- Amélie Lambert (CNAM)
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- Leo Liberti (CNRS & École Polytechnique)
- Ivana Ljubic (ESSEC)
- Evelyne Rayssac (École Polytechnique, Paris)
- Emiliano Traversi (U Paris XIII)
- Roberto Wolfler Calvo (U Paris XIII)

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Some polynomial special cases for the Minimum Gap Graph Partitioning Problem

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Abstract

We study various polynomial special cases for the problem of partitioning a vertex-weighted undirected graph into p connected subgraphs with minimum gap between the largest and the smallest vertex weight.

Keywords : *Graph partitioning, min-sum gap optimization, min-max gap optimization.*

1 Introduction

The *Minimum Gap Graph Partitioning Problem* (*MGGPP*) is a graph partitioning problem [1] introduced in [2]. Let $G = (V, E)$ be an undirected connected graph, w_v an integer *weight* coefficient defined on each vertex $v \in V$, and $p \leq |V|$ a positive integer. Given a vertex subset $U \subseteq V$, we denote by $m_U = \min_{u \in U} w_u$ and $M_U = \max_{u \in U} w_u$ the minimum and maximum weight in U, respectively, and define the *gap* of U as $\gamma_U = M_U - m_U$ (if U is a singleton, $\gamma_U = 0$). The *MGGPP* consists in partitioning G into p vertex-disjoint connected subgraphs $G_r = (V_r, E_r)$, $r = 1, \ldots, p$. We consider also the *nondegenerate* problem (*MGGPPnd*), in which all subgraphs must have at least two vertices. The *min-sum* version of both problems minimizes the sum of all gaps $f^{MS} = \sum_{r=1}^{p} \gamma_{V_r}$, while the *min-max* version minimizes $f^{MM} = \max_{r=1,\ldots,p} \gamma_{V_r}$. The *MGGPP* is related to uniform graph partitioning [4] which has applications, for example, in agriculture (divide a land into parcels with small height difference [7]) and in social network analysis.

The computational complexity and the approximability of the *MGGPP* are studied in [2]. A Tabu Search metaheuristic and a Mixed Integer Linear Programming (MILP) formulation for the *min-sum* version are proposed in [3]. We note that the *min-max MGGPP* can be seen as a special case of the following more general graph partitioning problem. Given a graph G and a $n \times n$ matrix of *dissimilarities* between any pair of vertices, find a partition of G into p connected components that minimizes the maximum dissimilarity of a pair of vertices in the same component. This problem is $\mathcal{NP}$-complete even on star graphs [5]. However, the *min-max MGGPP* might be easier because the dissimilarity for any given pair of vertices u and v is computed as $|w_u - w_v|$.

In this paper, we investigate some polynomial cases of the *MGGPP* concerning special graph topologies, such as paths, spiders, stars, caterpillars and complete graphs.

2 Polynomial cases

First of all, we introduce two useful properties of the *MGGPP* (but not of the *MGGPPnd*):

P1. the optimal value does not increase as the number p of components increases;

P2. given a partition π in p components with maximum gap equal to γ, another partition π' with $p' > p$ components and a gap less than or equal to γ always exists.

These properties hold for both objectives, because a singleton with zero gap can always be disconnected from a subgraph, increasing the number of components, but not the objective value. On the basis of the above properties we are able to solve the *min-max MGGPP* by a binary search over all the $O(n^2)$ possible values γ of the optimal maximum gap, that correspond to the differences between pair of vertices' weights. Therefore, we can follow an approach based on the solution of a polynomial number of instances of the following auxiliary problem.

Definition 1 (Feasibility problem) *Find, if any, a connected partition of G having the minimum number q of components and such that each component has gap at most γ.*

If q is greater than p, the value of γ must be increased; otherwise it must be decreased. At the end of the binary search, the partition having the maximum $q \leq p$ is considered and in case $q < p$, a partition in exactly p components with the same gap value is found by further dividing some components of the partition until p components are obtained. The binary search requires a pre-sorting of the $O(n^2)$ possible values of γ implying an overall time complexity of $O(n^2 \log n)$. This complexity can be reduced as follows. First sort the weights of the vertices, in $O(n \log n)$; then consider implicitly the ordered matrix of the differences between weights and apply the $O(n)$ time procedure for finding the k^{th} smallest value in a $n \times n$ matrix of reals with sorted rows and columns [6]. With this approach the whole solution procedure requires $O(n \log n + T_F(G) \log n)$ time, where $T_F(G)$ is the time for solving the feasibility problem. We apply this approach to paths and spiders.

2.1 Paths

If G is a path, we assume its vertices to be numbered progressively considering an arbitrary direction along the path: $P = \{v_1, \ldots, v_n\}$. Every feasible solution is a partition into p vertex-disjoint subpaths. Hence, the problem requires to identify and remove $p - 1$ edges from G.

Theorem 1 *When G is a path, the* min-max MGGPP *can be solved in $O(n \log n)$ time.*

Proof : For a given value γ, we find a partition of P into the minimum number of components such that the gap is less than or equal to γ by scanning P and removing some edges. Starting from one end vertex of P, say v_1, edge (v_{i-1}, v_i) is removed as soon as a vertex v_i is found such that $|w_{v_i} - w_{v_j}| > \gamma$ for at least one vertex v_j in the current subpath. This procedure is then repeated starting from vertex v_i. Exploiting the general binary search approach the overall time complexity is $O(n \log n)$. □

For the other versions, the following theorem provides a polynomial approach.

Theorem 2 *When G is a path, all versions of the* MGGPP *can be solved in $O(n^2 p)$ time.*

Proof : (Sketch) We build an auxiliary directed graph with a source node u_0, p nodes $u_{j,1}, \ldots, u_{j,p}$ for each vertex v_j of V, an arc from u_0 to each node $u_{j,1}$ and an arc $(u_{i,r-1}, u_{j,r})$ for each pair of vertices v_i and v_j with $i < j$ and for $r = 2, \ldots, p$. The arcs correspond to candidate subpaths and their costs to the corresponding gaps. Therefore, the optimum of the *MGGPP* on G is equal to the minimum cost of a path from u_0 to $u_{n,p}$ on the auxiliary graph. Since the graph is acyclic, this problem can be solved in $O(n^2 p)$. □

2.2 Spiders

A spider is a tree with at most one vertex of degree at least 3.

Theorem 3 *When G is a spider, the* min-max MGGPP *can be solved in $O(n \log^2 n)$ time.*

Proof : We apply the procedure described in Theorem 1 to each of the d paths connecting the leaves of G to the root (the only vertex of degree $d \geq 3$), visiting G bottom-up. Let $P_1, \dots, P_d$ be the last formed components in the partitions of such paths. We try to merge as many subpaths as possible so that their gap is $\leq \gamma$ by first ordering them w.r.t. their maximum and minimum vertex weights in $O(n \log n)$, and then checking the feasibility of the component under construction with a data structure that scans all the ordered minima and maxima in linear time. Considering the time required by the binary search, the overall time complexity is $O(n \log^2 n)$. □

2.3 Stars

A star is a connected graph with at most one vertex of degree at least 2.

Theorem 4 *When G is a star, the* MGGPP *admits only degenerate solutions, and can be solved in $O(n \log n)$ time.*

Proof : Any solution is a partition of the star where $p-1$ components are leaves (singletons) and one component contains all the other vertices. An $O(n \log n)$ time algorithm can be obtained as follows: i) sort the leaves by nonincreasing weights; ii) visit the leaves according to such an ordering to detect a non-singleton component with minimum gap. □

2.4 Caterpillars

A caterpillar is a tree formed by a central path with n' vertices and n'' leaves attached to it.

Theorem 5 *When G is a caterpillar, the* MGGPP *can be solved in $O(n^3 p^2 \log n)$ time; the* MGGPPnd *in $O(n^2 p)$ time.*

Proof : The *MGGPPnd* amounts to partitioning the central path $\{v_1, \dots, v_{n'}\}$. We build an auxiliary graph similar to that used in Theorem 2. An arc represents a feasible subgraph, i.e., a portion of the central path and all the attached leaves. Its cost is the gap of the subgraph. The optimal *min-sum* or *min-max* path from u_0 to $u_{n'p}$ identifies the optimal solution. Building the auxiliary graph, the cost function and detecting the optimal path take $O(n^2 p)$.

In the general case, the leaves can be isolated from the central path, but the auxiliary graph can be modified accordingly, i.e. introducing also arcs between non consecutive layers. Each arc represents a central subgraph including a portion of the central path and possibly some leaves, plus some isolated leaves. The cost of an arc is the gap of the central subgraph. For each arc, we determine the central subgraph with minimum gap adapting the $O(n \log n)$ algorithm used to solve the *MGGPP* on stars. The optimal *min-sum* or *min-max* path from u_0 to $u_{n'p}$ identifies the optimal solution. Detecting the optimal path takes $O(n^2 p^2)$, but computing the arc costs requires an overall $O(n^3 p^2 \log n)$ time. □

2.5 Complete graphs

In this case, the connectivity constraint is trivially satisfied. We sort the vertices by nondecreasing weights and rename them so that $i < j \Rightarrow w_{v_i} \leq w_{v_j}$. Then, we can restrict the search for the optimum to the solutions in which for every pair of subgraphs the vertices of one strictly precede the vertices of the other. In fact, every other feasible solution can be transformed into a non-worsening one of this family: for each pair of vertex subsets V' and V'' violating this condition, merge the two subsets and split the result in two, assigning the first $|V'|$ elements to the first subset and the last $|V''|$ elements to the second. Given the path that visits all vertices in nondecreasing weight order, the optimal solution can be detected as in Theorems 1 and 2. However, the vertex ordering allows more efficient algorithms for some cases.

Theorem 6 *When G is a complete graph, the* min-sum MGGPP *can be solved in $O(n \log n)$ time. The* min-sum MGGPPnd *can be solved in $O\left(\min\left(n\sqrt{n}\log\gamma_V, n^2\log n, n^2p\right)\right)$ time, where γ_V is the gap of the whole graph.*

Proof : Thanks to the ordering of the vertex weights, partitioning the auxiliary path into p subpaths of minimum total gap is equivalent to selecting $p-1$ edges such that the sum of the weight differences between their extreme vertices is maximum. This can be done by saving in a *max-heap* the weight differences between adjacent vertices and extracting from it the $p-1$ largest differences. The dominating time is given by the weight ordering.

In the nondegenerate case, it is forbidden to select two consecutive edges. The problem reduces to the search for a maximum weight matching of cardinality $p-1$ on the path. The Enhanced Capacity Scaling algorithm solves the problem in $O\left(n\sqrt{n}\log\gamma_V\right)$ time, while a reduction to the minimum cost flow problem solves it in $O\left(n^2\log n\right)$ time. Finally, the algorithm of Theorem 2 solves it in $O\left(n^2p\right)$ time. □

3 Conclusions and perspectives

Table 1 summarizes the special cases discussed in this paper: "NA" marks the non applicable cases (stars admit only degenerate solutions), "?" marks the open cases.

	min-max		*min-sum*	
Topology	*MGGPP*	*MGGPPnd*	*MGGPP*	*MGGPPnd*
Stars	$O(n\log n)$	NA	$O(n\log n)$	NA
Paths	$O(n\log n)$	$O(n^2p)$	$O(n^2p)$	$O(n^2p)$
Spiders	$O(n\log^2 n)$	?	?	?
Caterpillars	$O(n^3p^2\log n)$	$O(n^2p)$	$O(n^3p^2\log n)$	$O(n^2p)$
Complete	$O(n\log n)$	$O(n^2p)$	$O(n\log n)$	$O(min(n\sqrt{n}\log\gamma_V, n^2\log n, n^2p))$

TAB. 1: Summary of the computational complexity results

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