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Scattering of an inhomogeneous wave impinging on parallel stratified cylinders

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Abstract

The interplay between an inhomogeneous plane wave and unlimited parallel stratified cylinders is presented. A meticulous theoretical approach is employed to show the relative electromagnetic interaction. The generalized Vector Cylinder Harmonics (VCH) expansion application is used for investigating the electromagnetic scattering by a series of stratified cylinders. he commonly named complex-angle formalism is used in the exact mathematical model to depict the current scenario. This approach allows the obtaining of the superposition of the VCH and the Foldy-Lax Multiple Scattering Equations (FLMSE) for considering the effect of multi-scattering between the stratified cylinders.

1 Introduction

The literature provides several studies that dealt with the determination of Maxwell's equation to resolve the electromagnetic field scattered by intricate geometries. Simple scenarios were first investigated as cylinder, sphere, and symmetric objects to then expanding the research on ensembles of various structures of scatterers [1]-[12]. The internal characteristic of the scattering objects was a further factor to be explored, the aim was to learn their correlation with the analytical and physical behaviors. The search for the solution related to electromagnetic scattering problemby a a series of cylinders has been, in the last few years, a highly studied problem and this led to a multitude of exact mathematical models [5, 6, 9, 13, 14]. As mentioned above, the study introduces an accurate methodology to represent the inhomogeneous plane wave polarized as an expansion of VCH. Subsequently, the multi-scattering process of the stratified cylinders is addressed through the alleged T-matrix approach [15, 16]. Moreover, the continuity of the tangential components along the scatterers' surfaces is satisfied applying the FLMSEs [17, 18]. An electromagnetic wave represented as a general inhomogeneous wave drew attention from many researchers. A lossy medium, where an electromagnetic wave propagates, is represented by two vectors, one of phase and one of attenuation. The propagation through a lossless medium is only an utopian condition, because in nature every medium introduces losses. This study offers a new view on the potential, especially in terms of greater elaboration simplicity, of the approach of the complex-angle formulation to describe the incident field as Vector Cylinder Harmonics superposition [19]. This current methodology will be generalized for the scattering of a cylinder in a lossy medium developing this scenario on the more complex infinite number of parallel stratified cylinders hosted in a medium with loss along its results [20, 21]. The determination of the different formulations was realized in Matlab environment through a homemade code.

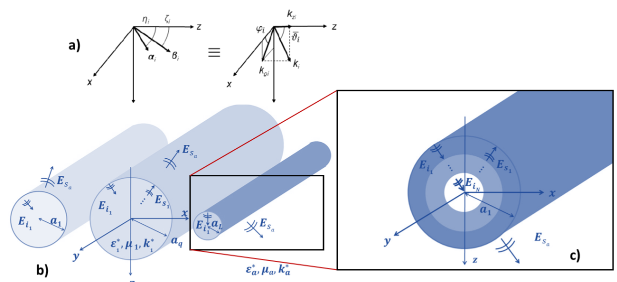

Figure 1. a) Representation of an inhomogeneous plane wave in complex-vector formalism (left) and in complex-angle formulation (right). b) Depiction of the problem. c) Zoom of b) to highlight the stratified nature of the cylinders.

2 Method and results

A lossy medium in which an inhomogeneous wave propagates has been investigated with two well-known formalisms. The Adler-Chu-Fano formulation, with better performance, is characterized by a vector with complex nature with $\boldsymbol{k}_i = \boldsymbol{\beta}_i + i\boldsymbol{\alpha}_i$, the terms $\boldsymbol{\beta}_i$, $\boldsymbol{\alpha}_i$ are the phase and the attenuation vectors, both real. The complex angle formulation is still characterized by a propagation vector with complex nature but in this case it is the superposition of real and imaginary parts $\boldsymbol{k_i} = k_{\mathbf{R}} + ik_{\mathbf{I}}$. The propagation vector creates an angle with a Cartesian axis with a complex nature too, $\bar{\vartheta}_i = \vartheta_{\mathbf{R}} + i\vartheta_{\mathbf{I}}$ (see Fig. 1). This research evinces that the electromagnetic field represented by superpositioned standard cylindrical waves using the complex-angle formalism is quite straightforward. The vectors $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ generating the angles η_i and ζ_i with z-axis laid on the (x, y), and a real angle φ is created with the x-axis (see Fig.1.a). The two formalisms, in the mentioned case, are linked as follows [20]:

$$\cos\vartheta_{\mathbf{R}} = \frac{k_{\mathbf{R}}\beta\cos\xi + k_{\mathbf{I}}\alpha\cos\eta}{\sqrt{k_{\mathbf{R}}^2\beta^2 - k_{\mathbf{I}}^2\alpha^2 + 2(k_{\mathbf{R}}k_{\mathbf{I}})^2}} \quad (1)$$

$$\sin\vartheta_{\mathbf{R}} = \frac{k_{\mathbf{R}}\beta\sin\xi + k_{\mathbf{I}}\alpha\sin\eta}{\sqrt{k_{\mathbf{R}}^2\beta^2 - k_{\mathbf{I}}^2\alpha^2 + 2(k_{\mathbf{R}}k_{\mathbf{I}})^2}} \quad (2)$$

$$\vartheta_{\mathbf{I}} = \frac{1}{2}\text{atanh}\left(\frac{2\beta\alpha}{k^2}\right) \quad (3)$$

Eqs. 1 and 2 aim to set a value to $\vartheta_{\mathbf{R}}$ preventing its indetermination. The assumption of $\varphi = 0$ is made for clearness but the case of $\varphi \neq 0$ can be absolutely expanded considering each plane. The formalism pioneered by Frezza et al. [20] can be used to describe a simple inhomogeneous plane, where any field obliquely elliptically polarized, may be expressed as the superposition of two components, a horizontal one (E_{hi} along h_0 direction) and a vertical one (E_{vi} along v_0 direction):

$$\mathbf{E}(\mathbf{r}) = \left[E_{hi}\mathbf{h}_0(\bar{\vartheta}_i,\varphi_i) + E_{vi}\mathbf{v}_0(\bar{\vartheta}_i,\varphi_i)\right]e^{i\mathbf{k}\cdot\mathbf{r}} = \\ = \sum_{m=-\infty}^{+\infty}\left[a_m\mathbf{M}_m(k^*\mathbf{r}) + b_m\mathbf{N}_m(k^*\mathbf{r})\right] \quad (4)$$

Setting the following definitions:

$$a_m = \frac{E_{hi}}{k_\rho}(-i)^{m-1}e^{-im\varphi_i} \quad (5)$$

$$b_m = -\frac{E_{vi}}{k_\rho}(-i)^m e^{-im\varphi_i} \quad (6)$$

$$\mathbf{k}_i = k^*\left(\sin\bar{\vartheta}_i\cos\varphi_i\mathbf{x}_0 + \sin\bar{\vartheta}_i\varphi_i\mathbf{y}_0 + \cos\bar{\vartheta}_i\mathbf{z}_0\right) \quad (7)$$

$$\mathbf{M}_m = \mathbf{m}_m e^{im\varphi}e^{ik_z z}e^{-i\omega t} \quad (8)$$

$$\mathbf{N}_m = \mathbf{n}_m e^{im\varphi}e^{ik_z z}e^{-i\omega t} \quad (9)$$

with:

$$\mathbf{m}_m = im\frac{Z_m(k_\rho\rho)}{\rho}\boldsymbol{\rho}_0 - k_\rho\frac{\partial\rho Z_m(k_\rho\rho)}{\partial\rho}\boldsymbol{\varphi}_0 \quad (10)$$

$$\mathbf{n}_m = i\frac{k_z k_\rho}{k}\frac{\partial\rho Z_m(k_\rho\rho)}{\partial\rho}\boldsymbol{\rho}_0 - \frac{mk_z}{k}\frac{Z_m(k_\rho\rho)}{\rho}\boldsymbol{\varphi}_0 + \\ + \frac{k_\rho^2}{k}Z_m(k_\rho\rho)\mathbf{z}_0 \quad (11)$$

$k_i{}^*$ is the wavenumber k complex conjugate, while ρ_0 and φ_0 are the unit vectors of the system of cylindrical coordinates. In free space, we can take into consideration a multitude of parallel cylinders and then resolve their scattering by the incident field defined above (Fig. 1.a).

Fig. 1.b displays the layout investigated. The scattering by parallel layered cylinders is considered, as depicted in Fig. 1.b,c. L cylinders are taken in the study immersed in a dissipative medium ($\varepsilon_i, \mu_i, \sigma_i \neq 0$) having infinite length and radii r_p. Each p-th cylinder is composed by many layers each characterized by a different relative dielectric constant ε_{pj}, with $p, j = 1,...,N$, and radii r_{pj}. Normally, an incident field to be considered is a wave polarized elliptically. The field laying on the q-th cylinder's surface has to be taken into account to apply the Foldy-Lax Multiple scattering equations; it is named exiting field and it represents the combination of the incident electric field with the fields outgoing from the scatterers: $\mathbf{E}_{ex}^q = \mathbf{E}_i + \sum_{p=1, p\neq q}^{L}\mathbf{E}_s^p$ [17, 18]. The incident field is represented by a vector cylindrical harmonics function, which is centered on the q-th cylinder [2, 3]:

$$\mathbf{E}_i(k_i\boldsymbol{\rho}_q) = \left[E_{v0}\boldsymbol{v} + E_{h0}\boldsymbol{h}\right]e^{i\mathbf{k}_i\cdot\boldsymbol{\rho}_q}e^{i\mathbf{k}_i\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_q)} = \\ = \sum_{m=-\infty}^{+\infty}\left[\tilde{a}_m\mathbf{M}_m^{(1)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_q) + \tilde{b}_m\mathbf{N}_m^{(1)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_q)\right] \quad (12)$$

with

$$\tilde{a}_m = a_m e^{i\mathbf{k}_i\cdot\boldsymbol{\rho}_q} \quad (13)$$

$$\tilde{b}_m = b_m e^{i\mathbf{k}_i\cdot\boldsymbol{\rho}_q} \quad (14)$$

being the incident coefficients converted on the q-sphere. The q-th cylinder has the following exiting field:

$$\mathbf{E}_{ex}^q(k_i\boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty}\left[w_m^q\mathbf{M}_m^{(1)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_q) + v_m^q\mathbf{N}_m^{(1)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_q)\right] \quad (15)$$

whereas, from $p \neq q$-th cylinder the dispersed electric field follows:

$$\mathbf{E}_s^p(k_i\boldsymbol{\rho}_p) = \sum_{m'=-\infty}^{+\infty}\left[T_{m'}^M w_{m'}^p\mathbf{M}_m^{(3)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_p) + \right. \\ \left. T_{m'}^N v_{m'}^p\mathbf{N}_m^{(3)}(k_i,\boldsymbol{\rho}-\boldsymbol{\rho}_p)\right] \quad (16)$$

$T_{m'}^M$ and $T_{m'}^N$ represent the T-matrix coefficients [15, 16] in the case of a stratified cylinder. In this specific scenario, the correct T-matrix coefficients will be calculated applying the alleged T-matrix method setting the boundary conditions on each cylindrical surface [2, 3]. This approach allows to obtain the coefficients T_m^M and T_m^N (See Appendix for further details). Also, the use of the Additional theorem on VCHs function leads to the following:

$$M_{m'}^{(1)}(k,\boldsymbol{\rho}_{0p}) = \sum_{m=-\infty}^{+\infty}B_{mm'}M_m^{(1)}(k,\boldsymbol{\rho}_{0q}) \quad (17)$$

$$N_{m'}^{(1)}(k,\boldsymbol{\rho}_{0p}) = \sum_{m=-\infty}^{+\infty}B_{mm'}N_m^{(1)}(k,\boldsymbol{\rho}_{0q}) \quad (18)$$

$$M_{m'}^{(3)}(k,\boldsymbol{\rho}_{0p} = \sum_{m=-\infty}^{+\infty}A_{mm'}M_m^{(1)}(k,\boldsymbol{\rho}_{0q}) \quad (19)$$

$$N_{m'}^{(3)}(k,\boldsymbol{\rho}_{0p}) = \sum_{m=-\infty}^{+\infty}A_{mm'}N_m^{(1)}(k,\boldsymbol{\rho}_{0q}) \quad (20)$$

with

$$A_{mm'} = H_{m-m'}^{(1)}(k|\boldsymbol{\rho}_{pq}|)e^{-i(m-m')\varphi_{pq}} \quad (21)$$

$$B_{mm'} = J_{m-m'}^{(1)}(k|\boldsymbol{\rho}_{pq}|)e^{-i(m-m')\varphi_{pq}} \quad (22)$$

and with $\boldsymbol{\rho}_{0p} = \boldsymbol{\rho} - \boldsymbol{\rho}_p$, $\boldsymbol{\rho}_{0q} = \boldsymbol{\rho} - \boldsymbol{\rho}_q$ and $\boldsymbol{\rho}_{pq} = \boldsymbol{\rho}_{\boldsymbol{p}} - \boldsymbol{\rho}_q$. All fields are implemented for the FLMSEs, the proprieties of orthogonality of the VCHs is also used to reach the linear system below:

$$w_m^q = \tilde{a}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^M w_{m'}^p \tag{23}$$

$$v_m^q = \tilde{b}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^N v_{m'}^p . \tag{24}$$

Next, the coefficients w_m^q and v_m^q can be determined after the resolution of the linear system. The scattered field of the q-th cylinder can be expressed as a superposition of VCHs:

$$\mathbf{E}_s^q = \sum_{m=-\infty}^{+\infty} \left[e_m^q \mathbf{M}_m^{(3)}(k_i, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + f_m^q \mathbf{N}_m^{(3)}(k_i, \boldsymbol{\rho} - \boldsymbol{\rho}_q) \right] \tag{25}$$

ultimately, finding the total scattered field: $\mathbf{E}_s = \sum_{q=1}^{L} \mathbf{E}_s^q$. The current investigated configuration is constituted by eight parallel two-layered cylinders along their centers located on the circumference of radius 200 nm (see Fig. 2.a). The wavevector and the electric vector connected with the incident field are contained on a plane where all the cylinders' centers fall. Moreover, for the visible range, the scattered field ($E_s(f) = \sqrt{E_{s_x}^2 + E_{s_y}^2 + E_{s_z}^2}$) for 5 distinct values of the relative permittivity of the shell cylinders as a function of the frequency is displayed. It is considered an inhomogeneous wave linearly polarized at the cylinders' axes direction, with $\bar{\vartheta}_i = \pi/6$, and with an intensity of 1 V/m. The solution is determined at $P = (2a, 0, 0)$, where each cylinder radius is $a_1 = 50$ nm and $a_2 = a_1/4$. Some assumptions on electromagnetic parameters have been made up: $\mu_i = \mu_1 = \mu_2$, $\sigma_i = \sigma_1 = \sigma_2 = 0$ S/m, $\varepsilon_i = 1$, $\varepsilon_1 = [2, 3, 4, 5, 6]$, and $\varepsilon_2 = 5$. Fig. 2.b displays how increasing the dielectric permittivity gets both a culmination of the resonance peaks and a red-shift. Ultimately, the electric field dispersed at 600 nm as incident wavelength and $\varepsilon_1 = 3$ is revealed; the other parameters remained the same (see Fig. 2.c).

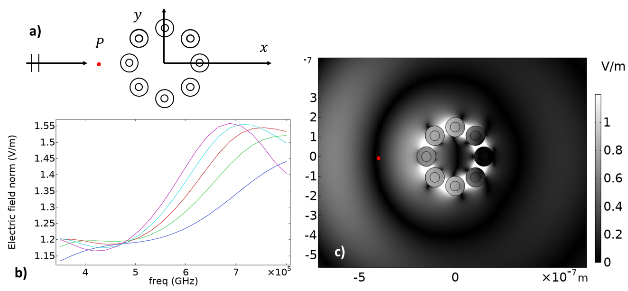

Figure 2. a) Depiction of the problem: eight parallel stratified cylinders distributed along a circumference of radius 200 nm. b) Scattered electric field determined on the point $P = (2a, 0, 0)$ at wavelength of 600 nm for 5 different dielectric constant values ($\varepsilon_1 = [2, 3, 4, 5, 6]$) of the external layer. c) The scattered electric field ($\sqrt{E_{s_x}^2 + E_{s_y}^2 + E_{s_z}^2}$).

Appendix

Let us assume again that the form of the incident electric fields is the following:

$$\mathbf{E}_i(k_i\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[a_m \mathbf{M}_m^{(1)}(k_i\mathbf{r}) + b_m \mathbf{N}_m^{(1)}(k_i\mathbf{r}) \right] \tag{26}$$

The scattered wave resulted can be expressed in a form of series of VCHs:

$$\mathbf{E}_s(k_i\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[e_m \mathbf{M}_m^{(3)}(k_i\mathbf{r}) + f_m \mathbf{N}_m^{(3)}(k_i\mathbf{r}) \right] \tag{27}$$

Moreover, the internal waves for each layer are constituted by traveling waves both inwards and outwards with respect to the center of the cylinder. The superposition of these waves can be applied thanks to the linearity of the problem and, again, they can be expressed as a superposition of VCHs. When the j-th layer is selected, the electric field into the thickness is:

$$\mathbf{E}_j(k_j\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[r_m^j \mathbf{M}_m^{(1)}(k_j\mathbf{r}) + s_m^j \mathbf{N}_m^{(1)}(k_j\mathbf{r}) \right] + \sum_{m=-\infty}^{+\infty} \left[u_m^j \mathbf{M}_m^{(2)}(k_j\mathbf{r}) + v_m^j \mathbf{N}_m^{(2)}(k_j\mathbf{r}) \right] \tag{28}$$

with $j = 1, \ldots, N-1$. Also, a combination of the Bessel cylindrical functions of the first two kinds have been used to express the j-th field, because the spot $\rho = 0$ is outside the domain under consideration. This is why the last internal field needs to be expressed as:

$$\mathbf{E}_N(k_N\mathbf{r}) = \sum_{m=-\infty}^{+\infty} \left[r_m^N \mathbf{M}_m^{(1)}(k_N\mathbf{r}) + s_m^N \mathbf{N}_m^{(1)}(k_N\mathbf{r}) \right] . \tag{29}$$

After the application of the continuity of the electric field on the j-th cylindrical surface, the field can be formulated in the following:

$$\begin{cases} s_m^{j+1} n_{\varphi m}^{(1)}(K_{j+1}) + v_m^{j+1} n_{\varphi m}^{(2)}(K_{j+1}) + r_m^{j+1} m_{\varphi m}^{(1)}(K_{j+1}) + \\ u_m^{j+1} m_{\varphi m}^{(2)}(K_{j+1}) = s_m^j n_{\varphi m}^{(1)}(K_j) + v_m^j n_{\varphi m}^{(2)}(K_j) + \\ r_m^j m_{\varphi m}^{(1)}(K_j) + u_m^j m_{\varphi m}^{(2)}(K_j) \\ k_{j+1}[-s_m^{j+1} m_{\varphi m}^{(1)}(K_{j+1}) - v_m^{j+1} m_{\varphi m}^{(2)}(K_{j+1}) + \\ r_m^{j+1} n_{\varphi m}^{(1)}(K_{j+1}) + u_m^{j+1} n_{\varphi m}^{(2)}(K_{j+1})] = k_j[-s_m^j m_{\varphi m}^{(1)}(K_j) - \\ v_m^j m_{\varphi m}^{(2)}(K_j) + r_m^j n_{\varphi m}^{(1)}(K_j) + u_m^j n_{\varphi m}^{(2)}(K_j)] \\ s_m^{j+1} n_{zm}^{(1)}(K_{j+1}) + v_m^{j+1} n_{zm}^{(2)}(K_{j+1}) = s_m^j n_{zm}^{(1)}(K_j) + \\ v_m^j n_{zm}^{(2)}(K_j) k_{j+1}[r_m^{j+1} n_{zm}^{(1)}(K_{j+1}) + u_m^{j+1} n_{zm}^{(2)}(K_{j+1})] = \\ k_j[r_m^j n_{zm}^{(1)}(K_j) + u_m^j n_{zm}^{(2)}(K_j)] \end{cases}$$

having indicated with $K_j = k_j \rho_j$ and with $K_{j+1} = k_{j+1} \rho_j$. At this point, the linear system using the formalism with matrices can be presented:

$$\begin{pmatrix} s_m^{j+1} \\ v_m^{j+1} \\ r_m^{j+1} \\ u_m^{j+1} \end{pmatrix} = \left[\; A^{-1}B \; \right] \begin{pmatrix} s_m^j \\ v_m^j \\ r_m^j \\ u_m^j \end{pmatrix} = \left[\; M \; \right] \begin{pmatrix} s_m^j \\ v_m^j \\ r_m^j \\ u_m^j \end{pmatrix} \tag{30}$$

The transmission between the layers can now be studied. With $v_m^N, u_m^N = 0$ as a result of the last layer $\rho = 0$, the linear system reached is:

$$\begin{cases} M_{11}e_m + M_{12}a_m + M_{13}f_m + M_{14}b_m = s_{mn}^N \\ M_{21}e_m + M_{22}a_m + M_{23}f_m + M_{24}b_m = 0 \\ M_{31}e_m + M_{32}a_m + M_{33}f_m + M_{34}b_m = r_{mn}^N \\ M_{41}e_m + M_{42}a_m + M_{43}f_m + M_{44}b_m = 0. \end{cases} \quad (31)$$

The resolution of the obtained linear system allows the determination of the unknown scattering coefficients e_m and f_m representing the two T-matrix coefficients $T_m^M = e_m/a_m$ and $T_m^N = f_m/b_m$.

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