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n-point correlators of twist-2 operators in $\mathrm{SU}(N)$ Yang-Mills theory to the lowest perturbative order

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Abstract: We compute, to the lowest perturbative order in $\mathrm{SU}(N)$ Yang-Mills theory, n-point correlators in the coordinate and momentum representation of the gauge-invariant twist-2 operators with maximal spin along the p_+ direction, both in Minkowskian and — by analytic continuation — Euclidean space-time. We also construct the corresponding generating functionals. Remarkably, they have the structure of the logarithm of a functional determinant of the identity plus a term involving the effective propagators that act on the appropriate source fields.

Keywords: Perturbative QCD, Renormalization Group

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1 Introduction and physics motivations

In the present paper we compute, to the lowest perturbative order in SU(N) Yang-Mills (YM) theory, n-point connected correlators, $G^{(n)}_{\text{conf}}(x_1,\dots,x_n)$, in the coordinate representation of the gauge-invariant twist-2 operators with maximal spin along the p_+ direction, both in Minkowskian and — by analytic continuation — Euclidean space-time.

In fact, our computation matches and extends the previous lowest-order perturbative computation of 2- and 3-point gluonic correlators of twist-2 operators in $\mathcal{N} = 4$ SUSY YM theory [1], by including the unbalanced[1] operators with collinear twist 2 in pure YM

[1] In our terminology 'unbalanced' and 'balanced' refers to either the different or the equal number of dotted and undotted indices that the aforementioned operators possess in the spinorial representation respectively. Unbalanced operators are referred to as 'asymmetric' in [2] and 'anisotropic' in [3].

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theory and, most importantly, by calculating all the n-point correlators in the balanced and unbalanced sectors separately, and the 3-point correlators in the mixed sector as well.

Our physics motivation is threefold.

Firstly, our lowest-order computation has an intrinsic interest in YM theory, and — according to [1] — in theories that extends it, such as its supersymmetric versions and QCD.

Secondly, our computation is preliminary to work out the ultraviolet (UV) asymptotics [4, 5] — based on the renormalization-group (RG) improvement of perturbation theory — of the above Euclidean n-point correlators.

Thirdly, our computation is an essential ingredient to test the prediction in section 3 of [6] that, by fundamental principles of the large-N 't Hooft expansion, the generating functional of the nonperturbative leading nonplanar contributions to the aforementioned Euclidean correlators must have the structure of the logarithm of a functional determinant [6] that sums the glueball one-loop diagrams.

Indeed, according to the philosophy of the asymptotically free bootstrap outlined in [6], the RG-improved correlators mentioned above must be asymptotic in the UV [6] to the corresponding nonperturbative correlators involving glueballs. Therefore, to the leading nonplanar order, the generating functional of the former must share with the one of the latter the very same structure of the logarithm of a functional determinant.

As an intermediate step for the program above, we construct the generating functionals of the aforementioned lowest-order n-point correlators.

Remarkably, they have the structure of the logarithm of a functional determinant of the identity plus a term involving the effective propagators that act on the appropriate source fields.

Hence, according to the argument above, our formulas for the generating functionals are as simple as they can be.

Incidentally, the generating functionals also allow us to compute straightforwardly the n-point correlators in the momentum representation, whose structure is slightly simpler than in the coordinate representation.

2 Main results

2.1 Balanced and unbalanced twist-2 conformal operators

We describe our calculation and the operators that enter it. We compute, to the lowest perturbative order in SU(N) YM theory, n-point connected correlators in Minkowskian spacetime of the gauge-invariant twist-2 operators with maximal spin along the p_+ direction:

$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle_{\text{lowest order}} = G^{(n)}_{\text{conf}}(x_1, \ldots, x_n) \tag{2.1}$$

It has been known for some time that, to the lowest order and the next one,[2] YM theory is conformal invariant [7], since the beta function only affects the solution of the Callan-Symanzik equation starting from the order of g^4. More recently, the exact conformal

[2] In fact, to the order of g^2, in the conformal subtraction scheme [7].

symmetry of QCD at the Wilson-Fisher critical point in $d = 4 - 2\epsilon$ dimensions has been exploited [8, 9][3] as a computational tool in higher orders of perturbation theory.

Therefore, following [7] we employ operators that have nice transformation properties with respect to the collinear conformal subgroup involving the coordinate x^+. Alternatively, operators can be also constructed with nice transformation properties [10, 11] with respect to the conformal group, whose suitably chosen components restrict to the aforementioned representations of the collinear conformal subgroup.

Primary conformal operators $\mathcal{O}_j(x)$, with collinear conformal spin $j = s + \frac{\tau}{2}$, where τ is the collinear twist and s the collinear spin, i.e., the spin projected along the p_+ direction, transform under the action of the generators [7] of the collinear conformal algebra $\mathrm{SL}(2,\mathbb{R})$:

$$\begin{aligned}[] [L_0, L_{\mp}] &= \mp L_{\mp} \\ [L_-, L_+] &= -2L_0 \end{aligned} \tag{2.2}$$

according [7] to:

$$\begin{aligned}[] [L_+, \mathcal{O}_j(x)] &= -\partial_+ \mathcal{O}_j(x) \\ [L_-, \mathcal{O}_j(x)] &= ({x^+}^2 \partial_+ + 2jx^+)\mathcal{O}_j(x) \\ [L_0, \mathcal{O}_j(x)] &= (x^+\partial_+ + j)\mathcal{O}_j(x) \end{aligned} \tag{2.3}$$

where in eq. (2.3) $x = (x^+, x^-, x^1, x^2)$ is restricted [7] to the line $x^- = x^1 = x^2 = 0$. Their conformal descendants, $\partial_+^i \mathcal{O}_j(x)$, are obtained by taking derivatives with respect to x^+, and have the same τ.

For a given canonical dimension $d = \tau + s$, the quasi-partonic [12] operators have minimum τ and maximum s, with nice mixing under renormalization and conformal properties as above [7, 10–15]. Their collinear twist τ does not necessarily coincide [7] with the twist $\mathcal{T}$ — defined by $d = \mathcal{T} + S$, where S is the spin — that refers to the conformal group [10, 11] instead of the collinear subgroup.

In general, local gauge-invariant operators with $\mathcal{T} = 2$ provide the leading contribution to the OPE of two vector currents in massless QCD-like theories[4] [16][5] in Minkowskian space-time near the light-cone.

An infinite family of quasi-partonic operators is constructed as follows. A composite gauge-covariant primary conformal operator, built by two elementary[6] gauge-covariant primary conformal operators Φ_{j_1}, Φ_{j_2}, with collinear conformal spins j_1, j_2, has the form [7]:

$$\mathcal{O}_l^{j_1 j_2}(x) = \Phi_{j_1}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^l P_l^{(2j_1-1,2j_2-1)}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\Phi_{j_2}(x) \tag{2.4}$$

where $P_l^{(2j_1-1,2j_2-1)}$ are Jacobi polynomials (appendix F), D_+ is the covariant derivative along the p_+ direction (appendix A), and the arrows denote the action of the derivative on

[3] And references therein.

[4] By massless QCD-like theories we mean asymptotically free gauge theories that are massless to all orders of perturbation theory, such as QCD with massless quarks.

[5] And references therein.

[6] In the present paper, we refer to the operators $\Phi_j(x)$ as elementary, since they play the role of elementary constituents, though they may actually be composite operators.

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the right or the left. The corresponding gauge-invariant object is obtained by taking the color trace.

The collinear conformal spin, j, of the operator, $\mathbb{O}_l^{j_1 j_2}(x)$, is $j = j_1 + j_2 + l$, where l is the power of the derivative in eq. (2.4). By working out the definition in eq. (2.4), we get:

$$\begin{aligned} \mathcal{O}_l^{j_1 j_2}(x) &= \sum_{k=0}^{l} \binom{l+2j_1-1}{k} \binom{l+2j_2-1}{k+2j_2-1} (-1)^{l-k} \Phi_{j_1}(x) \overleftarrow{D}_+^{l-k} \overrightarrow{D}_+^{k} \Phi_{j_2}(x) \\ &= \sum_{k=0}^{l} \mathcal{O}_{lk}^{j_1 j_2}(x) \end{aligned} \tag{2.5}$$

thus realizing the conformal operator $\mathcal{O}_l^{j_1 j_2}(x)$ as a sum of $l+1$ operators, $\mathcal{O}_{lk}^{j_1 j_2}(x)$, that are not necessarily conformal.

Hence, the composite operators depend on a choice of the elementary conformal operators Φ_{j_1}, Φ_{j_2}. We define the standard conformal basis for primary operators with collinear twist 2, where the elementary operators are $f_{11}, f_{\dot{1}\dot{1}}$ (section 4.1) with conformal spin $j = \frac{3}{2}$. In the standard basis the gluonic operators are classified as in [2, 10, 11]:

$$\begin{aligned} \mathbb{O}_s &= \operatorname{Tr} f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+} \right) f_{\dot{1}\dot{1}}(x) \qquad s = 2,4,6,\ldots \\ \tilde{\mathbb{O}}_s &= \operatorname{Tr} f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+} \right) f_{\dot{1}\dot{1}}(x) \qquad s = 3,5,7,\ldots \\ \mathbb{S}_s &= \frac{1}{\sqrt{2}} \operatorname{Tr} f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+} \right) f_{11}(x) \qquad s = 2,4,6,\ldots \\ \bar{\mathbb{S}}_s &= \frac{1}{\sqrt{2}} \operatorname{Tr} f_{\dot{1}\dot{1}}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+} \right) f_{\dot{1}\dot{1}}(x) \qquad s = 2,4,6,\ldots \end{aligned} \tag{2.6}$$

by restricting the appropriate conformal multiplet [10, 11] to the components along the p_+ direction, with C_l^{α} Gegenbauer polynomials (appendix F), which are a special case of Jacobi polynomials.

$\mathbb{O}_s$ and $\tilde{\mathbb{O}}_s$ are Hermitian balanced operators with $\tau = \mathcal{T} = 2$. They have an equal number of undotted and dotted spinor indices (appendices C and D):

$$\begin{aligned} \mathbb{O}_s &= \mathbb{O}_{1\dot{1}\ldots 1\dot{1}} \\ \tilde{\mathbb{O}}_s &= \tilde{\mathbb{O}}_{1\dot{1}\ldots 1\dot{1}} \end{aligned} \tag{2.7}$$

$\mathbb{S}_s$ and its Hermitian conjugate, $\bar{\mathbb{S}}_s$, denoted by the bar superscript, are unbalanced operators with $\tau = 2$. They have a different number of undotted and dotted spinor indices:

$$\begin{aligned} \mathbb{S}_s &= \mathbb{S}_{1111\ldots 1\dot{1}} \\ \bar{\mathbb{S}}_s &= \mathbb{S}_{\dot{1}\dot{1}\dot{1}\dot{1}\ldots \dot{1}1} \end{aligned} \tag{2.8}$$

Besides, we also define the extended conformal basis for primary operators with collinear twist 2, where the elementary operators are $D_+^{-1} f_{11}, D_+^{-1} f_{\dot{1}\dot{1}}$, with conformal spin $j = \frac{1}{2}$, which are nonlocal in general, but local (appendix E) in the light-cone gauge $A_+ = 0$. Clearly, gauge invariance ensures that all their correlators are local, as we verify explicitly.

The extended basis is natural in SUSY calculations [14], and includes (nonlocal) operators with $\tau = 2$ and $s = 0, 1$. We have chosen it in YM theory because of the simplicity of the results for the correlators. In the extended basis (section 4.2) the gluonic operators are:

$$\begin{aligned}
\mathbb{A}_s &= \mathrm{Tr}\, D_+^{-1} f_{11}(x)\,(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s = 0, 2, 4, \ldots \\
\tilde{\mathbb{A}}_s &= \mathrm{Tr}\, D_+^{-1} f_{11}(x)\,(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s = 1, 3, 5, \ldots \\
\mathbb{B}_s &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\, D_+^{-1} f_{11}(x)\,(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{11}(x) \qquad s = 0, 2, 4, \ldots \\
\bar{\mathbb{B}}_s &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\, D_+^{-1} f_{\dot{1}\dot{1}}(x)\,(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s = 0, 2, 4, \ldots
\end{aligned} \tag{2.9}$$

2.2 Minkowskian n-point correlators in the coordinate representation

2.2.1 Standard basis

We have normalized our operators in such a way that the 2-point correlators in the standard basis are equal for even s:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \langle \mathbb{S}_{s_1}(x)\bar{\mathbb{S}}_{s_2}(y)\rangle = \mathcal{C}_{s_1}(x,y)\delta_{s_1 s_2} \tag{2.10}$$

and for odd s:

$$\langle \tilde{\mathbb{O}}_{s_1}(x)\tilde{\mathbb{O}}_{s_2}(y)\rangle = \mathcal{C}_{s_1}(x,y)\delta_{s_1 s_2} \tag{2.11}$$

with:

$$\begin{aligned}
\mathcal{C}_s(x,y) &= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2} i^{2s-4}}{(4!)^2}(s+1)^2(s+2)^2\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}} \\
&\quad \sum_{k_1=0}^{s-2}\sum_{k_2=0}^{s-2}\binom{s}{k_1}\binom{s}{k_1+2}\binom{s}{k_2}\binom{s}{k_2+2}(-1)^{s-k_2+k_1} \\
&\quad (s-k_1+k_2)!(s+k_1-k_2)! \\
&= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2} i^{2s-4}}{(4!)^2}(s+1)^2(s+2)^2(2s)!\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}} \\
&\quad \sum_{k_1=0}^{s-2}\sum_{k_2=0}^{s-2}\binom{s}{k_1}\binom{s}{k_1+2}\binom{s}{k_2}\binom{s}{k_2+2}(-1)^{k_2+k_1}\frac{1}{\binom{2s}{k_1+k_2+2}}
\end{aligned} \tag{2.12}$$

where we omit the $i\epsilon$ prescription in the propagators in the coordinate representation, in such a way that (appendix A):

$$\frac{1}{|x|^2} \tag{2.13}$$

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should be read (appendix B):

$$\frac{1}{|x|^2 - i\epsilon} \tag{2.14}$$

The very same correlators are evaluated by a trick [1] (appendix H):

$$\mathcal{C}_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2}}{(4!)^2}(-1)^s(s-1)s(s+1)(s+2)(2s)!\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}} \tag{2.15}$$

Therefore, we have discovered the following — seemingly nontrivial — identity (section 5):

$$\frac{s_1(s_1-1)}{(s_1+1)(s_1+2)}\delta_{s_1s_2} = \sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}(-1)^{k_2+k_1}\frac{1}{\binom{s_1+s_2}{k_1+k_2+2}} \tag{2.16}$$

We have not found a direct proof of the above identity, but we have verified it numerically.

Moreover, the only nonvanishing 3-point correlators are:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = \langle \mathbb{O}_{s_1}(x)\mathbb{S}_{s_2}(y)\bar{\mathbb{S}}_{s_3}(z)\rangle = \mathcal{C}_{s_1s_2s_3}(x,y,z) \tag{2.17}$$

and:

$$\langle \mathbb{O}_{s_1}(x)\tilde{\mathbb{O}}_{s_2}(y)\tilde{\mathbb{O}}_{s_3}(z)\rangle = \mathcal{C}_{s_1s_2s_3}(x,y,z) \tag{2.18}$$

with:

$$\begin{aligned}\mathcal{C}_{s_1s_2s_3}(x,y,z) = &-\frac{1}{(4\pi^2)^3}(1+(-1)^{s_1+s_2+s_3})\left(\frac{2}{4!}\right)^3\frac{N^2-1}{8}i^{s_1+s_2+s_3}2^{s_1+s_2+s_3}\\ &(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_3+1)(s_3+2)\\ &\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\binom{s_3}{k_3}\binom{s_3}{k_3+2}\\ &(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\ &\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}}\end{aligned} \tag{2.19}$$

We also compute the n-point correlators. In the balanced sector, we get:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}} = &\frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}i^{\sum_{l=1}^n s_l}\\ &\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\ &\frac{(-1)^n}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!\\ &\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}\end{aligned} \tag{2.20}$$

The very same formula holds for an even number of operators $\tilde{\mathbb{O}}_s$, otherwise the correlators vanish. The nonvanishing correlators in the balanced sector are:

$$\begin{aligned}
&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}} \\
&= \frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}} 2^{\sum_{l=1}^{n+2m} s_l} i^{\sum_{l=1}^{n+2m} s_l} \frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_{n+2m}+3)}{\Gamma(5)\Gamma(s_{n+2m}+1)} \\
&\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2} \\
&\frac{(-1)^{n+2m}}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})! \\
&\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n+2m)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}}
\end{aligned} \tag{2.21}$$

In the unbalanced sector, we get:

$$\begin{aligned}
&\langle \mathbb{S}_{s_1}(x_1)\dots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\dots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle = \frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^{n} s_l+s'_l} i^{\sum_{l=1}^{n} s_l+s'_l} \\
&\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\frac{\Gamma(3)\Gamma(s'_1+3)}{\Gamma(5)\Gamma(s'_1+1)}\dots\frac{\Gamma(3)\Gamma(s'_n+3)}{\Gamma(5)\Gamma(s'_n+1)} \\
&\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2} \\
&\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n-2}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2} \\
&\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})! \\
&\dots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})! \\
&\frac{(x_{\sigma(1)}-y_{\rho(1)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left(|x_{\sigma(1)}-y_{\rho(1)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left(|y_{\rho(1)}-x_{\sigma(2)}|^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}} \\
&\dots\frac{(x_{\sigma(n)}-y_{\rho(n)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left(|x_{\sigma(n)}-y_{\rho(n)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left(|y_{\rho(n)}-x_{\sigma(1)}|^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}}
\end{aligned} \tag{2.22}$$

2.2.2 Extended basis

We normalize our operators in such a way that the 2-point correlators in the extended basis are equal for even s:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle = \langle \bar{\mathbb{B}}_{s_1}(x)\bar{\mathbb{B}}_{s_2}(y)\rangle = \mathcal{A}_{s_1}(x,y)\delta_{s_1s_2} \tag{2.23}$$

and for odd s:

$$\langle \tilde{\mathbb{A}}_{s_1}(x)\tilde{\mathbb{A}}_{s_2}(y)\rangle = \mathcal{A}_{s_1}(x,y)\delta_{s_1s_2} \tag{2.24}$$

with:

$$\mathcal{A}_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}i^{2s}\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}}$$
$$\sum_{k_1=0}^{s}\sum_{k_2=0}^{s}\binom{s}{k_1}\binom{s}{k_1}\binom{s}{k_2}\binom{s}{k_2}(-1)^{s-k_2+k_1}(s-k_1+k_2)!(s+k_1-k_2)!$$
$$= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}i^{2s}(2s)!\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}}$$
$$\sum_{k_1=0}^{s}\sum_{k_2=0}^{s}\binom{s}{k_1}\binom{s}{k_1}\binom{s}{k_2}\binom{s}{k_2}(-1)^{k_2+k_1}\frac{1}{\binom{2s}{k_1+k_2}} \tag{2.25}$$

The very same correlators are evaluated by a trick [1] (appendix H):

$$\mathcal{A}_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}(-1)^s(2s)!\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}} \tag{2.26}$$

Therefore, we have discovered the following — seemingly nontrivial — identity (section 5):

$$\delta_{s_1s_2} = \sum_{k_1=0}^{s_1}\sum_{k_2=0}^{s_2}\binom{s_1}{k_1}\binom{s_1}{k_1}\binom{s_2}{k_2}\binom{s_2}{k_2}(-1)^{k_2+k_1}\frac{1}{\binom{s_1+s_2}{k_1+k_2}} \tag{2.27}$$

We have not found a direct proof of the above identity, but we have verified it numerically.

Moreover, the only nonvanishing 3-point correlators are:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\mathbb{A}_{s_3}(z)\rangle = \langle \mathbb{A}_{s_1}(x)\mathbb{B}_{s_2}(y)\bar{\mathbb{B}}_{s_3}(z)\rangle = \mathcal{A}_{s_1s_2s_3}(x,y,z) \tag{2.28}$$

and:

$$\langle \mathbb{A}_{s_1}(x)\tilde{\mathbb{A}}_{s_2}(y)\tilde{\mathbb{A}}_{s_3}(z)\rangle = \mathcal{A}_{s_1s_2s_3}(x,y,z) \tag{2.29}$$

with:

$$\mathcal{A}_{s_1s_2s_3}(x,y,z) = -\frac{1}{(4\pi^2)^3}(1+(-1)^{s_1+s_2+s_3})\frac{N^2-1}{8}i^{s_1+s_2+s_3}2^{s_1+s_2+s_3}$$
$$\sum_{k_1=0}^{s_1}\sum_{k_2=0}^{s_2}\sum_{k_3=0}^{s_3}\binom{s_1}{k_1}\binom{s_1}{k_1}\binom{s_2}{k_2}\binom{s_2}{k_2}\binom{s_3}{k_3}\binom{s_3}{k_3}$$
$$(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!$$
$$\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}} \tag{2.30}$$

We also compute the n-point correlators. In the balanced sector, we get:

$$\langle \mathbb{A}_{s_1}(x_1)\dots\mathbb{A}_{s_n}(x_n)\rangle_{\text{conn}} = \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}i^{\sum_{l=1}^n s_l}$$
$$\sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\binom{s_1}{k_1}^2\dots\binom{s_n}{k_n}^2\frac{(-1)^n}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\cdots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}} \tag{2.31}$$

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The very same formula holds for an even number of operators $\tilde{\mathbb{A}}_s$, otherwise the correlators vanish. The nonvanishing correlators in the balanced sector are:

$$
\begin{aligned}
&\langle \mathbb{A}_{s_1}(x_1)\ldots\mathbb{A}_{s_n}(x_n)\tilde{\mathbb{A}}_{s_{n+1}}(x_{n+1})\ldots\tilde{\mathbb{A}}_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}}\\
&=\frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}}2^{\sum_{l=1}^{n+2m}s_l}i^{\sum_{l=1}^{n+2m}s_l}\sum_{k_1=0}^{s_1}\ldots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}^2\ldots\binom{s_{n+2m}}{k_{n+2m}}^2\\
&\quad\frac{(-1)^{n+2m}}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})!\\
&\quad\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\ldots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n+2m)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{2.32}
$$

In the unbalanced sector, we get:

$$
\begin{aligned}
&\langle \mathbb{B}_{s_1}(x_1)\ldots\mathbb{B}_{s_n}(x_n)\bar{\mathbb{B}}_{s'_1}(y_1)\ldots\bar{\mathbb{B}}_{s'_n}(y_n)\rangle=\frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^{n}s_l+s'_l}i^{\sum_{l=1}^{n}s_l+s'_l}\\
&\quad\sum_{k_1=0}^{s_1}\ldots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1-2}\ldots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}^2\ldots\binom{s_n}{k_n}^2\binom{s'_1}{k'_1}^2\ldots\binom{s'_n}{k'_n}^2\\
&\quad\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!\\
&\quad\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!\\
&\quad\frac{(x_{\sigma(1)}-y_{\rho(1)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left(|x_{\sigma(1)}-y_{\rho(1)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left(|y_{\rho(1)}-x_{\sigma(2)}|^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}\\
&\quad\ldots\frac{(x_{\sigma(n)}-y_{\rho(n)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left(|x_{\sigma(n)}-y_{\rho(n)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left(|y_{\rho(n)}-x_{\sigma(1)}|^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{2.33}
$$

2.3 Euclidean n-point correlators in the coordinate representation

2.3.1 Standard basis

After the Wick rotation (appendix A and section 8), we obtain in the standard basis:

$$
\begin{aligned}
\mathcal{C}^E_s(x,y)=&\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2}}{(4!)^2}(s+1)^2(s+2)^2\frac{(x-y)_z^{2s}}{((x-y)^2)^{2s+2}}\\
&\sum_{k_1=0}^{s-2}\sum_{k_2=0}^{s-2}\binom{s}{k_1}\binom{s}{k_1+2}\binom{s}{k_2+2}\binom{s}{k_2}(-1)^{s-k_2+k_1}\\
&(s-k_1+k_2)!(s+k_1-k_2)!
\end{aligned}
\tag{2.34}
$$

which is equivalent to:

$$
\mathcal{C}^E_s(x,y)=\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2}}{(4!)^2}(s-1)s(s+1)(s+2)(2s)!\frac{(x-y)_z^{2s}}{((x-y)^2)^{2s+2}}
\tag{2.35}
$$

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For the nonvanishing 3-point correlators, we get:

$$\begin{aligned}\mathcal{C}^E_{s_1s_2s_3}(x,y,z) = &\frac{1}{(4\pi^2)^3}(-1)^{s_1+s_2+s_3}(1+(-1)^{s_1+s_2+s_3})\left(\frac{2}{4!}\right)^3\frac{N^2-1}{8}2^{s_1+s_2+s_3}\\ &(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_3+1)(s_3+2)\\ &\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\binom{s_3}{k_3}\binom{s_3}{k_3+2}\\ &(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\ &\frac{(x-y)_z^{s_1-k_1+k_2}}{((x-y)^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_z^{s_2-k_2+k_3}}{((y-z)^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_z^{s_3-k_3+k_1}}{((z-x)^2)^{s_3+1-k_3+k_1}}\end{aligned} \tag{2.36}$$

Moreover, for the n-point correlators in the balanced sector, we obtain:

$$\begin{aligned}&\langle \mathbb{O}^E_{s_1}(x_1)\ldots\mathbb{O}^E_{s_n}(x_n)\rangle_{\text{conn}} = \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}(-1)^{\sum_{l=1}^n s_l}\\ &\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\cdots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\ &\frac{1}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!\\ &\frac{(x_{\sigma(1)}-x_{\sigma(2)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left((x_{\sigma(1)}-x_{\sigma(2)})^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\cdots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_z^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left((x_{\sigma(n)}-x_{\sigma(1)})^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}\end{aligned} \tag{2.37}$$

The very same formula holds for an even number of operators $\tilde{\mathbb{O}}^E_s$, otherwise the correlators vanish. The nonvanishing correlators in the balanced sector are:

$$\begin{aligned}&\langle \mathbb{O}^E_{s_1}(x_1)\ldots\mathbb{O}^E_{s_n}(x_n)\tilde{\mathbb{O}}^E_{s_{n+1}}(x_{n+1})\ldots\tilde{\mathbb{O}}^E_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}}\\ &=\frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}}2^{\sum_{l=1}^{n+2m}s_l}(-1)^{\sum_{l=1}^{n+2m}s_l}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\cdots\frac{\Gamma(3)\Gamma(s_{n+2m}+3)}{\Gamma(5)\Gamma(s_{n+2m}+1)}\\ &\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\\ &\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})!\\ &\frac{(x_{\sigma(1)}-x_{\sigma(2)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left((x_{\sigma(1)}-x_{\sigma(2)})^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\cdots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left((x_{\sigma(n+2m)}-x_{\sigma(1)})^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}}\end{aligned} \tag{2.38}$$

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In the unbalanced sector, we get:

$$
\begin{aligned}
&\langle \mathbb{S}^E_{s_1}(x_1)\dots\mathbb{S}^E_{s_n}(x_n)\bar{\mathbb{S}}^E_{s'_1}(y_1)\dots\bar{\mathbb{S}}^E_{s'_n}(y_n)\rangle = \frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^n s_l+s'_l}(-1)^{\sum_{l=1}^n s_l+s'_l}\\
&\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\frac{\Gamma(3)\Gamma(s'_1+3)}{\Gamma(5)\Gamma(s'_1+1)}\dots\frac{\Gamma(3)\Gamma(s'_n+3)}{\Gamma(5)\Gamma(s'_n+1)}\\
&\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\
&\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n-2}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}\\
&\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!\\
&\dots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!\\
&\frac{(x_{\sigma(1)}-y_{\rho(1)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\Big((x_{\sigma(1)}-y_{\rho(1)})^2\Big)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\Big((y_{\rho(1)}-x_{\sigma(2)})^2\Big)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}\\
&\dots\frac{(x_{\sigma(n)}-y_{\rho(n)})_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\Big((x_{\sigma(n)}-y_{\rho(n)})^2\Big)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_z^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\Big((y_{\rho(n)}-x_{\sigma(1)})^2\Big)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{2.39}
$$

2.3.2 Extended basis

We obtain in the extended basis:

$$
\begin{aligned}
\mathcal{A}^E_s(x,y) = &\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}\frac{(x-y)_z^{2s}}{((x-y)^2)^{2s+2}}\\
&\sum_{k_1=0}^{s}\sum_{k_2=0}^{s}\binom{s}{k_1}\binom{s}{k_1}\binom{s}{k_2}\binom{s}{k_2}(-1)^{s-k_2+k_1}(s-k_1+k_2)!(s+k_1-k_2)!
\end{aligned}
\tag{2.40}
$$

which is equivalent to:

$$
\mathcal{A}^E_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}(2s)!\frac{x_z^{2s}}{(x^2)^{2s+2}}
\tag{2.41}
$$

For the nonvanishing 3-point correlators, we get:

$$
\begin{aligned}
\mathcal{A}^E_{s_1s_2s_3}(x,y,z) = &\frac{1}{(4\pi^2)^3}(-1)^{s_1+s_2+s_3}(1+(-1)^{s_1+s_2+s_3})\frac{N^2-1}{8}2^{s_1+s_2+s_3}\\
&\sum_{k_1=0}^{s_1}\sum_{k_2=0}^{s_2}\sum_{k_3=0}^{s_3}\binom{s_1}{k_1}\binom{s_1}{k_1}\binom{s_2}{k_2}\binom{s_2}{k_2}\binom{s_3}{k_3}\binom{s_3}{k_3}\\
&(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\
&\frac{(x-y)_z^{s_1-k_1+k_2}}{((x-y)^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_z^{s_2-k_2+k_3}}{((y-z)^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_z^{s_3-k_3+k_1}}{((z-x)^2)^{s_3+1-k_3+k_1}}
\end{aligned}
\tag{2.42}
$$

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Moreover, for the n-point correlators in the balanced sector, we obtain:

$$\langle \mathbb{A}^E_{s_1}(x_1)\dots\mathbb{A}^E_{s_n}(x_n)\rangle_{\text{conn}} = \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}(-1)^{\sum_{l=1}^n s_l}$$
$$\sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\binom{s_1}{k_1}^2\dots\binom{s_n}{k_n}^2\frac{1}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-x_{\sigma(2)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left((x_{\sigma(1)}-x_{\sigma(2)})^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_z^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left((x_{\sigma(n)}-x_{\sigma(1)})^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}} \tag{2.43}$$

The very same formula holds for an even number of $\tilde{\mathbb{A}}^E_s$ operators, otherwise the correlators vanish. The nonvanishing correlators in the balanced sector are:

$$\langle \mathbb{A}^E_{s_1}(x_1)\dots\mathbb{A}^E_{s_n}(x_n)\tilde{\mathbb{A}}^E_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{A}}^E_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}}$$
$$= \frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}}2^{\sum_{l=1}^{n+2m} s_l}(-1)^{\sum_{l=1}^{n+2m} s_l}\sum_{k_1=0}^{s_1}\dots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}^2\dots\binom{s_{n+2m}}{k_{n+2m}}^2$$
$$\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-x_{\sigma(2)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left((x_{\sigma(1)}-x_{\sigma(2)})^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left((x_{\sigma(n+2m)}-x_{\sigma(1)})^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}} \tag{2.44}$$

In the unbalanced sector, we get:

$$\langle \mathbb{B}^E_{s_1}(x_1)\dots\mathbb{B}^E_{s_n}(x_n)\bar{\mathbb{B}}^E_{s'_1}(y_1)\dots\bar{\mathbb{B}}^E_{s'_n}(y_n)\rangle = \frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^n s_l+s'_l}(-1)^{\sum_{l=1}^n s_l+s'_l}$$
$$\sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}^2\dots\binom{s_n}{k_n}^2\binom{s'_1}{k'_1}^2\dots\binom{s'_n}{k'_n}^2$$
$$\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!$$
$$\dots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-y_{\rho(1)})_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left((x_{\sigma(1)}-y_{\rho(1)})^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left((y_{\rho(1)}-x_{\sigma(2)})^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}$$
$$\dots\frac{(x_{\sigma(n)}-y_{\rho(n)})_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left((x_{\sigma(n)}-y_{\rho(n)})^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_z^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left((y_{\rho(n)}-x_{\sigma(1)})^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}} \tag{2.45}$$

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2.4 Generating functional of n-point correlators in the coordinate representation

Remarkably, we find the following structure for the generating functional of correlators of balanced operators with $\mathcal{T} = 2$ to the lowest order:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \log \text{Det} \left(\mathbb{I} + \mathcal{D}^{-1}\mathcal{O}\right) \tag{2.46}$$

that, in the coordinate representation, is a compact notation for:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \log \text{Det} \left(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y)\right) \tag{2.47}$$

where the source fields $\mathcal{O}_{sk}(x)$ occur in the expansion of the source $\mathcal{O}_s(x)$, in analogy with eq. (2.5):

$$\mathcal{O}_s(x) = \sum_{k=0}^{l} \mathcal{O}_{sk}(x) \tag{2.48}$$

and, by a slight abuse of notation, we have employed for the source fields the very same notation as for the corresponding operators, with $l = s-2$ for the standard basis and $l = s$ for the extended basis.

Though the source fields are originally defined either for even or odd s respectively, to keep the notation simple in the following formulas, we extend their definition to all values of s, in such a way that they are 0 for either odd or even s respectively.

The argument of the determinant is the kernel:

$$K_{s_1k_1,s_2k_2}(x,y) = \delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y) \tag{2.49}$$

of the integral operator:

$$\psi_{s_1k_1}(x) = \sum_{s_2k_2} \int K_{s_1k_1,s_2k_2}(x,y)\phi_{s_2k_2}(y)d^4y \tag{2.50}$$

formally of Fredholm type.

$\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)$ — defined in the following — is the effective propagator associated to the source fields $\mathcal{O}_{sk}(x)$.

Then, the n-point correlators are computed by the functional derivatives:

$$\begin{aligned}\langle \mathcal{O}_{s_1}(x_1)\dots\mathcal{O}_{s_n}(x_n)\rangle_{\text{conn}} &= \frac{\delta}{\delta\mathcal{O}_{s_1}(x_1)}\cdots\frac{\delta}{\delta\mathcal{O}_{s_n}(x_n)}\Gamma_{\text{conf}}[\mathcal{O}] \\ &= \sum_{k_1=0}^{l_1}\dots\sum_{k_n=0}^{l_n}\frac{\delta}{\delta\mathcal{O}_{s_1k_1}(x_1)}\cdots\frac{\delta}{\delta\mathcal{O}_{s_nk_n}(x_n)}\Gamma_{\text{conf}}[\mathcal{O}]\end{aligned} \tag{2.51}$$

The generating functional — reported below — of correlators of the unbalanced operators with collinear twist $\tau = 2$ has a similar structure of the logarithm of a functional determinant.

In both cases, we verify that our ansatz reproduces the corresponding correlators.

2.4.1 Minkowskian standard basis

Specifically, we verify by direct computation (section 9) that:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O}] &= -(N^2-1)\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right)\\
\Gamma_{\text{conf}}[\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}+\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
&\quad-\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
&= -\frac{N^2-1}{2}\log\operatorname{Det}\left(\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}-2\mathcal{D}^{-1}\bar{\mathbb{S}}\mathcal{D}^{-1}\mathbb{S}\right)
\end{aligned}
\tag{2.52}
$$

in Minkowskian space-time.

By making explicit the continuous and discrete indices, the above equations read in the coordinate representation:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)(\mathbb{O}_{s_2k_2}(y)+\tilde{\mathbb{O}}_{s_2k_2}(y))\Big)\\
&\quad-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)(\mathbb{O}_{s_2k_2}(y)-\tilde{\mathbb{O}}_{s_2k_2}(y))\Big)\\
\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad-2\int d^4z\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(x-z)\bar{\mathbb{S}}_{sk}(z)\mathcal{D}^{-1}_{sk,s_2k_2}(z-y)\mathbb{S}_{s_2k_2}(y)\Bigg)
\end{aligned}
\tag{2.53}
$$

with:

$$
\begin{aligned}
\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y) &= \frac{i^{s_1+1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-\partial_+)^{s_1-k_1+k_2}\Box^{-1}(x-y)\\
&= \frac{i^{s_1}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-\partial_+)^{s_1-k_1+k_2}\frac{1}{|x-y|^2-i\epsilon}
\end{aligned}
\tag{2.54}
$$

2.4.2 Minkowskian extended basis

We also verify by direct computation (section 9) that:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{A}] &= -\,(N^2-1)\log\text{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{A}\right)\\
\Gamma_{\text{conf}}[\tilde{\mathbb{A}}] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-\mathcal{D}^{-1}\tilde{\mathbb{A}}\mathcal{D}^{-1}\tilde{\mathbb{A}}\right)\\
\Gamma_{\text{conf}}[\mathbb{A},\tilde{\mathbb{A}}] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{A}+\mathcal{D}^{-1}\tilde{\mathbb{A}}\right)\\
&\quad -\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{A}-\mathcal{D}^{-1}\tilde{\mathbb{A}}\right)\\
&= -\,\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{A}\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{A}}\mathcal{D}^{-1}\tilde{\mathbb{A}}\right)\\
\Gamma_{\text{conf}}[\mathbb{B},\bar{\mathbb{B}}] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}^{-1}\bar{\mathbb{B}}\mathcal{D}^{-1}\mathbb{B}\right)
\end{aligned}
\tag{2.55}
$$

in Minkowskian space-time.

By making explicit the continuous and discrete indices, the above equations read in the coordinate representation:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{A},\tilde{\mathbb{A}}] &= -\,\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad +\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)(\mathbb{A}_{s_2k_2}(y)+\tilde{\mathbb{A}}_{s_2k_2}(y))\Big)\\
&\quad -\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad +\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)(\mathbb{A}_{s_2k_2}(y)-\tilde{\mathbb{A}}_{s_2k_2}(y))\Big)\\
\Gamma_{\text{conf}}[\mathbb{B},\bar{\mathbb{B}}] &= -\,\frac{N^2-1}{2}\log\text{Det}\Bigg(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\
&\quad -2\int d^4z\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(x-z)\bar{\mathbb{B}}_{sk}(z)\mathcal{D}^{-1}_{sk,s_2k_2}(z-y)\mathbb{B}_{s_2k_2}(y)\Bigg)
\end{aligned}
\tag{2.56}
$$

with:

$$
\begin{aligned}
\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y) &= \frac{i^{s_1+1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-\partial_+)^{s_1-k_1+k_2}\Box^{-1}(x-y)\\
&= \frac{i^{s_1}}{8\pi^2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-\partial_+)^{s_1-k_1+k_2}\frac{1}{|x-y|^2-i\epsilon}
\end{aligned}
\tag{2.57}
$$

2.4.3 Euclidean standard basis

Similarly:

$$\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O}^E] &= -\,(N^2-1)\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{O}^E\right)\\
\Gamma_{\text{conf}}[\tilde{\mathbb{O}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\right)\\
\Gamma_{\text{conf}}[\mathbb{O}^E,\tilde{\mathbb{O}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{O}^E+\mathcal{D}^{-1}\tilde{\mathbb{O}}^E\right)\\
&\quad-\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{O}^E-\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\right)\\
&= -\,\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{O}^E\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{O}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\right)\\
\Gamma_{\text{conf}}[\mathbb{S}^E,\bar{\mathbb{S}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}_E^{-1}\bar{\mathbb{S}}^E\mathcal{D}_E^{-1}\mathbb{S}^E\right)
\end{aligned}\tag{2.58}$$

in Euclidean space-time, with:

$$\begin{aligned}
\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(x-y) &= \frac{(-i)^{-k_1+k_2}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}\partial_z^{s_1-k_1+k_2}\,\triangle^{-1}(x-y)\\
&= -\frac{(-i)^{-k_1+k_2}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}\partial_z^{s_1-k_1+k_2}\frac{1}{(x-y)^2}
\end{aligned}\tag{2.59}$$

2.4.4 Euclidean extended basis

Analogously:

$$\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{A}^E] &= -\,(N^2-1)\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{A}^E\right)\\
\Gamma_{\text{conf}}[\tilde{\mathbb{A}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\right)\\
\Gamma_{\text{conf}}[\mathbb{A}^E,\tilde{\mathbb{A}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{A}^E+\mathcal{D}^{-1}\tilde{\mathbb{A}}^E\right)\\
&\quad-\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{A}^E-\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\right)\\
&= -\,\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{A}^E\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{A}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\right)\\
\Gamma_{\text{conf}}[\mathbb{B}^E,\bar{\mathbb{B}}^E] &= -\,\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}_E^{-1}\bar{\mathbb{B}}^E\mathcal{D}_E^{-1}\mathbb{B}^E\right)
\end{aligned}\tag{2.60}$$

in Euclidean space-time, with:

$$\begin{aligned}
\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(x-y) &= \frac{(-i)^{-k_1+k_2}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}\partial_z^{s_1-k_1+k_2}\,\triangle^{-1}(x-y)\\
&= -\frac{(-i)^{-k_1+k_2}}{8\pi^2}\binom{s_1}{k_1}\binom{s_2}{k_2}\partial_z^{s_1-k_1+k_2}\frac{1}{(x-y)^2}
\end{aligned}\tag{2.61}$$

2.5 Generating functional and n-point correlators in the momentum representation

In the momentum representation the generating functionals are exactly the same, but for the fact that the corresponding source fields and effective propagators are Fourier transformed:

$$\mathcal{O}_s(p) = \int \mathcal{O}_s(x)\, e^{-ip\cdot x} d^4x \tag{2.62}$$

In the momentum representation the generating functional in eq. (2.46) is (section 10):

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \log \text{Det}\left(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(q_1-q_2)\right) \tag{2.63}$$

The argument of the determinant is the kernel:

$$K_{s_1k_1,s_2k_2}(q_1,q_2) = \delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(q_1-q_2) \tag{2.64}$$

of the integral operator:

$$\psi_{s_1k_1}(q_1) = \sum_{s_2k_2}\int K_{s_1k_1,s_2k_2}(q_1,q_2)\phi_{s_2k_2}(q_2)\frac{d^4q_2}{(2\pi)^4} \tag{2.65}$$

Accordingly, the n-point correlators in the momentum representation are computed by the functional derivatives:

$$\begin{aligned}\langle \mathcal{O}_{s_1}(p_1)\ldots\mathcal{O}_{s_n}(p_n)\rangle_{\text{conn}} &= \frac{\delta}{\delta\mathcal{O}_{s_1}(p_1)}\cdots\frac{\delta}{\delta\mathcal{O}_{s_n}(p_n)}\Gamma_{\text{conf}}[\mathcal{O}]\\ &= \sum_{k_1=0}^{l_1}\cdots\sum_{k_n=0}^{l_n}\frac{\delta}{\delta\mathcal{O}_{s_1k_1}(p_1)}\cdots\frac{\delta}{\delta\mathcal{O}_{s_nk_n}(p_n)}\Gamma_{\text{conf}}[\mathcal{O}]\end{aligned} \tag{2.66}$$

2.5.1 Minkowskian standard basis

In the momentum representation, we get in Minkowskian space-time (section 10):

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] = &-\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\ &+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{O}}_{s_2k_2}(q_1-q_2))\Big)\\ &-\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\ &+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{O}}_{s_2k_2}(q_1-q_2))\Big)\\ \Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] = &-\frac{N^2-1}{2}\log\text{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\ &-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(q_1)\bar{\mathbb{S}}_{sk}(q_1-q)\mathcal{D}^{-1}_{sk,s_2k_2}(q)\mathbb{S}_{s_2k_2}(q-q_2)\Bigg)\end{aligned} \tag{2.67}$$

with:

$$\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(p) = \frac{i^{s_1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-ip_+)^{s_1-k_1+k_2}\frac{-i}{|p|^2+i\epsilon} \tag{2.68}$$

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Explicitly, we obtain by direct computation the nonvanishing correlators in the balanced sector from their generating functional in the momentum representation:

$$
\begin{aligned}
&\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s_{n+2m}+1)}{\Gamma(3)\Gamma(s_{n+2m}+3)}\right)\\
&\quad\langle\mathbb{O}_{s_1}(p_1)\dots\mathbb{O}_{s_n}(p_n)\tilde{\mathbb{O}}_{s_{n+1}}(p_{n+1})\dots\tilde{\mathbb{O}}_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{n+2m}\delta^{(4)}(p_1+\dots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+q|^2}\\
&\quad\dots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_+^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{|\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q|^2}\frac{(q)_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}
\tag{2.69}
$$

In the unbalanced sector, we get:

$$
\begin{aligned}
&\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s_n+1)}{\Gamma(3)\Gamma(s_n+3)}\right)\left(\frac{8\pi}{N}\frac{\Gamma(5)\Gamma(s'_1+1)}{\Gamma(3)\Gamma(s'_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s'_n+1)}{\Gamma(3)\Gamma(s'_n+3)}\right)\\
&\quad\langle\mathbb{S}_{s_1}(p_1)\dots\mathbb{S}_{s_n}(p_n)\bar{\mathbb{S}}_{s'_1}(p'_1)\dots\bar{\mathbb{S}}_{s'_n}(p'_n)\rangle\\
&=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{2n}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)\\
&\quad\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\
&\quad\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}\\
&\quad\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+p'_{\rho(1)}+q|^2}\\
&\quad\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_+^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q|^2}\\
&\quad\dots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_+^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q|^2}\\
&\quad\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\\
&\quad\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{|\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}
\tag{2.70}
$$

2.5.2 Minkowskian extended basis

Similarly:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{A},\tilde{\mathbb{A}}] = &-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{A}}_{s_2k_2}(q_1-q_2)\Big)\\
&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{A}}_{s_2k_2}(q_1-q_2)\Big)\\
\Gamma_{\text{conf}}[\mathbb{B},\bar{\mathbb{B}}] = &-\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(q_1)\bar{\mathbb{B}}_{sk}(q_1-q)\mathcal{D}^{-1}_{sk,s_2k_2}(q)\mathbb{B}_{s_2k_2}(q-q_2)\Bigg)
\end{aligned}
\tag{2.71}
$$

with:

$$
\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(p)=\frac{i^{s_1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-ip_+)^{s_1-k_1+k_2}\frac{-i}{|p|^2+i\epsilon}
\tag{2.72}
$$

Explicitly, for the nonvanishing correlators of the balanced operators in the momentum representation, we obtain:

$$
\begin{aligned}
&\langle\mathbb{A}_{s_1}(p_1)\ldots\mathbb{A}_{s_n}(p_n)\tilde{\mathbb{A}}_{s_{n+1}}(p_{n+1})\ldots\tilde{\mathbb{A}}_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{n+2m}\delta^{(4)}(p_1+\ldots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1}\ldots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}\binom{s_1}{k_1}\ldots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+q|^2}\\
&\quad\ldots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_+^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{|\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q|^2}\frac{(q)_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}
\tag{2.73}
$$

In the unbalanced sector, we get:

$$\begin{aligned}
&\langle \mathbb{B}_{s_1}(p_1)\ldots\mathbb{B}_{s_n}(p_n)\bar{\mathbb{B}}_{s'_1}(p'_1)\ldots\bar{\mathbb{B}}_{s'_n}(p'_n)\rangle\\
&=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{2n}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)\\
&\sum_{k_1=0}^{s_1}\cdots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1}\cdots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}\binom{s_1}{k_1}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n}\\
&\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+p'_{\rho(1)}+q|^2}\\
&\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_+^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q|^2}\\
&\cdots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_+^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q|^2}\\
&\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\\
&\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{|\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}\tag{2.74}$$

2.5.3 Euclidean standard basis

The Euclidean generating functionals read in the momentum representation:

$$\begin{aligned}
\Gamma^E_{\text{conf}}[\mathbb{O}^E,\tilde{\mathbb{O}}^E]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{E\ \,s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{O}}^E_{s_2k_2}(q_1-q_2)\Big)\\
&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{E\ \,s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{O}}^E_{s_2k_2}(q_1-q_2)\Big)\\
\Gamma^E_{\text{conf}}[\mathbb{S}^E,\bar{\mathbb{S}}^E]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{E\ \,s_1k_1,sk}(q_1)\bar{\mathbb{S}}^E_{sk}(q_1-q)\mathcal{D}^{-1}_{E\ \,sk,s_2k_2}(q)\mathbb{S}^E_{s_2k_2}(q-q_2)\Bigg)
\end{aligned}\tag{2.75}$$

with:

$$\mathcal{D}^{-1}_{E\ \,s_1k_1,s_2k_2}(p)=\frac{i^{s_1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}p_z^{s_1-k_1+k_2}\frac{1}{p^2}\tag{2.76}$$

Explicitly, for the nonvanishing correlators of the balanced operators in the momentum representation, we obtain:

$$
\begin{aligned}
&\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\cdots\left(\frac{\Gamma(5)\Gamma(s_{n+2m}+1)}{\Gamma(3)\Gamma(s_{n+2m}+3)}\right)\\
&\quad\langle \mathbb{O}^E_{s_1}(p_1)\dots\mathbb{O}^E_{s_n}(p_n)\tilde{\mathbb{O}}^E_{s_{n+1}}(p_{n+1})\dots\tilde{\mathbb{O}}^E_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{\sum_{l=1}^{n+2m}s_l}\delta^{(4)}(p_1+\dots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+q)^2}\\
&\quad\dots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_z^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)^2}\frac{(q)_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}
\tag{2.77}
$$

In the unbalanced sector, we get:

$$
\begin{aligned}
&\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\cdots\left(\frac{\Gamma(5)\Gamma(s_n+1)}{\Gamma(3)\Gamma(s_n+3)}\right)\left(\frac{\Gamma(5)\Gamma(s'_1+1)}{\Gamma(3)\Gamma(s'_1+3)}\right)\cdots\left(\frac{\Gamma(5)\Gamma(s'_n+1)}{\Gamma(3)\Gamma(s'_n+3)}\right)\\
&\quad\langle \mathbb{S}^E_{s_1}(p_1)\dots\mathbb{S}^E_{s_n}(p_n)\bar{\mathbb{S}}^E_{s'_1}(p'_1)\dots\bar{\mathbb{S}}^E_{s'_n}(p'_n)\rangle\\
&=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{\sum_{l=1}^{n}s_l+s'_l}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)\\
&\quad\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\sum_{k'_1=0}^{s'_1-2}\cdots\sum_{k'_n=0}^{s'_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\
&\quad\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}\\
&\quad\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+p'_{\rho(1)}+q)^2}\\
&\quad\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_z^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)^2}\\
&\quad\dots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_z^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)^2}\\
&\quad\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\\
&\quad\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}
\tag{2.78}
$$

2.5.4 Euclidean extended basis

Analogously:

$$\begin{aligned}
\Gamma^E_{\text{conf}}[\mathbb{A}^E,\tilde{\mathbb{A}}^E] = &-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}^E_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{A}}^E_{s_2k_2}(q_1-q_2))\Big)\\
&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{A}}^E_{s_2k_2}(q_1-q_2))\Big)\\
\Gamma^E_{\text{conf}}[\mathbb{B}^E,\bar{\mathbb{B}}^E] = &-\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}_E^{-1}{}_{s_1k_1,sk}(q_1)\bar{\mathbb{B}}^E_{sk}(q_1-q)\mathcal{D}_E^{-1}{}_{sk,s_2k_2}(q)\mathbb{B}^E_{s_2k_2}(q-q_2)\Bigg)
\end{aligned} \tag{2.79}$$

with:

$$\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(p) = \frac{i^{s_1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}p_z^{s_1-k_1+k_2}\frac{1}{p^2} \tag{2.80}$$

Explicitly, for the nonvanishing correlators in the balanced sector, we obtain:

$$\begin{aligned}
&\langle\mathbb{A}^E_{s_1}(p_1)\ldots\mathbb{A}^E_{s_n}(p_n)\tilde{\mathbb{A}}^E_{s_{n+1}}(p_{n+1})\ldots\tilde{\mathbb{A}}^E_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{\sum_{l=1}^{n+2m}s_l}\delta^{(4)}(p_1+\ldots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1}\ldots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}\binom{s_1}{k_1}\ldots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+q)^2}\\
&\quad\ldots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_z^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)^2}\frac{(q)_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned} \tag{2.81}$$

In the unbalanced sector, we get:

$$
\begin{aligned}
&\langle \mathbb{B}^E_{s_1}(p_1)\dots\mathbb{B}^E_{s_n}(p_n)\bar{\mathbb{B}}^E_{s'_1}(p'_1)\dots\bar{\mathbb{B}}^E_{s'_n}(p'_n)\rangle \\
&= \frac{N^2-1}{2^{2n}}(2\pi)^4 i^{\sum_{l=1}^n s_l+s'_l}\delta^{(4)}\left(\sum_{l=1}^n p_l+p'_l\right) \\
&\quad \sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1}\dots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}\binom{s_1}{k_1}\dots\binom{s_n}{k_n}\binom{s_n}{k_n}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n} \\
&\quad \frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+p'_{\rho(1)}+q)^2} \\
&\quad \frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_z^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)^2} \\
&\quad \dots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_z^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)^2} \\
&\quad \frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2} \\
&\quad \frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}
\tag{2.82}
$$

3 Plan of the paper

In section 1 we outline our main results and physics motivations.

In section 2, after recalling some basic concepts about the employment of conformal symmetry in massless QCD-like theories, we display our results for the correlators and their generating functionals to the lowest perturbative order both in Minkowskian and Euclidean space-time.

In section 4 we review the classification and construction of the gluonic operators with collinear twist 2 in Minkowskian space-time both in the standard and extended basis.

In section 5 we compute the 2-point correlators in Minkowskian space-time both in the standard and extended basis.

In section 6 we compute the 3-point correlators in Minkowskian space-time both in the standard and extended basis.

In section 7 we compute the n-point correlators in Minkowskian space-time both in the standard and extended basis in the balanced and unbalanced sectors separately.

In section 8 we compute the n-point correlators in Euclidean space-time either by analytic continuation or by employing the corresponding Euclidean operators.

In section 9 we work out an ansatz for the generating functionals of the n-point correlators in the coordinate representation as the logarithm of a functional determinant, and we

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verify by direct computation that it actually reproduces the correlators in the coordinate representation.

In section 10 we work out the corresponding generating functionals in the momentum representation, and we employ them to actually compute the n-point correlators in the momentum representation.

In the appendices we fix the notation and provide ancillary computations.

In particular, in appendix G we verify that our results for the 2- and 3-point correlators of balanced operators with even collinear spin in Minkowskian space-time coincide with [1].

4 Twist-2 gluonic operators in Minkowskian space-time

We review the construction of the standard and extended conformal bases for gauge-invariant gluonic operators with $\tau = 2$. We also work out the dictionary between the spinorial, vectorial and complex bases (appendices D and E).

4.1 Standard basis

To construct gauge-invariant gluonic operators with $\tau = 2$ that are primary (section 2) for the collinear conformal group, we should find — according to eq. (2.4) — suitable gauge-covariant elementary conformal operators.

The local gauge-covariant operator with lowest canonical dimension, $d = 2$, in YM theory is the field-strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu + ig\,[A_\mu, A_\nu]$, where $A_\mu = A^a_\mu T^a$ is a traceless Hermitian matrix, with T^a the generators of the SU(N) Lie algebra:

$$[T^a, T^b] = if^{abc}T^c \tag{4.1}$$

normalized in the standard way:

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab} \tag{4.2}$$

It is convenient to write $F_{\mu\nu}$ in the spinorial representation [15] (appendix D):

$$F_{a\dot{a}b\dot{b}} = \sigma^\mu_{a\dot{a}}\sigma^\nu_{b\dot{b}}F_{\mu\nu} \tag{4.3}$$

It turns out [15] that:

$$F_{a\dot{a}b\dot{b}} = 2\left(f_{ab}\epsilon_{\dot{a}\dot{b}} - \epsilon_{ab}f_{\dot{a}\dot{b}}\right) \tag{4.4}$$

decomposes into the sum of two chiral representations, $f_{ab} \in (1,0)$ and $f_{\dot{a}\dot{b}} \in (0,1)$, of spin $S = 1$ (appendix D). In Minkowskian space-time $f_{\dot{a}\dot{b}} = \bar{f}_{ab}$. f_{11} and $f_{\dot{1}\dot{1}}$ have maximal collinear spin (appendix D), $s = 1$, along the p_+ direction. Therefore, they have $\tau = d - s = 1$ and $j = s + \frac{\tau}{2} = \frac{3}{2}$. Hence, they are well suited (section 2) to build 2-gluon twist-2 primary conformal operators [10, 11]. Taking the tensor product of the above representations, we get:

$$\begin{aligned}(1,0)\otimes(0,1) &= (1,1)_+ \oplus (1,1)_- \\ (1,0)\otimes(1,0) &= (2,0)\oplus\dots \\ (0,1)\otimes(0,1) &= (0,2)\oplus\dots\end{aligned} \tag{4.5}$$

where + and − label the parity, and the dots denote terms that do not contribute to the components with maximal collinear spin. Hence, there are four operators with maximal s that can be constructed by means of the corresponding bilinear operators:

$$
\begin{aligned}
O(x_1,x_2) &= f_{11}(x_1)f_{\dot{1}\dot{1}}(x_2) + f_{11}(x_2)f_{\dot{1}\dot{1}}(x_1) \\
O(x_1,x_2) &= f_{11}(x_1)f_{\dot{1}\dot{1}}(x_2) - f_{11}(x_2)f_{\dot{1}\dot{1}}(x_1) \\
S(x_1,x_2) &= \frac{1}{\sqrt{2}} f_{11}(x_1)f_{11}(x_2) \\
\bar{S}(x_1,x_2) &= \frac{1}{\sqrt{2}} f_{\dot{1}\dot{1}}(x_2)f_{\dot{1}\dot{1}}(x_1)
\end{aligned}
\tag{4.6}
$$

Following [7, 10, 11], we build (section 2) conformal operators with $\tau = 2$ and higher collinear spin inserting in eq. (4.6) the Gegenbauer polynomials (appendix F), $C_l^{\alpha}(v)$, in the derivatives and afterwards taking the local limit $x_1 = x_2$. l is the order of the polynomial, and its relation to the collinear conformal spin is $j = l + j_1 + j_2$, where j_1 and j_2 are the collinear conformal spins of the elementary operators, and $\alpha = 2j_1 - \frac{1}{2}$. The Gegenbauer polynomials are either symmetric or antisymmetric for the substitution $v \to -v$ for l even or odd respectively (appendix F). The corresponding primary conformal operators match precisely the ones in [2, 11, 13] up to perhaps the overall normalization:

$$
\begin{aligned}
\mathbb{O}_s &= \mathrm{Tr}\, f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) f_{\dot{1}\dot{1}}(x) \qquad & s = 2,4,6,\ldots \\
\tilde{\mathbb{O}}_s &= \mathrm{Tr}\, f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) f_{\dot{1}\dot{1}}(x) \qquad & s = 3,5,7,\ldots \\
\mathbb{S}_s &= \frac{1}{\sqrt{2}}\mathrm{Tr}\, f_{11}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) f_{11}(x) \qquad & s = 2,4,6,\ldots \\
\bar{\mathbb{S}}_s &= \frac{1}{\sqrt{2}}\mathrm{Tr}\, f_{\dot{1}\dot{1}}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C_{s-2}^{\frac{5}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) f_{\dot{1}\dot{1}}(x) \qquad & s = 2,4,6,\ldots
\end{aligned}
\tag{4.7}
$$

with $j = s+1$, $l = s-2$ and $\alpha = \frac{5}{2}$. For brevity, we define as in [1]:

$$
\mathcal{G}_l^{\alpha}\left(D_{x_1^+}, D_{x_2^+}\right) = i^l \left(\overrightarrow{D}_{x_2^+} + \overleftarrow{D}_{x_1^+}\right)^l C_l^{\alpha}\left(\frac{\overrightarrow{D}_{x_2^+} - \overleftarrow{D}_{x_1^+}}{\overrightarrow{D}_{x_2^+} + \overleftarrow{D}_{x_1^+}}\right) \tag{4.8}
$$

For $l = s-2$ and $\alpha = \frac{5}{2}$, we get in the light-cone gauge by means eq. (F.9):

$$
\begin{aligned}
\mathcal{G}_{s-2}^{\frac{5}{2}}\left(\partial_{x_1^+}, \partial_{x_2^+}\right) &= i^{s-2}\frac{\Gamma(s+3)\Gamma(3)}{\Gamma(5)\Gamma(s+1)}\sum_{k=0}^{s-2}\binom{s}{k}\binom{s}{k+2}(-1)^{s-k}\overleftarrow{\partial}_{x_1^+}^{s-k-2}\overrightarrow{\partial}_{x_2^+}^{k} \\
&= \frac{2i^{s-2}(s+1)(s+2)}{4!}\sum_{k=0}^{s-2}\binom{s}{k}\binom{s}{k+2}(-1)^{s-k}\overleftarrow{\partial}_{x_1^+}^{s-k-2}\overrightarrow{\partial}_{x_2^+}^{k}
\end{aligned}
\tag{4.9}
$$

4.2 Extended basis

There exists another choice of the basis for primary conformal operators with $\tau = 2$ involving the elementary operators $D_+^{-1}f_{11}$ and $D_+^{-1}f_{\dot{1}\dot{1}}$, which are nonlocal in general, but local in the light-cone gauge. Yet, gauge invariance ensures that the corresponding gauge-invariant correlators are local. Indeed, in the light-cone gauge (appendix E):

$$\begin{aligned}\partial_+^{-1} f_{11} &= -\bar{A}\\ \partial_+^{-1} f_{\dot{1}\dot{1}} &= -A\end{aligned} \tag{4.10}$$

where A has $d=1$, $s=0$, $j=\frac{1}{2}$ and $\tau=1$. The corresponding operators with $\tau=2$ read:

$$\begin{aligned}
\mathbb{A}_s &= \mathrm{Tr}\, D_+^{-1} f_{11}(x)\, (i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s=0,2,4,\dots\\
\tilde{\mathbb{A}}_s &= \mathrm{Tr}\, D_+^{-1} f_{11}(x)\, (i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s=1,3,5,\dots\\
\mathbb{B}_s &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\, D_+^{-1} f_{11}(x)\, (i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{11}(x) \qquad s=0,2,4,\dots\\
\bar{\mathbb{B}}_s &= \frac{1}{\sqrt{2}}\,\mathrm{Tr}\, D_+^{-1} f_{\dot{1}\dot{1}}(x)\, (i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} f_{\dot{1}\dot{1}}(x) \qquad s=0,2,4,\dots
\end{aligned} \tag{4.11}$$

with $j=s+1$, $l=s$ and $\alpha=\frac{1}{2}$. This basis naturally arises in SUSY calculations [14], and also includes (nonlocal) operators with $s=0,1$. We obtain in the light-cone gauge by means of eq. (F.10):

$$\mathcal{G}_s^{\frac{1}{2}}(\partial_{x_1^+}, \partial_{x_2^+}) = i^s \sum_{k=0}^{s} \binom{s}{k}\binom{s}{k} (-1)^{s-k}\, \overleftarrow{\partial}^{s-k}_{x_1^+} \overrightarrow{\partial}^{k}_{x_2^+} \tag{4.12}$$

5 2-point correlators of twist-2 gluonic operators

We compute to the lowest perturbative order the 2-point correlators of the operators in both bases.

5.1 Standard basis

In the light-cone gauge, the 2-point correlators of the balanced operators with even s are given by:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+}, \partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+}, \partial_{y_2^+})&\\ \langle \mathrm{Tr}\, f_{11}(x_1) f_{\dot{1}\dot{1}}(x_2)\ \mathrm{Tr}\, f_{11}(y_1) f_{\dot{1}\dot{1}}(y_2)\rangle\Big|^{y_1=y_2=y}_{x_1=x_2=x}&\end{aligned} \tag{5.1}$$

There is only one Wick contraction:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+}, \partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+}, \partial_{y_2^+})&\\ \langle \mathrm{Tr}\, f_{11}(x_1) f_{\dot{1}\dot{1}}(y_2)\rangle\langle \mathrm{Tr}\, f_{11}(y_1) f_{\dot{1}\dot{1}}(x_2)\rangle\Big|^{y_1=y_2=y}_{x_1=x_2=x}&\end{aligned} \tag{5.2}$$

By means of eq. (B.4), we get:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+}) \\ \partial^+_{x_1}\partial^+_{x_2}\partial^+_{y_1}\partial^+_{y_2}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-x_2|^2}\Big|^{y_1=y_2=y}_{x_1=x_2=x} \tag{5.3}$$

Now we substitute eq. (4.9) into the above equation:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^2 i^{s_1+s_2-4}}{(4!)^2}(s_1+1)(s_1+2)(s_2+1)(s_2+2) \\ \sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2} \\ (-\partial_{x_1^+})^{s_1-k_1-1}\partial^{k_1+1}_{x_2^+}(-\partial_{y_1^+})^{s_2-k_2-1}\partial^{k_2+1}_{y_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-x_2|^2}\Big|^{y_1=y_2=y}_{x_1=x_2=x} \tag{5.4}$$

We compute the derivatives:

$$\begin{aligned}\partial^i_{x^+}\partial^j_{y^+}\frac{1}{|x-y|^2} &= \partial^i_{x^+}\partial^j_{y^+}\frac{1}{2(x-y)_+(x-y)_- - (x-y)^2_\perp} \\ &= (-1)^j\partial^{i+j}_{x^+}\frac{1}{2(x-y)_+(x-y)_- - (x-y)^2_\perp} \\ &= (-1)^i(i+j)!\,2^{i+j}\frac{(x-y)^{i+j}_+}{(|x-y|^2)^{i+j+1}}\end{aligned} \tag{5.5}$$

by induction on the index i:

$$\begin{aligned}\partial^{i+1}_{x^+}\partial^j_{y^+}\frac{1}{|x-y|^2} &= (-1)^i(i+j)!\,2^{i+j}\partial_{x^+}\frac{(x-y)^{i+j}_+}{(|x-y|^2)^{i+j+1}} \\ &= (-1)^i(i+j)!\,2^{i+j}\partial_{x_-}\frac{(x-y)^{i+j}_+}{(2(x-y)_+(x-y)_- - (x-y)^2_\perp)^{i+j+1}} \\ &= (-1)^i(i+j)!\,2^{i+j}(x-y)^{i+j}_+\frac{-2(i+j+1)(x-y)_+}{(2(x-y)_+(x-y)_- - (x-y)^2_\perp)^{i+j+2}} \\ &= (-1)^{i+1}(i+j+1)!\,2^{i+j+1}\frac{(x-y)^{i+j+1}_+}{(|x-y|^2)^{i+j+2}}\end{aligned} \tag{5.6}$$

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We obtain:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^2 i^{s_1+s_2-4}}{(4!)^2}(s_1+1)(s_1+2)(s_2+1)(s_2+2)$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}$$
$$(-1)^{s_1-k_1-1}(-1)^{s_1-k_1-1}2^{s_1-k_1+k_2}(s_1-k_1+k_2)!\frac{(x_1-y_2)_+^{s_1-k_1+k_2}}{(|x_1-y_2|^2)^{s_1-k_1+k_2+1}}$$
$$(-1)^{s_2-k_2-1}(-1)^{s_2-k_2-1}2^{s_2+k_1-k_2}(s_2+k_1-k_2)!\frac{(y_1-x_2)_+^{s_2+k_1-k_2}}{(|y_1-x_2|^2)^{s_2+k_1-k_2+1}}\Big|_{x_1=x_2=x}^{y_1=y_2=y} \tag{5.7}$$

that becomes:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{s_1+s_2+2} i^{s_1+s_2-4}}{(4!)^2}(s_1+1)(s_1+2)(s_2+1)(s_2+2)$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}$$
$$(s_1-k_1+k_2)!(s_2+k_1-k_2)!\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1-k_1+k_2+1}}\frac{(y-x)_+^{s_2+k_1-k_2}}{(|y-x|^2)^{s_2+k_1-k_2+1}} \tag{5.8}$$

and simplifies as follows:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle &= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{s_1+s_2+2} i^{s_1+s_2-4}}{(4!)^2}\\ &\quad (s_1+1)(s_1+2)(s_2+1)(s_2+2)\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}}\\ &\quad \sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\\ &\quad (-1)^{s_2-k_2+k_1}(s_1-k_1+k_2)!(s_2+k_1-k_2)!\\ &= \mathcal{C}_{s_1}(x,y)\delta_{s_1 s_2}\end{aligned} \tag{5.9}$$

since the correlator is zero for $s_1 \neq s_2$ (appendix H). By setting $s = s_1 = s_2$, we get:

$$\mathcal{C}_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2} i^{2s-4}}{(4!)^2}(s+1)^2(s+2)^2\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}}$$
$$\sum_{k_1=0}^{s-2}\sum_{k_2=0}^{s-2}\binom{s}{k_1}\binom{s}{k_1+2}\binom{s}{k_2}\binom{s}{k_2+2}(-1)^{s-k_2+k_1}$$
$$(s-k_1+k_2)!(s+k_1-k_2)! \tag{5.10}$$

Moreover, by the substitution in eq. (5.9):

$$k_2' = s_2 - 2 - k_2 \tag{5.11}$$

we obtain:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{s_1+s_2+2}i^{s_1+s_2-4}}{(4!)^2}$$
$$(s_1+1)(s_1+2)(s_2+1)(s_2+2)\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}}$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2'=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{s_2-2-k_2'}\binom{s_2}{s_2-k_2'}(-1)^{s_2-s_2+2+k_2'+k_1}$$
$$(s_1-k_1+s_2-2-k_2')!(s_2+k_1-s_2+2+k_2')! \tag{5.12}$$

that becomes:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{s_1+s_2+2}i^{s_1+s_2-4}}{(4!)^2}$$
$$(s_1+1)(s_1+2)(s_2+1)(s_2+2)\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}}$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2+2}\binom{s_2}{k_2}(-1)^{k_2+k_1}$$
$$(s_1+s_2-k_1-k_2-2)!(k_1+k_2+2)!$$
$$= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{s_1+s_2+2}i^{s_1+s_2-4}}{(4!)^2}$$
$$(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_1+s_2)!\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}}$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2+2}\binom{s_2}{k_2}(-1)^{k_2+k_1}\frac{1}{\binom{s_1+s_2}{k_1+k_2+2}} \tag{5.13}$$

Besides, according to the trick in [1] (appendix H), we get:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \delta_{s_1s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s_1+2}}{(4!)^2}(-1)^{s_1}$$
$$(s_1-1)s_1(s_1+1)(s_1+2)(2s_1)!\frac{(x-y)_+^{2s_1}}{(|x-y|^2)^{2s_1+2}} \tag{5.14}$$

Hence, comparing eqs. (5.13) and (5.14), we can virtually perform the sums in eq. (5.13) to obtain the identity:

$$\delta_{s_1s_2}\frac{s_1(s_1-1)}{(s_1+1)(s_1+2)} = \sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}(-1)^{k_2+k_1}\frac{1}{\binom{s_1+s_2}{k_1+k_2+2}} \tag{5.15}$$

Similarly, for the balanced operators with odd s, we get as well:

$$\langle \tilde{\mathbb{O}}_{s_1}(x)\tilde{\mathbb{O}}_{s_2}(y)\rangle = \mathcal{C}_{s_1}(x,y)\delta_{s_1s_2} \tag{5.16}$$

where the definition of $\mathcal{C}_s(x,y)$ in eq. (5.10) has been extended to odd s. Correspondingly, eq. (5.15) extends to odd s_1, s_2 as well.

Now we compute the only correlators of two unbalanced operators that are nonzero:

$$\langle \mathbb{S}_{s_1}(x)\bar{\mathbb{S}}_{s_2}(y)\rangle = \frac{1}{2}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+}) \langle \mathrm{Tr}\, f_{11}(x_1)f_{11}(x_2)\ \mathrm{Tr}\, f_{\dot{1}\dot{1}}(y_1)f_{\dot{1}\dot{1}}(y_2)\rangle\Big|^{y_1=y_2=y}_{x_1=x_2=x} \tag{5.17}$$

There are two Wick contractions but an extra factor of $\frac{1}{2}$ in the normalization of the operators, in such a way that the result is the same as for the correlators of balanced operators with even s:

$$\begin{aligned}\langle \mathbb{S}_{s_1}(x)\bar{\mathbb{S}}_{s_2}(y)\rangle &= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+}) \\ &\quad \partial^+_{x_1}\partial^+_{x_2}\partial^+_{y_1}\partial^+_{y_2}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-x_2|^2}\Big|^{y_1=y_2=y}_{x_1=x_2=x} \\ &= \mathcal{C}_{s_1}(x,y)\delta_{s_1s_2}\end{aligned} \tag{5.18}$$

All the remaining 2-point correlators vanish.

5.2 Extended basis

Similarly, employing eq. (4.12), we obtain the correlators in the extended basis:

$$\begin{aligned}\langle \mathbb{A}_s(x)\mathbb{A}_s(y)\rangle =& \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}2^{2s}i^{2s}\frac{(x-y)^{2s}_+}{(|x-y|^2)^{2s+2}} \\ & \sum_{k_1=0}^{s}\sum_{k_2=0}^{s}\binom{s}{k_1}\binom{s}{k_1}\binom{s}{k_2}\binom{s}{k_2}(-1)^{s-k_2+k_1}(s-k_1+k_2)!(s+k_1-k_2)! \\ =& \mathcal{A}_s(x,y)\end{aligned} \tag{5.19}$$

We extend the above definition of $\mathcal{A}_s(x,y)$ to odd s. We get for even s:

$$\langle \mathbb{B}_{s_1}(x)\bar{\mathbb{B}}_{s_2}(y)\rangle = \mathcal{A}_{s_1}(x,y)\delta_{s_1s_2} \tag{5.20}$$

and for odd s:

$$\langle \tilde{\mathbb{A}}_{s_1}(x)\tilde{\mathbb{A}}_{s_2}(y)\rangle = \mathcal{A}_{s_1}(x,y)\delta_{s_1s_2} \tag{5.21}$$

We obtain (appendix H):

$$\mathcal{A}_s(x,y) = \frac{1}{(4\pi^2)^2}(-1)^s\frac{N^2-1}{4}2^{2s}(2s)!\frac{(x-y)^{2s}_+}{(|x-y|^2)^{2s+2}} \tag{5.22}$$

Similarly, it follows the identity:

$$\delta_{s_1s_2} = \sum_{k_1=0}^{s_1}\sum_{k_2=0}^{s_2}\binom{s_1}{k_1}\binom{s_1}{k_1}\binom{s_2}{k_2}\binom{s_2}{k_2}(-1)^{k_2+k_1}\frac{1}{\binom{s_1+s_2}{k_1+k_2}} \tag{5.23}$$

All the remaining 2-point correlators vanish.

6 3-point correlators of twist-2 gluonic operators

We compute to the lowest perturbative order the 3-point correlators of the operators in both bases.

6.1 Standard basis

In the light-cone gauge, for the 3-point correlators of the balanced operators with even s, we obtain:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = {}& \mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})\\ &\langle \mathrm{Tr}\, f_{11}(x_1)f_{\dot{1}\dot{1}}(x_2)\ \mathrm{Tr}\, f_{11}(y_1)f_{\dot{1}\dot{1}}(y_2)\ \mathrm{Tr}\, f_{11}(z_1)f_{\dot{1}\dot{1}}(z_2)\rangle\Big|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x}\end{aligned} \tag{6.1}$$

Since there are two Wick contractions, employing eq. (B.4), we get:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = {}& \frac{1}{(4\pi^2)^3}\frac{N^2-1}{8}\Bigg(\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})\\ &(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\\ &+\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})\\ &(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-z_2|^2}\frac{1}{|z_1-y_2|^2}\frac{1}{|y_1-x_2|^2}\Bigg)\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x}\end{aligned} \tag{6.2}$$

In the second term of eq. (6.2) we may conveniently relabel the coordinates, $x_1 \to x_2$, $y_1 \to y_2$, $z_1 \to z_2$, and vice versa, because they coincide in the local limit:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = {}& \frac{1}{(4\pi^2)^3}\frac{N^2-1}{8}\Bigg(\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})\\ &(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\\ &+\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_2^+},\partial_{x_1^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_2^+},\partial_{y_1^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_2^+},\partial_{z_1^+})\\ &(-\partial_{x_2^+})\partial_{x_1^+}(-\partial_{y_2^+})\partial_{y_1^+}(-\partial_{z_2^+})\partial_{z_1^+}\frac{1}{|x_2-z_1|^2}\frac{1}{|z_2-y_1|^2}\frac{1}{|y_2-x_1|^2}\Bigg)\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x}\end{aligned} \tag{6.3}$$

In the second term above, we employ the property of the Gegenbauer polynomials (appendix F):

$$\mathcal{G}^{\alpha}_{l}(\partial_{x_1^+},\partial_{x_2^+}) = (-1)^l\mathcal{G}^{\alpha}_{l}(\partial_{x_2^+},\partial_{x_1^+}) \tag{6.4}$$

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to get:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = \frac{1}{(4\pi^2)^3}\frac{N^2-1}{8}\Bigg(\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})$$
$$(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}$$
$$+(-1)^{s_1+s_2+s_3}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})$$
$$(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\Bigg)\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x} \tag{6.5}$$

Therefore:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = \frac{1}{(4\pi^2)^3}(1+(-1)^{s_1+s_2+s_3})\frac{N^2-1}{8}$$
$$\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})$$
$$(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x} \tag{6.6}$$

Since the collinear spins are all even, $1+(-1)^{s_1+s_2+s_3}=2$, so that:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = \frac{1}{(4\pi^2)^3}2\frac{N^2-1}{8}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})$$
$$(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x} \tag{6.7}$$

Eq. (6.6) also holds for the 3-point correlators of $\mathbb{O}_s$, $\tilde{\mathbb{O}}_s$, $\mathbb{S}_s$ and $\bar{\mathbb{S}}_s$ below, with the factor of $1+(-1)^{s_1+s_2+s_3}$ selecting which of these correlators yield a nonzero result. Therefore, by defining:

$$\mathcal{C}_{s_1s_2s_3}(x,y,z) = \frac{1}{(4\pi^2)^3}(1+(-1)^{s_1+s_2+s_3})\frac{N^2-1}{8}$$
$$\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\mathcal{G}^{\frac{5}{2}}_{s_3-2}(\partial_{z_1^+},\partial_{z_2^+})$$
$$(-\partial_{x_1^+})\partial_{x_2^+}(-\partial_{y_1^+})\partial_{y_2^+}(-\partial_{z_1^+})\partial_{z_2^+}\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\Bigg|^{y_1=y_2=y,z_1=z_2=z}_{x_1=x_2=x} \tag{6.8}$$

the nonvanishing correlators are:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = \langle \mathbb{O}_{s_1}(x)\mathbb{S}_{s_2}(y)\bar{\mathbb{S}}_{s_3}(z)\rangle = \mathcal{C}_{s_1s_2s_3}(x,y,z) \tag{6.9}$$

and:

$$\langle \mathbb{O}_{s_1}(x)\tilde{\mathbb{O}}_{s_2}(y)\tilde{\mathbb{O}}_{s_3}(z)\rangle = \mathcal{C}_{s_1s_2s_3}(x,y,z) \tag{6.10}$$

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After substituting the definition of the Gegenbauer polynomials in eq. (4.9), we obtain:

$$\mathcal{C}_{s_1s_2s_3}(x,y,z) = \frac{1}{(4\pi^2)^3}\left(1+(-1)^{s_1+s_2+s_3}\right)\left(\frac{2}{4!}\right)^3 \frac{N^2-1}{8} i^{s_1+s_2+s_3-6}$$
$$(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_3+1)(s_3+2)$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\binom{s_3}{k_3}\binom{s_3}{k_3+2}$$
$$(-\partial_{x_1^+})^{s_1-k_1-1}\partial_{x_2^+}^{k_1+1}(-\partial_{y_1^+})^{s_2-k_2-1}\partial_{y_2^+}^{k_2+1}(-\partial_{z_1^+})^{s_3-k_3-1}\partial_{z_2^+}^{k_3+1}$$
$$\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-z_2|^2}\frac{1}{|z_1-x_2|^2}\Bigg|_{x_1=x_2=x}^{y_1=y_2=y,z_1=z_2=z} \tag{6.11}$$

Employing eq. (5.5), we get:

$$\mathcal{C}_{s_1s_2s_3}(x,y,z) = -\frac{1}{(4\pi^2)^3}\left(1+(-1)^{s_1+s_2+s_3}\right)\left(\frac{2}{4!}\right)^3 \frac{N^2-1}{8} i^{s_1+s_2+s_3} 2^{s_1+s_2+s_3} \tag{6.12}$$
$$(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_3+1)(s_3+2)$$
$$\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\binom{s_3}{k_3}\binom{s_3}{k_3+2}$$
$$(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!$$
$$\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}}$$

All the remaining 3-point correlators vanish.

6.2 Extended basis

The 3-point correlators in the extended basis are computed analogously.

The nonvanishing correlators are:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\mathbb{A}_{s_3}(z)\rangle = \langle \mathbb{A}_{s_1}(x)\mathbb{B}_{s_2}(y)\bar{\mathbb{B}}_{s_3}(z)\rangle = \mathcal{A}_{s_1s_2s_3}(x,y,z) \tag{6.13}$$

and:

$$\langle \mathbb{A}_{s_1}(x)\tilde{\mathbb{A}}_{s_2}(y)\tilde{\mathbb{A}}_{s_3}(z)\rangle = \mathcal{A}_{s_1s_2s_3}(x,y,z) \tag{6.14}$$

with:

$$\mathcal{A}_{s_1s_2s_3}(x,y,z) = -\frac{1}{(4\pi^2)^3}\left(1+(-1)^{s_1+s_2+s_3}\right)\frac{N^2-1}{8} i^{s_1+s_2+s_3} 2^{s_1+s_2+s_3}$$
$$\sum_{k_1=0}^{s_1}\sum_{k_2=0}^{s_2}\sum_{k_3=0}^{s_3}\binom{s_1}{k_1}\binom{s_1}{k_1}\binom{s_2}{k_2}\binom{s_2}{k_2}\binom{s_3}{k_3}\binom{s_3}{k_3}$$
$$(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!$$
$$\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}} \tag{6.15}$$

All the remaining 3-point correlators vanish.

7 n-point correlators of twist-2 gluonic operators

We compute to the lowest perturbative order n-point correlators of the operators in both bases.

7.1 Standard basis

Given the bilocal operators:

$$O(x_i^A, x_i^B) = \mathrm{Tr}\, f_{11}(x_i^A) f_{\bar{1}\bar{1}}(x_i^B) = \frac{1}{2} f_{11}^a(x_i^A) f_{\bar{1}\bar{1}}^a(x_i^B) \tag{7.1}$$

in the light-cone gauge, the corresponding n-point correlator that is connected in the local limit, $x_i^A = x_i^B = x_i$, takes the form:

$$\begin{aligned}\langle O(x_1^A, x_1^B) \ldots O(x_n^A, x_n^B)\rangle = \frac{1}{n}\frac{1}{2^n} \sum_{i_1=1}^{n} \cdots \sum_{i_n=1}^{n} \Big|_{i_1 \neq i_2 \neq \ldots \neq i_n} \langle f_{11}^{a_{i_1}}(x_{i_1}^A) f_{\bar{1}\bar{1}}^{a_{i_2}}(x_{i_2}^B)\rangle \\ \langle f_{11}^{a_{i_2}}(x_{i_2}^A) f_{\bar{1}\bar{1}}^{a_{i_3}}(x_{i_3}^B)\rangle \ldots \langle f_{11}^{a_{i_n}}(x_{i_n}^A) f_{\bar{1}\bar{1}}^{a_{i_1}}(x_{i_1}^B)\rangle\end{aligned} \tag{7.2}$$

The factor of $\frac{1}{n}$ arises because, if the first index — for example $i_1 = 1$ — is kept fixed, there are only $(n-1)!$ Wick contractions that contribute to the connected correlator. A nicer — but completely equivalent — formula is written in terms of permutations. If we denote by P_n the set of permutations of $1 \ldots n$, it follows identically:

$$\begin{aligned}\langle O(x_1^A, x_1^B) \ldots O(x_n^A, x_n^B)\rangle = \frac{1}{n}\frac{1}{2^n} \sum_{\sigma \in P_n} \langle f_{11}^{a_{\sigma(1)}}(x_{\sigma(1)}^A) f_{\bar{1}\bar{1}}^{a_{\sigma(2)}}(x_{\sigma(2)}^B)\rangle \\ \langle f_{11}^{a_{\sigma(2)}}(x_{\sigma(2)}^A) f_{\bar{1}\bar{1}}^{a_{\sigma(3)}}(x_{\sigma(3)}^B)\rangle \ldots \langle f_{11}^{a_{\sigma(n)}}(x_{\sigma(n)}^A) f_{\bar{1}\bar{1}}^{a_{\sigma(1)}}(x_{\sigma(1)}^B)\rangle\end{aligned} \tag{7.3}$$

Besides, eq. (B.4) reads:

$$\langle f_{11}^a(x_i) f_{\bar{1}\bar{1}}^b(x_j)\rangle = -\partial_{x_i^+} \partial_{x_j^+} \frac{\delta^{ab}}{4\pi^2 |x_i - x_j|^2} \tag{7.4}$$

Hence, for the balanced operators with even s, we get in the light-cone gauge:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x_1) \ldots \mathbb{O}_{s_n}(x_n)\rangle = \frac{1}{2^n} \mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^A+}, \partial_{x_1^B+}) \ldots \mathcal{G}^{\frac{5}{2}}_{s_n-2}(\partial_{x_n^A+}, \partial_{x_n^B+}) \\ \langle f_{11}^{a_1}(x_1^A) f_{\bar{1}\bar{1}}^{a_1}(x_1^B) \ldots f_{11}^{a_n}(x_n^A) f_{\bar{1}\bar{1}}^{a_n}(x_n^B)\rangle \Big|_{A=B}\end{aligned} \tag{7.5}$$

where:

$$\begin{aligned}\mathcal{G}^{\frac{5}{2}}_{s-2}(\partial_{x_i^+}, \partial_{x_j^+}) &= \frac{i^{s-2}\Gamma(3)\Gamma(s+3)}{\Gamma(5)\Gamma(s+1)} \sum_{k=0}^{s-2} \binom{s}{k} \binom{s}{k+2} (-1)^{s-k} \overleftarrow{\partial}_{x_i^+}^{s-k-2} \overrightarrow{\partial}_{x_j^+}^{k} \\ &= \frac{2 i^{s-2}(s+1)(s+2)}{4!} \sum_{k=0}^{s-2} \binom{s}{k} \binom{s}{k+2} (-1)^{s-k} \overleftarrow{\partial}_{x_i^+}^{s-k-2} \overrightarrow{\partial}_{x_j^+}^{k}\end{aligned} \tag{7.6}$$

It follows from eq. (5.5) that, correspondingly, the n-point correlator contains factors of the form:

$$
\begin{aligned}
&-\partial^{s_i-k_i-2}_{x_i^+}\partial^{k_j}_{x_j^+}\partial_{x_i^+}\partial_{x_j^+}\frac{1}{4\pi^2|x_i-x_j|^2}\\
&=\frac{1}{4\pi^2}(-1)^{s_i-k_i}(s_i-k_i+k_j)!\,2^{s_i-k_i+k_j}\frac{(x_i-x_j)_+^{s_i-k_i+k_j}}{(|x_i-x_j|^2)^{s_i-k_i+k_j+1}}
\end{aligned}
\tag{7.7}
$$

Therefore, we get:

$$
\begin{aligned}
&\langle \mathbb{O}_{s_1}(x_1)\ldots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}}=\frac{(-1)^n}{(4\pi^2)^n}\frac{N^2-1}{2^n}i^{\sum_{l=1}^n s_l}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\cdots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\frac{1}{n}\sum_{\sigma\in P_n}\\
&\sum_{k_{\sigma(1)}=0}^{s_{\sigma(1)}-2}\cdots\sum_{k_{\sigma(n)}=0}^{s_{\sigma(n)}-2}\binom{s_{\sigma(1)}}{k_{\sigma(1)}}\binom{s_{\sigma(1)}}{k_{\sigma(1)}+2}(-1)^{s_{\sigma(1)}-k_{\sigma(1)}}\cdots\binom{s_{\sigma(n)}}{k_{\sigma(n)}}\binom{s_{\sigma(n)}}{k_{\sigma(n)}+2}(-1)^{s_{\sigma(n)}-k_{\sigma(n)}}\\
&2^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}(-1)^{s_{\sigma(1)}-k_{\sigma(1)}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\\
&\ldots 2^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}(-1)^{s_{\sigma(n)}-k_{\sigma(n)}}(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{7.8}
$$

where we have set $x_i^A = x_i^B = x_i$ in order to implement the local limit of the bilocal operators. The color factor comes from the contraction of the n Kronecker delta:

$$
N^2-1=\delta^{a_{\sigma(1)}a_{\sigma(2)}}\delta^{a_{\sigma(2)}a_{\sigma(3)}}\ldots\delta^{a_{\sigma(n)}a_{\sigma(1)}}
\tag{7.9}
$$

The overall factor of $(-1)^n$ occurs because of the factor of i^{-2}, which is a partial factor of i^{s-2} in eq. (7.6).

After cancelling between themselves the pairs of factors of the kind $(-1)^{s_a-k_a}$ in eq. (7.8), and moving out of the sum over the permutations the product of the binomial coefficients, since it is independent of the permutations, we obtain:

$$
\begin{aligned}
&\langle \mathbb{O}_{s_1}(x_1)\ldots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}}=\frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}i^{\sum_{l=1}^n s_l}\\
&\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\cdots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\
&\frac{(-1)^n}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!\\
&\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\cdots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{7.10}
$$

Actually, if n is even, eq. (7.10) also holds for the n-point correlator of the operators $\tilde{\mathbb{O}}_s$, with the only difference that their collinear spin is odd.

Otherwise, if n is odd, the correlator vanishes. To verify it, it suffices to notice that, in the sum over the permutations, for every permutation the inverse permutation also occurs with the opposite sign. For example, for 3-point correlators we get pairs of terms of the kind:

$$\begin{aligned}
&\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\dots(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\
&(x_1-x_2)^{s_1-k_1+k_2}(x_2-x_3)^{s_2-k_2+k_3}(x_3-x_1)^{s_3-k_3+k_1}\\
&+\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\dots(s_2-k_2+k_1)!(s_3-k_3+k_2)!(s_1-k_1+k_3)!\\
&(x_2-x_1)^{s_2-k_2+k_1}(x_3-x_2)^{s_3-k_3+k_2}(x_1-x_3)^{s_1-k_1+k_3}
\end{aligned}\tag{7.11}$$

Employing the substitution $k'_i = s_i - 2 - k_i$, we obtain for the last term above:

$$\begin{aligned}
&\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\dots(-1)^{s_1+s_2+s_3}(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\
&(x_1-x_2)^{s_1-k_1+k_2}(x_2-x_3)^{s_2-k_2+k_3}(x_3-x_1)^{s_3-k_3+k_1}
\end{aligned}\tag{7.12}$$

that cancels the first term in eq. (7.11).

The same reasoning applies to the $n+2m+1$-point correlators of balanced operators:

$$\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{n+2m+1}}(x_{n+2m+1})\rangle_{\text{conn}} = 0 \tag{7.13}$$

Otherwise, we get:

$$\begin{aligned}
&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}}\\
&=\frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}}2^{\sum_{l=1}^{n+2m}s_l}i^{\sum_{l=1}^{n+2m}s_l}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_{n+2m}+3)}{\Gamma(5)\Gamma(s_{n+2m}+1)}\\
&\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\\
&\frac{(-1)^{n+2m}}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})!\\
&\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n+2m)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}}
\end{aligned}\tag{7.14}$$

For the correlators of the unbalanced operators, we obtain:

$$\begin{aligned}
&\langle \mathbb{S}_{s_1}(x_1)\dots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\dots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle\\
&=\frac{1}{2^{2n}}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^A+},\partial_{x_1^B+})\dots\mathcal{G}^{\frac{5}{2}}_{s_n-2}(\partial_{x_n^A+},\partial_{x_n^B+})\mathcal{G}^{\frac{5}{2}}_{s'_1-2}(\partial_{y_1^A+},\partial_{y_1^B+})\dots\mathcal{G}^{\frac{5}{2}}_{s'_n-2}(\partial_{y_n^A+},\partial_{y_n^B+})\\
&\frac{1}{2^n}\langle f^{a_1}_{11}(x_1^A)f^{a_1}_{11}(x_1^B)\dots f^{a_n}_{11}(x_n^A)f^{a_n}_{11}(x_n^B)f^{b_1}_{\dot{1}\dot{1}}(y_1^A)f^{b_1}_{\dot{1}\dot{1}}(y_1^B)\dots f^{b_n}_{\dot{1}\dot{1}}(y_n^A)f^{b_n}_{\dot{1}\dot{1}}(y_n^B)\rangle\Big|_{A=B}
\end{aligned}\tag{7.15}$$

The factor of $\frac{1}{2^{2n}}$ arises from the normalization of the color trace in eq. (7.1), while the factor of $\frac{1}{2^n}$ comes from the normalization of the operators.

We get the very same correlator by exchanging A and B in all the couples (x_i^A, x_i^B) and (y_k^A, y_k^B) simultaneously:

$$\begin{aligned}
&\langle \mathbb{S}_{s_1}(x_1)\dots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\dots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle \\
&= \frac{1}{2^n}\mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^B+},\partial_{x_1^A+})\dots\mathcal{G}^{\frac{5}{2}}_{s_n-2}(\partial_{x_n^B+},\partial_{x_n^A+})\mathcal{G}^{\frac{5}{2}}_{s'_1-2}(\partial_{y_1^B+},\partial_{y_1^A+})\dots\mathcal{G}^{\frac{5}{2}}_{s'_n-2}(\partial_{y_n^B+},\partial_{y_n^A+}) \\
&\quad \frac{1}{2^{2n}}\langle f^{a_1}_{11}(x_1^B)f^{a_1}_{11}(x_1^A)\dots f^{a_n}_{11}(x_n^B)f^{a_n}_{11}(x_n^A)f^{b_1}_{\dot{1}\dot{1}}(y_1^B)f^{b_1}_{\dot{1}\dot{1}}(y_1^A)\dots f^{b_n}_{\dot{1}\dot{1}}(y_n^B)f^{b_n}_{\dot{1}\dot{1}}(y_n^A)\rangle\Big|_{A=B}
\end{aligned} \tag{7.16}$$

Indeed, in eq. (7.17) we may conveniently relabel the coordinates, $x_i^A \to x_i^B$, $y_k^A \to y_k^B$, and vice versa for each i and k, since they coincide in the local limit. Moreover, according to eq. (6.4), $\mathcal{G}^{\frac{5}{2}}_{s-2}(\partial_{x^A+},\partial_{x^B+})$ in eqs. (7.15) and (7.16) is symmetric for the exchange of its arguments, because the collinear spin is even.

We evaluate the Wick contractions:

$$\langle f^{a_1}_{11}(x_1^A)\dots f^{a_n}_{11}(x_n^A)f^{b_1}_{\dot{1}\dot{1}}(y_1^A)\dots f^{b_n}_{\dot{1}\dot{1}}(y_n^A)f^{a_1}_{11}(x_1^B)\dots f^{a_n}_{11}(x_n^B)f^{b_1}_{\dot{1}\dot{1}}(y_1^B)\dots f^{b_n}_{\dot{1}\dot{1}}(y_n^B)\rangle\Big|_{A=B} \tag{7.17}$$

We exploit the symmetry above: we only perform the Wick contractions involving the pairing of x_i^A with y_k^A and of x_i^B with y_k^B for any i, k, since all the remaining contractions provide the very same result due to the symmetry, and can be taken into account by a symmetry factor that we compute momentarily.

Besides, since we only are interested in the connected correlator, once x_i^A has been contracted with some y_k^A, x_i^B cannot be contracted with y_k^B, because the corresponding contribution to the correlator is not connected.

Hence, we construct the correlator as follows: we contract all the x_i^A with the y_k^A and all the $x_{i'}^B$ with the $y_{k'}^B$ with $i \neq i'$ if $k = k'$and $k \neq k'$if $i = i'$, in such a way that we build a single connected loop.

This is realized by summing over two sets of independent permutations arranged in such a way that no disconnected piece may be created: firstly, we contract $x_{i_1}^A$ with $y_{k_1}^A$, secondly, we contract $y_{k_1}^B$ with $x_{i_2}^B$ for $i_1 \neq i_2$, then, we contract $x_{i_2}^A$ with $y_{k_2}^A$ for $k_2 \neq k_1$, afterwards, we contract $y_{k_2}^B$ with $x_{i_3}^B$ for $i_3 \neq i_2 \neq i_1$ and so on, until we arrive at $x_{i_1}^B$, which we contract with the last remaining $y_{k_n}^A$ with $k_n \neq k_{n-1} \neq \dots \neq k_1$, in order to close the loop. We end up with a chain that looks like:

$$\begin{aligned}
&\sum_{i_1\neq i_2\neq i_3\dots\neq i_n}\ \sum_{k_1\neq k_2\neq k_3\dots\neq k_n} \langle f^{a_{i_1}}_{11}(x_{i_1}^A)f^{b_{k_1}}_{\dot{1}\dot{1}}(y_{k_1}^A)\rangle\langle f^{b_{k_1}}_{\dot{1}\dot{1}}(y_{k_1}^B)f^{a_{i_2}}_{11}(x_{i_2}^B)\rangle \\
&\langle f^{a_{i_2}}_{11}(x_{i_2}^A)f^{b_{k_2}}_{\dot{1}\dot{1}}(y_{k_2}^A)\rangle\langle f^{b_{k_2}}_{\dot{1}\dot{1}}(y_{k_2}^B)f^{a_{i_3}}_{11}(x_{i_3}^B)\rangle\dots\langle f^{a_{i_n}}_{11}(x_{i_n}^A)f^{b_{k_n}}_{\dot{1}\dot{1}}(y_{k_n}^A)\rangle\langle f^{b_{k_n}}_{\dot{1}\dot{1}}(y_{k_n}^B)f^{a_{i_1}}_{11}(x_{i_1}^B)\rangle
\end{aligned} \tag{7.18}$$

Yet, now we are creating a redundancy, since we also are summing on the possible n choices of the starting point of the loop. Therefore, we divide the sum by a factor of n:

$$\frac{1}{n}\sum_{i_1\neq i_2\neq i_3\ldots\neq i_n}\sum_{k_1\neq k_2\neq k_3\ldots\neq k_n}\langle f_{11}^{a_{i_1}}(x^A_{i_1})f_{\text{ii}}^{b_{k_1}}(y^A_{k_1})\rangle\langle f_{\text{ii}}^{b_{k_1}}(y^B_{k_1})f_{11}^{a_{i_2}}(x^B_{i_2})\rangle \tag{7.19}$$
$$\langle f_{11}^{a_{i_2}}(x^A_{i_2})f_{\text{ii}}^{b_{k_2}}(y^A_{k_2})\rangle\langle f_{\text{ii}}^{b_{k_2}}(y^B_{k_2})f_{11}^{a_{i_3}}(x^B_{i_3})\rangle\ldots\langle f_{11}^{a_{i_n}}(x^A_{i_n})f_{\text{ii}}^{b_{k_n}}(y^A_{k_n})\rangle\langle f_{\text{ii}}^{b_{k_n}}(y^B_{k_n})f_{11}^{a_{i_1}}(x^B_{i_1})\rangle$$

A nicer — but completely equivalent — formula is written in terms of permutations. It follows identically:

$$\frac{1}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\langle f_{11}^{a_{\sigma_1}}(x^A_{\sigma_1})f_{\text{ii}}^{b_{\rho_1}}(y^A_{\rho_1})\rangle\langle f_{\text{ii}}^{b_{\rho_1}}(y^B_{\rho_1})f_{11}^{a_{\sigma_2}}(x^B_{\sigma_2})\rangle\langle f_{11}^{a_{\sigma_2}}(x^A_{\sigma_2})f_{\text{ii}}^{b_{\rho_2}}(y^A_{\rho_2})\rangle\langle f_{\text{ii}}^{b_{\rho_2}}(y^B_{\rho_2})f_{11}^{a_{\sigma_3}}(x^B_{\sigma_3})\rangle$$
$$\ldots\langle f_{11}^{a_{\sigma_n}}(x^A_{\sigma_n})f_{\text{ii}}^{b_{\rho_n}}(y^A_{\rho_n})\rangle\langle f_{\text{ii}}^{b_{\rho_n}}(y^B_{\rho_n})f_{11}^{a_{\sigma_1}}(x^B_{\sigma_1})\rangle \tag{7.20}$$

All the remaining contractions are obtained from this formula by exchanging the coordinates in each couple, (x_i^A, x_i^B) and (y_k^A, y_k^B), for each i and k. There are 2^{2n} of such exchanges.

However, the actual degeneration factor is 2^{2n-1}. Indeed, the extra factor of $\frac{1}{2}$ comes from the fact that the simultaneous exchange of the coordinates in each couple, (x_i^A, x_i^B) and (y_k^A, y_k^B), yields a contraction that has already been counted due to the symmetry of eqs. (7.15) and (7.16) with respect of the simultaneous exchange of A with B in all the coordinate pairs.

Hence, by combining the degeneration factor of 2^{2n-1} with the factor of $\frac{1}{2^n}$ from the normalization of the operators, the overall factor of 2^{n-1} survives. It follows:

$$\langle\mathbb{S}_{s_1}(x_1)\ldots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\ldots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle=\frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^n s_l+s'_l}i^{\sum_{l=1}^n s_l+s'_l}$$
$$\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\cdots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\frac{\Gamma(3)\Gamma(s'_1+3)}{\Gamma(5)\Gamma(s'_1+1)}\cdots\frac{\Gamma(3)\Gamma(s'_n+3)}{\Gamma(5)\Gamma(s'_n+1)}$$
$$\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n+2}$$
$$\sum_{k'_1=0}^{s'_1-2}\cdots\sum_{k'_n=0}^{s'_n-2}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}$$
$$\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!$$
$$\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-y_{\rho(1)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left(|x_{\sigma(1)}-y_{\rho(1)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left(|y_{\rho(1)}-x_{\sigma(2)}|^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}$$
$$\ldots\frac{(x_{\sigma(n)}-y_{\rho(n)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left(|x_{\sigma(n)}-y_{\rho(n)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left(|y_{\rho(n)}-x_{\sigma(1)}|^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}} \tag{7.21}$$

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7.2 Extended basis

Similarly, in the extended basis, we get:

$$\langle \mathbb{A}_{s_1}(x_1)\ldots\mathbb{A}_{s_n}(x_n)\rangle = \frac{1}{2^n}\mathcal{G}^{\frac{1}{2}}_{s_1}(\partial_{x_1^{A+}},\partial_{x_1^{B+}})\ldots\mathcal{G}^{\frac{1}{2}}_{s_n}(\partial_{x_n^{A+}},\partial_{x_n^{B+}})$$
$$\langle \partial^{-1}_{x_1^{A+}} f^{a_1}_{11}(x_1^A)\partial^{-1}_{x_1^{B+}} f^{a_1}_{\bar{1}\bar{1}}(x_1^B)\ldots\partial^{-1}_{x_n^{A+}} f^{a_n}_{11}(x_n^A)\partial^{-1}_{x_n^{B+}} f^{a_n}_{\bar{1}\bar{1}}(x_n^B)\rangle\Big|_{A=B} \tag{7.22}$$

in the light-cone gauge, where:

$$\mathcal{G}^{\frac{1}{2}}_{s}(\partial_{x_1^+},\partial_{x_2^+}) = i^s\sum_{k=0}^{s}\binom{s}{k}\binom{s}{k}(-1)^{s-k}\overleftarrow{\partial}^{s-k}_{x_1^+}\overrightarrow{\partial}^{k}_{x_2^+} \tag{7.23}$$

It follows from eqs. (B.5) and (5.5) that, correspondingly, the n-point correlator contains factors of the form:

$$-\partial^{s_i-k_i}_{x_i^+}\partial^{k_j}_{x_j^+}\frac{1}{4\pi^2|x_i-x_j|^2}$$
$$= -\frac{1}{4\pi^2}(-1)^{s_i-k_i}(s_i-k_i+k_j)!\,2^{s_i-k_i+k_j}\frac{(x_i-x_j)_+^{s_i-k_i+k_j}}{(|x_i-x_j|^2)^{s_i-k_i+k_j+1}} \tag{7.24}$$

Therefore:

$$\langle \mathbb{A}_{s_1}(x_1)\ldots\mathbb{A}_{s_n}(x_n)\rangle_{\text{conn}} = \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}i^{\sum_{l=1}^n s_l}$$
$$\sum_{k_1=0}^{s_1}\ldots\sum_{k_n=0}^{s_n}\binom{s_1}{k_1}^2\ldots\binom{s_n}{k_n}^2\frac{(-1)^n}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\ldots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}} \tag{7.25}$$

where now the overall factor of $(-1)^n$ occurs because of the extra minus sign in eq. (7.24) with respect to eq. (7.7).

The very same formula holds for an even number of operators $\tilde{\mathbb{A}}_s$, otherwise the correlators vanish. We obtain as well:

$$\langle \mathbb{A}_{s_1}(x_1)\ldots\mathbb{A}_{s_n}(x_n)\tilde{\mathbb{A}}_{s_{n+1}}(x_{n+1})\ldots\tilde{\mathbb{A}}_{s_{n+2m}}(x_{n+2m})\rangle_{\text{conn}}$$
$$= \frac{1}{(4\pi^2)^{n+2m}}\frac{N^2-1}{2^{n+2m}}2^{\sum_{l=1}^{n+2m} s_l}i^{\sum_{l=1}^{n+2m} s_l}\sum_{k_1=0}^{s_1}\ldots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}^2\ldots\binom{s_{n+2m}}{k_{n+2m}}^2$$
$$\frac{(-1)^{n+2m}}{n+2m}\sum_{\sigma\in P_{n+2m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\ldots(s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)})!$$
$$\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\ldots\frac{(x_{\sigma(n+2m)}-x_{\sigma(1)})_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n+2m)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}+1}} \tag{7.26}$$

Analogously, for the unbalanced operators in the extended basis, we get:

$$
\begin{aligned}
&\langle \mathbb{B}_{s_1}(x_1)\ldots\mathbb{B}_{s_n}(x_n)\bar{\mathbb{B}}_{s'_1}(y_1)\ldots\bar{\mathbb{B}}_{s'_n}(y_n)\rangle = \frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^n s_l+s'_l} i^{\sum_{l=1}^n s_l+s'_l}\\
&\sum_{k_1=0}^{s_1}\cdots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1-2}\cdots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}^2\cdots\binom{s_n}{k_n}^2\binom{s'_1}{k'_1}^2\cdots\binom{s'_n}{k'_n}^2\\
&\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!\\
&\ldots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!\\
&\frac{(x_{\sigma(1)}-y_{\rho(1)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left(|x_{\sigma(1)}-y_{\rho(1)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left(|y_{\rho(1)}-x_{\sigma(2)}|^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}\\
&\ldots\frac{(x_{\sigma(n)}-y_{\rho(n)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left(|x_{\sigma(n)}-y_{\rho(n)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left(|y_{\rho(n)}-x_{\sigma(1)}|^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}} \qquad (7.27)
\end{aligned}
$$

8 *n*-point correlators and twist-2 gluonic operators in Euclidean space-time

8.1 Analytic continuation of *n*-point correlators to Euclidean space-time

The Minkowskian n-point correlators can be analytically continued to Euclidean space-time by substituting (appendix A):

$$
x_+ \to -ix_z \qquad (8.1)
$$

and:

$$
\frac{1}{|x|^2} \to -\frac{1}{x^2} \qquad (8.2)
$$

We describe the effect of the analytic continuation about various numerical factors.

In the standard basis, for the $(n+2m)$-point correlators of balanced operators, the extra factor of $(-i)^{\sum_{l=1}^{n+2m} s_l}$, which arises from the substitution of eq. (8.1) into the numerators of eq. (2.21), cancels out the factor of $i^{\sum_{l=1}^{n+2m} s_l}$ — already present in eq. (2.21) — that comes from the definition of the Minkowskian operators, but the extra factor of $(-1)^{\sum_{l=1}^{n+2m}(s_l+1)} = (-1)^{\sum_{l=1}^{n+2m} s_l}(-1)^n$ arises, which comes from the substitution of eq. (8.2) into the denominators of eq. (2.21). It combines with the factor of $(-1)^n$ already present in eq. (2.21), in such a way that only the factor of $(-1)^{\sum_{l=1}^{n+2m} s_l}$ survives after the analytic continuation.

Similarly, for the $2n$-point correlators of unbalanced operators, the factor of $(-i)^{\sum_{l=1}^n s_l+s'_l}$, which arises from the substitution of eq. (8.1) into the numerators of eq. (2.22), cancels out the factor of $i^{\sum_{l=1}^n s_l+s'_l}$ — already present in eq. (2.22) — that comes from the definition of the Minkowskian operators. Thus, only the factor of $(-1)^{\sum_{l=1}^n(s_l+1+s'_l+1)} = (-1)^{\sum_{l=1}^n s_l+s'_l}$, which comes from the substitution of eq. (8.2) into the denominators of eq. (2.22), survives after the analytic continuation. Exactly the same factors survive in the analytic continuation of the corresponding correlators in the extended basis.

8.2 Twist-2 gluonic operators in Euclidean space-time

Alternatively, the correlators may be computed by first defining the Euclidean operators and afterwards evaluating them in Euclidean space-time. Of course, the two procedures must furnish identical results, as we verify momentarily.

By performing the Wick rotation (appendix A) to Euclidean space-time, the operators get rotated as follows. The derivative along the p_+ direction transforms as:

$$\partial_+ \to i\partial_z \tag{8.3}$$

Correspondingly, for the elementary operators in the light-cone gauge (appendix E), we get:

$$\begin{aligned} f_{11} &= -\partial_+ \bar{A} \longrightarrow f_{11}^E = -i\partial_z \bar{A}^E \\ f_{\dot{1}\dot{1}} &= -\partial_+ A \longrightarrow f_{\dot{1}\dot{1}}^E = -i\partial_z A^E \end{aligned} \tag{8.4}$$

and:

$$\begin{aligned} \partial_+^{-1} f_{11} &= -\bar{A} \longrightarrow -i\partial_z^{-1} f_{11}^E = -\bar{A}^E \\ \partial_+^{-1} f_{\dot{1}\dot{1}} &= -A \longrightarrow -i\partial_z^{-1} f_{\dot{1}\dot{1}}^E = -A^E \end{aligned} \tag{8.5}$$

We observe that the structure and the sign of the propagators (appendix B) of the Euclidean elementary operators, $f_{11}^E, f_{\dot{1}\dot{1}}^E$ and $\partial_z^{-1} f_{11}^E, \partial_z^{-1} f_{\dot{1}\dot{1}}^E$, are the same as for the Minkowskian operators, $f_{11}, f_{\dot{1}\dot{1}}$ and $\partial_+^{-1} f_{11}, \partial_+^{-1} f_{\dot{1}\dot{1}}$, respectively.

Therefore, the change of the numerical factors in the Euclidean correlators with respect to the Minkowskian correlators may only arise from the change of the numerical factors in the definition of the Euclidean composite operators in terms of the Euclidean elementary operators, $f_{11}^E, f_{\dot{1}\dot{1}}^E$ and $\partial_z^{-1} f_{11}^E, \partial_z^{-1} f_{\dot{1}\dot{1}}^E$.

In the standard basis, we get:

$$\begin{aligned} \mathbb{O}_s &\to (-1)^s \operatorname{Tr} f_{11}^E(x) (\overrightarrow{D}_z + \overleftarrow{D}_z)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_z - \overleftarrow{D}_z}{\overrightarrow{D}_z + \overleftarrow{D}_z} \right) f_{\dot{1}\dot{1}}^E(x) = \mathbb{O}_s^E \\ \tilde{\mathbb{O}}_s &\to (-1)^s \operatorname{Tr} f_{11}^E(x) (\overrightarrow{D}_z + \overleftarrow{D}_z)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_z - \overleftarrow{D}_z}{\overrightarrow{D}_z + \overleftarrow{D}_z} \right) f_{\dot{1}\dot{1}}^E(x) = \tilde{\mathbb{O}}_s^E \\ \mathbb{S}_s &\to \frac{1}{\sqrt{2}} (-1)^s \operatorname{Tr} f_{11}^E(x) (\overrightarrow{D}_z + \overleftarrow{D}_z)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_z - \overleftarrow{D}_z}{\overrightarrow{D}_z + \overleftarrow{D}_z} \right) f_{11}^E(x) = \mathbb{S}_s^E \\ \bar{\mathbb{S}}_s &\to \frac{1}{\sqrt{2}} (-1)^s \operatorname{Tr} f_{\dot{1}\dot{1}}^E(x) (\overrightarrow{D}_z + \overleftarrow{D}_z)^{s-2} C_{s-2}^{\frac{5}{2}} \left(\frac{\overrightarrow{D}_z - \overleftarrow{D}_z}{\overrightarrow{D}_z + \overleftarrow{D}_z} \right) f_{\dot{1}\dot{1}}^E(x) = \bar{\mathbb{S}}_s^E \end{aligned} \tag{8.6}$$

since the factor of i^{s-2}, which comes from the substitution of eq. (8.3) into eq. (2.6), combines with the already present factor of i^{s-2} in eq. (2.6) to produce the factor of $(-1)^s$.

As a consequence, for all the Euclidean n-point correlators in the standard basis, a factor of $i^{\sum_{l=1}^n (s_l-2)} = i^{\sum_{l=1}^n s_l} (-1)^n$ disappears with respect to the Minkowskian correlators, because it disappears from the definition of the Euclidean operators, but a factor of

$(-1)^{\sum_{l=1}^n s_l}$ takes its place, in such a way that only the latter survives, thus providing the same result as in the preceding discussion about the analytic continuation.

Similarly, in the extended basis, we get:

$$
\begin{aligned}
\mathbb{A}_s &\to (-1)^{s+1}\,\mathrm{Tr}\, D_z^{-1} f_{11}^E(x)(\overrightarrow{D}_z+\overleftarrow{D}_z)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_z-\overleftarrow{D}_z}{\overrightarrow{D}_z+\overleftarrow{D}_z}\right) D_z^{-1} f_{\bar{1}\bar{1}}^E(x)=\mathbb{A}_s^E\\
\tilde{\mathbb{A}}_s &\to (-1)^{s+1}\,\mathrm{Tr}\, D_z^{-1} f_{11}^E(x)(\overrightarrow{D}_z+\overleftarrow{D}_z)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_z-\overleftarrow{D}_z}{\overrightarrow{D}_z+\overleftarrow{D}_z}\right) D_z^{-1} f_{\bar{1}\bar{1}}^E(x)=\tilde{\mathbb{A}}_s^E\\
\mathbb{B}_s &\to \frac{1}{\sqrt{2}}(-1)^{s+1}\,\mathrm{Tr}\, D_z^{-1} f_{11}^E(x)(\overrightarrow{D}_z+\overleftarrow{D}_z)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_z-\overleftarrow{D}_z}{\overrightarrow{D}_z+\overleftarrow{D}_z}\right) D_z^{-1} f_{11}^E(x)=\mathbb{B}_s^E\\
\bar{\mathbb{B}}_s &\to \frac{1}{\sqrt{2}}(-1)^{s+1}\,\mathrm{Tr}\, D_z^{-1} f_{\bar{1}\bar{1}}^E(x)(\overrightarrow{D}_z+\overleftarrow{D}_z)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{D}_z-\overleftarrow{D}_z}{\overrightarrow{D}_z+\overleftarrow{D}_z}\right) D_z^{-1} f_{\bar{1}\bar{1}}^E(x)=\bar{\mathbb{B}}_s^E
\end{aligned}
\tag{8.7}
$$

since an extra minus sign with respect to the operators in the standard basis comes from the analytic continuation of the two operators D_+^{-1}, which contribute $(-i)^2=-1$.

Correspondingly, for all the Euclidean n-point correlators in the extended basis, a factor of $i^{\sum_{l=1}^n s_l}$ disappears with respect to the Minkowskian correlators, because it disappears from the definition of the Euclidean operators, but a factor of $(-1)^{\sum_{l=1}^n s_l}(-1)^n$ takes its place, which combines with the factor of $(-1)^n$ already present in the Minkowskian correlators, in such a way that only the factor of $(-1)^{\sum_{l=1}^n s_l}$ survives, thus providing the same result as in the preceding discussion about the analytic continuation.

9 Generating functional of n-point correlators in the coordinate representation

We work out an ansatz for the generating functional of the correlators in the coordinate representation and verify by direct computation that it reproduces the n-point correlators computed in the previous sections.

The basic idea is that a conformal operator $\mathcal{O}_s(x)$ of spin s is actually the sum of — not necessarily conformal — operators $\mathcal{O}_{sk}(x)$ according to eq. (2.5):

$$
\mathcal{O}_s(x)=\sum_{k=0}^{l}\mathcal{O}_{sk}(x) \tag{9.1}
$$

where $l=s-2$ for the standard basis and $l=s$ for the extended basis.

Hence, the generating functional should be expressed in terms of the corresponding source fields, which by a slight abuse of notation we label by the very same symbols, $\mathcal{O}_s(x)$ and $\mathcal{O}_{sk}(x)$.

Originally, the source fields are defined either for even or odd s respectively but, to keep the notation simple in the following formulas, it is convenient to extend their definition to all values of s, in such a way that they are 0 for either odd or even s respectively.

9.1 Minkowskian standard basis

Our ansatz for the generating functional of correlators of balanced operators with even spin is:

$$\Gamma_{\text{conf}}[\mathbb{O}] = -(N^2-1)\log \operatorname{Det}\left(\mathbb{I} + \mathcal{D}^{-1}\mathbb{O}\right) \tag{9.2}$$

where:

$$\begin{aligned}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y) &= \frac{i^{s_1+1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-\partial_+)^{s_1-k_1+k_2}\Box^{-1}(x-y)\\ &= \frac{i^{s_1}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-\partial_+)^{s_1-k_1+k_2}\frac{1}{|x-y|^2-i\epsilon}\end{aligned} \tag{9.3}$$

with:

$$\Box = g^{\mu\nu}\partial_\mu\partial_\nu = \partial_0^2 - \sum_{i=1}^{3}\partial_i^2 \tag{9.4}$$

and:

$$\frac{1}{4\pi^2}\frac{1}{|x-y|^2-i\epsilon} = i\Box^{-1}\delta^{(4)}(x-y) \tag{9.5}$$

The corresponding correlators are computed by the functional derivatives:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}} &= \frac{\delta}{\delta\mathbb{O}_{s_1}(x_1)}\cdots\frac{\delta}{\delta\mathbb{O}_{s_n}(x_n)}\Gamma_{\text{conf}}[\mathbb{O}]\\ &= \sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\frac{\delta}{\delta\mathbb{O}_{s_1k_1}(x_1)}\cdots\frac{\delta}{\delta\mathbb{O}_{s_nk_n}(x_n)}\Gamma_{\text{conf}}[\mathbb{O}]\end{aligned} \tag{9.6}$$

Explicitly:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{O}] = &-(N^2-1)\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\ &+\frac{i^{s_1}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-\partial_+)^{s_1-k_1+k_2}\frac{1}{|x-y|^2-i\epsilon}\mathbb{O}_{s_2k_2}(y)\Bigg)\end{aligned} \tag{9.7}$$

From now on, we skip the $i\epsilon$ in the effective propagators.

Expanding the logarithm of the determinant:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{O}] = -(N^2-1)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n\sum_{s_1k_1}\cdots\sum_{s_nk_n}\\ \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x_1-x_2)\mathbb{O}_{s_2k_2}(x_2)\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(x_n-x_1)\mathbb{O}_{s_1k_1}(x_1)\end{aligned} \tag{9.8}$$

and performing the functional derivatives in eq. (9.6), we obtain:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}} = (N^2-1)\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\frac{(-1)^n}{n}\sum_{\sigma\in P_n}\\ \mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n)}-x_{\sigma(1)})\end{aligned} \tag{9.9}$$

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Employing eqs. (9.3) and (5.5), we get:

$$
\begin{aligned}
\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\rangle_{\text{conn}} = {} & \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n} 2^{\sum_{l=1}^n s_l} i^{\sum_{l=1}^n s_l}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)} \\
& \sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\frac{(-1)^n}{n}\sum_{\sigma\in P_n} \\
& (s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})! \\
& \frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}
\end{aligned}
\tag{9.10}
$$

which coincides exactly with the correlator in the coordinate representation in eq. (2.20).

Now, in the whole balanced sector, we demonstrate that:

$$
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] = -\frac{N^2-1}{2}\log\operatorname{Det}\left(\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right) \tag{9.11}
$$

Of course, setting $\mathbb{O}$ or $\tilde{\mathbb{O}}$ to zero, we recover the generating functionals for the single operators:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O}] &= -(N^2-1)\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right) \\
\Gamma_{\text{conf}}[\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)
\end{aligned}
\tag{9.12}
$$

We rewrite eq. (9.11) as:

$$
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] = -\frac{N^2-1}{2}\log\operatorname{Det}\left[\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}+\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\right] \tag{9.13}
$$

Thus:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] = {} & -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}+\mathcal{D}^{-1}\tilde{\mathbb{O}}\right) \\
& -\frac{N^2-1}{2}\log\operatorname{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)
\end{aligned}
\tag{9.14}
$$

By expanding the logarithm of the determinant, it follows that:

$$
\begin{aligned}
\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}] = {} & -\frac{N^2-1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n\sum_{s_1k_1}\dots\sum_{s_nk_n} \\
& \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x_1-x_2)(\mathbb{O}_{s_2k_2}(x_2)+\tilde{\mathbb{O}}_{s_2k_2}(x_2)) \\
& \dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(x_n-x_1)(\mathbb{O}_{s_1k_1}(x_1)+\tilde{\mathbb{O}}_{s_1k_1}(x_1)) \\
& -\frac{N^2-1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n\sum_{s_1k_1}\dots\sum_{s_nk_n} \\
& \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x_1-x_2)(\mathbb{O}_{s_2k_2}(x_2)-\tilde{\mathbb{O}}_{s_2k_2}(x_2)) \\
& \dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(x_n-x_1)(\mathbb{O}_{s_1k_1}(x_1)-\tilde{\mathbb{O}}_{s_1k_1}(x_1))
\end{aligned}
\tag{9.15}
$$

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The correlators read:

$$\begin{aligned}&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{m+n}}(x_{m+n})\rangle_{\text{conn}}\\&=\frac{\delta}{\delta\mathbb{O}_{s_1}(x_1)}\dots\frac{\delta}{\delta\mathbb{O}_{s_n}(x_n)}\frac{\delta}{\delta\tilde{\mathbb{O}}_{s_1}(x_{n+1})}\dots\frac{\delta}{\delta\tilde{\mathbb{O}}_{s_n}(x_{n+m})}\Gamma_{\text{conf}}[\mathbb{O},\tilde{\mathbb{O}}]\end{aligned}\tag{9.16}$$

Performing the functional derivatives yields:

$$\begin{aligned}&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{m+n}}(x_{m+n})\rangle_{\text{conn}}\\&=-\frac{N^2-1}{2}\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+m}=0}^{s_{n+m}-2}\frac{(-1)^{n+m+1}}{n+m}\sum_{\sigma\in P_{n+m}}\\&\quad\mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n+m)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n+m)}-x_{\sigma(1)})\\&\quad-(-1)^m\frac{N^2-1}{2}\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+m}=0}^{s_{n+m}-2}\frac{(-1)^{n+m+1}}{n+m}\sum_{\sigma\in P_{n+m}}\\&\quad\mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n+m)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n+m)}-x_{\sigma(1)})\\&=-(1+(-1)^m)\frac{N^2-1}{2}\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+m}=0}^{s_{n+m}-2}\frac{(-1)^{n+m+1}}{n+m}\sum_{\sigma\in P_{n+m}}\\&\quad\mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n+m)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n+m)}-x_{\sigma(1)})\end{aligned}\tag{9.17}$$

Hence, the correlators are nonzero only if m is even, as it should be (section 7). Therefore:

$$\begin{aligned}&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{m+n}}(x_{m+n})\rangle_{\text{conn}}\\&=(1+(-1)^m)\frac{N^2-1}{2}\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+m}=0}^{s_{n+m}-2}\frac{(-1)^{n+m}}{n+m}\sum_{\sigma\in P_{n+m}}\mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\\&\quad\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n+m)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n+m)}-x_{\sigma(1)})\end{aligned}\tag{9.18}$$

which matches eq. (2.21) once everything is made explicit by means of eqs. (9.3) and (5.5):

$$\begin{aligned}&\langle \mathbb{O}_{s_1}(x_1)\dots\mathbb{O}_{s_n}(x_n)\tilde{\mathbb{O}}_{s_{n+1}}(x_{n+1})\dots\tilde{\mathbb{O}}_{s_{m+n}}(x_{m+n})\rangle_{\text{conn}}=\frac{(1+(-1)^m)}{2}\frac{1}{(4\pi^2)^{n+m}}\\&\quad\frac{N^2-1}{2^{n+m}}2^{\sum_{l=1}^{n+m}s_l}i^{\sum_{l=1}^{n+m}s_l}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_{m+n}+3)}{\Gamma(5)\Gamma(s_{m+n}+1)}\\&\quad\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{m+n}=0}^{s_{m+n}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_{m+n}}{k_{m+n}}\binom{s_{m+n}}{k_{m+n}+2}\\&\quad\frac{(-1)^{n+m}}{n+m}\sum_{\sigma\in P_{n+m}}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n+m)}-k_{\sigma(n+m)}+k_{\sigma(1)})!\\&\quad\frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n+m)}-x_{\sigma(1)})_+^{s_{\sigma(n+m)}-k_{\sigma(n+m)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n+m)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n+m)}-k_{\sigma(n+m)}+k_{\sigma(1)}+1}}\end{aligned}\tag{9.19}$$

In the unbalanced sector, we demonstrate that:

$$\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] = -\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I} - 2\mathcal{D}^{-1}\bar{\mathbb{S}}\mathcal{D}^{-1}\mathbb{S}\right) \tag{9.20}$$

which, more explicitly, reads:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] = -\frac{N^2-1}{2}\log\text{Det}\Bigg(&\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\\ &-2\int d^4z\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(x-z)\bar{\mathbb{S}}_{sk}(z)\mathcal{D}^{-1}_{sk,s_2k_2}(z-y)\mathbb{S}_{s_2k_2}(y)\Bigg)\end{aligned} \tag{9.21}$$

By expanding the logarithm of the determinant, it follows that:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}] = \frac{N^2-1}{2}\sum_{n=1}^{\infty}\frac{2^n}{n}\int d^4x_1\dots d^4x_nd^4y_1\dots d^4y_n\sum_{s_1k_1}\dots\sum_{s_nk_n}\sum_{s'_1k'_1}\dots\sum_{s'_nk'_n}\\ \mathcal{D}^{-1}_{s_1k_1,s'_1k'_1}(x_1-y_1)\bar{\mathbb{S}}_{s'_1k'_1}(y_1)\mathcal{D}^{-1}_{s'_1k'_1,s_2k_2}(y_1-x_2)\mathbb{S}_{s_2k_2}(x_2)\dots\\ \dots\mathcal{D}^{-1}_{s_nk_n,s'_nk'_n}(x_n-y_n)\bar{\mathbb{S}}_{s'_nk'_n}(y_n)\mathcal{D}^{-1}_{s'_nk'_n,s_1k_1}(y_n-x_1)\mathbb{S}_{s_1k_1}(x_1)\end{aligned} \tag{9.22}$$

As a consequence, we obtain the $2n$-point correlator:

$$\begin{aligned}&\langle\mathbb{S}_{s_1}(x_1)\dots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\dots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle =\\ &\frac{\delta}{\delta\mathbb{S}_{s_1}(x_1)}\dots\frac{\delta}{\delta\mathbb{S}_{s_n}(x_n)}\frac{\delta}{\delta\bar{\mathbb{S}}_{s'_1}(y_1)}\dots\frac{\delta}{\delta\bar{\mathbb{S}}_{s'_n}(y_n)}\Gamma_{\text{conf}}[\mathbb{S},\bar{\mathbb{S}}]\end{aligned} \tag{9.23}$$

by means of eqs. (9.3) and (5.5):

$$\begin{aligned}&\langle\mathbb{S}_{s_1}(x_1)\dots\mathbb{S}_{s_n}(x_n)\bar{\mathbb{S}}_{s'_1}(y_1)\dots\bar{\mathbb{S}}_{s'_n}(y_n)\rangle = \frac{1}{(4\pi^2)^{2n}}\frac{N^2-1}{2^{2n}}2^{\sum_{l=1}^n s_l+s'_l}i^{\sum_{l=1}^n s_l+s'_l}\\ &\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\dots\frac{\Gamma(3)\Gamma(s_n+3)}{\Gamma(5)\Gamma(s_n+1)}\frac{\Gamma(3)\Gamma(s'_1+3)}{\Gamma(5)\Gamma(s'_1+1)}\dots\frac{\Gamma(3)\Gamma(s'_n+3)}{\Gamma(5)\Gamma(s'_n+1)}\\ &\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\\ &\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n-2}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}\\ &\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)})!(s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)})!\\ &\dots(s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)})!(s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)})!\\ &\frac{(x_{\sigma(1)}-y_{\rho(1)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{\left(|x_{\sigma(1)}-y_{\rho(1)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}+1}}\frac{(y_{\rho(1)}-x_{\sigma(2)})_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{\left(|y_{\rho(1)}-x_{\sigma(2)}|^2\right)^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}+1}}\\ &\dots\frac{(x_{\sigma(n)}-y_{\rho(n)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{\left(|x_{\sigma(n)}-y_{\rho(n)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}+1}}\frac{(y_{\rho(n)}-x_{\sigma(1)})_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{\left(|y_{\rho(n)}-x_{\sigma(1)}|^2\right)^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}+1}}\end{aligned} \tag{9.24}$$

which coincides exactly with the correlator in eq. (2.22).

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9.2 Minkowskian extended basis

We demonstrate by direct computation that:

$$\Gamma_{\text{conf}}[\mathbb{A}] = -(N^2-1)\log\text{Det}\left(\mathbb{I} + \mathcal{D}^{-1}\mathbb{A}\right) \tag{9.25}$$

with:

$$\begin{aligned}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y) &= \frac{i^{s_1+1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-\partial_+)^{s_1-k_1+k_2}\Box^{-1}(x-y)\\ &= \frac{i^{s_1}}{8\pi^2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-\partial_+)^{s_1-k_1+k_2}\frac{1}{|x-y|^2-i\epsilon}\end{aligned} \tag{9.26}$$

Expanding the logarithm of the determinant:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{A}] = -(N^2-1)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n\sum_{s_1k_1}\dots\sum_{s_nk_n}\\ \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x_1-x_2)\mathbb{A}_{s_2k_2}(x_2)\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(x_n-x_1)\mathbb{A}_{s_1k_1}(x_1)\end{aligned} \tag{9.27}$$

and performing the functional derivatives, we get:

$$\begin{aligned}\langle\mathbb{A}_{s_1}(x_1)\dots\mathbb{A}_{s_n}(x_n)\rangle_{\text{conn}} = (1-N^2)\sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\frac{(-1)^{n+1}}{n}\sum_{\sigma\in P_n}\\ \mathcal{D}^{-1}_{s_{\sigma(1)}k_{\sigma(1)},s_{\sigma(2)}k_{\sigma(2)}}(x_{\sigma(1)}-x_{\sigma(2)})\dots\mathcal{D}^{-1}_{s_{\sigma(n)}k_{\sigma(n)},s_{\sigma(1)}k_{\sigma(1)}}(x_{\sigma(n)}-x_{\sigma(1)})\end{aligned} \tag{9.28}$$

Employing eq. (9.26), we obtain:

$$\begin{aligned}\langle\mathbb{A}_{s_1}(x_1)\dots\mathbb{A}_{s_n}(x_n)\rangle_{\text{conn}} = \frac{1}{(4\pi^2)^n}\frac{N^2-1}{2^n}2^{\sum_{l=1}^n s_l}i^{\sum_{l=1}^n s_l}\sum_{k_1=0}^{s_1}\dots\sum_{k_n=0}^{s_n}\binom{s_1}{k_1}^2\dots\binom{s_n}{k_n}^2\\ \frac{(-1)^n}{n}\sum_{\sigma\in P_n}(s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)})!\dots(s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)})!\\ \frac{(x_{\sigma(1)}-x_{\sigma(2)})_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{\left(|x_{\sigma(1)}-x_{\sigma(2)}|^2\right)^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}+1}}\dots\frac{(x_{\sigma(n)}-x_{\sigma(1)})_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{\left(|x_{\sigma(n)}-x_{\sigma(1)}|^2\right)^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}+1}}\end{aligned} \tag{9.29}$$

which coincides exactly with eq. (2.31).

Moreover, in analogy with eqs. (9.11) and (9.20), we get:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{A},\tilde{\mathbb{A}}] &= -\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{A}\right)^2 - \mathcal{D}^{-1}\tilde{\mathbb{A}}\mathcal{D}^{-1}\tilde{\mathbb{A}}\right)\\ \Gamma_{\text{conf}}[\mathbb{B},\bar{\mathbb{B}}] &= -\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}^{-1}\bar{\mathbb{B}}\mathcal{D}^{-1}\mathbb{B}\right)\end{aligned} \tag{9.30}$$

9.3 Euclidean standard basis

Similarly, the generating functionals of the Euclidean correlators are:

$$\Gamma^{E}_{\text{conf}}[\mathbb{O}^E,\tilde{\mathbb{O}}^E] = -\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{O}^E\right)^2-\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{O}}^E\right)$$
$$\Gamma^{E}_{\text{conf}}[\mathbb{S}^E,\bar{\mathbb{S}}^E] = -\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}_E^{-1}\bar{\mathbb{S}}^E\mathcal{D}_E^{-1}\mathbb{S}^E\right) \tag{9.31}$$

where the kernel in eq. (9.3) is analytically continued to Euclidean space-time:

$$\begin{aligned}\mathcal{D}^{-1}_{E\ s_1k_1,s_2k_2}(x-y) &= \frac{(-i)^{-k_1+k_2}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}\partial_z^{s_1-k_1+k_2}\triangle^{-1}(x-y)\\ &= -\frac{(-i)^{-k_1+k_2}}{8\pi^2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}\partial_z^{s_1-k_1+k_2}\frac{1}{(x-y)^2}\end{aligned} \tag{9.32}$$

with:

$$\triangle = \delta_{\mu\nu}\partial_\mu\partial_\nu = \partial_4^2+\sum_{i=1}^{3}\partial_i^2 \tag{9.33}$$

and:

$$\frac{1}{4\pi^2}\frac{1}{(x-y)^2} = -\triangle^{-1}\delta^{(4)}(x-y) \tag{9.34}$$

9.4 Euclidean extended basis

Analogously:

$$\Gamma^{E}_{\text{conf}}[\mathbb{A}^E,\tilde{\mathbb{A}}^E] = -\frac{N^2-1}{2}\log\text{Det}\left(\left(\mathbb{I}+\mathcal{D}_E^{-1}\mathbb{A}^E\right)^2-\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\mathcal{D}_E^{-1}\tilde{\mathbb{A}}^E\right)$$
$$\Gamma^{E}_{\text{conf}}[\mathbb{B}^E,\bar{\mathbb{B}}^E] = -\frac{N^2-1}{2}\log\text{Det}\left(\mathbb{I}-2\mathcal{D}_E^{-1}\bar{\mathbb{B}}^E\mathcal{D}_E^{-1}\mathbb{B}^E\right) \tag{9.35}$$

where the kernel in eq. (9.26) is analytically continued to Euclidean space-time:

$$\begin{aligned}\mathcal{D}^{-1}_{E\ s_1k_1,s_2k_2}(x-y) &= \frac{(-i)^{-k_1+k_2}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}\partial_z^{s_1-k_1+k_2}\triangle^{-1}(x-y)\\ &= -\frac{(-i)^{-k_1+k_2}}{8\pi^2}\binom{s_1}{k_1}\binom{s_2}{k_2}\partial_z^{s_1-k_1+k_2}\frac{1}{(x-y)^2}\end{aligned} \tag{9.36}$$

10 Generating functional and n-point correlators in the momentum representation

The generating functionals of correlators in the momentum representation are obtained from the corresponding generating functionals in the coordinate representation (section 9) as follows.

Given the generating functional in the coordinate representation:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \log\text{Det}\left(\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y)\right) \tag{10.1}$$

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the argument of the determinant is the kernel:

$$K_{s_1k_1,s_2k_2}(x,y) = \delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y) \tag{10.2}$$

of the integral operator:

$$\psi_{s_1k_1}(x) = \sum_{s_2k_2}\int K_{s_1k_1,s_2k_2}(x,y)\phi_{s_2k_2}(y)d^4y \tag{10.3}$$

In order to obtain the kernel in the momentum representation, we perform the Fourier transform of the l.h.s.:

$$\psi_{s_1k_1}(q) = \int \psi_{s_1k_1}(x)e^{-iq\cdot x}d^4x = \sum_{s_2k_2}\int K_{s_1k_1,s_2k_2}(x,y)\phi_{s_2k_2}(y)e^{-iq\cdot x}\,d^4x\,d^4y \tag{10.4}$$

and write the r.h.s. in terms of the Fourier-transformed fields:

$$\psi_{s_1k_1}(q) = \sum_{s_2k_2}\int K_{s_1k_1,s_2k_2}(x,y)\phi_{s_2k_2}(p)e^{ip\cdot y}e^{-iq\cdot x}\,d^4x\,d^4y\frac{d^4p}{(2\pi)^4} \tag{10.5}$$

By substituting the kernel in eq. (10.2):

$$\begin{aligned}\psi_{s_1k_1}(q) = \sum_{s_2k_2}\int &\delta_{s_1k_1,s_2k_2}\delta^{(4)}(x-y)\phi_{s_2k_2}(p)e^{ip\cdot y}e^{-iq\cdot x} \\ &+ \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y)\phi_{s_2k_2}(p)e^{ip\cdot y}e^{-iq\cdot x}\,d^4x\,d^4y\frac{d^4p}{(2\pi)^4}\end{aligned} \tag{10.6}$$

we get:

$$\begin{aligned}\psi_{s_1k_1}(q) = &\sum_{s_2k_2}\int \delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(p-q)\phi_{s_2k_2}(p)\frac{d^4p}{(2\pi)^4} \\ &+\sum_{s_2k_2}\int \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y)\mathcal{O}_{s_2k_2}(y)\phi_{s_2k_2}(p)e^{ip\cdot y}e^{-iq\cdot x}\,d^4x\,d^4y\frac{d^4p}{(2\pi)^4}\end{aligned} \tag{10.7}$$

The second line in the above equation becomes:

$$\sum_{s_2k_2}\int \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(k_1)\mathcal{O}_{s_2k_2}(k_2)\phi_{s_2k_2}(p)e^{ik_1\cdot(x-y)}e^{ik_2\cdot y}e^{ip\cdot y}e^{-iq\cdot x}\,d^4x\,d^4y\frac{d^4p}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \tag{10.8}$$

which simplifies to:

$$\sum_{s_2k_2}\int \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q)\mathcal{O}_{s_2k_2}(q-p)\phi_{s_2k_2}(p)\frac{d^4p}{(2\pi)^4} \tag{10.9}$$

Therefore, the kernel in the momentum representation reads:

$$K_{s_1k_1,s_2k_2}(q_1,q_2) = \delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2) + \mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(q_1-q_2) \tag{10.10}$$

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which defines the integral operator:

$$\psi_{s_1k_1}(q_1) = \sum_{s_2k_2}\int K_{s_1k_1,s_2k_2}(q_1,q_2)\phi_{s_2k_2}(q_2)\frac{d^4q_2}{(2\pi)^4} \tag{10.11}$$

Equivalently, we may expand the logarithm of the functional determinant in the coordinate representation:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n \sum_{s_1k_1}\dots\sum_{s_nk_n}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x_1-x_2)\mathcal{O}_{s_2k_2}(x_2)$$
$$\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(x_n-x_1)\mathcal{O}_{s_1k_1}(x_1) \tag{10.12}$$

and express the source fields and effective propagators in terms of their Fourier transforms $\mathcal{O}_{sk}(p)$:

$$\mathcal{O}_{sk}(x) = \int\frac{d^4p}{(2\pi)^4}\mathcal{O}_{sk}(p)\,e^{ip\cdot x} \tag{10.13}$$

and $\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q)$;

$$\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(x-y) = \int\frac{d^4q}{(2\pi)^4}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q)\,e^{iq\cdot(x-y)} \tag{10.14}$$

We get:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4x_1\dots d^4x_n\frac{d^4q_1}{(2\pi)^4}\dots\frac{d^4q_n}{(2\pi)^4}\frac{d^4p_1}{(2\pi)^4}\dots\frac{d^4p_n}{(2\pi)^4}$$
$$\sum_{s_1k_1}\dots\sum_{s_nk_n}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(p_1)\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(q_n)\mathcal{O}_{s_1k_1}(p_n)$$
$$e^{iq_1\cdot(x_1-x_2)}\dots e^{iq_n\cdot(x_n-x_1)}\,e^{ip_1\cdot x_2}\dots e^{ip_n\cdot x_1} \tag{10.15}$$

Performing the product of the exponentials above:

$$e^{-ix_1\cdot(q_n-q_1-p_n)}e^{-ix_2\cdot(q_1-q_2-p_1)}e^{-ix_3\cdot(q_2-q_3-p_2)}\dots e^{-ix_n\cdot(q_{n-1}-q_n-p_{n-1})} \tag{10.16}$$

and employing:

$$\frac{1}{(2\pi)^4}\int d^4x\,e^{-iq\cdot x} = \delta^{(4)}(q) \tag{10.17}$$

we obtain:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int\frac{d^4q_1}{(2\pi)^4}\dots\frac{d^4q_n}{(2\pi)^4}\sum_{s_1k_1}\dots\sum_{s_nk_n}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(q_1-q_2)$$
$$\mathcal{D}^{-1}_{s_2k_2,s_3k_3}(q_2)\mathcal{O}_{s_3k_3}(q_2-q_3)\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(q_n)\mathcal{O}_{s_1k_1}(q_n-q_1) \tag{10.18}$$

Either way, the generating functional in the momentum representation reads:

$$\Gamma_{\text{conf}}[\mathcal{O}] \sim \log\text{Det}\left(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathcal{O}_{s_2k_2}(q_1-q_2)\right) \tag{10.19}$$

and the corresponding correlators are obtained by the functional derivatives:

$$\langle\mathcal{O}_{s_1}(p_1)\dots\mathcal{O}_{s_n}(p_n)\rangle_{\text{conn}} = \frac{\delta}{\delta\mathcal{O}_{s_1}(p_1)}\dots\frac{\delta}{\delta\mathcal{O}_{s_n}(p_n)}\Gamma_{\text{conf}}[\mathcal{O}]$$
$$= \sum_{k_1=0}^{l_1}\dots\sum_{k_n=0}^{l_n}\frac{\delta}{\delta\mathcal{O}_{s_1k_1}(p_1)}\dots\frac{\delta}{\delta\mathcal{O}_{s_nk_n}(p_n)}\Gamma_{\text{conf}}[\mathcal{O}] \tag{10.20}$$

10.1 Minkowskian standard basis

The generating functionals in the coordinate representation in Minkowskian space-time read (section 9.1):

$$\begin{aligned}
\Gamma_{\rm conf}[\mathbb{O}] &= -(N^2-1)\log\mathrm{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right)\\
\Gamma_{\rm conf}[\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\mathrm{Det}\left(\mathbb{I}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
\Gamma_{\rm conf}[\mathbb{O},\tilde{\mathbb{O}}] &= -\frac{N^2-1}{2}\log\mathrm{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}+\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
&\quad-\frac{N^2-1}{2}\log\mathrm{Det}\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}-\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
&= -\frac{N^2-1}{2}\log\mathrm{Det}\left(\left(\mathbb{I}+\mathcal{D}^{-1}\mathbb{O}\right)^2-\mathcal{D}^{-1}\tilde{\mathbb{O}}\mathcal{D}^{-1}\tilde{\mathbb{O}}\right)\\
\Gamma_{\rm conf}[\mathbb{S},\bar{\mathbb{S}}] &= -\frac{N^2-1}{2}\log\mathrm{Det}\left(\mathbb{I}-2\mathcal{D}^{-1}\bar{\mathbb{S}}\mathcal{D}^{-1}\mathbb{S}\right)
\end{aligned}\tag{10.21}$$

By making explicit the continuous and discrete indices, the above equations read in the momentum representation:

$$\begin{aligned}
\Gamma_{\rm conf}[\mathbb{O}] =& -(N^2-1)\log\mathrm{Det}\left(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)\mathbb{O}_{s_2k_2}(q_1-q_2)\right)\\
\Gamma_{\rm conf}[\tilde{\mathbb{O}}] =& -\frac{N^2-1}{2}\log\mathrm{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(q_1)\tilde{\mathbb{O}}_{sk}(q_1-q)\mathcal{D}^{-1}_{sk,s_2k_2}(q)\tilde{\mathbb{O}}_{s_2k_2}(q-q_2)\Bigg)\\
\Gamma_{\rm conf}[\mathbb{O},\tilde{\mathbb{O}}] =& -\frac{N^2-1}{2}\log\mathrm{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{O}}_{s_2k_2}(q_1-q_2))\Big)\\
&-\frac{N^2-1}{2}\log\mathrm{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{O}}_{s_2k_2}(q_1-q_2))\Big)\\
\Gamma_{\rm conf}[\mathbb{S},\bar{\mathbb{S}}] =& -\frac{N^2-1}{2}\log\mathrm{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(q_1)\bar{\mathbb{S}}_{sk}(q_1-q)\mathcal{D}^{-1}_{sk,s_2k_2}(q)\mathbb{S}_{s_2k_2}(q-q_2)\Bigg)
\end{aligned}\tag{10.22}$$

with:

$$\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(p) = \frac{i^{s_1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}(-ip_+)^{s_1-k_1+k_2}\frac{-i}{|p|^2+i\epsilon}\tag{10.23}$$

where the Fourier transform of the effective propagator in eq. (9.3) is computed by:

$$\int\frac{d^4p}{(2\pi)^4}\,e^{ip\cdot(x-y)}\,\frac{-i}{|p|^2+i\epsilon} = \frac{1}{4\pi^2}\frac{1}{|x-y|^2-i\epsilon}\tag{10.24}$$

Expanding the logarithm of the determinant and defining $p_i = q_i - q_{i+1}$, we obtain:

$$\Gamma^E_{\text{conf}}[\mathbb{O}] = -(N^2-1)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int d^4p_1\dots d^4p_n\,(2\pi)^4\delta^{(4)}(p_1+p_2+\dots+p_n)$$
$$\int\frac{d^4q}{(2\pi)^4}\sum_{s_1k_1}\dots\sum_{s_nk_n}\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q+p_n)\mathbb{O}_{s_2k_2}(p_1)\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q+p_1+p_n)\mathbb{O}_{s_2k_2}(p_2)$$
$$\dots\mathcal{D}^{-1}_{s_nk_n,s_1k_1}(q)\mathbb{O}_{s_1k_1}(p_n) \tag{10.25}$$

The functional derivatives yield:

$$\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s_n+1)}{\Gamma(3)\Gamma(s_n+3)}\right)\langle\mathbb{O}_{s_1}(p_1)\dots\mathbb{O}_{s_n}(p_n)\rangle_{\text{conn}}$$
$$=\frac{N^2-1}{2^n}(2\pi)^4 i^n\delta^{(4)}(p_1+\dots+p_n)\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}$$
$$\frac{1}{n}\sum_{\sigma\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+q|^2}$$
$$\dots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+q)_+^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k_{\sigma(n)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+q|^2}\frac{(q)_+^{s_{\sigma(n)}-k_{\sigma(n)}+k_{\sigma(1)}}}{|q|^2} \tag{10.26}$$

All the remaining correlators are obtained in a similar way.

For the nonvanishing correlators in the balanced sector, we get:

$$\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s_{n+2m}+1)}{\Gamma(3)\Gamma(s_{n+2m}+3)}\right)$$
$$\langle\mathbb{O}_{s_1}(p_1)\dots\mathbb{O}_{s_n}(p_n)\tilde{\mathbb{O}}_{s_{n+1}}(p_{n+1})\dots\tilde{\mathbb{O}}_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}$$
$$=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{n+2m}\delta^{(4)}(p_1+\dots+p_{n+2m})$$
$$\sum_{k_1=0}^{s_1-2}\dots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}$$
$$\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+q|^2}$$
$$\dots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_+^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{|\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q|^2}\frac{(q)_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{|q|^2} \tag{10.27}$$

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In the unbalanced sector, we obtain:

$$\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\ldots\left(\frac{\Gamma(5)\Gamma(s_n+1)}{\Gamma(3)\Gamma(s_n+3)}\right)\left(\frac{\Gamma(5)\Gamma(s'_1+1)}{\Gamma(3)\Gamma(s'_1+3)}\right)\ldots\left(\frac{\Gamma(5)\Gamma(s'_n+1)}{\Gamma(3)\Gamma(s'_n+3)}\right)$$
$$\langle \mathbb{S}_{s_1}(p_1)\ldots\mathbb{S}_{s_n}(p_n)\bar{\mathbb{S}}_{s'_1}(p'_1)\ldots\bar{\mathbb{S}}_{s'_n}(p'_n)\rangle$$
$$=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{2n}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)$$
$$\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_n=0}^{s_n-2}\sum_{k'_1=0}^{s'_1-2}\cdots\sum_{k'_n=0}^{s'_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n+2}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}$$
$$\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+p'_{\rho(1)}+q|^2}$$
$$\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_+^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q|^2}$$
$$\cdots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_+^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q|^2}$$
$$\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}$$
$$\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{|\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{|q|^2} \tag{10.28}$$

10.2 Minkowskian extended basis

Similarly:

$$\begin{aligned}\Gamma_{\text{conf}}[\mathbb{A},\tilde{\mathbb{A}}]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{A}}_{s_2k_2}(q_1-q_2)\Big)\\&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&+\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{A}}_{s_2k_2}(q_1-q_2)\Big)\\\Gamma_{\text{conf}}[\mathbb{B},\bar{\mathbb{B}}]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{s_1k_1,sk}(q_1)\bar{\mathbb{B}}_{sk}(q_1-q)\mathcal{D}^{-1}_{sk,s_2k_2}(q)\mathbb{B}_{s_2k_2}(q-q_2)\Bigg)\end{aligned} \tag{10.29}$$

with:

$$\mathcal{D}^{-1}_{s_1k_1,s_2k_2}(p)=\frac{i^{s_1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}(-ip_+)^{s_1-k_1+k_2}\frac{-i}{|p|^2+i\epsilon} \tag{10.30}$$

Explicitly, for the nonvanishing correlators in the balanced sector, we obtain in the momentum representation:

$$\begin{aligned}
&\langle \mathbb{A}_{s_1}(p_1)\ldots\mathbb{A}_{s_n}(p_n)\tilde{\mathbb{A}}_{s_{n+1}}(p_{n+1})\ldots\tilde{\mathbb{A}}_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{n+2m}\delta^{(4)}(p_1+\ldots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1}\cdots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}\binom{s_1}{k_1}\cdots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+q|^2}\\
&\quad\cdots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_+^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{|\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q|^2}\frac{(q)_+^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}\tag{10.31}$$

In the unbalanced sector, we get:

$$\begin{aligned}
&\langle \mathbb{B}_{s_1}(p_1)\ldots\mathbb{B}_{s_n}(p_n)\bar{\mathbb{B}}_{s'_1}(p'_1)\ldots\bar{\mathbb{B}}_{s'_n}(p'_n)\rangle\\
&=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{2n}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)\\
&\quad\sum_{k_1=0}^{s_1}\cdots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1}\cdots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}\binom{s_1}{k_1}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n}\\
&\quad\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_+^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{|p_{\sigma(1)}+q|^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_+^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{|p_{\sigma(1)}+p'_{\rho(1)}+q|^2}\\
&\quad\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_+^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q|^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_+^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{|p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q|^2}\\
&\quad\cdots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_+^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q|^2}\\
&\quad\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{|\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\\
&\quad\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_+^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{|\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q|^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{|q|^2}
\end{aligned}\tag{10.32}$$

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10.3 Euclidean standard basis

The Euclidean generating functionals read in the standard basis:

$$
\begin{aligned}
\Gamma^{E}_{\text{conf}}[\mathbb{O}^E,\tilde{\mathbb{O}}^E] = &-\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{E\ \ s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{O}}^E_{s_2k_2}(q_1-q_2)\Big)\\
&-\frac{N^2-1}{2}\log\text{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&+\mathcal{D}^{-1}_{E\ \ s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{O}}^E_{s_2k_2}(q_1-q_2)\Big)\\
\Gamma^{E}_{\text{conf}}[\mathbb{S}^E,\bar{\mathbb{S}}^E] = &-\frac{N^2-1}{2}\log\text{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\
&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}^{-1}_{E\ \ s_1k_1,sk}(q_1)\bar{\mathbb{S}}^E_{sk}(q_1-q)\mathcal{D}^{-1}_{E\ \ sk,s_2k_2}(q)\mathbb{S}^E_{s_2k_2}(q-q_2)\Bigg)
\end{aligned}
\tag{10.33}
$$

with:

$$
\mathcal{D}^{-1}_{E\ \ s_1k_1,s_2k_2}(p) = \frac{i^{s_1}}{2}\frac{\Gamma(3)\Gamma(s_1+3)}{\Gamma(5)\Gamma(s_1+1)}\binom{s_1}{k_1}\binom{s_2}{k_2+2}p_z^{s_1-k_1+k_2}\frac{1}{p^2}
\tag{10.34}
$$

For the nonvanishing correlators on the balanced sector, we obtain:

$$
\begin{aligned}
&\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\cdots\left(\frac{\Gamma(5)\Gamma(s_{n+2m}+1)}{\Gamma(3)\Gamma(s_{n+2m}+3)}\right)\\
&\quad\langle\mathbb{O}^E_{s_1}(p_1)\ldots\mathbb{O}^E_{s_n}(p_n)\tilde{\mathbb{O}}^E_{s_{n+1}}(p_{n+1})\ldots\tilde{\mathbb{O}}^E_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{\sum_{l=1}^{n+2m}s_l}\delta^{(4)}(p_1+\ldots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1-2}\cdots\sum_{k_{n+2m}=0}^{s_{n+2m}-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\cdots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}+2}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+q)^2}\\
&\quad\cdots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_z^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)^2}\frac{(q)_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}
\tag{10.35}
$$

In the unbalanced sector, we get:

$$\left(\frac{\Gamma(5)\Gamma(s_1+1)}{\Gamma(3)\Gamma(s_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s_n+1)}{\Gamma(3)\Gamma(s_n+3)}\right)\left(\frac{8\pi}{N}\frac{\Gamma(5)\Gamma(s'_1+1)}{\Gamma(3)\Gamma(s'_1+3)}\right)\dots\left(\frac{\Gamma(5)\Gamma(s'_n+1)}{\Gamma(3)\Gamma(s'_n+3)}\right)$$
$$\langle\mathbb{S}^E_{s_1}(p_1)\dots\mathbb{S}^E_{s_n}(p_n)\bar{\mathbb{S}}^E_{s'_1}(p'_1)\dots\bar{\mathbb{S}}^E_{s'_n}(p'_n)\rangle$$
$$=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{\sum_{l=1}^n s_l+s'_l}\delta^{(4)}\left(\sum_{l=1}^n p_l+p'_l\right)$$
$$\sum_{k_1=0}^{s_1-2}\dots\sum_{k_n=0}^{s_n-2}\sum_{k'_1=0}^{s'_1-2}\dots\sum_{k'_n=0}^{s'_n-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\dots\binom{s_n}{k_n}\binom{s_n}{k_n+2}$$
$$\binom{s'_1}{k'_1}\binom{s'_1}{k'_1+2}\dots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n+2}$$
$$\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+p'_{\rho(1)}+q)^2}$$
$$\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_z^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)^2}$$
$$\dots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_z^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)^2}$$
$$\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}$$
$$\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{(q)^2} \tag{10.36}$$

10.4 Euclidean extended basis

Similarly, in the extended basis we get:

$$\begin{aligned}\Gamma^E_{\text{conf}}[\mathbb{A}^E,\tilde{\mathbb{A}}^E]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&+\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(q_1)(\mathbb{A}^E_{s_2k_2}(q_1-q_2)+\tilde{\mathbb{A}}^E_{s_2k_2}(q_1-q_2))\Big)\\&-\frac{N^2-1}{2}\log\operatorname{Det}\Big(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&+\mathcal{D}_E^{-1}{}_{s_1k_1,s_2k_2}(q_1)(\mathbb{O}^E_{s_2k_2}(q_1-q_2)-\tilde{\mathbb{A}}^E_{s_2k_2}(q_1-q_2))\Big)\\\Gamma^E_{\text{conf}}[\mathbb{B}^E,\bar{\mathbb{B}}^E]=&-\frac{N^2-1}{2}\log\operatorname{Det}\Bigg(\delta_{s_1k_1,s_2k_2}(2\pi)^4\delta^{(4)}(q_1-q_2)\\&-2\int\frac{d^4q}{(2\pi)^4}\sum_{sk}\mathcal{D}_E^{-1}{}_{s_1k_1,sk}(q_1)\bar{\mathbb{B}}^E_{sk}(q_1-q)\mathcal{D}_E^{-1}{}_{sk,s_2k_2}(q)\mathbb{B}^E_{s_2k_2}(q-q_2)\Bigg)\end{aligned} \tag{10.37}$$

with:

$$\mathcal{D}^{-1}_{E\ s_1k_1,s_2k_2}(p) = \frac{i^{s_1}}{2}\binom{s_1}{k_1}\binom{s_2}{k_2}p_z^{s_1-k_1+k_2}\frac{1}{p^2} \tag{10.38}$$

Explicitly, for the nonvanishing correlators in the balanced sector, we obtain in the momentum representation:

$$\begin{aligned}
&\langle \mathbb{A}^E_{s_1}(p_1)\ldots\mathbb{A}^E_{s_n}(p_n)\tilde{\mathbb{A}}^E_{s_{n+1}}(p_{n+1})\ldots\tilde{\mathbb{A}}^E_{s_{n+2m}}(p_{n+2m})\rangle_{\text{conn}}\\
&=\frac{N^2-1}{2^{n+2m}}(2\pi)^4 i^{\sum_{l=1}^{n+2m}s_l}\delta^{(4)}(p_1+\ldots+p_{n+2m})\\
&\quad\sum_{k_1=0}^{s_1}\cdots\sum_{k_{n+2m}=0}^{s_{n+2m}}\binom{s_1}{k_1}\binom{s_1}{k_1}\cdots\binom{s_{n+2m}}{k_{n+2m}}\binom{s_{n+2m}}{k_{n+2m}}\frac{1}{n+2m}\sum_{\sigma\in P_{n+2m}}\\
&\quad\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+q)^2}\\
&\quad\ldots\frac{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)_z^{s_{\sigma(n+2m-1)}-k_{\sigma(n+2m-1)}+k_{\sigma(n+2m)}}}{(\sum_{l=1}^{n+2m-1}p_{\sigma(l)}+q)^2}\frac{(q)_z^{s_{\sigma(n+2m)}-k_{\sigma(n+2m)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}\tag{10.39}$$

In the unbalanced sector, we get:

$$\begin{aligned}
&\langle \mathbb{B}^E_{s_1}(p_1)\ldots\mathbb{B}^E_{s_n}(p_n)\bar{\mathbb{B}}^E_{s'_1}(p'_1)\ldots\bar{\mathbb{B}}^E_{s'_n}(p'_n)\rangle\\
&=\frac{N^2-1}{2^{2n}}(2\pi)^4 i^{\sum_{l=1}^{n}s_l+s'_l}\delta^{(4)}\left(\sum_{l=1}^{n}p_l+p'_l\right)\\
&\quad\sum_{k_1=0}^{s_1}\cdots\sum_{k_n=0}^{s_n}\sum_{k'_1=0}^{s'_1}\cdots\sum_{k'_n=0}^{s'_n}\binom{s_1}{k_1}\binom{s_1}{k_1}\cdots\binom{s_n}{k_n}\binom{s_n}{k_n}\binom{s'_1}{k'_1}\binom{s'_1}{k'_1}\cdots\binom{s'_n}{k'_n}\binom{s'_n}{k'_n}\\
&\quad\frac{2^{n-1}}{n}\sum_{\sigma\in P_n}\sum_{\rho\in P_n}\int\frac{d^4q}{(2\pi)^4}\frac{(p_{\sigma(1)}+q)_z^{s_{\sigma(1)}-k_{\sigma(1)}+k'_{\rho(1)}}}{(p_{\sigma(1)}+q)^2}\frac{(p_{\sigma(1)}+p'_{\rho(1)}+q)_z^{s'_{\rho(1)}-k'_{\rho(1)}+k_{\sigma(2)}}}{(p_{\sigma(1)}+p'_{\rho(1)}+q)^2}\\
&\quad\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)_z^{s_{\sigma(2)}-k_{\sigma(2)}+k'_{\rho(2)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+q)^2}\frac{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)_z^{s'_{\rho(2)}-k'_{\rho(2)}+k_{\sigma(3)}}}{(p_{\sigma(1)}+p_{\sigma(2)}+p'_{\rho(1)}+p'_{\rho(2)}+q)^2}\\
&\quad\ldots\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)_z^{s_{\sigma(n-1)}-k_{\sigma(n-1)}+k'_{\rho(n-1)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-2}p'_{\rho(l)}+q)^2}\\
&\quad\frac{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s'_{\rho(n-1)}-k'_{\rho(n-1)}+k_{\sigma(n)}}}{(\sum_{l=1}^{n-1}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\\
&\quad\frac{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)_z^{s_{\sigma(n)}-k_{\sigma(n)}+k'_{\rho(n)}}}{(\sum_{l=1}^{n}p_{\sigma(l)}+\sum_{l=1}^{n-1}p'_{\rho(l)}+q)^2}\frac{(q)_+^{s'_{\rho(n)}-k'_{\rho(n)}+k_{\sigma(1)}}}{(q)^2}
\end{aligned}\tag{10.40}$$

A Notation and Wick rotation

We mostly follow the notation in [7]. We define the Minkowskian metric as:

$$(g_{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1) \tag{A.1}$$

The light-cone coordinates are:

$$x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}} = x_{\mp} \tag{A.2}$$

The corresponding Minkowskian (squared) distance is:

$$|x|^2 = 2x^+x^- - x_\perp^2 \tag{A.3}$$

where:

$$x_\perp^2 = (x^1)^2 + (x^2)^2 \tag{A.4}$$

We denote the derivative with respect to x^+ by:

$$\partial_+ = \frac{\partial}{\partial x^+} = \partial_{x^+} = \frac{\partial}{\partial x_-} = \partial_{x_-} \tag{A.5}$$

We define the light-like vectors n^μ and $\bar{n}^\mu$:

$$n_\mu n^\mu = \bar{n}_\mu \bar{n}^\mu = 0 \qquad n_\mu \bar{n}^\mu = 1 \tag{A.6}$$

that can be parametrized as $(n^\mu) = \frac{1}{\sqrt{2}}(1, 0, 0, 1)$ and $(\bar{n}^\mu) = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$.

The Minkowskian metric can be decomposed into orthogonal and longitudinal parts with respect to the light-like vectors:

$$g_{\mu\nu} = g^\perp_{\mu\nu} + n_\mu \bar{n}_\nu + n_\nu \bar{n}_\mu \tag{A.7}$$

The Euclidean metric is:

$$(\delta_{\mu\nu}) = \mathrm{diag}(1, 1, 1, 1) \tag{A.8}$$

The corresponding Euclidean (squared) distance is:

$$x^2 = 2x^z x^{\bar{z}} + x_\perp^2 \tag{A.9}$$

with:

$$x^z = \frac{x^4 + ix^3}{\sqrt{2}} = \frac{x_4 + ix_3}{\sqrt{2}} = x_{\bar{z}} \tag{A.10}$$

and:

$$x^{\bar{z}} = \frac{x^4 - ix^3}{\sqrt{2}} = \frac{x_4 - ix_3}{\sqrt{2}} = x_z \tag{A.11}$$

We define the Wick rotation by:

$$x^0 = x_0 \to -ix^4 = -ix_4 \tag{A.12}$$

and:

$$p_0 = p^0 \to ip_4 = ip^4 \tag{A.13}$$

Eq. (A.12) ensures that $\exp(iS_M) \to \exp(-S_E)$, where S_M and S_E are the Minkowskian and Euclidean actions respectively, with S_E positive definite.

By defining $p \cdot x = p_\mu x^\mu$ and $\langle px \rangle = p_\mu x^\mu$ in Minkowskian and Euclidean space-time respectively, eq. (A.13) ensures that, by the Wick rotation, $p \cdot x \to \langle px \rangle$, in such a way that the pairings $p \cdot x$ and $\langle px \rangle$ are actually independent of the Minkowskian and Euclidean metric respectively.

Therefore, by a slight abuse of notation, we also write $p \cdot x$ in Euclidean space-time, instead of $\langle px \rangle$. Besides, $|x|^2 \to -x^2$ and $|p|^2 \to -p^2$.

As a consequence, the Wick rotation of the scalar propagator of mass m in Minkowskian space-time:

$$\langle \phi(x)\phi(y) \rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{i}{|p|^2 - m^2 + i\epsilon} \tag{A.14}$$

reads in Euclidean space-time:

$$\langle \phi^E(x)\phi^E(y) \rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{i^2}{-p^2 - m^2} = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{1}{p^2 + m^2} \tag{A.15}$$

as it should be. Moreover, the Wick rotation of the light-cone coordinates is:

$$x^+ = x_- \to -ix^z = -ix_{\bar z} \tag{A.16}$$

and:

$$x^- = x_+ \to -ix^{\bar z} = -ix_z \tag{A.17}$$

Correspondingly, the Wick rotation of the derivative with respect to x^+ is:

$$\partial_+ \to i\partial_z = i\frac{\partial}{\partial x^z} \tag{A.18}$$

B Minkowskian and Euclidean propagators

The gluon propagator in the light-cone gauge, $n \cdot A = A_+ = 0$, is:

$$\langle A^a_\mu(x) A^b_\nu(y) \rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{-i\,\delta^{ab}}{|p|^2 + i\epsilon} \left(g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{p \cdot n} \right) \tag{B.1}$$

and in the Feynman gauge:

$$\langle A^a_\mu(x) A^b_\nu(y) \rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{-i\,\delta^{ab}}{|p|^2 + i\epsilon}\, g_{\mu\nu} = \frac{\delta^{ab}}{4\pi^2} \frac{g_{\mu\nu}}{|x-y|^2 - i\epsilon} \tag{B.2}$$

Hence, in the light-cone gauge the transverse propagator is:

$$\begin{aligned} \langle A^a(x) A^b(y) \rangle &= 0 \\ \langle \bar A^a(x) \bar A^b(y) \rangle &= 0 \\ \langle A^a(x) \bar A^b(y) \rangle &= -\frac{\delta^{ab}}{4\pi^2} \frac{1}{|x-y|^2 - i\epsilon} \end{aligned} \tag{B.3}$$

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We employ eq. (E.2) to work out the 2-point correlators in the light-cone gauge:

$$\begin{aligned}
&\langle f^a_{11}(x) f^b_{11}(y)\rangle = 0 \\
&\langle f^a_{\dot{1}\dot{1}}(x) f^b_{\dot{1}\dot{1}}(y)\rangle = 0 \\
&\langle f^a_{11}(x) f^b_{\dot{1}\dot{1}}(y)\rangle = -\frac{\delta^{ab}}{4\pi^2}\partial_{x^+}\partial_{y^+}\frac{1}{|x-y|^2 - i\epsilon}
\end{aligned} \tag{B.4}$$

and:

$$\begin{aligned}
&\langle \partial_+^{-1} f^a_{11}(x) \partial_+^{-1} f^b_{11}(y)\rangle = 0 \\
&\langle \partial_+^{-1} f^a_{\dot{1}\dot{1}}(x) \partial_+^{-1} f^b_{\dot{1}\dot{1}}(y)\rangle = 0 \\
&\langle \partial_+^{-1} f^a_{11}(x) \partial_+^{-1} f^b_{\dot{1}\dot{1}}(y)\rangle = -\frac{\delta^{ab}}{4\pi^2}\frac{1}{|x-y|^2 - i\epsilon}
\end{aligned} \tag{B.5}$$

The Euclidean propagator in the Feynman gauge follows from the Wick rotation (appendix A):

$$\langle A^{Ea}_\mu(x) A^{Eb}_\nu(y)\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip\cdot(x-y)}\, \frac{\delta^{ab}}{p^2}\,\delta_{\mu\nu} = \frac{\delta^{ab}}{4\pi^2}\frac{\delta_{\mu\nu}}{(x-y)^2} \tag{B.6}$$

Moreover, by performing the Wick rotation of eq. (B.4) and (B.5), we obtain in Euclidean space-time:

$$\begin{aligned}
&\langle f^{Ea}_{11}(x) f^{Eb}_{11}(y)\rangle = 0 \\
&\langle f^{Ea}_{\dot{1}\dot{1}}(x) f^{Eb}_{\dot{1}\dot{1}}(y)\rangle = 0 \\
&\langle f^{Ea}_{11}(x) f^{Eb}_{\dot{1}\dot{1}}(y)\rangle = -\frac{\delta^{ab}}{4\pi^2}\partial_{x^z}\partial_{y^z}\frac{1}{(x-y)^2}
\end{aligned} \tag{B.7}$$

and:

$$\begin{aligned}
&\langle \partial_z^{-1} f^{Ea}_{11}(x) \partial_z^{-1} f^{Eb}_{11}(y)\rangle = 0 \\
&\langle \partial_z^{-1} f^{Ea}_{\dot{1}\dot{1}}(x) \partial_z^{-1} f^{Eb}_{\dot{1}\dot{1}}(y)\rangle = 0 \\
&\langle \partial_z^{-1} f^{Ea}_{11}(x) \partial_z^{-1} f^{Eb}_{\dot{1}\dot{1}}(y)\rangle = -\frac{\delta^{ab}}{4\pi^2}\frac{1}{(x-y)^2}
\end{aligned} \tag{B.8}$$

C Identities involving σ^μ and $\bar{\sigma}^\mu$

We define the matrix $(\sigma^\mu_{a\dot{a}})$:

$$(\sigma^\mu) = (1, \vec{\sigma}) \tag{C.1}$$

by means of the Pauli matrices that satisfy:

$$\begin{aligned}
&\left[\sigma^i, \sigma^j\right] = 2i\epsilon^{ijk}\sigma^k \\
&\{\sigma^i, \sigma^j\} = 2\delta^{ij}\mathbb{I}
\end{aligned} \tag{C.2}$$

We also define:

$$(\bar{\sigma}^\mu) = (1, -\vec{\sigma}) \tag{C.3}$$

and:

$$\sigma^+ = \frac{1+\sigma_3}{\sqrt{2}} \quad \sigma^- = \frac{1-\sigma_3}{\sqrt{2}} \qquad\qquad (\sigma_\perp^\mu) = (\sigma^1, \sigma^2) \quad (\bar\sigma_\perp^\mu) = (-\sigma^1, -\sigma^2) \tag{C.4}$$

By means of $(\sigma^\mu_{a\dot a})$ we may represent a vector, V_μ, in matrix form:

$$\mathbb{V} = V_\mu \sigma^\mu = \sqrt{2}\begin{pmatrix} V_+ & \bar V \\ V & V_- \end{pmatrix} \tag{C.5}$$

with:

$$\begin{aligned} V_+ &= \frac{V_0+V_3}{\sqrt{2}} & V_- &= \frac{V_0-V_3}{\sqrt{2}} \\ V &= \frac{V_1+iV_2}{\sqrt{2}} & \bar V &= \frac{V_1-iV_2}{\sqrt{2}} \end{aligned} \tag{C.6}$$

in such a way that:

$$\mathrm{Det}(\mathbb{V}_{a\dot a}) = 2(V_+V_- - V\bar V) = V^\mu V_\mu \tag{C.7}$$

Hence, the Lorentz group is embedded into $\mathrm{SL}(2,\mathbb{C})$, and a Lorentz transformation acts as:

$$\mathbb{V}' = L\mathbb{V}\bar L \tag{C.8}$$

with $L \in \mathrm{SL}(2,\mathbb{C})$, leaving the determinant invariant.

We introduce the antisymmetric symbols $\epsilon_{ab}, \epsilon^{ab}$ [17]:

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1 \tag{C.9}$$

with:

$$\begin{aligned} \epsilon_{ac}\epsilon^{cb} &= \delta_a^{\ b} \\ \epsilon^{ab} &= \epsilon^{ac}\epsilon_{cd}\epsilon^{db} \end{aligned} \tag{C.10}$$

that are employed to lower and rise the spinor indices respectively. For example:

$$\psi_a = \epsilon_{ab}\psi^b \qquad \Phi^{ab} = \epsilon^{ac}\epsilon^{bd}\Phi_{cd} \tag{C.11}$$

The following identities [17] hold:

$$\begin{aligned} \bar\sigma^{\mu\,\dot a a} &= \epsilon^{ab}\epsilon^{\dot a\dot b}\sigma^\mu_{b\dot b} \\ \sigma^\mu_{a\dot a}\bar\sigma_\mu^{\dot b b} &= 2\delta_a^{\ b}\delta^{\dot b}_{\ \dot a} \\ \sigma^\mu_{a\dot a}\sigma_{\mu\, b\dot b} &= 2\epsilon_{ab}\epsilon_{\dot a\dot b} \\ \bar\sigma^{\mu\,\dot a a}\bar\sigma_\mu^{\dot b b} &= 2\epsilon^{ab}\epsilon^{\dot a\dot b} \end{aligned} \tag{C.12}$$

Besides, we define [17]:

$$\begin{aligned} (\sigma^{\mu\nu})_a^{\ b} &= \frac{i}{4}\left(\sigma^\mu_{a\dot c}\bar\sigma^{\nu\,\dot c b} - \sigma^\nu_{a\dot c}\bar\sigma^{\mu\,\dot c b}\right) \\ (\bar\sigma^{\mu\nu})^{\dot a}_{\ \dot b} &= \frac{i}{4}\left(\bar\sigma^{\mu\,\dot a c}\sigma^\nu_{c\dot b} - \bar\sigma^{\nu\,\dot a c}\sigma^\mu_{c\dot b}\right) \end{aligned} \tag{C.13}$$

with vanishing traces:

$$\begin{aligned}\operatorname{Tr}\sigma^{\mu\nu} &= \epsilon_{ab}(\sigma^{\mu\nu})^{ab} = 0 \\ \operatorname{Tr}\bar{\sigma}^{\mu\nu} &= \epsilon_{\dot{a}\dot{b}}(\bar{\sigma}^{\mu\nu})^{\dot{a}\dot{b}} = 0\end{aligned} \tag{C.14}$$

$\sigma_{\mu\nu}$ and $\bar{\sigma}_{\mu\nu}$ satisfy the duality relations:

$$\begin{aligned}\sigma^{\mu\nu} &= -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} \\ \bar{\sigma}^{\mu\nu} &= \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma}\end{aligned} \tag{C.15}$$

where $\epsilon^{0123} = 1$. Moreover, the following identities [17] hold:

$$\begin{aligned}&(\sigma^{\mu\nu})_b{}^a(\sigma_{\mu\nu})_d{}^c = 2\delta_b{}^c\delta_d{}^a - \delta_b{}^a\delta_d{}^c \\ &(\bar{\sigma}^{\mu\nu})^{\dot{a}}{}_{\dot{b}}(\bar{\sigma}_{\mu\nu})^{\dot{c}}{}_{\dot{d}} = 2\delta^{\dot{c}}{}_{\dot{b}}\delta^{\dot{a}}{}_{\dot{d}} - \delta^{\dot{a}}{}_{\dot{b}}\delta^{\dot{c}}{}_{\dot{d}} \\ &(\sigma^{\mu\nu})_b{}^a(\bar{\sigma}_{\mu\nu})^{\dot{c}}{}_{\dot{d}} = 0\end{aligned} \tag{C.16}$$

D Relation between the spinorial and vectorial bases in Minkowskian space-time

The components of $F_{\mu\nu}$, with their s, j, τ assignments, are:

$$\begin{aligned}F_{+\mu} &= F^{\alpha\beta}n_\alpha g^\perp_{\beta\mu} && \mu,\nu = 1,2 && s=1, \quad j=\frac{3}{2}, \tau=1 \\ F_{-\mu} &= F^{\alpha\beta}\bar{n}_\alpha g^\perp_{\beta\mu} && \mu,\nu = 1,2 && s=-1, j=\frac{1}{2}, \tau=3 \\ F_{\mu\nu} &= F^{\alpha\beta}g^\perp_{\alpha\mu}g^\perp_{\beta\nu} && \mu,\nu = 1,2 && s=0, \quad j=1, \ \ \tau=2 \\ F_{+-} &= F^{\alpha\beta}n_\alpha\bar{n}_\beta && && s=0, \quad j=1, \ \ \tau=2\end{aligned} \tag{D.1}$$

The component with maximal s, $F_{+\mu}$, is well suited (section 2) to build twist-2 operators that are primary [10, 11] for the collinear conformal subgroup. In the light-cone gauge:

$$F_{+\mu} = \partial_+ A_\mu \tag{D.2}$$

with $\mu = 1, 2$. Similarly, twist-2 primary conformal operators can also be built by means of $\tilde{F}_{+\mu}$:

$$\begin{aligned}\tilde{F}_{+\mu} &= \tilde{F}^{\alpha\beta}n_\alpha g^\perp_{\beta\mu} = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}n_\alpha g^\perp_{\beta\mu}F_{\rho\sigma} = \epsilon^{-\beta+\sigma}g^\perp_{\beta\mu}F_{+\sigma} \\ &= \epsilon^{-\beta+\sigma}g^\perp_{\beta\mu}g^\perp_{\sigma\nu}F_+^{\ \nu} = -\epsilon^{\sigma\beta+-}g^\perp_{\beta\mu}g^\perp_{\sigma\nu}F_+^{\ \nu} = \epsilon^{\beta\sigma+-}g^\perp_{\beta\mu}g^\perp_{\sigma\nu}F_+^{\ \nu}\end{aligned} \tag{D.3}$$

with $s = 1$, $j = \frac{3}{2}$ and $\tau = 1$, where:

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \tag{D.4}$$

We define:

$$\epsilon_{\mu\nu} = \epsilon^{\alpha\beta\rho\sigma}g^\perp_{\alpha\mu}g^\perp_{\beta\nu}\bar{n}_\rho n_\sigma = \epsilon^{\alpha\beta+-}g^\perp_{\alpha\mu}g^\perp_{\beta\nu} \tag{D.5}$$

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with $\mu, \nu = 1, 2$ and $\epsilon_{12} = 1$. Hence:

$$\tilde{F}_{+\mu} = \epsilon_{\mu\nu} F_+^{\ \nu} \tag{D.6}$$

and:

$$\tilde{F}_{+1} = -F_{+2} \qquad \tilde{F}_{+2} = F_{+1} \tag{D.7}$$

In the spinorial representation [15]:

$$F_{a\dot{a}b\dot{b}} = \sigma^{\mu}_{a\dot{a}} \sigma^{\nu}_{b\dot{b}} F_{\mu\nu} \tag{D.8}$$

It turns out that $F_{\mu\nu}$ decomposes [15] into the $(1,0) \oplus (0,1)$ representation of the Lorentz group:

$$F_{a\dot{a}b\dot{b}} = 2(f_{ab}\epsilon_{\dot{a}\dot{b}} - \epsilon_{ab} f_{\dot{a}\dot{b}}) \tag{D.9}$$

where:[7]

$$f_{ab} = \frac{i}{2}(\sigma^{\mu\nu})_{ab} F_{\mu\nu} \tag{D.10}$$

and:

$$f_{\dot{a}\dot{b}} = -\frac{i}{2}(\bar{\sigma}^{\mu\nu})_{\dot{a}\dot{b}} F_{\mu\nu} \tag{D.11}$$

with:

$$\bar{f}_{ab} = f_{\dot{a}\dot{b}} \tag{D.12}$$

Indeed, since $\epsilon^{\dot{a}\dot{b}} f_{\dot{a}\dot{b}} = 0$ and $\epsilon_{\dot{a}\dot{b}}\epsilon_{\dot{a}\dot{b}} = -2$, we get from eqs. (D.8) and (D.9):

$$\epsilon^{\dot{a}\dot{b}} \sigma^{\mu}_{a\dot{a}} \sigma^{\nu}_{b\dot{b}} F_{\mu\nu} = 2 f_{ab} \epsilon^{\dot{a}\dot{b}} \epsilon_{\dot{a}\dot{b}} \tag{D.13}$$

that coincides with eq. (D.10) by the antisymmetry of $F_{\mu\nu}$ and the definition of $\sigma^{\mu\nu}_{ab}$ in eq. (C.13). Similarly, we obtain eq. (D.11). It follows that:

$$f_{11} = \frac{i}{2}(\sigma^{\mu\nu})_{11} F_{\mu\nu} \tag{D.14}$$

where:

$$\begin{aligned}
(\sigma^{\mu\nu})_{11} &= \frac{i}{4}\epsilon^{\dot{c}\dot{d}}(\sigma^{\mu}_{1\dot{c}}\sigma^{\nu}_{1\dot{d}} - \sigma^{\nu}_{1\dot{c}}\sigma^{\nu}_{1\dot{d}}) \\
&= \frac{i}{4}(\sigma^{\mu}_{1\dot{1}}\sigma^{\nu}_{1\dot{2}} - \sigma^{\mu}_{1\dot{2}}\sigma^{\nu}_{1\dot{1}} - \sigma^{\nu}_{1\dot{1}}\sigma^{\mu}_{1\dot{2}} + \sigma^{\nu}_{1\dot{2}}\sigma^{\mu}_{1\dot{1}}) \\
&= \frac{i}{2}(\sigma^{\mu}_{1\dot{1}}\sigma^{\nu}_{1\dot{2}} - \sigma^{\mu}_{1\dot{2}}\sigma^{\nu}_{1\dot{1}})
\end{aligned} \tag{D.15}$$

From the definition of the matrices $(\sigma^{\mu}_{a\dot{a}})$ (appendix C):

$$\begin{aligned}
(\sigma^{+}_{a\dot{a}}) &= \frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} = \frac{2}{\sqrt{2}}\,(\delta_{a1}\delta_{\dot{a}\dot{1}}) \\
(\sigma^{-}_{a\dot{a}}) &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} = \frac{2}{\sqrt{2}}\,(\delta_{a2}\delta_{\dot{a}\dot{2}}) \\
(\sigma^{1}_{a\dot{a}}) &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\delta_{a1}\delta_{\dot{a}\dot{2}} + \delta_{a2}\delta_{\dot{a}\dot{1}}) \\
(\sigma^{2}_{a\dot{a}}) &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i\,(-\delta_{a1}\delta_{\dot{a}\dot{2}} + \delta_{a2}\delta_{\dot{a}\dot{1}})
\end{aligned} \tag{D.16}$$

[7] We write eqs. (D.10) and (D.11) in the notation of appendix C, as opposed to the one in [15].

it follows that $\sigma^{\mu}_{1\dot{1}}$ is nonvanishing only for $\mu = +$, and $\sigma^{\mu}_{1\dot{2}}$ is nonvanishing only for $\mu = 1, 2$. Hence, by employing the antisymmetry of $F_{\mu\nu}$, we obtain:

$$f_{11} = \frac{i}{2}\left(2\frac{i}{2}\sigma^{+}_{1\dot{1}}\sigma^{\nu}_{1\dot{2}}\right) F_{+\nu} = \frac{i}{2}2\frac{i}{2}\frac{2}{\sqrt{2}}\left(\sigma^{1}_{1\dot{2}}F_{+1} + \sigma^{2}_{1\dot{2}}F_{+2}\right) \tag{D.17}$$

Therefore:

$$\begin{aligned} f_{11} &= -\frac{1}{\sqrt{2}}\left(F_{+1} - iF_{+2}\right) \\ f_{\dot{1}\dot{1}} &= -\frac{1}{\sqrt{2}}\left(F_{+1} + iF_{+2}\right) \end{aligned} \tag{D.18}$$

We can now build the dictionary from the spinorial to the vectorial basis of the twist-2 operators:

$$\begin{aligned} f_{11}f_{\dot{1}\dot{1}} &= \frac{1}{2}\left(F_{+1} - iF_{+2}\right)\left(F_{+1} + iF_{+2}\right) \\ &= \frac{1}{2}\Big(F_{+1}F_{+1} + F_{+2}F_{+2} - i(F_{+1}F_{+2} - F_{+2}F_{+1})\Big) \\ &= -\frac{1}{2}\Big(g_{\perp}^{\mu\nu}F_{+\mu}F_{+\nu} + i\epsilon^{\mu\nu}F_{+\mu}F_{+\nu}\Big) \end{aligned} \tag{D.19}$$

and:

$$\begin{aligned} f_{11}f_{11} &= \frac{1}{2}\left(F_{+1} - iF_{+2}\right)\left(F_{+1} - iF_{+2}\right) \\ &= \frac{1}{2}\left(F_{+1} + i\tilde{F}_{+1}\right)\left(F_{+1} + i\tilde{F}_{+1}\right) \end{aligned} \tag{D.20}$$

In principle, the unbalanced operators with $\tau = 2$ in the vectorial basis should be constructed by means of the tensor:

$$\frac{1}{2}\left(F_{+\mu} + i\tilde{F}_{+\mu}\right)\left(F_{+\nu} + i\tilde{F}_{+\nu}\right) \tag{D.21}$$

and its Hermitian conjugate, with $\mu, \nu = 1, 2$. However, a simple computation shows that all the components of the operators above are actually proportional to $f_{11}f_{11}$ and its Hermitian conjugate respectively. Indeed:

$$\begin{aligned} \frac{1}{2}\left(F_{+2} + i\tilde{F}_{+2}\right)\left(F_{+2} + i\tilde{F}_{+2}\right) &= \frac{1}{2}\left(F_{+2} + iF_{+1}\right)\left(F_{+2} + iF_{+1}\right) \\ &= -\frac{1}{2}\left(F_{+1} - iF_{+2}\right)\left(F_{+1} - iF_{+2}\right) = -f_{11}f_{11} \end{aligned} \tag{D.22}$$

and:

$$\frac{1}{2}\left(F_{+1} + i\tilde{F}_{+1}\right)\left(F_{+2} + i\tilde{F}_{+2}\right) = \frac{i}{2}\left(F_{+1} - iF_{+2}\right)\left(F_{+1} - iF_{+2}\right) = if_{11}f_{11} \tag{D.23}$$

It follows that in the standard basis:

$$\mathbb{O}_s = -\frac{1}{2} g_\perp^{\mu\nu} \operatorname{Tr} F_{+\mu}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) F_{+\nu}(x)$$
$$\tilde{\mathbb{O}}_s = -\frac{i}{2} \epsilon^{\mu\nu} \operatorname{Tr} F_{+\mu}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) F_{+\nu}(x)$$
$$\mathbb{S}_s = \frac{1}{2\sqrt{2}} \operatorname{Tr} \left(F_{+1} + i\tilde{F}_{+1}\right)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\left(F_{+1} + i\tilde{F}_{+1}\right)$$
$$\bar{\mathbb{S}}_s = \frac{1}{2\sqrt{2}} \operatorname{Tr} \left(F_{+1} - i\tilde{F}_{+1}\right)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\left(F_{+1} - i\tilde{F}_{+1}\right) \tag{D.24}$$

and in the extended basis:

$$\mathbb{A}_s = -\frac{1}{2} g_\perp^{\mu\nu} \operatorname{Tr} D_+^{-1} F_{+\mu}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s} C^{\frac{1}{2}}_{s}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} F_{+\nu}(x)$$
$$\tilde{\mathbb{A}}_s = -\frac{i}{2} \epsilon^{\mu\nu} \operatorname{Tr} D_+^{-1} F_{+\mu}(x)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s} C^{\frac{1}{2}}_{s}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1} F_{+\nu}(x)$$
$$\mathbb{B}_s = \frac{1}{2\sqrt{2}} \operatorname{Tr} D_+^{-1}\left(F_{+1} + i\tilde{F}_{+1}\right)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s} C^{\frac{1}{2}}_{s}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1}\left(F_{+1} + i\tilde{F}_{+1}\right)$$
$$\bar{\mathbb{B}}_s = \frac{1}{2\sqrt{2}} \operatorname{Tr} D_+^{-1}\left(F_{+1} - i\tilde{F}_{+1}\right)(i\overrightarrow{D}_+ + i\overleftarrow{D}_+)^{s} C^{\frac{1}{2}}_{s}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right) D_+^{-1}\left(F_{+1} - i\tilde{F}_{+1}\right) \tag{D.25}$$

E Complex basis

The complex basis is defined by means of:

$$A = \frac{A_1 + iA_2}{\sqrt{2}} \qquad \bar{A} = \frac{A_1 - iA_2}{\sqrt{2}} \tag{E.1}$$

In the light-cone gauge, it follows from eq. (D.18) that:

$$f_{11} = -\partial_+ \bar{A}$$
$$f_{\bar{1}\bar{1}} = -\partial_+ A \tag{E.2}$$

Hence, the operators in the standard basis are:

$$\mathbb{O}_s = \operatorname{Tr} \partial_+ \bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) \partial_+ A(x)$$
$$\tilde{\mathbb{O}}_s = \operatorname{Tr} \partial_+ \bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) \partial_+ A(x)$$
$$\mathbb{S}_s = \frac{1}{\sqrt{2}} \operatorname{Tr} \partial_+ \bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) \partial_+ \bar{A}(x)$$
$$\bar{\mathbb{S}}_s = \frac{1}{\sqrt{2}} \operatorname{Tr} \partial_+ A(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) \partial_+ A(x) \tag{E.3}$$

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in the light-cone gauge, and analogously in the extended basis:

$$\mathbb{A}_s = \mathrm{Tr}\,\bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) A(x)$$
$$\tilde{\mathbb{A}}_s = \mathrm{Tr}\,\bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) A(x)$$
$$\mathbb{B}_s = \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,\bar{A}(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) \bar{A}(x)$$
$$\bar{\mathbb{B}}_s = \frac{1}{\sqrt{2}}\,\mathrm{Tr}\,A(x)(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^s C_s^{\frac{1}{2}}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right) A(x) \tag{E.4}$$

F Jacobi and Gegenbauer polynomials

We work out the formulas for the Jacobi and Gegenbauer polynomials that are employed in the present paper.

For x real, the Jacobi polynomials, $P_l^{(\alpha,\beta)}(x)$, admit the representation [18]:

$$P_l^{(\alpha,\beta)}(x) = \sum_{k=0}^{l}\binom{l+\alpha}{k}\binom{l+\beta}{k+\beta}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{l-k} \tag{F.1}$$

with α, β real and l a natural number. Moreover, they satisfy the symmetry property:

$$P_l^{(\alpha,\beta)}(-x) = (-1)^l P_l^{(\beta,\alpha)}(x) \tag{F.2}$$

The Gegenbauer polynomials, $C_l^{\alpha'}(x)$, are a special case of the Jacobi polynomials:

$$C_l^{\alpha'}(x) = \frac{\Gamma(l+2\alpha')\Gamma(\alpha'+\frac{1}{2})}{\Gamma(2\alpha')\Gamma(l+\alpha'+\frac{1}{2})}P_l^{(\alpha'-\frac{1}{2},\alpha'-\frac{1}{2})}(x) \tag{F.3}$$

Therefore, they satisfy the symmetry property:

$$C_l^{\alpha'}(-x) = (-1)^l C_l^{\alpha'}(x) \tag{F.4}$$

From now on, we set:

$$x = \frac{b-a}{a+b} \tag{F.5}$$

in such a way that:

$$\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{l-k} = (-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \tag{F.6}$$

Hence, eq. (F.1) becomes:

$$P_l^{(\alpha,\beta)}(x) = \sum_{k=0}^{l}\binom{l+\alpha}{k}\binom{l+\beta}{k+\beta}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \tag{F.7}$$

Besides, setting $l = J - \alpha' + \frac{1}{2}$ in eq. (F.3) and $\alpha = \beta = \alpha' - \frac{1}{2}$ in eq. (F.1), we obtain:

$$C^{\alpha'}_{J-\alpha'+\frac{1}{2}}(x) = \frac{\Gamma(J+\frac{1}{2}+\alpha')\Gamma(\alpha'+\frac{1}{2})}{\Gamma(2\alpha')\Gamma(J+1)} \sum_{k=0}^{J-\alpha'+\frac{1}{2}} \binom{J}{k}\binom{J}{k+\alpha'-\frac{1}{2}}(-1)^{J-\alpha'+\frac{1}{2}-k}\frac{a^{J-\alpha'+\frac{1}{2}-k}b^k}{(a+b)^{J-\alpha'+\frac{1}{2}}} \tag{F.8}$$

Specializing the above equation to $J = s$ and $\alpha' = \frac{5}{2}$, we get:

$$C^{\frac{5}{2}}_{s-2}(x) = \frac{\Gamma(s+3)\Gamma(3)}{\Gamma(5)\Gamma(s+1)} \sum_{k=0}^{s-2}\binom{s}{k}\binom{s}{k+2}(-1)^{s-k}\frac{a^{s-k-2}b^k}{(a+b)^{s-2}} \tag{F.9}$$

Moreover, for $J = s$ and $\alpha' = \frac{1}{2}$, we obtain:

$$C^{\frac{1}{2}}_{s}(x) = \sum_{k=0}^{s}\binom{s}{k}\binom{s}{k}(-1)^{s-k}\frac{a^{s-k}b^k}{(a+b)^s} \tag{F.10}$$

From now on, we restrict α, β to the natural numbers and, correspondingly, α' to the positive half-integers and J to the natural numbers.

By employing the identity:

$$\binom{l+\alpha}{k}\binom{l+\beta}{k+\beta} = \frac{(l+\beta)!(l+\alpha)!}{l!(l+\alpha+\beta)!}\binom{l}{k}\binom{l+\beta+\alpha}{k+\beta} \tag{F.11}$$

it follows from eq. (F.7) that:

$$P^{(\alpha,\beta)}_l(x) = \frac{(l+\beta)!(l+\alpha)!}{l!(l+\alpha+\beta)!}\sum_{k=0}^{l}\binom{l}{k}\binom{l+\beta+\alpha}{k+\beta}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \tag{F.12}$$

Corrispondingly, eq. (F.3) reads:

$$C^{\alpha'}_l(x) = \frac{\Gamma(l+2\alpha')\Gamma(\alpha'+\frac{1}{2})}{\Gamma(2\alpha')\Gamma(l+\alpha'+\frac{1}{2})}\frac{(l+\alpha'-\frac{1}{2})!(l+\alpha'-\frac{1}{2})!}{l!(l+2\alpha'-1)!} \sum_{k=0}^{l}\binom{l}{k}\binom{l+2\alpha'-1}{k+\alpha'-\frac{1}{2}}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \tag{F.13}$$

which reduces to:

$$C^{\alpha'}_l(x) = \frac{\Gamma(\alpha'+\frac{1}{2})\Gamma(l+\alpha'+\frac{1}{2})}{\Gamma(2\alpha')\Gamma(l+1)}\sum_{k=0}^{l}\binom{l}{k}\binom{l+2\alpha'-1}{k+\alpha'-\frac{1}{2}}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \tag{F.14}$$

Specializing the above equation to $\alpha' = \frac{5}{2}$, we obtain:

$$\begin{aligned} C^{\frac{5}{2}}_l(x) &= \frac{\Gamma(l+3)\Gamma(3)}{\Gamma(5)\Gamma(l+1)}\sum_{k=0}^{l}\binom{l}{k}\binom{l+4}{k+2}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \\ &= \frac{2(l+1)(l+2)}{4!}\sum_{k=0}^{l}\binom{l}{k}\binom{l+4}{k+2}(-1)^{l-k}\frac{a^{l-k}b^k}{(a+b)^l} \end{aligned} \tag{F.15}$$

to be compared with eq. (F.9).

G Matching 2- and 3-point Minkowskian correlators with [1]

We verify that our results for the 2- and 3-point Minkowskian correlators of the balanced operators with even collinear spin in the standard basis coincide with the ones in [1] up to the different normalization of the operators.

Starting from eq. (2.15):

$$\mathcal{C}_s(x,y) = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s+2}}{(4!)^2}(-1)^s(s-1)s(s+1)(s+2)(2s)!\frac{(x-y)_+^{2s}}{(|x-y|^2)^{2s+2}} \tag{G.1}$$

we get:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \delta_{s_1 s_2}\frac{1}{(8\pi^2)^2}(N^2-1)2^{2s_1}(-1)^{s_1}\frac{1}{2^4}\frac{\Gamma(s_1+3)}{3^2\Gamma(s_1-1)}\frac{\Gamma(2s_1+2)}{2s_1+1}\frac{(x-y)_+^{2s_1}}{(|x-y|^2)^{2s_1+2}} \tag{G.2}$$

in terms of gamma functions. Besides, we rewrite the above correlator in terms of $j = s-1$,[8] to match the notation of eq. (2.11) in [1]:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \delta_{j_1 j_2}\frac{1}{(8\pi^2)^2}(-1)^{j_1+1}(N^2-1)2^{2j_1-3}\frac{\Gamma(j_1+4)}{3^2\Gamma(j_1)}\frac{\Gamma(2j_1+4)}{j_1+\frac{3}{2}}\frac{(x-y)_+^{2j_1+2}}{(|x-y|^2)^{2j_1+4}} \tag{G.3}$$

This is the very same result in [1] up to the overall factor of $\sigma_{j_1}\sigma_{j_2}$, which is missing as — contrary to eq. (2.2) in [1] — we have defined the operators $\mathbb{O}_s$ in eq. (4.7) without the factor of σ_j in front.

Our 3-point correlators of balanced operators with even collinear spin read in eq. (2.19):

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle = &-\frac{1}{(4\pi^2)^3}2\left(\frac{2}{4!}\right)^3\frac{N^2-1}{8}i^{s_1+s_2+s_3}2^{s_1+s_2+s_3}\\ &(s_1+1)(s_1+2)(s_2+1)(s_2+2)(s_3+1)(s_3+2)\\ &\sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1}{k_1}\binom{s_1}{k_1+2}\binom{s_2}{k_2}\binom{s_2}{k_2+2}\binom{s_3}{k_3}\binom{s_3}{k_3+2}\\ &(s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\ &\frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}}\end{aligned} \tag{G.4}$$

Employing:

$$\binom{s}{k}\binom{s}{k+2} = \frac{s(s-1)}{(s+2)(s+1)}\binom{s-2}{k}\binom{s+2}{k+2} \tag{G.5}$$

[8] $j = s-1$ in this section should not be confused with the conformal spin in the rest of the present paper.

we obtain:

$$
\begin{aligned}
\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle =& -\frac{1}{(4\pi^2)^3}2\left(\frac{2}{4!}\right)^3\frac{N^2-1}{8}i^{s_1+s_2+s_3}2^{s_1+s_2+s_3}\\
& s_1(s_1-1)s_2(s_2-1)s_3(s_3-1)\\
& \sum_{k_1=0}^{s_1-2}\sum_{k_2=0}^{s_2-2}\sum_{k_3=0}^{s_3-2}\binom{s_1-2}{k_1}\binom{s_1+2}{k_1+2}\binom{s_2-2}{k_2}\binom{s_2+2}{k_2+2}\binom{s_3-2}{k_3}\binom{s_3+2}{k_3+2}\\
& (s_1-k_1+k_2)!(s_2-k_2+k_3)!(s_3-k_3+k_1)!\\
& \frac{(x-y)_+^{s_1-k_1+k_2}}{(|x-y|^2)^{s_1+1-k_1+k_2}}\frac{(y-z)_+^{s_2-k_2+k_3}}{(|y-z|^2)^{s_2+1-k_2+k_3}}\frac{(z-x)_+^{s_3-k_3+k_1}}{(|z-x|^2)^{s_3+1-k_3+k_1}}
\end{aligned}
\tag{G.6}
$$

that in terms of j_1, j_2, j_3 reads:

$$
\begin{aligned}
\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\mathbb{O}_{s_3}(z)\rangle =& -\frac{1}{(8\pi^2)^3}\frac{1}{2^2 3^3}(N^2-1)i^{j_1+j_2+j_3+3}2^{j_1+j_2+j_3}\\
& j_1(j_1+1)j_2(j_2+1)j_3(j_3+1)\\
& \sum_{k_1=0}^{j_1-1}\sum_{k_2=0}^{j_2-1}\sum_{k_3=0}^{j_3-1}\binom{j_1-1}{k_1}\binom{j_1+3}{k_1+2}\binom{j_2-1}{k_2}\binom{j_2+3}{k_2+2}\binom{j_3-1}{k_3}\binom{j_3+3}{k_3+2}\\
& (j_1+1-k_1+k_2)!(j_2+1-k_2+k_3)!(j_3+1-k_3+k_1)!\\
& \frac{(x-y)_+^{j_1+1-k_1+k_2}}{(|x-y|^2)^{j_1+2-k_1+k_2}}\frac{(y-z)_+^{j_2+1-k_2+k_3}}{(|y-z|^2)^{j_2+2-k_2+k_3}}\frac{(z-x)_+^{j_3+1-k_3+k_1}}{(|z-x|^2)^{j_3+2-k_3+k_1}}
\end{aligned}
\tag{G.7}
$$

This is the very same result of eq. (2.22) in [1] up to the overall factor of $\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}$, which is missing because of the aforementioned different normalization of the operators.

H Summation trick for 2-point correlators

We compute the 2-point correlators by means of the technique in [1].

H.1 Standard basis

In the standard basis, we get:

$$
\begin{aligned}
\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle =& \mathcal{G}^{\frac{5}{2}}_{s_1-2}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{5}{2}}_{s_2-2}(\partial_{y_1^+},\partial_{y_2^+})\\
& \langle \mathrm{Tr}\, f_{11}(x_1)f_{\dot{1}\dot{1}}(y_2)\rangle\langle \mathrm{Tr}\, f_{11}(y_1)f_{\dot{1}\dot{1}}(x_2)\rangle\Big|^{y_1=y_2=y}_{x_1=x_2=x}
\end{aligned}
\tag{H.1}
$$

in the light-cone gauge. We restrict the correlators to $(x-y)_\perp = 0$, so that $|x-y|^2 = 2(x-y)_+(x-y)_-$. For $x_- > y_-$, we obtain:

$$
\frac{\Gamma(k)}{(x-y)_-^k} = \int_0^\infty d\tau\, \tau^{k-1}e^{-\tau(x-y)_-}
\tag{H.2}
$$

By the above formula, we convert derivatives into multiplications:

$$
\begin{aligned}
\partial_{x_1^+} &\to -\tau_1 \qquad & \partial_{x_2^+} &\to -\tau_2\\
\partial_{y_1^+} &\to \tau_2 \qquad & \partial_{y_2^+} &\to \tau_1
\end{aligned}
\tag{H.3}
$$

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By eq. (F.4):

$$C^{\frac{5}{2}}_{s_1-2}\left(-\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)=(-1)^{s_1}C^{\frac{5}{2}}_{s_1-2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)$$
$$C^{\frac{5}{2}}_{s_2-2}\left(\frac{\tau_1-\tau_2}{\tau_2+\tau_1}\right)=(-1)^{s_2}C^{\frac{5}{2}}_{s_2-2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right) \tag{H.4}$$

we obtain:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}i^{s_1+s_2-4}\frac{1}{4(x-y)_+^2}(-1)^{s_1+s_2} \tag{H.5}$$
$$\int_0^\infty d\tau_1 d\tau_2\,(\tau_1+\tau_2)^{s_1+s_2-4}\tau_1^2\tau_2^2 e^{-(\tau_1+\tau_2)(x-y)_-}C^{\frac{5}{2}}_{s_1-2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)C^{\frac{5}{2}}_{s_2-2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)$$

From the substitution:

$$\tau_1=\tau\gamma \qquad \tau_2=\tau(1-\gamma) \tag{H.6}$$

it follows:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}i^{s_1+s_2-4}\frac{1}{4(x-y)_+^2}(-1)^{s_1+s_2} \tag{H.7}$$
$$\int_0^\infty d\tau\int_0^1 d\gamma\,\tau^{s_1+s_2+1}\gamma^2(1-\gamma)^2e^{-\tau(x-y)_-}C^{\frac{5}{2}}_{s_1-2}(1-2\gamma)C^{\frac{5}{2}}_{s_2-2}(1-2\gamma)$$

The τ integral is:

$$\int_0^\infty d\tau\,\tau^{s_1+s_2+1}e^{-\tau(x-y)_-}=\frac{\Gamma(s_1+s_2+2)}{(x-y)_-^{s_1+s_2+2}} \tag{H.8}$$

while:

$$\int_0^1 d\gamma\,\gamma^2(1-\gamma)^2C^{\frac{5}{2}}_{s_1-2}(1-2\gamma)C^{\frac{5}{2}}_{s_2-2}(1-2\gamma) \tag{H.9}$$

is rewritten as:

$$\int_{-1}^1\frac{du}{2}\left(\frac{1-u^2}{4}\right)^2C^{\frac{5}{2}}_{s_1-2}(u)C^{\frac{5}{2}}_{s_2-2}(u) \tag{H.10}$$

with $u=1-2\gamma$. The orthogonality of Gegenbauer polynomials reads:

$$\int_{-1}^1 dz\,(1-z^2)^{\alpha'-\frac{1}{2}}C^{\alpha'}_{n_1}(z)C^{\alpha'}_{n_2}(z)=\delta_{n_1n_2}\frac{\pi 2^{1-2\alpha'}\Gamma(n_1+2\alpha')}{n_1!(n_1+\alpha')\Gamma(\alpha')^2} \tag{H.11}$$

Hence, for $\alpha'=\frac{5}{2}$:

$$\int_{-1}^1 du\,\left(1-u^2\right)^2C^{\frac{5}{2}}_{s_1-2}(u)C^{\frac{5}{2}}_{s_2-2}(u)=\delta_{s_1s_2}2^{-4}\frac{16(s_1+2)!}{9(s_1-2)!(2s_1+1)} \tag{H.12}$$

Collecting all the factors, we get:

$$\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle = \delta_{s_1s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}i^{s_1+s_2-4}(-1)^{s_1+s_2}\frac{1}{2^4}\frac{1(s_1+2)!}{3^2(s_1-2)!}\frac{\Gamma(s_1+s_2+2)}{2s_1+1}$$
$$\frac{1}{4(x-y)_+^2(x-y)_-^{s_1+s_2+2}} \tag{H.13}$$

Going back outside the plane $(x-y)_\perp = 0$:

$$\frac{1}{4(x-y)_+^2(x-y)_-^{s_1+s_2+2}} \to 2^{s_1+s_2}\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}} \tag{H.14}$$

we obtain:

$$\begin{aligned}\langle \mathbb{O}_{s_1}(x)\mathbb{O}_{s_2}(y)\rangle &= \delta_{s_1s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}i^{s_1+s_2-4}2^{s_1+s_2}(-1)^{s_1+s_2}\frac{1}{2^43^2}\\ &\quad \frac{(s_1+2)!}{(s_1-2)!}\frac{\Gamma(s_1+s_2+2)}{2s_1+1}\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}}\\ &= \delta_{s_1s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\frac{2^{2s_1+2}}{(4!)^2}(-1)^{s_1}\\ &\quad (s_1-1)s_1(s_1+1)(s_1+2)(2s_1)!\frac{(x-y)_+^{2s_1}}{(|x-y|^2)^{2s_1+2}}\end{aligned} \tag{H.15}$$

Analogously, the above equation extends to the balanced operators $\tilde{\mathbb{O}}_s$ with odd s.

H.2 Extended basis

In the extended basis, we get:

$$\begin{aligned}\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle &= \mathcal{G}^{\frac{1}{2}}_{s_1}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{1}{2}}_{s_2}(\partial_{y_1^+},\partial_{y_2^+})\\ &\quad \langle \mathrm{Tr}\,\partial_+^{-1}f_{11}(x_1)\partial_+^{-1}f_{\dot{1}\dot{1}}(y_2)\rangle\langle \mathrm{Tr}\,\partial_+^{-1}f_{11}(y_1)\partial_+^{-1}f_{\dot{1}\dot{1}}(x_2)\rangle\Big|^{y_1=y_2=y}_{x_1=x_2=x}\end{aligned} \tag{H.16}$$

in the light-cone gauge. Performing the Wick contractions, we obtain:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}\mathcal{G}^{\frac{1}{2}}_{s_1}(\partial_{x_1^+},\partial_{x_2^+})\mathcal{G}^{\frac{1}{2}}_{s_2}(\partial_{y_1^+},\partial_{y_2^+})\frac{1}{|x_1-y_2|^2}\frac{1}{|y_1-x_2|^2}\Big|^{y_1=y_2=y}_{x_1=x_2=x} \tag{H.17}$$

Similarly, we restrict the correlators to $(x-y)_\perp = 0$. By eq. (F.4):

$$\begin{aligned}C^{\frac{1}{2}}_{s_1}\left(-\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right) &= (-1)^{s_1}C^{\frac{1}{2}}_{s_1}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)\\ C^{\frac{1}{2}}_{s_2}\left(\frac{\tau_1-\tau_2}{\tau_2+\tau_1}\right) &= (-1)^{s_2}C^{\frac{1}{2}}_{s_2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)\end{aligned} \tag{H.18}$$

we get:

$$\begin{aligned}\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle &= \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}i^{s_1+s_2}\frac{1}{4(x-y)_+^2}(-1)^{s_1+s_2}\\ &\quad \int_0^\infty d\tau_1 d\tau_2\,(\tau_1+\tau_2)^{s_1+s_2}e^{-(\tau_1+\tau_2)(x-y)_-}C^{\frac{1}{2}}_{s_1}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)C^{\frac{1}{2}}_{s_2}\left(\frac{\tau_2-\tau_1}{\tau_1+\tau_2}\right)\end{aligned} \tag{H.19}$$

Changing variables:

$$\tau_1 = \tau\gamma, \qquad \tau_2 = \tau(1-\gamma) \tag{H.20}$$

we obtain:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle = \frac{1}{(4\pi^2)^2}\frac{N^2-1}{4} i^{s_1+s_2}\frac{1}{4(x-y)_+^2}(-1)^{s_1+s_2} \int_0^\infty d\tau \int_0^1 d\gamma\, \tau^{s_1+s_2+1} e^{-\tau(x-y)_-} C_{s_1}^{\frac{1}{2}}(1-2\gamma) C_{s_2}^{\frac{1}{2}}(1-2\gamma) \tag{H.21}$$

The τ integral is:

$$\int_0^\infty d\tau\, \tau^{s_1+s_2+1} e^{-\tau(x-y)_-} = \frac{\Gamma(s_1+s_2+2)}{(x-y)_-^{s_1+s_2+2}} \tag{H.22}$$

while:

$$\int_0^1 d\gamma\, C_{s_1}^{\frac{1}{2}}(1-2\gamma) C_{s_2}^{\frac{1}{2}}(1-2\gamma) \tag{H.23}$$

is rewritten as:

$$\int_{-1}^1 \frac{du}{2} C_{s_1}^{\frac{1}{2}}(u) C_{s_2}^{\frac{1}{2}}(u) \tag{H.24}$$

with $u = 1-2\gamma$. The orthogonality of the Gegenbauer polynomials reads:

$$\int_{-1}^1 dz\, (1-z^2)^{\alpha'-\frac{1}{2}} C_{s_1}^{\alpha'}(z) C_{s_2}^{\alpha'}(z) = \delta_{s_1 s_2} \frac{\pi 2^{1-2\alpha'}\Gamma(s_1+2\alpha')}{s_1!(s_1+\alpha')\Gamma(\alpha')^2} \tag{H.25}$$

Hence, for $\alpha' = \frac{1}{2}$:

$$\int_{-1}^1 \frac{du}{2} C_{s_1}^{\frac{1}{2}}(u) C_{s_2}^{\frac{1}{2}}(u) = \delta_{s_1 s_2}\frac{1}{2s_1+1} \tag{H.26}$$

Collecting all the factors, we get:

$$\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle = \delta_{s_1 s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4} i^{s_1+s_2}(-1)^{s_1+s_2}\frac{\Gamma(s_1+s_2+2)}{2s_1+1} \frac{1}{4(x-y)_+^2 (x-y)_-^{s_1+s_2+2}} \tag{H.27}$$

Going back outside the plane $(x-y)_\perp = 0$:

$$\frac{1}{4(x-y)_+^2 (x-y)_-^{s_1+s_2+2}} \to 2^{s_1+s_2}\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}} \tag{H.28}$$

we obtain:

$$\begin{aligned}\langle \mathbb{A}_{s_1}(x)\mathbb{A}_{s_2}(y)\rangle &= \delta_{s_1 s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4} i^{s_1+s_2}(-1)^{s_1+s_2}2^{s_1+s_2}\frac{\Gamma(s_1+s_2+2)}{2s_1+1}\frac{(x-y)_+^{s_1+s_2}}{(|x-y|^2)^{s_1+s_2+2}} \\ &= \delta_{s_1 s_2}\frac{1}{(4\pi^2)^2}\frac{N^2-1}{4}(-1)^{s_1}2^{2s_1}(2s_1)!\frac{(x-y)_+^{2s_1}}{(|x-y|^2)^{2s_1+2}}\end{aligned} \tag{H.29}$$

Analogously, the above equation extends to the balanced operators $\tilde{\mathbb{A}}_s$ with odd s.

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