



# Analysis of the prediction of the 2021 time-evolution of the Covid-19 pandemic in Italy using a Planck's distribution

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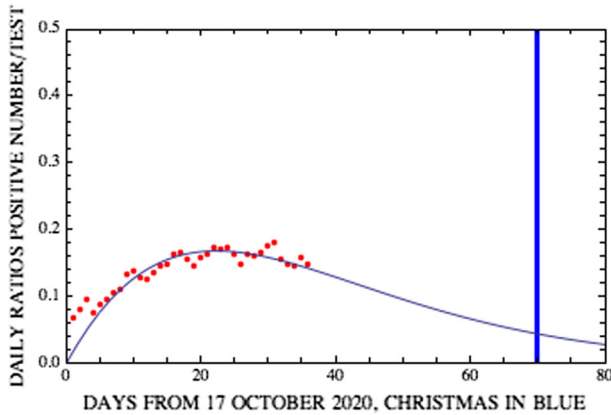
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**Abstract** In a previous paper, we studied the time-evolution of the Covid-19 pandemic in Italy during the first wave of 2020 using a number of distribution laws. We concluded that the best distribution law to predict the evolution of the pandemic is a distribution of the type of Planck's law with three parameters, provided that the basic conditions of the pandemic (such as social distancing, vaccination campaigns and new Covid variants) do not appreciably change the spread of the pandemic. In our 2020 study, we did not use the number of daily positive cases in Italy but the ratio of daily positive cases per number of daily tests, ratio today sometimes referred to as: "*positivity rate*." We showed that if basic conditions do not change, the Planck's distribution with three parameters provides very good predictions of the *positivity rate* about 1 month in advance. In a second paper, using the Planck's distribution with three parameters, we predicted, about 1 month in advance, the spread of the pandemic in Italy during the Christmas 2020 holidays. Here we show that indeed in our second paper the spread of the pandemic in Italy, after 1 month, was well predicted using the Planck's distribution with an error of a few percent only. We then studied the present (October 2021) evolution of the pandemic in Italy, and we showed that the Planck's distribution, based on the data of July and August, predicted well the evolution of the pandemic. We then show that the peak of the *positivity rate* was approximately reached during the middle of August. However, the end of the Italian holidays and the start of all the activities including schools, intensive use of public transportation and further distancing measures may change again the trend of the *positivity rate* of the pandemic.

## 1 Discussion of previous analysis

In a previous paper [1], in the attempt to mathematically predict the evolution of the pandemic in Italy, we fit the ratio of daily cases per daily tests, or "*positivity rate*," from February 26, 2020, to the beginning of April 2020, using a number of different distributions (the number of daily cases and daily tests were taken from [2]). We considered distributions of the type of Gauss, Beta, Gamma, Weibull, lognormal and two ones of the type of the Planck's black body radiation law. The number of independent parameters chosen for some distributions

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**Fig. 1** Points in red are the ratios of positive daily cases-to-tests (positivity rates) from October 17, 2020, included, to November 21, 2020, included (36 points). The solid line is the Planck’s law fitting function extrapolated a few weeks after the last data point of 21 November. This figure corresponds Fig. 4 of ref. [4]

was between two and three. It turned out that a distribution of the type of the Planck’s law with three independent parameters, to be experimentally fitted, provided the best prediction capability up to the end of April 2020, i.e., about one month in advance. The Planck-type distribution law we used is:

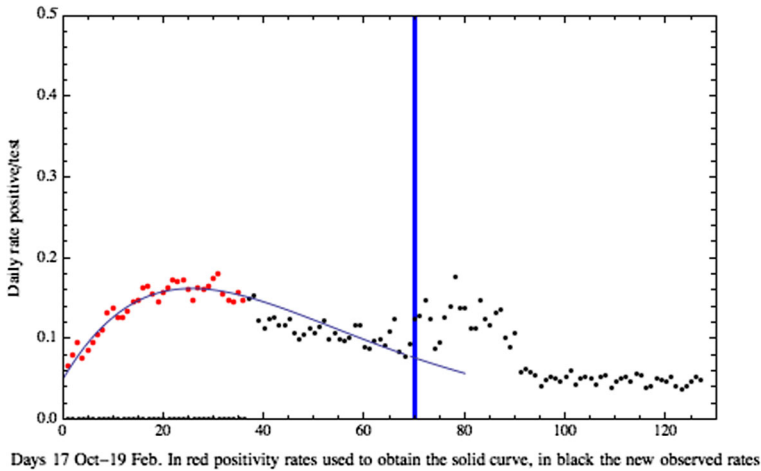
$$\frac{a(t+c)^2}{e^{b(t+c)} - 1}$$

where  $a$ ,  $b$  and  $c$  are three parameters to be fitted for. Planck’s law is one of the outstanding laws of physics [3]; it describes the energy of the radiation, emitted by a black body at thermal equilibrium with temperature  $T$ , as a function of the frequency or of the wavelength. For example, the number of photons emitted per unit area, unit time and unit frequency  $\nu$  is:

$$\frac{8\pi\nu^2}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

where  $h$ ,  $c$  and  $k_B$  are Planck constant, speed of light and Boltzmann constant, respectively. Derived in 1900 by Max Planck, with the idea of energy quantization, it marked the beginning of Quantum Mechanics. A distribution of the type of the Planck’s law with three parameters showed a good prediction potentiality over a period of about one month also in the second wave of Covid 19.

Indeed, in [4], using 36 days starting from October 17, 2020, up to November 21, 2020 (included), we predicted, at the end of November 2020, the behavior of the second wave of the pandemic in Italy using the Planck’s distribution with three parameters. Figure 1 shows the prediction (corresponding to Fig. 4 published in [4]), of the daily cases-to-test ratios (positivity rate). The continuous curve is the best fit of the 36 data using a three parameter Planck’s law. Christmas 2020 is marked with the vertical blue line. This analysis [4] posted on December 9, 2020, in the medRxiv, predicted quite well the behavior of the pandemic in Italy during the Christmas 2020 holidays (see Fig. 2), after approximately one month from the last experimental data of November 21, 2020 (included). For example, after 31 days, on December 22, 2020, the difference between our prediction and the observed positivity rate has shown an error of less than 2% only. Figure 2 shows our prediction (solid curve), based on the fit of the circles in red (positivity rates of 36 days from October 17, 2020, included,



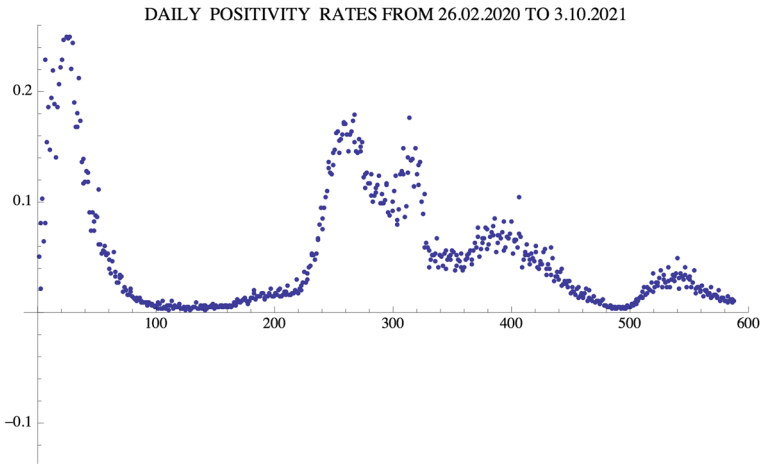
**Fig. 2** This figure is the same of Fig. 1 where, however, we added, with black circles, the positivity rates (i.e., the ratios of positive daily cases-to-tests) after November 21, 2020, rates not known to us and thus not used to determine the parameters of the Planck's distribution (solid line). Indeed, the Planck's distribution fitting function was based on the red points only. The fitting function extrapolated after about one month shows a very good agreement with the observed positivity rates. For example, after 31 days, on December 22, 2020, the difference between the positivity rate predicted by the fitting Planck's distribution and the real observed positivity rate is less than 2% only

until November 21, 2020) with the Planck's distribution, together with the positivity rates later observed (which were not known to us at the end of November 2020) which are shown by the black circles. The percent error of less than 2% only in the prediction of the spread of the pandemic is calculated (Fig. 2) as difference between the solid curve and the real cases on December 22, 2020. Approximately from December 25, 2020, there was a second peak in the positivity rates, probably due to the attenuation of social distancing during the Christmas holiday season. However, approximately after such second peak lasting about three weeks from Christmas, the Planck's curve is still fitting quite well the subsequent positivity rates up to, e.g., February 19, 2021.

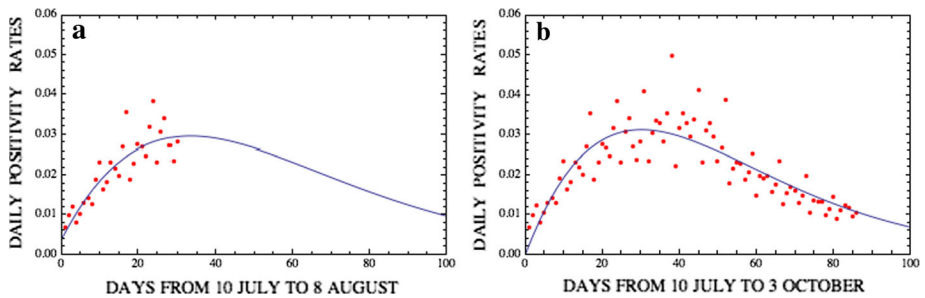
## 2 Prediction of the new wave of pandemic during summer 2021 in Italy

In spite of the diffusion of the Covid vaccines in Italy, probably due to the Covid Delta Variant, starting approximately during the mid of June, it was observed a new wave of pandemic in Italy. In Fig. 3, we show the *positivity rates* (i.e., the ratios of positive daily cases-to-tests) from February 26, 2020, until October 3, 2021. In this figure, the new wave of the pandemic in Italy during summer 2021 can be observed after approximately the point 500 of the x-axis.

On August 9, 2021, we analyzed and fitted with the Planck's distribution, with three parameters, the first 30 positivity rates from July 10, 2021 (included), until August 8, 2021 (included) [5]. The result is displayed in Fig. 4a, which shows a peak approximately around the middle of August. The agreement of the prediction of this Planck's function (fitting up to July 10, 2021) and the positivity rate observed after 8 weeks, on September 12, 2021 (the last red point, number 86, of Figs. 4b and 5), is extremely good. Eight weeks later, we then repeated the analysis and the fit using the next 56 days of daily positivity rates,



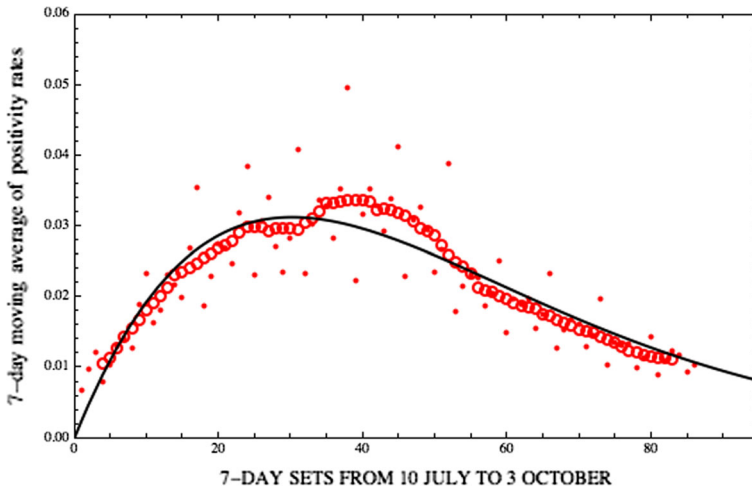
**Fig. 3** Ratios of positive daily cases-to-tests (*positivity rates*) from February 26, 2020, until October 3, 2021, in Italy. It shows various waves of the pandemic



**Fig. 4 a** Ratios of positive daily cases-to-tests (*daily positivity rates*) from July 10, 2021, included, until August 8, 2021, included (30 red points) [5]. The solid line is the best fit Planck’s distribution function through the 30 red points. The agreement of the prediction of this Planck’s fitting function and the positivity rate observed after 8 weeks, on October 3, 2021 (the last red point, number 86, of Figs. 4b and 5), is extremely good. **b** Ratios of positive daily cases-to-tests (*daily positivity rates*) from July 10, 2021, included, until October 3, 2021, included (86 red points). The solid line is the best fit Planck’s distribution function through the red points

i.e., until October 3, 2021 (included). The result is shown in Fig. 4b. Even though the daily positivity rates are somehow oscillating, the extended fit of Fig. 4b confirms that the peak of the pandemic occurred approximately around the middle of August 2021, in very good agreement with our prediction.

To take into account the oscillating nature of the daily positivity rates during the period middle of July–beginning of October, we took the 7-day moving average of the daily positivity rates. The result is shown in Fig. 5, where the red circles represent the 7-day moving averages of the daily positivity rates, the red points the daily positivity rates, and the continuous solid black line is the Planck’s fitting function of the daily positivity rates. Figure 5 confirms that the peak was reached approximately around the middle of August and that we are now (beginning of October 2021) in the decreasing phase of the summer/autumn 2021 wave of the pandemic. This curve predicts that the positivity rate will reduce to about the value of July 10, 2021, in the last week of October with the hypothesis that the basic conditions of the pandemic, such as schools, intensive use of public transportation, social distancing, mask



**Fig. 5** Red points are the ratios of positive daily cases-to-tests (daily positivity rates) from July 10, 2021, until October 3, 2021, and the black solid line is the best fit Planck's function through these red points. The red circles are the 7-day moving averages of the daily positivity rates

wearing, vaccination rate and new potential Covid variants will not change appreciably the spread of the pandemic.

### 3 Conclusions

In a previous paper [1], we carried out an extensive study of a number of distributions to possibly fit the daily positivity rates (ratios of positive daily cases-to-tests) of the Covid-19 pandemic. We concluded that the best fitting function is of the type of the Planck's law with three parameters. In November 2020, we then applied such fitting function to predict the behavior of the pandemic more than one month later, i.e., during the Christmas holidays 2020, in Italy. Here we show that our prediction, performed in November using the Planck's distribution, was accurate after over 1 month, with an error of a few percent only. For example, after 31 days, the difference between our prediction and the observed positivity rate was less than 2%. We finally applied the Planck's distribution, with three parameters, to study the latest wave of the pandemic occurring in Italy during summer 2021. Using the data up to August 8, 2021, only, we concluded that the peak of the wave of the pandemic during summer 2021 would be reached approximately around the middle of August [5], in very good agreement with later observations. Therefore, we are now, at the beginning of October, in the decreasing phase of the summer 2021 wave of the pandemic in Italy, as confirmed by the fit of the positivity rates up to October 3, 2021.

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