Volterra models of magnetorheological dampers and their application to vibrating systems

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Abstract

Magnetorheological dampers (MRD) are nowadays among the most promising semi-active devices used in automotive and structural engineering since they have small power requirements and low cost. Several models are studied to simulate the behavior of magnetorheological fluids which are characterized by strongly nonlinear viscoelastic hysteresis phenomena. These nonlinearities make many control logics ineffective because they fail when interfaced with complex models. The aim of this work is to optimize the damper behavior by controlling the mechanical vibrations of the system by a novel control algorithm. The magnetorheological behavior is modelled by a Volterra's equation which allows the use of an indirect optimal feedback control algorithm, named Proportional-Nth-order-Derivatives or PD(N). Numerical results show how the PD(N) achieves excellent results when compared with conventional controls.

1 Introduction

Hysteresis is a phenomenon that has always fascinated mathematicians and physicists and it is still a subject of great interest today. The hysteresis, which was originally formulated by Greeks to indicate a *lag in* arrival, was first studied in 1882 by Ewing J. A. to describe the magnetic properties of metals. Nowadays, the mathematical models of hysteresis are used in many fields, from chemistry, to biology, engineering, etc. The physical models describe properly the phenomena such as ferromagnetism, ferroelectricity, superconductivity, friction and shape memory materials.

A systematic analysis of this hysteresis phenomena under a strict mathematical point of view began 40 years ago, with the seminal work of Bouc R. [1]. Later, several Russian mathematicians studied systematically the concept of hysteresis operator [2]. In the eighties, other contributions emerged in connection with partial differential equations and applicative problems [3].

Magnetorheological fluids (MR) are materials with rheological properties that depend on the magnetic field. Hysteresis behavior is an intrinsic nonlinear property of MR fluids that makes them very complex to control, limiting their application in turn.

The MR fluid consists of a mixture of a fluid with dispersed metal particles, the equivalent viscosity of which depends on an external magnetic field, for example generated by a solenoid excited by an electric current. Understanding the working mechanism of the hysteresis rheological fluids characteristics is significant for studying the MR actuators and implementing high-efficiency control systems.

The MRD are usually studied by different mathematical models, and among them Bouc–Wen, Duhem, Dahal and LuGre models [19-24]. According to Naser [25], the hysteretic behaviour of causal systems can be described analytically by the so-called "hysteresis loops", involving fractional calculus. Naser shows how these systems can be modelled with Riemann–Liouville integral, an approximated form of which , in some cases, is provided by the integrals of Volterra, as shown in [7].

Magnetorheological fluids are nowadays applied to dissipative actuators, clutches and brakes [4, 5] and experimental tests show that the hysteresis only occurs about the direction-change points under sinusoidal displacement excitations [6].

In this paper is a novel model is presented, recently proposed by the authors in the context of optimal control of integral differential equations (IDE), including Volterra-type integrals, and called Proportional-Nth-order-Derivatives or PD(N) [7]. This kind of control belongs to the category of Variational Feedback Controls (VFC) [8-10]. The PD(N) is based on the classical variational control approach formulated by Pontryagin, implying the solution of a two-boundary values problem. Usually, the integro-differential equations (IDE) are solved by direct methods, except for those cases in which closed form analytical solutions are obtainable thanks to special properties of the kernel [11-13]. Direct methods imply the discretization and solution of the problem passing through a nonlinear programming [14, 15]. Instead, the proposed indirect method permits to solve the problem minimizing the computational efforts that usually grows at a nonlinear rate with the number of grid points used for the quantization [16-18].

The present paper shows how to control the hysteresis of a system including Magnetorheological dampers (MRD) that presents Bouc-Wen hysteresis and approximated by a convolution integral of Volterra. The optimal feedback PD(N) control is applied to a quarter, car model the suspension of which includes a MRD, exhibiting excellent performances.

2 Optimal control of Volterra equation

In this section the optimal control algorithm is derived for a prototypal Volterra integro-differential equation as:

$$\dot{x} = ax + K * x + bu \tag{1}$$

where a and b are constants, u is a scalar control variable, and K is the kernel which includes the hysteresis behaviour of the damper.

The optimal control law is based on the minimization of a generic quadratic cost function J which depends on the state and the control variables together with the Lagrange multiplier λ that permits to introduce a compact form of the minimization constrained problem:

$$J(x,u,\lambda) = \int_0^T \frac{1}{2}qx^2 + \frac{1}{2}ru^2 + \lambda[\dot{x} - ax - K * x - bu]dt \tag{2}$$

The variational calculus finds a solution to the stated problem by using the stationary condition $\delta J(x,u,\lambda) = 0$, that produces:

$$\begin{aligned} &\dot{\lambda} = qx - a\lambda - K \diamond \lambda \\ &\dot{x} = ax + bu + K * x \\ &u = \frac{b}{r}\lambda \\ &\lambda(T) = 0\,, x(0) = x_0 \end{aligned} \tag{3}$$

where $K \diamond \lambda$ is a term borne from the variations of the convolution term $K * x$. In fact, in [16] the authors demonstrate that, for a causal kernel, i.e. if $k(t-\tau) = 0$ for $t < 0$ and $\tau > t$, it follows:

$$\delta \int_0^T x(t) \int_0^t K(t-\tau)x(\tau)d\tau\, dt = \int_0^T \int_t^T K(\tau - t)x(\tau)d\tau\, \delta x dt = \int_0^T K \diamond x\, \delta x\, dt \tag{4}$$

The problem reduces to a pair of coupled integral-differential equations for the optimal state x and costate λ, and a single algebraic equation involving the control u. The optimal solution is constrained with two-points boundary conditions. The existence of a trasversality condition λ(T)=0, makes the problem difficult to solve precluding, in general, the chance of a direct feedback control. Nevertheless, the authors in [16] proposed a technique based on an iterative MPC-based algorithm that solves the problem in a satisfactory way.

The present paper attacks the problem of the infinite time horizon that engineers could prefer in many ways. In this case, and in the absence of the integral contributions, the classical infinite time-horizon permits a feedback solution $\lambda = Px$, where P can be set constant.

In the present case, however, a simple solution as $\lambda = Px$ is not permitted and an alternative solution technique has to be determined.

We can limit our attention to the case in which the kernel has the form $(t) = \sum \alpha_i e^{-c_i t}$. In this case:

$$\begin{aligned} \mathcal{L}\{K * x\} &= X(s)\sum \frac{\alpha_i}{s+c_i} = X(s)\frac{P(N-1)(s)}{D(N)(s)} \\ \mathcal{L}\{K \diamond x\} &= X(s)\sum \frac{\alpha_i}{s-c_i} = -X(s)\frac{P(N-1)(-s)}{D(N)(-s)} \end{aligned} \tag{5}$$

indicating with P and D general N-1 and N order polynomials, respectively. With these properties, the Laplace transform of equations (3) produces:

$$\begin{aligned} sX - x_0 &= aX + \frac{b^2}{r}\Lambda + \frac{P(N-1)(s)}{D(N)(s)} \\ s\Lambda - \lambda_0 &= qX - a\Lambda + \frac{P(N-1)(-s)}{D(N)(-s)}\Lambda \end{aligned} \tag{6}$$

or rearranging:

$$\begin{aligned} [(s-a)D(N)(s) - P(N-1)(s)]X - x_0 D(N)(s) - \frac{b^2}{r}D(N)(s)\Lambda = 0 \\ -qXD(N)(-s) + [(s-a)D(N)(-s) - P(N-1)(-s)]\Lambda - \lambda_0 D(N)(-s) = 0 \end{aligned} \tag{7}$$

Backward transform to the time domain, produces:

$$\begin{aligned} \textstyle\sum_{i=0}^{N+1} C_i x^{(i)} + \sum_{i=0}^{N} D_i \lambda^{(i)} = 0 \\ \textstyle\sum_{i=0}^{N+1} F_i \lambda^{(i)} + \sum_{i=0}^{N} E_i x^{(i)} = 0 \end{aligned} \tag{8}$$

where $x^{(i)} = \frac{d^i x}{dt^i}$, and the problem is reduced to a pure differential equation set, linear and time-invariant. Collecting the variables into $\boldsymbol{\xi} = \left[x \dots x^{(i)} \dots x^{(N)}\right]$ and $\boldsymbol{\eta} = \left[\lambda \dots \lambda^{(i)} \dots \lambda^{(N)}\right]$, equations (8) can be reduced to a first order normal form differential problem:

$$\begin{bmatrix} \dot{\boldsymbol{\xi}} \\ \dot{\boldsymbol{\eta}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{\xi\xi} & \boldsymbol{H}_{\xi\eta} \\ \boldsymbol{H}_{\eta\xi} & \boldsymbol{H}_{\eta\eta} \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} \tag{9}$$

The general solution of system (9) can be expressed, in function of its 2N eigenvectors $\boldsymbol{\psi_k}\ \boldsymbol{\theta_k}$ and eigenvalues $\boldsymbol{p} = [p_1, \dots\ p_k, \dots, p_{2N}]$, as follow:

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} = \textstyle\sum_{k=1}^{2N} c_k \begin{bmatrix} \boldsymbol{\psi_k} \\ \boldsymbol{\theta_k} \end{bmatrix} e^{p_k t} \tag{10}$$

where the c_k are the unknown coefficients founded by imposing the boundary conditions. Rearranging the eigenvalues p_k between R positive and J-negative real part, equation (10) becomes:

$$\boldsymbol{q} = \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \textstyle\sum_{k=1}^{R} c_k^{(R)} \begin{bmatrix} \boldsymbol{\psi_k^{(R)}} \\ \boldsymbol{\theta_k^{(R)}} \end{bmatrix} e^{p_k^{(R)} t} + \sum_{k=1}^{J} c_k^{(J)} \begin{bmatrix} \boldsymbol{\psi_k^{(J)}} \\ \boldsymbol{\theta_k^{(J)}} \end{bmatrix} e^{p_k^{(J)} t} \tag{11}$$

where $R + J = 2N$ and the superscript (R) identifies the set of values $\boldsymbol{\psi_k}$, $\boldsymbol{\theta_k}$ and c_k referred to the positive real parts Â$\{p_k\} > 0$ and vice versa the superscript (J) refers to the set of with Â$\{p_k\} < 0$.

The system in (11) can be solved in two different ways based on the number of eigenvalues of matrix $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{\xi\xi} & \boldsymbol{H}_{\xi\eta} \\ \boldsymbol{H}_{\eta\xi} & \boldsymbol{H}_{\eta\eta} \end{bmatrix}$ with Â$\{p_k\} > 0$.

In the first case, $R > J$, where the number of eigenvalues p_k with negative real part are more than the others the relation between the $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ vectors can be found as

$$\boldsymbol{\eta} = \boldsymbol{\Theta\Psi^{-1}\xi} \tag{12}$$

where $\boldsymbol{\Theta}, \boldsymbol{\Psi}$ are the eigenvalues and eigenvectors of matrix $\boldsymbol{H}$, respectively. On the other hand, when $R < J$, the relation become:

$$\boldsymbol{\eta} = \boldsymbol{\Theta\Psi^{+}\xi} \tag{13}$$

where Ψ^+ is pseudo transpose eigenvectors matrix.

In general, the control law and its derivative, $\boldsymbol{\eta}$, is proportional to the state and its derivatives, $\boldsymbol{\xi}$ as follows:

$$\boldsymbol{\eta} = \boldsymbol{P\xi} = \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix} \boldsymbol{\xi} \tag{14}$$

Finally, the analytical form of the control law can be found by combining the third equation of (3) and (14) as

$$u = \frac{b}{r} \sum_{i=1}^{N} P_{Ni} x^{(i-1)} \tag{15}$$

which depends on the coefficients and order of the kernel function, on the state variable and on combination of state derivatives of N-order. For these reasons, the proposed control algorithm is called Proportional-hyper-derivative, PD(N).

The obtained control law is not a casual one, because it is depending on the derivatives of the state $x^{(i)}$. However, by computing a direct expression of $x^{(i)}$ starting from the Laplace formulation of equation (1), after some manipulations, the $x^{(i)}$ can be expressed by the integral of the state and of the control. For sake of simplicity, this general mathematics is omitted, and the complete formulation can be found in [26].

3 Active control of MR damper

The system chosen to test the PD(N) behavior is a simple quarter-car model. The sprung mass M is the quarter vehicle mass and the unsprang mass m represents the wheel mass, both are controlled by an active actuator u, which considers the presence of the MR damper with F_d force. The MR damper has been modelled to consider the hysteresis effects typical in magnetorheological fluids.

The prototypal system in Figure 1 shows a 2-Dof system including the vertical displacements z, z_t on which the road roughness z_r is acting as an external disturbances. k and k_t are stiffness of the suspension and tyre, respectively. The equation of motion of the quarter-car model follows as:

$$\begin{aligned} M\ddot{z} + k(z - z_t) + F_d &= u \\ m\ddot{z}_t + k(z_t - z) - F_d + k_t(z_t - z_r) &= -u \end{aligned} \tag{16}$$

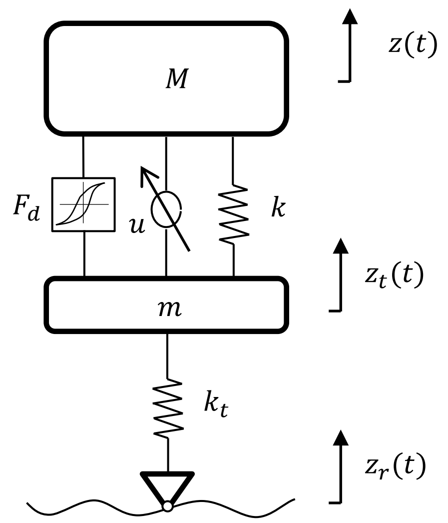


Figure 1: Quarter-car model with active control in parallel with a MR damper.

Many mathematical models are employed to describe the hysteresis phenomena, but the specific magnetorheological behavior is mainly studied by the Bouc model. This start from the representation of a hysteresis force F_d. Bouc proposes, under the rate-independent assumption, the following model [23]:

$$\frac{d}{dt}F_d = g\left(\Delta, F_d, sign(\dot{\Delta})\right)\dot{\Delta} \tag{17}$$

for some given initial conditions $F_d(t_0)$ and $\Delta(t_0)$. Due to the nonlinearities of the g function, Bouc proposed a hysteresis formulation based on the solution of the following Riemann-Stieltjes integral:

$$F_d(t) = c\Delta(t) + \int_\tau^t f(V_s^t)\mathrm{d}\Delta(\mathrm{s}) \tag{18}$$

where c is a parameter and τ is the time instant after which the displacement and force are defined and the term V_s^t is the total variation of Δ in the time interval $[s-t]$. The function f is called the hereditary kernel and it takes into account hysteretic phenomena. In the special case of an exponential kernel $f(x) = Ae^{-\beta x}$with $, \beta > 0$, a differential formulation of the second terms of (18) can be easily deduced:

$$\dot{r} = A\dot{\Delta} - \beta r|\dot{\Delta}| \tag{19}$$

indicating $r = \int_\beta^t f(V_s^t)\mathrm{d}\Delta(\mathrm{s})$. The derivation of these equations is detailed in Reference [23]. Equation (19) has been extended in reference [22] to describe restoring forces with hysteresis in the following form:

$$\dot{r} = A\dot{\Delta} - \beta r^n|\dot{\Delta}| - \gamma\dot{\Delta}|r^n| \tag{20}$$

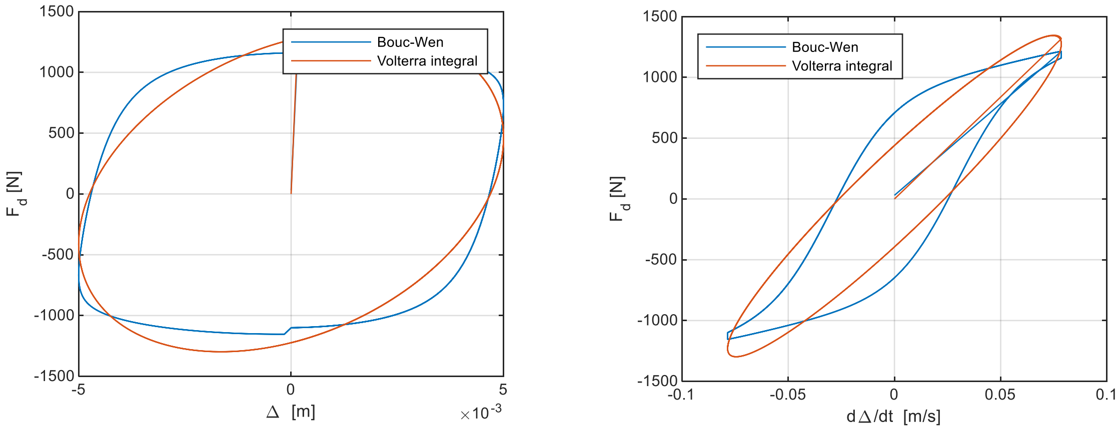
with n odd and γ an additional tuning parameter. The equation (20) is called Bouc-Wen model and it was modified by the contributions of several authors that has proposed different variants [27].

The system dynamics (16) can host a simplified form of the hysteresis force (18). Linearizing the Bouc-Wen model, one obtains for F_d:

$$F_d \approx \mathrm{c}\Delta + \mathrm{K} * \Delta \tag{21}$$

where $K(t) = \sum \alpha_i e^{-c_i t}$.

The parameters α_i and c_i can be set to match the cycle of hysteresis of the original Bouc-Wen model. This last depends on the current injected into the MRD coils [28], that means a best fitting of α_i and c_i can be determined for any flowing current. An example of this approximation is show in Figure 2 where the behaviour of a MR with LORD MR fluid of RD-1005-3 Serie, is excited by an harmonic signal at 2.5Hz, 5mm amplitude and a current of 1.5A, (see [28] for more details). In this case a second order kernel expansion has been employed, $c = 16.7e3;\ \alpha_1 = 9.6e6;\ \alpha_2 = 73e3;\ c_1 = 130; c_2 = 0.05$.



(a) (b)

Figure 2: Comparison of MR damper between the Bouc-Wen model and the Volterra integral approximation.

4 Numerical simulation

The following numerical results concern the simulation of the quarter-car model passing on a bump in order to control the car speed in urban streets. The bump is characterized by 7 cm height and 60 cm length. The crossing car velocity has been chosen constant as 15 m/s. Figure 3 - Figure 6 show the numerical results comparing the passive suspension with the controlled one when the PD(N) algorithm is applied and also with the classical LQR method. In Table 1 the simulation and control parameters are reported.

Table 1: Simulation parameters

Description	**Parameters**	**Value**
Quarter-car stiffness [N/m]	k	$2.2 \cdot 10^4$
Tyre stiffness [N/m]	k_t	$2.5 \cdot 10^5$
Tyre mass [kg]	m	35
Quarter-car mass [kg]	M	380
Control Gains	$\boldsymbol{Q}$	$\begin{bmatrix} 70 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & 0.03 \end{bmatrix} \cdot 10^7$
	r	0.01
	$\boldsymbol{K}_{lqr}$	$[2.4 \quad 0.34 \quad -6.3 \quad -0.0078] \cdot 10^5$

Starting from equation (16) and reducing it at the first order, it is possible to derive the dynamic system in the state space form as follow:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{K} * \boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{d} \tag{22}$$

where $\boldsymbol{x} = [z, \dot{z}, z_t, \dot{z}_t]$, $\boldsymbol{A}$ is the dynamic matrix 4x4, $\boldsymbol{B}$ is the control vector 4x1, $\boldsymbol{d}$ is the external disturb including the road profile z_r and the convolution integral $\boldsymbol{K} * \boldsymbol{x}$ represents the hysteresis memory effects with $\boldsymbol{K}$ a 4x4 matrix. The passive suspension behavior is simulated by imposing $\boldsymbol{u} = \boldsymbol{0}$ in equation (22). The LQR control is found solving the classical Algebraic Riccati Equation with the stationary conditions, $\boldsymbol{u} = -\boldsymbol{K_{lqr}}\boldsymbol{x}$, where the LQR control gain $\boldsymbol{K_{lqr}}$ is only depending on $\boldsymbol{A}$ and $\boldsymbol{B}$, neglecting the convolution term, as it is unable to incorporate it into the solution. Differently, the PD(N) algorithm starts from the cost function that can consider the memory effects:

$$J(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = \int_0^T \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \frac{1}{2}\boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u} + \boldsymbol{\lambda}^T[\dot{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{x} - \boldsymbol{K} * \boldsymbol{x} - \boldsymbol{B}\boldsymbol{u}]dt \tag{23}$$

where the $\boldsymbol{Q}$ and $\boldsymbol{R}$ gain matrices are the same for both tested controls so that the results in terms of performance and cost function can be compared. Starting form (23) and following the same mathematical approach illustrated in Section 2, which for reasons of clarity has been exposed for a scalar problem so as not to burden the mathematical formulation, the PD(N) optimal feedback control can be expressed as:

$$\boldsymbol{u} = \boldsymbol{R}^{-1}\boldsymbol{B}^T \sum_{i=1}^{N} \boldsymbol{P}_{Ni}\, \boldsymbol{x}^{(i-1)} \tag{24}$$

Eq. (24) has the same structure of (15) where the $\boldsymbol{P}$ matrix is now composed by submatrices $\boldsymbol{P}_{Ni}$. For all the simulations a kernel function $K(t)$ characterized by the parameters $c = 8e3$; $\alpha_1 = 9.6e6$; $c_1 = 130$ has been chosen.

The following Figure 3 - Figure 6 show the numerical results in term of masses displacement z, z_t and tyre deflection. This last parameter, which is the handling measure $z_t - z_r$, is important to guarantee a good stability and maneuverability of the car. In fact, when a vehicle crosses asymmetric potholes or bumps in the road, it can skid or lose stability due to a bad tyre-to-road contact. The quality of this contact can then be assessed based on the tyre bending.

Figure 3 and Figure 4 show the time evolution of the car and tyre displacement, respectively, when the system is uncontrollable (passive) and when it is controlled by the LQR and PD(N) algorithms. Both the controller can minimize the car oscillations due to the external bump on the road in comparison to the passive system. Moreover, the novel PD(N) solution can overcome the bump peak better than LQR, managing to improve the attenuation of the car and tyre displacement even for times after the peak itself.

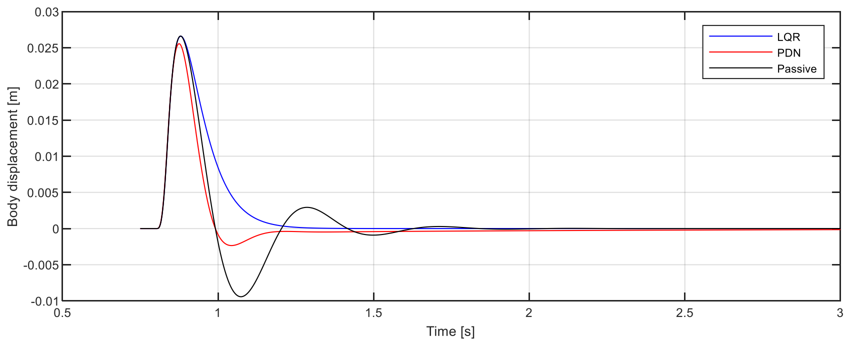


Figure 3 Car-body displacement in time

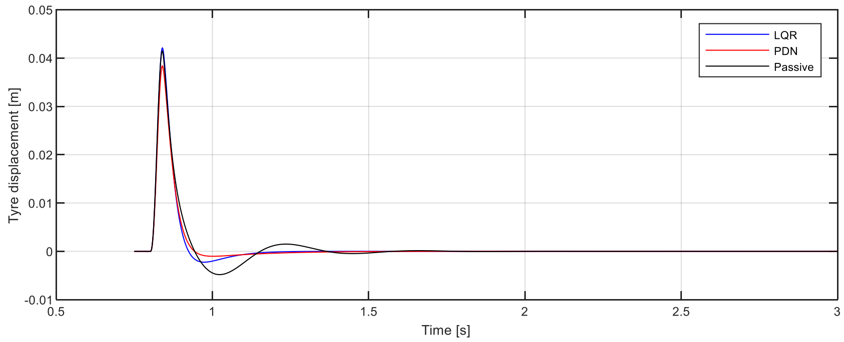


Figure 4 Tyre displacement in time

At the same level of forces applied, lower tyre deflections has been obtained with the PD(N) control compared to the LQR one, because the standard method does not take into account memory effects due to the hysteresis phenomena of the damper.

The better PDN performance targeted are evidenced also by the presence of a faster and smoother control law (Figure 5) and a lower value of cost function in time (Figure 6) respect to the LQR.

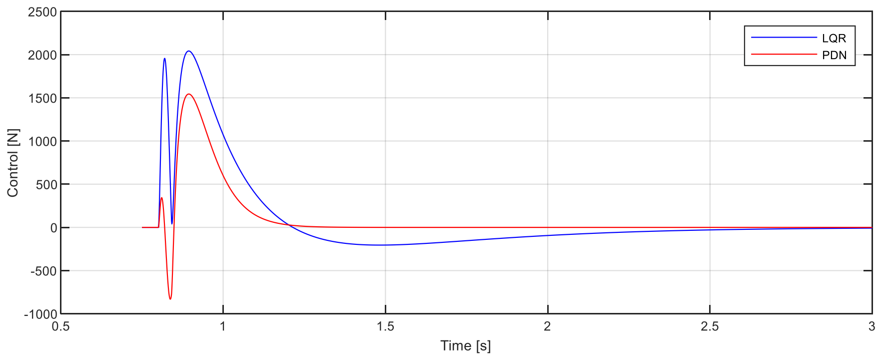


Figure 5 Control law evolution in time

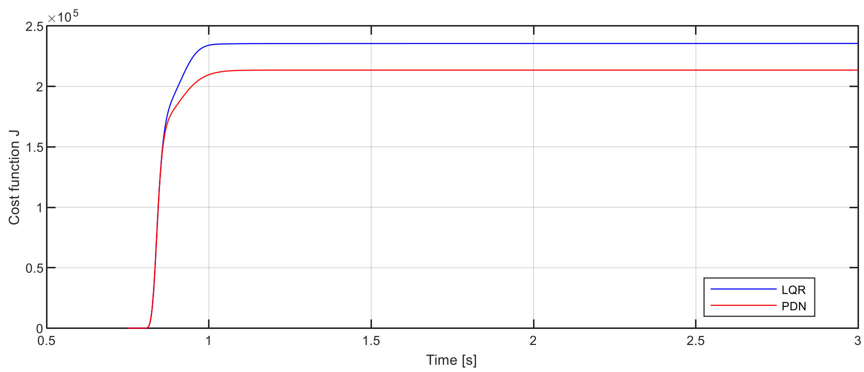


Figure 6 Cost function in time

Conclusions

In this work a novel feedback control has been formulated to control dynamic system with hysteresis effects. The analytical solution of the optimal control of integral-differential Volterra equations, allows to control the memory effects typical of ferromagnetic fluids such as the MRD. These dampers are normally represented by nonlinear mathematical models that make many control logics ineffective because they fail when interfaced with these complex models. In this preliminary study, the damper has been approximated by a convolution integral allowing to apply the Proportional-Nth-order-Derivatives or PD(N) to a quarter-car model passing on a bump. The numerical results show how the proposed PD(N) controller has better performance, in terms of minimization of mass displacement and tyre bending, compared to the standard LQR method.

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