Stochastic Fairness and Language-Theoretic Fairness in Planning on Nondeterministic Domains

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Abstract

We address two central notions of fairness in the literature of planning on nondeterministic fully observable domains. The first, which we call stochastic fairness, is classical, and assumes an environment which operates probabilistically using possibly unknown probabilities. The second, which is language-theoretic, assumes that if an action is taken from a given state infinitely often then all its possible outcomes should appear infinitely often (we call this state-action fairness). While the two notions coincide for standard reachability goals, they diverge for temporally extended goals. This important difference has been overlooked in the planning literature, and we argue has led to confusion in a number of published algorithms which use reductions that were stated for state-action fairness, for which they are incorrect, while being correct for stochastic fairness. We remedy this and provide an optimal sound and complete algorithm for solving stateaction fair planning for LTL/LTLf goals, as well as a correct proof of the lower bound of the goal-complexity (our proof is general enough that it provides new proofs also for the nofairness and stochastic-fairness cases). Overall, we show that stochastic fairness is better behaved than state-action fairness.

1 Introduction

Nondeterminism in planning captures uncertainty that the agent has at planning time about the effects of its actions. For instance, "remove block A from the table" may either succeed, resulting in "block A is not on the table", or fail, resulting in "block A is on the table". Plans in nondeterministic environments are not simply sequences of actions as in classical planning; rather, the next action may depend on the sequences of actions (and observations¹) so far, and are captured by policies (also known as strategies and controllers).

Broadly speaking, nondeterminism manifests in one of two ways, stochastic- and adversarial-environments.

Stochastic environments Nondeterministic environments with probabilities are often modeled as Markov Decision Processes (MDPs) in planning. These are state-transition systems in which the probability of an effect depends only on the current state and action. However,

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sometimes the probabilities of action effects are not available, or are non stationary, or are hard to estimate, e.g., a robot may encounter an unexpected obstacle, or an exogenous event or failure occurs. A long thread in this setting aims to understand what it means to plan in such an environment (Daniele, Traverso, and Vardi 1999; Pistore and Traverso 2001; Cimatti et al. 2003; Ghallab, Nau, and Traverso 2016;

D'Ippolito, Rodríguez, and Sardiña 2018). One mon intuition is that the goal should be achievable by trial-and-error while expecting only a finite amount of bad luck (Cimatti et al. 2003), e.g., a policy that repeats the action "remove block A from the table" would eventually succeed under this assumption. This amounts to assuming that some unknown distribution assigns a non-zero probability to each of the alternative effects.² Thus, although there are no explicit probabilities, the stochastic principle is still in place, and we call such assumptions stochastic fairness. Plans in such a setting are called strong-cyclic, and their importance is evidenced by the fact that there are several tools for finding strong-cyclic policies, e.g., NDP (Alford et al. 2014), FIP (Fu et al. 2016), myND (Mattmüller et al. 2010), (Kissmann and Edelkamp 2011), Gamer PRP (Muise, McIlraith, and Beck 2012), **GRENADE** (Ramírez and Sardiña 2014). FOND-SAT and (Geffner and Geffner 2018). Such policies also correspond to ensuring that the goal holds with probability one (Ghallab, Nau, and Traverso 2016; Geffner and Bonet 2013).

Adversarial environments Nondeterministic environments without probabilities are often modeled as fully observable nondeterministic planning domains (FOND). These are state-transition systems in which the effect of an action is a set of possible states, rather than a single state as in classical planning. Policies that guarantee success, i.e., the goal is achieved no matter how the nondeterminism is resolved, are called strong solutions. When handling adversarial nondeterminism it is often reasonable to require that a policy should guarantee success under some additional assumptions about the environment. For instance, a typical

¹In this paper we assume there is no uncertainty about the current state of the system, i.e., environments are fully observable.

²Although reinforcement-learning also makes a similar assumption, it is out of the scope of this work which focuses on non-determinism in model-based control and in planning in particular.

assumption is that repeating an action in a given state results in all possible effects, e.g., repeating the action "remove block A from the table" would eventually succeed (as well as eventually fail). Note that this can be expressed as a property of traces, and so for the purpose of this paper, we call such notions language-theoretic fairness. We focus on one central such notion which we call state-action fairness and which says, of a trace, that if an action a is taken from a state s infinitely often in the trace, and if s' is a possible effect of a from s, then infinitely often in the trace s' is the resulting effect of action a from state s. Although there are many notions of fairness, this particular notion has been identified as providing sufficient assumptions that guarantee the success of solutions that repeatedly retry; see (D'Ippolito, Rodríguez, and Sardiña 2018) where the notion is called state strong fairness.

What is the relationship between fairness in an adversarial setting and fairness in a stochastic setting? On the one hand, the two notions of fairness are similar. Indeed, planning assuming either notion of fairness means that the policy can ignore some traces, which are guaranteed not to be produced by the environment.³ Also, it turns out that when planning for reachability goals (i.e., eventually reach a certain target set of states) the two notions of fairness are interchangeable. More precisely, a policy achieves the reachability goal assuming stochastic fairness (i.e., it is a strongcyclic solution) if and only if it achieves the reachability goal assuming state-action fairness (i.e., the target set is reached on all state-action fair traces). On the other hand, it turns out that the two notions of fairness are not generally interchangeable for planning for temporally extended goals (such as those expressed in linear temporal logic LTL or its finitetrace variant LTL $_f$). It is the purpose of this paper to clarify this fact and study its consequences.

Outline of the paper and contributions In Section 3 we point out the distinction between stochastic fairness and state-action fairness in the context of planning. Once this distinction has been noted, one realizes that there are algorithms (published in IJCAI) for fair planning for temporally-extended goals that, although stated for state-action fairness, are actually correct for stochastic fairness (but do not address state-action fairness at all). The relevant parts of these algorithms are discussed in Section 4. To remedy this, the focus of the rest of the paper is on algorithms and the computational complexity of planning for temporally-extended goals assuming state-action fairness.

In Section 5 we provide a new algorithm for this problem that does not conflate the two notions. We go on to show that the complexity in the goal is in 2EXPTIME, while the complexity in the domain is in 1NEXPTIME.

In Section 6 we provide a proof of the matching 2EXPTIME lower-bound for the goal-complexity. We also discuss the domain-complexity: it is 1EXPTIME-hard already

for reachability goals, leaving a gap between deterministic and nondeterministic exponential time. We also show that our lower bound is proved using a technique that is general enough to give new proofs of the 2EXPTIME-hardness for the goal complexity also for the no-fairness and stochastic-fairness cases.

2 Fair Planning Problems

In this section we define planning domains, temporally extended goals, and isolate the two notions of fairness.

Planning Domains A nondeterministic planning domain is a tuple (St, Act, s_0, Tr) where St is a finite set of states, Act is a finite set of actions, s_0 is an initial state, and $Tr \subseteq St \times Act \times St$ is a transition relation. We will sometimes write Tr in functional form, i.e., $Tr(s,a) \subseteq St$. We say that the action a is applicable in state s if $Tr(s,a) \neq \emptyset$. We assume, by adding a dummy action and state if needed, that for every state there is an applicable action.

For a finite set X let Dbn(X) denote the set of (probability) distributions over X, i.e., functions $d:X\to [0,1]$ such that $\sum_{x\in X}d(x)=1$. An element x is in the support of d if d(x)>0. A stochastic planning domain is a tuple (D,Pr) where $D=(St,Act,s_0,Tr)$ is called the induced nondeterministic planning domain, and Pr, called the probabilistic transition function, is a partial function $Pr:St\times Act\to Dbn(St)$ defined only for pairs (s,a) where a is applicable in s, satisfying that the support of Pr(s,a) is equal to Tr(s,a). Note that stochastic domains are variants of Markov Decision Processes (MDPs). However, MDPs typically have Markovian rewards, while stochastic planning problems may have goals that depend on the history.

We will refer to both nondeterministic and stochastic planning domains simply as domains. Unless otherwise stated, domains are compactly represented, e.g., in variants of the Planning Domain Description Language (PDDL), and thus can usually be represented with a number of bits which is poly-logarithmic in the number of states and actions. In particular, the states are encoded as assignments to Boolean variables \mathcal{F} called *fluents*, thus we have that $St = 2^{\mathcal{F}}$. For symmetry, also the actions are encoded as assignments to Boolean variables A that are disjoint from F, thus we have that $Act = 2^{A}$. Although the literature also contains formalisms for compactly representing stochastic domains (such as Probabilistic PDDL), here we will not be concerned with a detailed formalization of probabilistic transition functions since it is known (as we will later discuss) that probabilities essentially play no role in the stochastic-fair planning problem (formally defined below).

Traces and Policies Let D be a domain. A $trace \ au$ of D is a finite or infinite sequence $(s_0 \cup a_0)(s_1 \cup a_1) \cdots$ over the alphabet $(St \cup Act) = 2^{\mathcal{F} \cup \mathcal{A}}$ where s_0 is the initial state, and $(s_{i-1}, a_{i-1}, s_i) \in Tr$ for all i with $1 \le i < |\tau|$. Moreover, the sequence $s_0s_1 \cdots$ of states is called the path induced by τ . A policy is a function $f: (St)^+ \to Act$ such that for every $u \in (St)^+$ the action f(u) is applicable in the last state of u. Note that policies are history dependent in this paper. A trace τ is generated by f, or simply called an f-

³In the language-theoretic setting, the policy need not succeed on traces that do not satisfy the fairness property; while in the stochastic setting the policy need not succedd on any set of traces whose probability measure is zero.

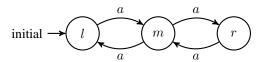


Figure 1: A drawing of the domain *D* from Example 1, used in counterexamples. States are labeled by fluents, and there is a single applicable action.

trace, if for every finite prefix $(s_0 \cup a_0) \cdots (s_i \cup a_i)$ of τ we have that $f(s_0 s_1 \cdots s_i) = a_i$.

A finite-state representation of a policy f is a finite-state input/output automaton that, on reading $u \in (St)^+$ as input, outputs the action f(u). A finite-state policy is one having a finite-state representation.

A stochastic domain D combined with a policy f induces a (possibly infinite-state) Markov chain, denoted (D, f), in the usual way, which gives rise to a probability distribution over the set of infinite f-traces in D (Vardi 1985).

The following domain, illustrated in Figure 1, will be used in counterexamples.

Example 1 Define the domain $D = (St, Act, s_0, Tr)$ where $St = 2^{\mathcal{F}}$ with $\mathcal{F} = \{l, m, r\}$, $Act = 2^{\mathcal{A}}$ with $\mathcal{A} = \{a\}$, $s_0 = \{l\}$, and Tr consist of the triples $(\{l\}, \{a\}, \{m\}), (\{m\}, \{a\}, \{l\}), (\{m\}, \{a\}, \{r\}))$ and $(\{r\}, \{a\}, \{m\})$. Note that the only applicable action (from any state) is $\{a\}$, only three states are reachable from the initial state using this action (i.e., $\{l\}, \{m\}$ and $\{r\}$), and there is only one policy available (it always does the action $\{a\}$). Define the trace τ as $(\{l,a\}\{m,a\}\{r,a\}\{m,a\})^{\omega}$. Note that this trace takes each of the transitions $m \xrightarrow{a} r$ and $m \xrightarrow{a} l$ infinitely often.

Linear Temporal Logic Linear Temporal Logic (LTL) is a formalism that was introduced into the verification literature for describing computations of programs without the use of explicit time-stamps (Pnueli 1977). The logic has since been used in planning as a language for specifying temporally extended goals and for expressing search control, see, e.g., (Fainekos, Kress-Gazit, and Pappas 2005; Bacchus and Kabanza 2000).

The syntax of LTL consists of atoms AP, and is closed under the Boolean operations \neg and \land , and the temporal operators \bigcirc (read "next") and \mathcal{U} (read "until"):

$$\psi ::= p \mid (\neg \psi) \mid (\psi_1 \wedge \psi_2) \mid (\bigcirc \psi) \mid (\psi_1 \mathcal{U} \psi_2)$$

with p varying over the elements of AP.

We use the usual short-hands, e.g., false $:= p \land \neg p$, $\psi_1 \supset \psi_2 := \neg \psi_1 \lor \psi_2$, $\diamondsuit \psi := \mathsf{true} \, \mathcal{U} \, \psi$ (read "eventually ψ "), and $\Box \psi := \neg \diamondsuit \neg \psi$ (read "always ψ ").

Formulas of LTL are interpreted over infinite sequences $\tau = \tau_0 \tau_1 \cdots$ over the alphabet 2^{AP} . Define $\tau, j \models \psi$ inductively on the structure of ψ , simultaneously for all time points $j \geq 0$, as follows:

$$-\tau, j \models p \text{ if } p \in \tau_j,$$

- $-\tau, j \models \psi_1 \land \psi_2 \text{ if } \tau, j \models \psi_i \text{ for } i = 1, 2,$
- $-\tau, j \models \bigcirc \psi \text{ if } \tau, j+1 \models \psi,$
- $-\tau, j \models \psi_1 \mathcal{U} \psi_2 \text{ if } \tau, k \models \psi_2 \text{ for some } k \geq j, \text{ and } \tau, i \models \psi_1 \text{ for all } i \in [j, k).$

We also consider the variant LTL_f of LTL interpreted over finite sequences. It has the same syntax and semantics as LTL except that τ is a finite sequence and that one defines \bigcirc as follows, cf. (Bacchus and Kabanza 2000; Baier and McIlraith 2006; De Giacomo and Vardi 2013):

 $-\tau, j \models \bigcirc \psi$ if $j+1 \leq last(\tau)$ and $\tau, j+1 \models \psi$ where $last(\tau)$ is the last position of τ , i.e., $last(\tau) = |\tau| - 1$ since sequences start with position 0.

If ψ is an LTL (resp. LTL_f) formula and τ is an infinite (resp. finite) sequence over AP, we write $\tau \models \psi$, and say that τ satisfies ψ , iff $\tau, 0 \models \psi$.

We also make the following useful convention that allows us to interpret LTL_f formulas over infinite traces: if τ is infinite and ψ is an LTL_f formula, then $\tau \models \psi$ is defined to mean that some finite prefix of τ satisfies ψ .

In the context of a planning domain D, we will take AP to be $\mathcal{F} \cup \mathcal{A}$ (this is for convenience; some papers take $AP = \mathcal{F}$). We write $(D, f) \models A\psi$, and say that f enforces ψ , if every infinite f-trace of D satisfies ψ .

Planning Problems A goal G is a set of infinite traces of D. A planning problem $\langle D, G \rangle$ consists of a domain D and a goal G. Solving the planning problem is to decide, given D (compactly represented) and G (suitably represented), if there is a policy f such that every infinite f-trace satisfies G (i.e., is in G). In this paper, goals will typically be represented by LTL/LTL f formulas.

Fair Planning Problems We now define the two types of fair planning problems mentioned in the introduction.

A trace τ of a domain D is *state-action fair* if for every transition (s, a, s') of D, if s, a occurs infinitely often in τ then s, a, s' occurs infinitely often in τ . This can be expressed by the following LTL formula:

$$\phi_{D,fair} := \bigwedge_{(s,a,s') \in Tr} \left(\Box \Diamond (s \land a) \supset \Box \Diamond (s \land a \land \bigcirc s') \right) \right).$$

A policy f solves the *state-action-fair planning problem* $\langle D, \psi \rangle$ if every state-action-fair f-trace satisfies ψ , written $(D, f) \models A^{\text{sa-fair}} \psi$.

For a stochastic domain D, we write $(D,f) \models A^{=1}\psi$ to mean that the probability that an f-trace satisfies ψ is equal to 1, and we say that f almost surely enforces ψ . It is known that $(D,f) \models A^{=1}\psi$ does not depend on the probabilistic transition function of D, but only on its induced nondeterministic domain; indeed, it does not depend on the exact distributions Pr(s,a) but only on their supports, which are specified by the transition relation Tr of the induced nondeterministic domain, cf. (Vardi and Wolper 1986). Hence, we can actually extend this probabilistic notion of enforcing also to nondeterministic domains, as follows. For a nondeterministic domain D, we write $(D,f) \models A^{=1}\psi$ to mean that $(D',f) \models A^{=1}\psi$ where D' is any stochastic domain whose induced nondeterministic domain is D. Thus, for a

⁴For a finite string u, we write u^{ω} for the infinite string $uuu \dots$

domain D (nondeterministic or stochastic), we say that a policy f solves the *stochastic-fair planning problem* $\langle D, \psi \rangle$ if $(D, f) \models A^{=1}\psi$.

Connection with Planning for Reachability Goals The classic goal in planning is reachability, typically represented as a Boolean combination target of fluents, i.e., it can be expressed by an LTL/LTL_f formula $\diamond target$. A policy f enforcing $\diamond target$ is known as a strong solution (Cimatti et al. 2003) or an acyclic safe solution (Ghallab, Nau, and Traverso 2016)). A policy enforcing $\diamond target$ assuming state-action fairness is known as a strong cyclic solution (Cimatti et al. 2003) or a cyclic safe solution (Ghallab, Nau, and Traverso 2016)).

Computational Complexity Planning problems have two inputs: the domain (represented compactly) and the goal (typically represented as a formula). Combined complexity measures the complexity in terms of the size of both inputs, while goal complexity (resp. domain complexity) only measures the complexity in the size of the goal (resp. domain). Formally, we say that the *goal complexity is in a complexity* class \mathcal{C} if for every domain D, the complexity of the problem that takes as input a goal ψ and decides if there is a solution to the planning problem $\langle D, \psi \rangle$, is in \mathcal{C} ; and we say that the goal complexity is hard for C if there is a domain D such that the complexity of the problem that takes as input a goal ψ and decides if there is a solution to the planning problem $\langle D, \psi \rangle$ is C-hard. Similar definitions hold for domain complexity. Such measures were first introduced in database theory (Vardi 1982).

Automata-theoretic approach to planning A typical approach for solving planning problems with temporally-extended goals is to use an automata-theoretic approach. Here we recall just enough for our needs in Sections 4 and 5.

A deterministic automaton is a tuple $M = (\Sigma, Q, q_0, \delta, \mathcal{C})$ where Σ is the *input alphabet*, Q is a finite set of *states*, $q_0 \in Q$ is the *initial state*, $\delta: Q \times \Sigma \to Q$ is the *transi*tion function, and C is the acceptance condition (described later). A (finite or infinite) input word $u = u_0 u_1 \cdots$ determines a run, i.e., the sequence $q_0q_1\cdots$ of states starting with the initial state and respecting the transition function, i.e., $\delta(q_{i-1}, u_{i-1}) = q_i$ for all $1 \le i < |u|$. A word is accepted by M if its run satisfies the acceptance condition C. There are a variety of different ways to define the acceptance condition. If M is to accept only finite words, then we typically have $\mathcal{C} \subseteq Q$; and we say that a finite run satisfies \mathcal{C} if its last state is in C. Such an automaton is called a *deterministic* finite word automaton (DFW). If M is to accept only infinite words, then there are a number choices for C. We will not be concerned with the specific choice until Section 5.

The synchronous product of a domain D and a deterministic automaton A over the input alphabet $2^{\mathcal{F} \cup A}$ is a domain, denoted $D \times A$, whose states are pairs (d,q) where d is a state of D and q is a state of A, and that can transition from state (d,q) to state (d',q') on action a if $(d,a,d') \in Tr$ and the automaton can go from q reading $d \cup a$ to q'. Intuitively, $D \times A$ simulates both D and A simultaneously. Such products are used in algorithms for planning with LTL/LTL f

goals in Section 4 and Section 5. We remark that the product is sometimes also compactly represented, although the details depend on the context and will not concern us.

3 Stochastic Fairness $\not\equiv$ State-action Fairness

In this section we compare the two notions of fairness in the context of planning. It turns out that they are equivalent for reachability goals, but not for general LTL/LTL_f goals. The first principle is known, e.g. (Rintanen 2004; Ghallab, Nau, and Traverso 2016), and is repeated here for completeness.

Proposition 1 Let D be a (nondeterministic or stochastic) domain and let target be a Boolean combination of fluents. The following are equivalent for every finite-state policy f:

- 1. $(D, f) \models A^{sa\text{-fair}}(\lozenge target)$, i.e., the target is reached on state-action fair traces.
- 2. $(D, f) \models A^{=1}(\diamondsuit target)$, i.e., the target is reached with probability one.

Proof. Assume that $(D,f) \models A^{\operatorname{sa-fair}}(\diamond target)$. Observe that the state-action fair traces have probability 1, cf. (Vardi and Wolper 1986), and thus, by definition, $(D, f) \models A^{=1}(\lozenge target)$. For the other direction, assume by way of contradiction that 2. holds but 1. doesn't, and pick an infinite state-action fair f-trace τ that doesn't satisfy $\Diamond target$. Let M be the finite-state Markov chain induced by D and f, viewed as a directed graph, and let π be the path in M induced by τ . Since τ is state-action fair, π reaches a bottom strongly connected component C of M, and visits every state in C. By the assumption that $\tau \not\models \Diamond target, \pi$ contains no state in which target holds. Let ρ be some (fixed) prefix of π that ends in a state in C, and consider the set E of infinite f-traces whose induced paths have ρ as a prefix. Observe that the probability of E is positive, and none of the traces in E satisfy $\diamond target$. This contradicts 2.

We now turn to goals expressed as LTL/LTL_f formulas. Unfortunately, in this case the analogue of Proposition 1 does not hold. Indeed, only the forward direction holds.

Proposition 2 Let D be a domain, ψ and LTL/LTL_f formula, and f a finite-state policy. If $(D, f) \models A^{sa\text{-}fair}(\psi)$ then $(D, f) \models A^{=1}(\psi)$.

Proof. As in Proposition 1, simply use the fact that the set of infinite state-action fair f-traces has probability 1.

The next proposition shows that the converse of Proposition 2 does not hold. Intuitively, the reason is that, assuming stochastic fairness, every finite trace that is enabled infinitely often appears with probability 1, while assuming state-action fairness, this is only true for traces of length one.

For the next proposition, recall Example 1.

Proposition 3 There is a domain D, a finite-state policy f, and an LTL_f goal ψ such that $(D, f) \models A^{=1}(\psi)$, but for no policy g does it hold that $(D, g) \models A^{sa-fair}(\psi)$.

Proof. Let D be the domain from Example 1. Let ψ be the LTL/LTL $_f$ formula $\Diamond(l \land \bigcirc \bigcirc l)$ (i.e., eventually l and two steps afterwards l again). There is only one policy f

available: it always chooses the action $\{a\}$. Observe that $(D,f) \models A^{=1}(\psi)$, but that $(D,f) \not\models A^{\operatorname{sa-fair}}(\psi)$ as witnessed by the trace $\tau := (\{l,a\}\{m,a\}\{r,a\}\{m,a\})^{\omega}$.

4 Confusion in the literature

Certain algorithms in the literature for solving state-action fair planning problems with temporally extended goals rely on a reduction to another state-action fair planning problem, that, as we prove, is complete but not sound. The papers, in order of publication, are (Patrizi, Lipovetzky, and Geffner 2013)[Theorem 3], (De Giacomo and Rubin 2018)[Theorem 4] and (Camacho and McIlraith 2019)[Theorem 2]. If one assume stochastic-fairness instead of state-action fairness, then the reduction is both sound and complete. This suggests that the cited algorithms are correct if one assumes stochastic fairness instead of state-action fairness.

The reduction We begin by describing the reduction without any mention of fairness. From a planning problem $\langle D, \psi \rangle$, first define a deterministic automaton A_{ψ} that recognizes exactly the traces that satisfy ψ . Second, define the domain $D' = D \times A_{\psi}$ as the synchronous product of D and A_{ψ} . Finally, define the planning problem $\langle D', Acc \rangle$ where Acc is a goal that captures the acceptance condition of A_{ψ} , i.e., Acc consists of those traces of D' whose first components are traces of D that are accepted by A_{ψ} .

Analsis of the reduction If this reduction is to be used to give an exact algorithm for planning assuming state-action fairness, it should be sound and complete, i.e., $\langle D, \psi \rangle$ is solvable assuming state-action fairness iff $\langle D', Acc \rangle$ is solvable assuming state-action fairness. The reduction is indeed complete because every state-action fair trace in the product domain D' projects to a state-action fair trace in D (this follows immediately from the definition of state-action fairness and of the synchronous product). On the other hand, the reduction is not sound because there may be fair traces in D that do not induce any fair trace in D' (intuitively, this is due to synchronization in D' between the domain D and the automaton A_{ψ}). We formalise this in the following theorem which actually shows that the reduction is not sound no matter which deterministic automaton A_{ψ} for ψ is used.

Theorem 1 There is a domain D, and an LTL/LTL $_f$ goal ψ , s.t. a) there is no solution to the state-action fair planning problem $\langle D, \psi \rangle$, but b) for every deterministic automaton A_{ψ} accepting exactly the traces that satisfy ψ , there is a solution to the state-action fair planning problem $\langle D', Acc \rangle$, where D' is the product of D and A_{ψ} , and Acc captures the acceptance condition of A_{ψ} .

Proof. Let D (resp. au) be the domain (resp. trace) from Example 1, let ψ be the formula $\neg l \lor \Diamond (l \land \bigcirc \bigcirc \neg r) \lor \Diamond (l \land \bigcirc \bigcirc \bigcirc \bigcirc \neg l)$, and observe that all traces of D satisfy ψ except for the trace τ .

There is a single policy f available in D, i.e., always perform the single applicable action. However, τ is a state-action-fair f-trace that does not satisfy ψ . Thus, there is no solution to the state-action fair problem $\langle D, \psi \rangle$.

We claim that the single policy available in D' is a solution to $\langle D', Acc \rangle$. For this, it is enough to show that every state-action fair trace in D' induces in D a trace that satisfies ψ , i.e., a trace other than τ . Let τ' be a trace in D' that induces τ . To see that τ' is not state-action fair, let (m,s) be a state that appears in τ' infinitely often after a state of the form (l,?). Note that (m,s) never appears as a source of a transition to a state of the form (l,?). Indeed, since l occurs on τ exactly every four steps, the source of such a transition is only reached three steps after reading an l; and while reading τ , A_{ψ} is always in a different state than s three steps after reading an l (so not to confuse occurrences of $\neg l$ four steps after an l with ones two steps after it). Thus, some successor (l,q) of (m,s) is enabled infinitely often but never taken.

We note, however, that if one uses stochastic fairness instead of state-action fairness then the reduction above is sound and complete. This is because stochastic-fairness is preserved by taking a product with a deterministic automaton, a fact which is exploited in the automata-theoretic approach to verification of probabilistic systems (Vardi 1985; Courcoubetis and Yannakakis 1995; Bianco and de Alfaro 1995; Bollig and Leucker 2004):

Theorem 2 Let $\langle D, \psi \rangle$ be a planning problem, and let $\langle D', Acc \rangle$ be a planning problem constructed as in the reduction above. There is a policy solving $\langle D, \psi \rangle$ assuming stochastic fairness iff there is a policy solving $\langle D', Acc \rangle$ assuming stochastic fairness.

In summary, we conjecture that some errors in the proofs and algorithms for state-action fair planning in the literature arise from the mistaken intuition that state-action fairness always behaves like stochastic fairness, which it does not in the presence of even simple LTL/LTL_f formulas (that are not reachability formulas).

5 Algorithm for State-action Fair Planning

In the previous section we showed that some algorithms in the literature for state-action fair planning for temporally extended goals use complete but unsound reductions. In this section, we provide a sound and complete reduction to the problem of solving *Rabin games* (defined below).

Theorem 3 The combined (and thus goal) complexity of solving planning with LTL/LTL_f goals assuming state-action fairness is in 2EXPTIME, and the domain complexity is in 1NEXPTIME (in the size of a compactly represented domain).

The main approach to solving such a problem is to use, explicitly or implicitly, an automata-theoretic approach. However, as we now remark, naive applications of this approach yield a 3EXPTIME domain-complexity (which we then show how to lower to 1NEXPTIME), a 3EXPTIME combined-complexity (which we then show how to lower to 2EXPTIME), and a 2EXPTIME goal complexity.

 $^{^5 \}text{The representation of } Acc$ is induced by the acceptance condition of A_{ψ} , but here the specific representation of Acc is not relevant

Remark 1 The problem of solving the state-action fair planning problem $\langle D, \psi \rangle$ where ψ is an LTL/LTL $_f$ formula is equivalent to solving the planning problem $\langle D, \phi_{D,fair} \supset \psi \rangle$ where

$$\phi_{D,fair} := \bigwedge_{(s,a,s') \in Tr} (\Box \Diamond (s \wedge a) \supset \Box \Diamond (s \wedge a \wedge \bigcirc s')))$$

is an LTL formula expressing state-action fairness in the domain D (for more on this equivalence see (Aminof et al. 2019)). However, the size of $\phi_{D,fair}$ is exponential in the size of D (compactly represented). Thus, we have reduced the problem to solving planning for an LTL goal of size exponential in the size of D and linear in the size of ψ . In turn, there are algorithms that solve planning with LTL goals (no fairness assumptions) that run in 1Exptime in the size of the domain and 2Exptime in the size of the goal (Aminof et al. 2019; Camacho, Bienvenu, and McIlraith 2019). Putting this together results in an algorithm for the state-action-fair planning problem that runs in 3Exptime in the size of the domain D and 2Exptime in the size of the formula ψ .

The main insight that achieves the complexities in Theorem 3 is that one should use Rabin conditions.

A Rabin condition over a set X is a set \mathcal{R} of pairs of the form (I,F) with $I,F\subseteq X$. The pairs are called Rabin pairs. An infinite sequence τ over the alphabet X is said to satisfy the Rabin condition \mathcal{R} if there is a pair $(I,F)\in\mathcal{R}$ such that some $x\in I$ appears infinitely often in τ and no $x\in F$ appears infinitely often in τ . Below we use Rabin conditions in two ways: as acceptance conditions (for automata) and as winning conditions (in games).

Rabin Automata A Deterministic Rabin Word (DRW) automaton is an automaton $M=(\Sigma,Q,q_0,\delta,\mathcal{R})$ where the acceptance condition \mathcal{R} is a Rabin condition over Q. The size of a DRW is the number of its states and its index is the number of pairs in \mathcal{R} .

The reader may be wondering why we chose the Rabin acceptance condition instead of some other acceptance condition. The reason is: they can capture very general properties, including LTL/LTL_f ; they are naturally closed under union; they can naturally express that a trace is not state-action fair.

Theorem 4 [cf. (Vardi 1995b)] Given an LTL/LTL_f formula ψ one can build a DRW M_{ψ} that accepts exactly the infinite traces satisfying ψ . Moreover, M_{ψ} has size 2exp and index 1exp in $|\psi|$.

Lemma 1 Given a domain D one can build a DRW $M_{D,unfair}$ that accepts exactly the infinite traces of D that are not state-action fair. Moreover, $M_{D,unfair}$ has size and index 1exp in the size of D (compactly represented).

To see this, let the states of the DRW store the last state-action and last state-action-state of D, and the Rabin pairs are of the form $(\{sa\}, \{sas'\})$ for Tr(s, a, s').

Lemma 2 Given DRW M_1 , M_2 one can build a DRW M, denoted $M_1 \vee M_2$, that accepts the words accepted by M_1 or M_2 . The size of M is the product of the sizes of the M_i s, and the index of M is the sum of the indices of the M_i s.

To see this, if $M_i = (\Sigma, Q_i, q_i, \delta_i, \mathcal{R}_i)$ define $M = (\Sigma, Q_1 \times Q_2, (q_1, q_2), \delta', \mathcal{R}')$ where $\delta'((s_1, s_2), \sigma) = (\delta_1(s_1, \sigma), \delta_2(s_2, \sigma))$, and \mathcal{R} consists of all pairs of the form $(Q_1 \times I, Q_1 \times F)$ for $(I, F) \in \mathcal{R}_2$ and all pairs of the form $(I \times Q_2, F \times Q_2)$ for $(I, F) \in \mathcal{R}_1$.

Rabin Games The other use for the Rabin condition is to give winning conditions in games. A *Rabin game* is an explicitly represented planning problem whose goal is expressed as a Rabin condition \mathcal{R} over the set St of states.

Theorem 5 (Buhrke, Lescow, and Vöge 1996;

Emerson and Jutla 1988) There is an algorithm that solves Rabin games in time $O(d!n^dm)$ where d is the number of Rabin pairs, n is the number of states, and m is the number of transitions. In addition, solving Rabin games is NP-complete (in the size of the explicit representation).

Reduction and Algorithm We can now describe the algorithm promised in Theorem 3. Given a state-action fair planning problem $\langle D, \psi \rangle$, reduce it to the problem of solving the Rabin game G = (Ar, Acc) constructed as follows. The arena Ar is defined as the synchronous product of the domain D, explicitly represented, and the DRW $M = M_{D,unfair} \vee M_{\psi}$. The Rabin winning condition Acc is induced by the Rabin acceptance condition \mathcal{R} of M, i.e., Acc consists of all pairs of the form $(S \times I, S \times F)$ for $(I,F) \in \mathcal{R}$.

This completes the description of the reduction. To see that it is sound and complete, simply note that a policy f solves the state-action fair planning problem $\langle D, \psi \rangle$ iff every fair f-trace in D is accepted by the DRW M iff every trace (fair and not-fair) in G generated by the strategy that maps $(s_0,q_0)(s_1,q_1)\cdots(s_n,q_n)$ to the action $f(s_0s_1\cdots s_n)$ satisfies the Rabin condition Acc. The first iff is due to Theorem 4, and the second iff follows from the definition of Rabin condition and of the synchronous product.

For the complexity analysis, simply note that the DRW $M_{D,unfair} \lor M_{\psi}$ has size 1exp in the size of D (compactly represented) and 2exp in $|\psi|$, and index 1exp in D (compactly represented) and 1exp in $|\psi|$. Now apply Theorem 5 to get the stated goal, combined, and domain complexities.

6 Lower bounds for state-action fair planning

We showed that state-action fair planning for temporally extended goals has 2EXPTIME combined-complexity and goal-complexity, and 1NEXPTIME domain complexity. In this section we study lower bounds for the problem and show that we can match the 2EXPTIME goal complexity (with a technique that also supplies new proofs of 2EXPTIME goal complexity for the cases of no-fairness and stochastic fairness). For domain-complexity, we observe that existing results show the problem is 1EXPTIME-hard. This leaves open whether the domain complexity can be lowered from 1NEXPTIME to 1EXPTIME.

⁶The reader might find it helpful to read the Rabin condition in LTL notation: $\bigvee_{(I,F)\in\mathcal{R}}\Box\Diamond I\wedge\neg\Box\Diamond F$.

⁷Recall that we define that an infinite trace satisfies an LTL_f formula ψ if some prefix of it satisfies ψ .

Domain-complexity It is not hard to establish a 1EXPTIME lower-bound for the domain complexity. Indeed, one can reduce the problem of stochasticfair planning with reachability goals, which is be 1EXPTIME-complete (Littman 1997; known to Rintanen 2004). Indeed, introduce a fresh fluent p and fix the goal $\Diamond p$. Then, for a stochastic-fair planning problem with domain D and reachability goal $\Diamond target$, build a new domain D_p from D by adding the fluent p and a new action with precondition target and postcondition p. Then the stochastic-fair problem $\langle D, \diamondsuit target \rangle$ has a solution iff the stochastic-fair problem $\langle D_p, \Diamond p \rangle$ has a solution. Moreover, the latter holds iff it has a finite state solution. By Proposition 1, this is equivalent to the fact that the state-action fair problem $\langle D_p, \Diamond p \rangle$ has a solution.

Goal complexity The contribution of this section is a proof of the following theorem.⁸

Theorem 6 The goal complexity (and therefore, also combined complexity) of planning for LTL/LTL $_f$ goals assuming state-action fairness is 2EXPTIME-hard.

Inspired by (Courcoubetis and Yannakakis 1995),we provide a polynomial-time construction that, given an alternating EXPSPACE Turing machine M and an input word x, produces a probabilistic domain D (explicitly represented) and an LTL formula Φ such that M accepts x iff $\exists f.(D,f) \models A\Phi$. Note that to handle the goal complexity, the domain D will be independent of M and x.

Notation. An alternating Turing machine is a tuple $(Q, \Sigma, \Delta, q_0, q_a, q_r)$ where Q is the set of states partitioned into Q_\exists and Q_\forall (called the existential and universal modes), Σ is the tape-alphabet, $\Delta \subseteq (\Sigma \times Q)^2 \times \{L, R, N\}$ is the transition relation, and $q_0 \in Q$ is the initial state, $q_a, q_r \in Q$ are the accepting and rejecting states. A configuration is a string matching the expression $\Sigma^* \cdot (\Sigma \times Q) \cdot \Sigma^*$; it is initial (resp. accepting, rejecting) if the state is q_0 (resp. q_a, q_r). A computation of M is a sequence of configurations, starting in an initial configuration, respecting the transition relation, and ending in an accepting or rejecting state. Wlog, we assume that the existential and universal modes of M strictly alternate, with the existential going first.

Say M runs in space $2^{p(|x|)}$ for some polynomial $p(\cdot)$. In particular, a configuration of M running on x has length at most $2^{p(|x|)}$. Let n:=p(|x|). Intuitively, the domain D ensures that the agent and the environment generate strings of the form

$$C_0 \cdot (\# \cdot T_1 \cdot \#' \cdot C_1 \cdot \#'' \cdot K_1) \cdot (\# \cdot T_2 \cdot \#' \cdot C_2 \cdot \#'' \cdot K_2)$$
$$\cdots (\# \cdot T_j \cdot \#' \cdot C_j \cdot \#'' \cdot K_j) \cdot \# \cdot \bot \cdot \bot \cdot \bot \cdot \bot \cdots$$

where the C_i s are arbitrary strings over $\{0, 1, \%, \$\}$, the T_i s and K_i s are arbitrary strings over $\{0, 1\}$, and \bot is a special symbol. Intuitively, the C_i s will encode configurations

of M and are generated by the agent, the T_i s will encode transitions of M and are generated by the agent for odd iand the environment for even i, and the K_i s are generated by the environment and encode a position/index $k \in [1, 2^n]$ on the tape that the environment wants to check. Finally, \perp holds in a sink of the domain that the agent can go to when it is done. Note that this allows the agent to never go to the sink, but such traces will be rejected by the goal formula. We define the LTL_f goal $\Phi := \Phi_{Env} \supset \Phi_{Ag}$. Intuitively, Φ will enforce that as long as the environment encodes its parts correctly (i.e., Φ_{Env} holds), then so does the agent, and the accepting state is reached (i.e., Φ_{Aq} holds). The formula $\Phi_{Ag} := \Phi_{conf} \wedge \Phi_{tran}^{odd} \wedge \Phi_{chal} \wedge \Phi_{acc}$, and $\Phi_{Env} := \Phi_{num} \wedge \Phi_{tran}^{even}$, where Φ_{conf} says that each C_i encodes a configuration, with C_0 encoding the initial configuration; Φ_{tran}^{odd} (resp. Φ_{tran}^{even}) says that each T_i with i odd (resp. even) encodes a transition of M; Φ_{num} says that each K_i encodes a number in $[1, 2^n]$; and Φ_{acc} says that an accepting configuration is reached; Φ_{chal} (think of it as a "challenge") is used to check that the kth letter in the configuration encoded by C_i is the result of applying transition T_i to the configuration C_{i-1} . Intuitively, since the environment is adversarial, all possible positions will be challenged and all universal transitions will be taken. Thus, the agent will be able to enforce the goal iff M accepts x. We now provide details on how to write the subformulas of Φ .

Choose a sufficiently large integer m to encode all members of Q and $\Sigma \times Q$ as a binary string of length exactly m. Let SYM denote a set of binary strings of length m that encode either a tape-letter l or a tape-letter/state pair (l, q). Let bin(i) denote the binary string of length n whose numeric value is i. The possible configurations of M are encoded by the strings of the form $s(\% \cdot bin(0) \cdot \$ \cdot SYM) \cdot (\% \cdot$ $bin(1) \cdot \$ \cdot SYM) \cdots (\% \cdot bin(2^n - 1) \cdot \$ \cdot SYM)$ which have exactly one symbol encoding a tape-letter/state pair (l, q). The reason for the bin(i)s is they allow the formula to check if an encoding of one configuration can be reached in one step of M from an encoding of another. We call the substring $(\% \cdot bin(i) \cdot \$ \cdot w)$ the *i*th block, where *i* is the *block* number and w is the block symbol. One can write an LTL_f formula conf, of size linear in n and the size of M, that enforces this structure. Indeed, using a standard encoding of the binary counter on n-bit strings, the formula says that exactly one symbol encodes a tape-letter/state pair, and all the other symbols encode just tape-letters. It also says that $bin(0) = 0^n$, $bin(2^n - 1) = 1^n$, and for every $j \le n$, the jth bit in a block is flipped in the next block iff all bits strictly lower in this block are 1s. Thus, the formula Φ_{conf} can be defined as init $\wedge \square(\#' \supset \bigcirc conf)$ where init is a formula that encodes the initial configuration (which can be hard-coded by a polynomial sized formula by explicitly specifying the first |x| blocks, and that the rest of the blocks in the configuration contain the encoding of the blank tape symbol). Writing linearly-sized LTL $_f$ formulas $\Phi_{tran}^{odd}, \Phi_{tran}^{even}, \Phi_{num}$, and Φ_{acc} poses no particular problem.

It remains to show how to build the formula Φ_{chal} . It will be the conjunction of two formulas Φ^1_{chal} and Φ^2_{chal} . The first handles the first challenge, and the second handles all

⁸The result is stated in (De Giacomo and Rubin 2018) for LTL_f but with an incorrect proof. The error there is concluding that every f-trace visits each state at most once. This is true for memoryless strategies, but need not be true for other strategies which might be required when planning for temporally extended goals.

the rest. We now show how to build the second (the first is similar). Define Φ_{chal}^2 as:

$$\Box \bigwedge \left[\left(\operatorname{cha} \wedge \operatorname{cur}_x \wedge \operatorname{nx}_y \wedge \operatorname{nxnx}_z \wedge \operatorname{tr}_t \right) \supset \operatorname{img}_{y'} \right]$$

where the conjunction is over tuples (x,y,z,t,y') such that applying the transition t to the triple of tape-contents xyz (including a possible state) results in the tape content y' of the middle cell (e.g., for t=(q,l,q',l',R), if x=(l,q) then y'=(y,q'), if x=l then y'=y, etc.). Intuitively, cha expresses that we are currently at the start of a block of a configuration, say C_i , whose number is one less than the challenge number encoded by K_{i+1} ; the formula cur_x expresses that the symbol in the current block is x; the formula nx_y expresses that the symbol in the next block is y; the formula nx_x expresses that the symbol in the block after that is z; the formula tr_t says that T_{i+1} encodes the transition t; and the formula $\operatorname{img}_{y'}$ says that the block whose number is encoded by K_{i+1} in the configuration C_{i+1} is y'.

We use the following shorthand, that can scan the string for patterns: define $\phi_1 \mathcal{J}^1 \phi_2 := (\neg \phi_1) \mathcal{U}(\phi_1 \wedge \bigcirc \phi_2)$ and $\phi_1 \mathcal{J}^2 \phi_2 := \phi_1 \mathcal{J}^1(\phi_1 \mathcal{J}^1 \phi_2)$. Intuitively, $\phi_1 \mathcal{J}^i \phi_2$ means ϕ_2 holds one step after the *i*th occurrence of ϕ_1 .

Formally, define cha as:

$$\% \wedge \left[(\%) \mathcal{J}^2 \left(\bigwedge_{i \in [0,n)} \bigwedge_{b \in \{0,1\}} \left[\bigcirc^i b \iff \#'' \mathcal{J}^2 \bigcirc^i b \right] \right) \right].$$

Define cur_x as: $(\$)\mathcal{J}^1(\wedge_{i:0\leq i< m}\bigcirc^i b_i)$ where $b_1b_2\cdots b_m$ encodes the symbol x, and nxn_x as imilarly. Define tr_t as: $(\#)\mathcal{J}^1(\wedge_{i:0\leq i< m}\bigcirc^i t_i)$ where $t_1t_2\cdots t_m$ encodes the symbol t. Define $\operatorname{img}_{y'}$ as:

$$[\#']\,\mathcal{J}^1\left[(\mathrm{match}\supset\bigcirc^n\wedge_{i:0\leq i< m}\bigcirc^i y_i')\,\mathcal{U}\,\#''\right]$$
 where $y_1'y_2'\cdots y_m'$ encodes the symbol y' , and match is

$$\bigwedge_{i \in [0,n)} \bigwedge_{b \in \{0,1\}} (\bigcirc^i b \iff (\#'') \mathcal{J}^1(\bigcirc^i b)).$$

Intuitively, it says that in the next configuration, if a block number equals the challenge number, then the block symbol should be y'. This completes the construction of the goal Φ . This completes the proof for the case of no fairness.

For stochastic and state-action fairness, observe that a) if M accepts x then, already with no fairness assumptions, there is a solution, and b) if M rejects x, then for every policy f, the environment can, within a finite number of steps, prevent any hope of satisfying the goal: either by exposing that the agent is cheating in the simulation, or by reaching a rejecting configuration. Since every finite f-trace can be extended to a fair infinite f-trace, the policy f is not a solution to the state-action fair planning problem, nor is it a solution to the stochastic fair planning problem since the set of infinite f-traces that extend this finite f-trace has positive probability.

7 Related Work and Discussion

We have discussed how the distinction between stochasticand state-action fairness is so-far missing from the planning/AI literature. On the other hand, as we now discuss, this distinction is present in the verification literature. Related work in verification Early work in verification was motivated by the problem of providing formal methods (such as proof-systems or model-checking algorithms) to reason about probabilistic concurrent systems. As such, some effort was made to abstract probabilities and capture stochastic fairness by language-theoretic properties. In fact, sophisticated forms of language-theoretic fairness were introduced to do this (Pnueli and Zuck 1993; Baier and Kwiatkowska 1998), since simple language-theoretic notions (similar to state-action fairness) were known not to capture stochastic fairness (Pnueli 1983).

A comprehensive study of fairness in reactive systems is provided in (Völzer and Varacca 2012) where fairness is characterized language-theoretically, game-theoretically, topologically, and probabilistically. Fairness is used in verification of concurrent systems in order to prove liveness properties, i.e., that something good will eventually happen. The limitations of fairness for proving liveness properties, as well as ways to overcome these limitations, are analysed in (van Glabbeek and Höfner 2019).

The verification literature on probabilistic concurrent programs typically considers policies as schedulers. In particular, the central decision problem there is different to the planning problem: it asks whether *every* (rather than *some*) policy *f* almost-surely enforces the temporally-extended goal (Vardi 1985; Pnueli and Zuck 1993; Bianco and de Alfaro 1995; Courcoubetis and Yannakakis 1995). Just as we used Rabin conditions to capture state-action unfair traces, one can use the dual Street condition to capture stochastically-fair traces (Vardi 1985).

Generalizations of strong-cyclic solutions, other than those that use state-action fairness, have been studied using automata-theory. For instance, (Pistore and Vardi 2007) consider that f is a solution if every finite f-trace can be extended to an infinite f-trace satisfying the given LTL formula. However, such a notion is different from a solution assuming state-action fairness (the example in Proposition 3 shows this). Also, (Vardi 1995a) studies a variation of LTL synthesis assuming a fair scheduler, where the transitions over a given set of states are assumed to be implicitly encoded in LTL.

Discussion While stochastic fairness admits well-behaved algorithms, it is not clear that language-theoretic fairness does. For the moment, even for the case of LTL_f goals, our algorithm (Section 5) requires automata over infinite traces to deal with state-action fairness, which itself is a property of infinite traces. Unfortunately, algorithms for automata over infinite traces are not as easy to implement as for finite traces (Fogarty et al. 2013). Nonetheless, we hope that our new algorithm, which suggests the importance of planning for Rabin goals, spurs the planning community to devise translations and heuristics for solving these.

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