Can future observation of the living partner post-tag the past decayed state in entangled neutral K mesons?

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Entangled neutral K mesons allow for the study of their correlated dynamics at interference and decoherence times not accessible in any other system. We find novel quantum phenomena associated to a correlation in time between the two partners: The past state of the first decayed kaon, when it was entangled before its decay, is post-tagged by the result and the time of the future observation of the second decay channel. This surprising "from future to past" effect is fully observable and leads to the unique experimental tag of the K_S state, an unsolved problem since the discovery of CP violation.

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I. INTRODUCTION

Since long ago, several authors have stated the crucial role that the neutral kaon system has played for understanding the intricacies of the quantum world. In particular, the words of Feynman [1], Lee [2], and Okun [3] all emphasize the uniqueness of this system as a jewel donated to us by nature. They were referring to the peculiar properties of single neutral kaon states, which display several rare phenomena like the strangeness oscillation, the tiny mass splitting and the large difference in lifetimes of the physical states, the violation of the fundamental discrete symmetries charge parity (CP) and time reversal (T), the regeneration when traversing a slab of material.

The present research is related to another peculiar character of neutral kaons: the "strange entanglement," i.e., the entanglement that is specific to two neutral kaon systems with all the interconnections with the above properties. It is worth reminding here that the entanglement is one of the most striking features of quantum mechanics, as stressed by Schrödinger [4], in reply to the famous argument by Einstein, Podolsky, and Rosen [5] (EPR) based on local realism.

Several tests of quantum mechanics and searches for possible decoherence and CPT violation effects that can exploit strange entanglement of neutral kaons have been proposed [6–38]. The experimental investigation of strange entanglement started with the CPLEAR experiment [39] and continued with the KLOE and KLOE-2 experiments [40–42] at DAΦNE [43–45], yielding several precision results [46–54].

The characteristic behavior of strange entanglement, with the peculiar properties of neutral kaons not found in any other system, makes possible the exploration of novel phenomena: the surviving correlation in time from the observation of the future decay of the living partner at a given time to the identification of the past kaon state leading to the first decay. This *from the future to the past* information in a system with nontrivial time evolution, entering into times in which the system was still entangled, could contribute to unveil the kind of reality to be associated to each part of the system.

The methodology that we follow consists in comparing the description of the double decay distribution at times t_1, t_2 with $\Delta t = t_2 - t_1 > 0$, using (i) the formalism of the two decay times state first introduced by Lee and Yang (LY) [55–58] with (ii) the time history (TH) of the entangled state from the coherent correlated neutral kaon system until its fate. The quantum consistency of the two approaches and the t_1, t_2 symmetry of the first approach, with no special role of one of the two decay times, naturally demand the study of a novel problem: Is it possible to infer the initial kaon state previous to the first decay at t_1 from the observation of the second decay at time $t_2 > t_1$, i.e., a correlation able to provide information from the future to the past? Contrary to the information from the past to the future, i.e., the prediction of the kaon state at time t_2 from the observation of the first decay at time t_1, the question formulated in this paper involves information on a

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part of the system at times in which the state was still fully entangled, i.e., before the first decay, when asking *which is which* is considered to be *unspeakable* in John Bell's terminology [59].

In the following, first we analyze the correlation from past to future, i.e., which is the state before the second decay at t_2, from the observation of the first decay channel at t_1. Then we infer the correlation from future to past, i.e., which is the state before the first decay at t_1, from the observation of the second decay channel at t_2. We identify the decoherence region in Δt in which the surviving correlation tells us K_L at t_2 and K_S at t_1, providing the unique way to tag a K_S experimentally. We summarize the results presenting some final remarks and our conclusion.

II. FROM PAST TO FUTURE

We consider an entangled two-body neutral kaon system, as actually realized at DAΦNE, with $\phi \to K^0\bar{K}^0$ decays, the source of EPR coherent $K^0\bar{K}^0$ pairs in the $C = -1$ antisymmetric state: $|i\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\}$.

Under particle exchange, we call particle-one the first one to decay at time t_1 and particle-two the last to decay at time t_2. We remind the reader that quantum entanglement is associated to nonseparability in two aspects: (i) We cannot identify which is which due to indistinguishability, and (ii) we cannot specify the two parts of the system that are not definite, showing that the parts have no local physical reality.

In fact, the antisymmetric state $|i\rangle$ is unique and therefore identically given in terms of any two generic linearly independent neutral kaon states, orthogonal or not [60]. As a particular case, it can be written in terms of the K_S, K_L states with definite time evolution [61]: $|i\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle\}$ with $|\mathcal{N}|^2 = (1 - |\langle K_S|K_L\rangle|^2)^{-1} \simeq 1$. As a consequence, the entangled state $|i\rangle$ at any time t after its production remains unaltered, even in the presence of $K^0 - \bar{K}^0$ mixing:

$$|i(t)\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|K_S\rangle e^{-i\lambda_S t}|K_L\rangle e^{-i\lambda_L t} - |K_L\rangle e^{-i\lambda_L t}|K_S\rangle e^{-i\lambda_S t}\} = e^{-i(\lambda_S+\lambda_L)t}|i\rangle. \quad (1)$$

If nothing is registered after the observation of the first decay at time t_1 (i.e., integrating over all subsequent decays at times t_2 of particle-two), the *survival probability* of the entangled state is necessarily characterized by the total width $\Gamma = \Gamma_S + \Gamma_L$ of the system [62]: $P(t_1) = \||i(t = t_1)\rangle\|^2 = e^{-\Gamma t_1}$. This also holds for any decay channel t_1-distribution with no other subsequent observation.

A. Two decay times state formalism (LY)

Following the LY approach of the two decay times entangled state (1), the correlated state of the two partners decaying at times t_1 and t_2 can be formally written as [55–58]

$$|i_{t_1,t_2}\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|K_S\rangle e^{-i\lambda_S t_1}|K_L\rangle e^{-i\lambda_L t_2} - |K_L\rangle e^{-i\lambda_L t_1}|K_S\rangle e^{-i\lambda_S t_2}\}. \quad (2)$$

The two decay times formalism defines in the combined two terms of the entangled state (1) what one calls particle-one—the first one to decay—and particle-two—the second one to decay. The (formal) use as evolution times is justified because they are disjointed, and there is no overlap between them: t_1 before, and t_2 after, the performed measurement and its associated projection. Accordingly, the decay amplitude of the initial state $|i\rangle$ to channel f_1 at time t_1 for particle-one and channel f_2 at time t_2 for particle-two and the corresponding observable double differential decay rate $I(f_1, t_1; f_2, t_2)$ can be readily calculated [6,63–65]:

$$\begin{aligned} I(f_1, t_1; f_2, t_2)_{\rm LY} &= |\langle f_1(t_1) f_2(t_2)|T|i(t)\rangle|^2 \\ &= |\langle f_1 f_2|T|i_{t_1,t_2}\rangle|^2 \\ &= C_{12}\{|\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} \\ &\quad - 2|\eta_1||\eta_2| e^{-\frac{(\Gamma_S+\Gamma_L)}{2}(t_1+t_2)} \\ &\quad \times \cos[\Delta m \Delta t + \phi_1 - \phi_2]\}, \end{aligned} \quad (3)$$

with $\langle f_i|T|K_S\rangle$ and $\langle f_i|T|K_L\rangle$ the decay amplitudes to the f_i channel of K_S and K_L, $\eta_i \equiv |\eta_i|e^{i\phi_i} = \frac{\langle f_i|T|K_L\rangle}{\langle f_i|T|K_S\rangle}$, and $C_{12} = \frac{|\mathcal{N}|^2}{2}|\langle f_1|T|K_S\rangle\langle f_2|T|K_S\rangle|^2$.

As a corollary of the above approach, one can notice that at an intermediate step of the calculation—after the first decay at time t_1—the state of the surviving kaon (particle-two) immediately before its decay at time t_2 is expressed as

$$\begin{aligned} |K^{(2)}(t = t_2)\rangle &= \langle f_1|T|i_{t_1,t_2}\rangle \\ &= \frac{\mathcal{N}}{\sqrt{2}}\langle f_1|T|K_S\rangle e^{-i(\lambda_S+\lambda_L)t_1} \\ &\quad \times [e^{-i\lambda_L \Delta t}|K_L\rangle - \eta_1 e^{-i\lambda_S \Delta t}|K_S\rangle]. \end{aligned} \quad (4)$$

Keeping t_1 and f_1 fixed—the observation—and renormalizing the state at time $t_2 = t_1$, it corresponds to the evolution from time t_1 to time t_2 of the pure state,

$$|K^{(2)}(t = t_1)\rangle = \mathcal{N}_2[|K_L\rangle - \eta_1|K_S\rangle], \quad (5)$$

with $\mathcal{N}_2$ a suitable normalization factor. This is precisely the state of the living particle-two that cannot decay to f_1, as a result of the projection by the decay of particle-one at t_1 as a filtering measurement—see Eqs. (7) and (8) below.

It is worth noting here that due to $\Delta\Gamma = \Gamma_S - \Gamma_L \neq 0$, two regimes can be identified in the time evolution of state (5): (i) the generic interference region and (ii) the decoherence region, with the relative weight of the K_S component negligible when the following condition is satisfied:

$$|\eta_1| e^{-\Delta\Gamma\Delta t/2} \ll 1 \quad [K_L - \text{tag}]. \tag{6}$$

At long enough Δt—depending on what f_1 was—the living partner is always a $|K_L\rangle$. This property is well understood, and it has been used in the past in order to have K_L beams "for all practical purposes" (FAPP) in Bell's terminology [59].

B. Time history (TH)

It is worth it to point out that the result (5) for the living partner is in agreement with the EPR instantaneous information due to the first decay when following the time history of strange entanglement, which we are now going to study in detail.

We first notice that in the case of decay processes, any initial state has some probability per unit time to decay to a given decay channel f except that with zero probability. In particular, the linear combination,

$$|K_{\nrightarrow f}\rangle = \mathcal{N}_{\nrightarrow f}[|K_L\rangle - \eta_f |K_S\rangle], \tag{7}$$

having a vanishing decay amplitude $\langle f|T|K_{\nrightarrow f}\rangle = 0$, cannot decay to f. This state is the one tagged for the unmeasured particle as a consequence of the projection imposed by the decay of the observed particle. For the first decay to f_1 at time t_1, the tagged state of the surviving partner is given by Eq. (7), with $f = f_1$. In other words, the measured decay on one side prepares, in the quantum mechanical sense, its partner on the other side as a single kaon particle at a starting time $t = t_1$. Then the $|K_{\nrightarrow f}\rangle$ state freely evolves in time—and in this sense, the information is from past to future—until its decay time at t_2; see Eq. (4). We may ask whether this information constrains the past state of the decayed particle at t_1, which was undefined in the entangled system. This is a question that, for different scenarios, is being debated in the literature—see, for example, Refs. [66–69]. In our case, any state linearly independent to Eq. (7), orthogonal or not, leads to the same decay probability. This "filtering identity" [70] is saying that the orthogonal component $|K^{\perp}_{\nrightarrow f}\rangle$ is filtered from the past undefined state by the decay. The decay acts as a filtering measurement and, for calculation purposes, it is convenient to rewrite the entangled state at t_1, in terms of these two orthogonal states, as

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^{\perp}_{\nrightarrow f}\rangle|K_{\nrightarrow f}\rangle - |K_{\nrightarrow f}\rangle|K^{\perp}_{\nrightarrow f}\rangle\}. \tag{8}$$

In this way, we may use the concept of transition probabilities at the different relevant times in the history of the system.

In summary, four sequential steps are present in the time history of the entangled state $|i\rangle$:

(1) The time evolution of the state $|i\rangle$ from time $t = 0$ to time $t = t_1$, with definite total width Γ;

(2) The projection of the state $|i(t = t_1)\rangle$ onto the orthogonal pair $|K^{\perp}_{\nrightarrow f_1}\rangle|K_{\nrightarrow f_1}\rangle$, filtered by the decay f_1, times the decay amplitude of the state $|K^{\perp}_{\nrightarrow f_1}\rangle$ into the f_1 channel;

(3) The time evolution of the surviving (single) kaon state $|K_{\nrightarrow f_1}\rangle$ from time $t = t_1$ to time $t = t_2$;

(4) The projection at time $t = t_2$ of the evolved state $|K_{\nrightarrow f_1}(\Delta t)\rangle$ onto the state $|K^{\perp}_{\nrightarrow f_2}\rangle$ filtered by the decay f_2, times the decay amplitude of the state $|K^{\perp}_{\nrightarrow f_2}\rangle$ into the f_2 channel.

These steps straightforwardly lead to the calculation of the observable double differential decay rate by factorizing the amplitudes as follows:

$$I(f_1, t_1; f_2, t_2)_{\mathrm{TH}} = |\langle f_2|T|K^{\perp}_{\nrightarrow f_2}\rangle\langle K^{\perp}_{\nrightarrow f_2}|K_{\nrightarrow f_1}(\Delta t)\rangle \times \langle f_1|T|K^{\perp}_{\nrightarrow f_1}\rangle\langle K^{\perp}_{\nrightarrow f_1} K_{\nrightarrow f_1}|i(t = t_1)\rangle|^2. \tag{9}$$

One can easily verify that the TH approach is fully consistent with the LY approach [71]: $I(f_1, t_1; f_2, t_2)_{\mathrm{TH}} = I(f_1, t_1; f_2, t_2)_{\mathrm{LY}} \equiv I(f_1, t_1; f_2, t_2)$.

III. FROM FUTURE TO PAST

As already pointed out, the state (5) evaluated from expression (4) in the LY approach coincides with the state $|K_{\nrightarrow f_1}\rangle$ of the surviving kaon after the first decay in the TH approach. The t_1, t_2 symmetry of the correlated state in the LY approach—Eq. (2)—with no special role of one of the two decay times, demands the exploration of its implications when projecting it instead onto the f_2 channel at time t_2. With this information, the resulting past decayed state at time t_1 is

$$\boxed{|K^{(1)}(t = t_1)\rangle = \langle f_2|T|i_{t_1,t_2}\rangle = \frac{\mathcal{N}}{\sqrt{2}}\langle f_2|T|K_S\rangle\{e^{-i\lambda_S t_1}[\eta_2 e^{-i\lambda_L t_2}|K_S\rangle] - e^{-i\lambda_L t_1}[e^{-i\lambda_S t_2}|K_L\rangle]\}.} \tag{10}$$

Expression (10) corresponds to the state of the decayed kaon (particle-one) immediately before its decay at time t_1 once t_2 and f_2 are fixed for the future "fate" of its partner. Keeping t_2 and f_2 fixed—the observation—and varying the first decay time t_1, it corresponds to the single kaon evolved state, before the first decay, from time $t = 0$ to time $t = t_1$ of the state

$$|K^{(1)}(t=0)\rangle = \mathcal{N}_1\{\eta_2 e^{-i\lambda_L t_2}|K_S\rangle - e^{-i\lambda_S t_2}|K_L\rangle\}, \quad (11)$$

with $\mathcal{N}_1$ a suitable renormalization factor. *Contrary to Eq. (5), which is independent on the past t_1 decay time, Eq. (11) shows a dependence not only on the decay channel f_2, but also on the future t_2 decay time.*

This is a striking result that clearly involves a correlation in time from the future observation at time t_2 to the past, inferring the initial kaon state *before* its first decay at t_1. It becomes well defined during the time evolution of the entangled state $|i\rangle$ described by Eq. (1) when the state of particle-one (and particle-two) should have been undefined in the absence of any observation. We insist that the *post-tagging* implied by Eq. (10) is not an artifact of the formalism but a factual observable accessible to experimental studies, and thus, it is fully physical. In a time history from future to past, the future observation at time t_2 tags particle-one at the time $t_1 = t_2$ into the state proportional to $\{\eta_2|K_S\rangle - |K_L\rangle\}$, the state not decaying to f_2. Keeping t_2 and η_2 fixed—the observation—the backward evolution of this tagged unobserved state to $t_1 < t_2$, leads to Eq. (10).

A. The interference and decoherence regimes: the K_S tag

As a counterpart of the observability of the *predicted* Eq. (4) through the t_2 time distribution of the second decay, once the first decay to the f_1 decay channel at t_1 is fixed, the t_1 time distribution of the first decay as *postdicted* in Eq. (10) is also observable once the second decay channel f_2 and the decay time t_2 are fixed. As function of t_1, two different regimes can be identified: the generic interference region, in which the t_2 dependence of Eq. (10) is apparent, and the decoherence region, in which the relative weight of the K_L component is negligible. Decoherence is reached for large Δt, satisfying the condition,

$$\boxed{e^{-\Delta\Gamma\Delta t/2}/|\eta_2| \ll 1 \quad [K_S - \text{tag}],} \quad (12)$$

leading to a pure K_S beam before the first decay. This consequence of the surviving correlation in time is most rewarding. Due to CP violation and the nonorthogonality of the stationary states $\langle K_L|K_S\rangle \neq 0$, there is no decay channel able to tag either K_S or K_L on an event-by-event basis. While it is relatively easy to prepare FAPP pure K_L beams, fulfilment of condition (12) constitutes the only known FAPP method to actually *postpare* a K_S beam (i.e., the short-lived stationary state) with arbitrary high purity (depending on Δt and η_2), preparation otherwise impossible with other methods. As an illustration of the observables in the two different regimes, Fig. 1 shows the decay rate distribution into a generic channel f_1 of state (10) as a function of t_1 in two cases: either observed at $t_2 = 3\tau_S$ (interference region) or when condition (12) is satisfied (decoherence region), with $f_2 = f_1$ to maximize the interference effects and make visible the difference between the two cases. This choice $f_2 = f_1$ also emphasizes the differing results as due to the dependence on the time of the future observation. *Whereas the decoherence case shows a definite width Γ_S, the future observation in the interference region leads to a t_1-distribution with no definite lifetime*. In the latter case, the t_1 distribution does depend on the decay channel. All these results differ from the time distribution, given by the total width Γ, in the absence of any future observation [78,79].

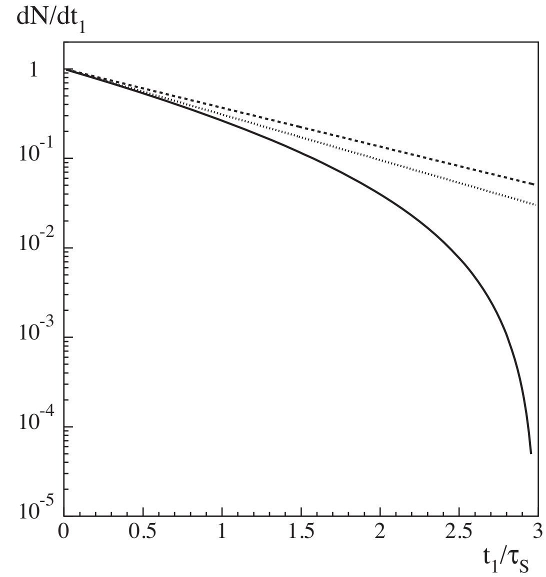

FIG. 1. The decay rate distribution into a generic channel f_1 of state (10) as a function of t_1 for the future observation at $t_2 = 3\tau_S$ (solid line) and when condition (12) for decoherence is satisfied (dashed line), with $f_2 = f_1$. The last shows a definite lifetime τ_S and does not depend on the decay channel f_1. They differ both from the t_1-distribution (dotted line) in the absence of a future measurement (in this case, Γ_L has been multiplied by a factor 100 to appreciate graphically the difference between dotted and dashed lines). All distributions are normalized to unity at $t_1 = 0$.

IV. REMARKS AND CONCLUSIONS

In the case of entangled neutral mesons in the $C = (-)$ state, the dynamics before the first decay was considered to be trivial, even with mixing, as corresponding to a definite time evolution with the total width of the system. Hence, in the past, the experimental studies were concentrated in the observation of the single kaon decay rate distribution between the two decays depending on Δt. The first decay acts as a filtering preparation of the *initial* single state of the living partner. Our paper demonstrates the consistency of this description in terms of observables such as "the state of the living partner at the time t_2 of the second decay," a well defined—speakable—question, because at t_2 the system is no longer entangled, after the measurement of the first decay channel f_1 at t_1. Asking about "the state of the first decayed kaon," however, was considered to be unspeakable, because before the first decay, the system was entangled. Our study from future to past leads to the conclusion that this last contention is only valid as long as no future observation is made. In our case of a *future* measurement, the independence on the reference frame is assured when a timelike interval between the two decays is considered.

The present research has gone indeed a step further and seems to recognize that the correlation between the two partners survives their explicit dynamics, with a transition from the quantum correlation of entanglement to a classical correlation of separable K_S, K_L states. The information from a measurement on the living partner—f_2 decay channel at t_2—to the state of the decayed meson at t_1 is most surprising. Entering into the entangled region, i.e., just before the first decay, is not an artifact of our formalism but a precise experimental observable through the t_1 distribution of decays in any channel, as shown in Fig. 1. This is so for even situations of decoherence, Eq. (12), a physical situation only reachable for strange entanglement with the two very different lifetimes of K_L and K_S . The relevance of this result for particle physics is outstanding: the unique way to tag *what a* K_S *is*, i.e., the solution of an open problem since the discovery of CP violation.

Our results seem to confirm the counterintuitive feature of time in quantum mechanics. The surviving correlation in time found here goes beyond other phenomena, like delayed choice experiments, quantum erasers, or teleportation, discussed for photons [80–84]. In the case of delayed choice designs, the system is stationary at all times, and the choice of the outcome can be made by either advanced or delayed observation with the result unchanged [85]. In the effect discussed here, the tagged state of the past decayed kaon has a nontrivial time dependence with a result depending on the decay time of the future observation (post-tagging), as depicted in Fig. 1. The result is not symmetric when comparing the outcomes in the two senses of "from past to future," Eq. (4), and "from future to past," Eq. (10). This is characteristic of the neutral K-meson system with flavor mixing, $\Delta\Gamma > 0$ and $\langle K_L|K_S\rangle \neq 0$, leading at decoherence times to the unique experimental K_S tag. These predictions are fully observable through the measurement of t_1-distributions as the ones shown in Fig. 1, with no analog in other physical systems.

Our results demonstrate that the correlation in time is definite between the outcome at a given time of the observed decay and the state of the unobserved partner. This correlation also survives when the observation is made in the future, when the system is no longer entangled after the first decay, post-tagging the past state of the unobserved decayed partner depending on the result and the time of the future observation. The nontrivial time evolution for the neutral K-meson system leads to a result that is nonsymmetric in time. As a consequence, it opens the way to a novel kind of experimental studies, not envisaged before.

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[60] In fact, the antisymmetric state $|i\rangle$ can be expressed as: $|i\rangle = \frac{\mathcal{N}_{\alpha\beta}}{\sqrt{2}}\{|K_\alpha\rangle|K_\beta\rangle - |K_\beta\rangle|K_\alpha\rangle\}$, with $|K_\alpha\rangle$ and $|K_\beta\rangle$ two arbitrary linearly independent superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states, and $|\mathcal{N}_{\alpha\beta}|^2 = (1 - |\langle K_\alpha|K_\beta\rangle|^2)^{-1}$.

[61] The eigenstates of the effective Hamiltonian describing a single neutral kaon are the short- and long-lived physical states $|K_S\rangle$ and $|K_L\rangle$. They have definite masses $m_{S,L}$ and lifetimes $\tau_{S,L}$ and evolve as a function of the kaon proper time t as pure exponentials $|K_{S,L}(t)\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}\rangle$ with $\lambda_{S,L} = m_{S,L} - i\Gamma_{S,L}/2$, and $\Gamma_{S,L} = (\tau_{S,L})^{-1}$. They are usually expressed in terms of the flavor eigenstates $|K^0\rangle$, $|\bar{K}^0\rangle$ as: $|K_{S,L}\rangle \propto [(1+\epsilon_{S,L})|K^0\rangle \pm (1-\epsilon_{S,L})|\bar{K}^0\rangle]$ with ϵ_S and ϵ_L two small complex parameters describing the CP impurity in the physical states, making them non-orthogonal, $\langle K_S|K_L\rangle \simeq \epsilon_L + \epsilon_S^\star$. One can equivalently define $\epsilon \equiv (\epsilon_S + \epsilon_L)/2$, and $\delta \equiv (\epsilon_S - \epsilon_L)/2$; adopting a suitable phase convention $\epsilon \neq 0$ implies T violation, $\delta \neq 0$ implies CPT violation, while $\delta \neq 0$ or $\epsilon \neq 0$ implies CP violation. It is worth noting that one of the most stringent tests of the fundamental CPT symmetry is performed comparing the measured CP violation in K_S and K_L states [6,51].

[62] This property holds only for the $C = -1$ antisymmetric state, but not for the $C = +1$ symmetric state in which the time evolution would induce K^0K^0 and $\bar{K}^0\bar{K}^0$ terms due to $K^0 - \bar{K}^0$ mixing by weak interactions.

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[79] The KLOE/KLOE-2 experiment has already used the method based on condition (12) to select a pure K_S beam to study its semileptonic decay [51,53], to search for its rare CP violating decay [50] into $3\pi^0$, and to study the K_S decay time distribution and $K_{S,L}$ interference with $f_1 = f_2 = \pi^+\pi^-$ [46–49,54]. In light of these results, the KLOE-2 experiment is in an unique position to perform a full experimental verification of the surprising phenomenon discussed in this paper about the information from future to past implied by result (10).

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