Absorption and invisibility of a Transparent Radial Anisotropy (TRA) cylinder

Batool Sidra *, Nisar Mehwish *, Mangini Fabio †, Frezza Fabrizio *
*Department of Information Engineering, Electronics and Telecommunications,
“La Sapienza” University of Rome, Via Eudossiana 18, 00184 Rome, Italy
†Department of Information Engineering, University of Brescia,
Via Branze 38, 25123 Brescia, Italy
*sidra.batool@uniroma1.it

***Abstract*—This article presents the inspection of a Transparent Radially Anisotropic (TRA) cylinder with permittivity dyadic, in the electrostatic settings. The quasistatic polarizability of the TRA cylinder with the values and signs of permittivity components offering different interesting features, involving, absorbance, anomalous losses, resonant singularities, invisibility. In this manuscript, we have a special focus on the electrostatic response of a circular cylinder with a certain condition of anisotropic parameters. We study the stimulating behaviour of polarizability of a TRA cylinder in quadrant (plane geometry). We have also computed the TRA cylinder behavior as a cloak using suitable choices of the anisotropic parameters.**

***Index Terms*—Anomalous losses, resonant singularities, invisibility**

I. Introduction

Countless research work has been done in the field of electrostatic problems involving a TRA cylinder in the recent year [1], [2], [3], [4], [5]. The electromagnetic invisibility using the anisotropic material distribution of the constitutive parameters were initially suggested by Pendry *et. al.* [6]. Alu *et. al.* [7] purposed the isotropic plasmonic coatings to make invisible objects involving the Spatial Transformation Process (STP). Since, the cloaking invisibility using anisotropic materials [8] and scattering from metamaterials [9], [10], have also been considered in several papers [11]. The full-wave electromagnetic scattering by infinitely long cylinders with anisotropic coatings has been inspected in a metamaterials research work [12]. Tarn et al. reported the realization of heat conduction in a cylindrically anisotropic circular tube. They studied the limits of the effective (heat) conductivity with the anisotropic materials [13].

Kettunen *et. al.* found the use of positive permittivity only extreme anisotropic components, which would design an invisible Faraday shield that could easily hide any objects regardless of their geometry or material properties [14]. Hyperbolic metamaterials are usually defined as simple Cartesian geometries. The hyperbolic RA cylinder shows anomalous lossy responses in both dynamic and static cases. Qiu *et. al.* have also described the anomalous enhancement of absorption, or damping effects [15]. In the theory, anomalous resonances entirely assemble plasmonic nanostructures, can lead to an infinite efficiencies of absorption or gain over the transmission capacity. Alu *et. al.* studied the analogous absorption efficiency using an analytical solution of conjoined half-cylinder [16].

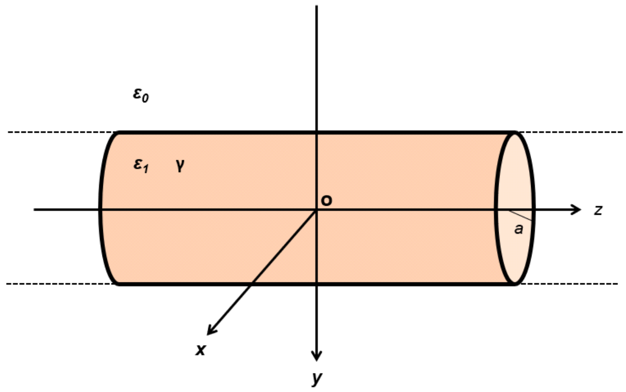

Fig. 1. Anisotropic cylinder is present in a free space

In this manuscript, we study the authentic description for finding anomalous absorption, resonances, and invisibility cloaking in a static case of a TRA cylinder. We have also visualized the imaginary losses and cloaking of a TRA cylinder using COMSOL Multiphysics 5.4.

II. Methodology

A. Electrostatic solution

The generalized Laplace's equation for anisotropic source free medium is [14]

$$\nabla.(\bar{\bar{\varepsilon}}.\nabla\phi) = 0 \quad (1)$$

whose solution can be obtained for any usual coordinate system using separation of variables. Permittivity that is given in $2D$ polar coordinates is

$$\bar{\bar{\varepsilon}} = \varepsilon_0(\varepsilon_\rho \mathbf{u}_\rho \mathbf{u}_\rho + \varepsilon_\phi \mathbf{u}_\phi \mathbf{u}_\phi) \quad (2)$$

where the permittivity of the tangential and azimuthal components are ε_ρ, ε_ρ. In the case of circular/axial symmetric RA cylinder shown in Fig. 1, general solution to the above equation is

$$\phi(\rho, \varphi) = R(\rho)T(\varphi) \quad (3)$$

where the component functions are

$$R(\rho) = A\rho^{\gamma} + B\rho^{-\gamma} \tag{4}$$

$$T(\varphi) = C\cos(m\varphi) + D\sin(m\varphi) \quad \text{m=0,1,2,....} \tag{5}$$

and

$$\gamma = \sqrt{\frac{\varepsilon_t}{\varepsilon_r}} \tag{6}$$

Generally, constants A, B, C, and D can be determined using boundary conditions at the interface and an infinite sum over $m = 0, 1, 2...$. For the isotropic case ($\varepsilon_t = \varepsilon_r$), we get $\gamma = 1$.

B. Intact RA cylinder

Let us derive the solution for an intact RA cylinder assuming positive permittivity. For this, consider a circular RA cylinder of radius a placed in an external uniform x-polarized electric field. The corresponding external potential is [17], [18]

$$\phi_o(\rho,\varphi) = -E_o\frac{\rho}{a}\cos\varphi \tag{7}$$

The required solution in our case is

$$\phi(\rho,\varphi) = \begin{cases} \phi_{in}(\rho,\varphi), & \rho < a \\ \phi_s(\rho,\varphi) + \phi_i(\rho,\varphi), & \rho > a \end{cases} \tag{8}$$

where the internal and scattered potentials are

$$\phi_{in}(\rho,\varphi) = A(\frac{\rho}{a})^{\gamma}\cos\varphi \tag{9}$$

$$\phi_s(\rho,\varphi) = B(\frac{\rho}{a})^{-1}\cos\varphi \tag{10}$$

Applying boundary conditions at $\rho = a$ yields

$$A = \frac{-2E_o}{\gamma\varepsilon_r + 1} \qquad B = \frac{(\gamma\varepsilon_r - 1)E_o}{\gamma\varepsilon_r + 1} \tag{11}$$

Let's define here 2D normalized polarizability as the ratio between induced dipole moment p and the external electric field $\mathbf{E_o}$. The scattered potential ϕ_s is equivalent to potential given rise by x-polarized line dipole having dipole moment $p = 2\pi a\varepsilon_o B$ at the origin. We can obtain the polarizability using coefficient B given by Eq. 10 as

$$\alpha = 2\frac{B}{E_o} \tag{12}$$

The normalized polarizability of RA circular cylinder can be written as

$$\alpha = 2\frac{\varepsilon_{eff} - 1}{\varepsilon_{eff} + 1} \tag{13}$$

C. Numerical results and Discussion

Let us define the solution of effective permittivity of RA cylinder [3].

$$\varepsilon_{eff} = \varepsilon_r\gamma = \sqrt{\varepsilon_r\varepsilon_t} \tag{14}$$

This solution is straight forward, if both permittivity components ε_r and ε_t are real and positive even when both are real and negative as well. However, for the special case $\varepsilon_{eff} = -1$, where the polarizability should be infinity. Fig. 2, represents the solution of anisotropic ratio γ as a function of the positive and negative permittivity components. For the case of real part, when the ratio of $\varepsilon_t/\varepsilon_r < 0$, the parameter γ and $-\gamma$ are equal to zero. Similarly, when $\varepsilon_t/\varepsilon_r > 0$ the imaginary part of γ and $-\gamma$ are always zero. So for our independent choice of singularity in potential function is suitable at the origin. When anyone of the two permittivity components is negative hence, the parameter γ is complex. The anisotropy of γ and $-\gamma$ having an opposite sign, which means they are complex conjugates of each other, that undergoes to complex conjugate pairs of ε_{eff}. In this way, we can easily choose the passive (lossy) solution as compare to active one. However, polarizability becomes complex, when the γ is purely imaginary for the hyperbolic case of $\varepsilon_t/\varepsilon_r < 0$.

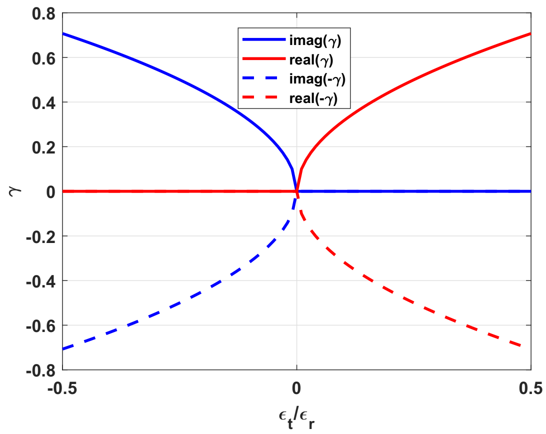

Fig. 2. The exponent γ as a function of ϵ_t/ϵ_r. The dashed line is the inverted solution $-\gamma$.

Let's study the normalized polarizability α given by Eq 12, for any real values of ε_r and ε_t and observe interesting behavior of $\varepsilon_r\varepsilon_t$ in a (plane geometry). The polarizability is singular, when $\varepsilon_r < 0$, corresponding to the III (-; -) quadrant in a (plane geometry), Indeed, $\alpha = 0$ implies the well defined solution, which can be represented in the first I (+; +), quadrant by taking positive real values of ε_r and ε_t in a plane geometry illustrated in Fig. 3.

Let us study the remarkable role of the anisotropy ratio γ with the corresponding radial and tangential permittivities. For that, we deal with a single anisotropic cylinder present in a free space using COMSOL (Multiphysics 5.4) illustrated in Fig. 4. Let us consider the real positive values of $\varepsilon_r = 5$; $\varepsilon_t = 3/5$. In this case, the value of $\gamma = 0.346$ is very small. We have investigated the response of electrostatic potential. The potential is strong at the center of the cylinder. Field lines were easily transmitted from the TRA cylinder. Similarly, When we have also considered the negative values of $\varepsilon_r = -5$; $\varepsilon_t = 3/5$. The value of anisotropy ratio is imaginary $\gamma = 0.346i$. We have observed more dense lines and somehow losses shown in Fig. 5.

So, the potential of the cylindrical core corresponds to the element ρ^{γ}, which allows an imaginary part does not approach zero at the center of the cylinder. But we have inspected high oscillation at $\gamma \to 0$. At the origin, electric field becomes

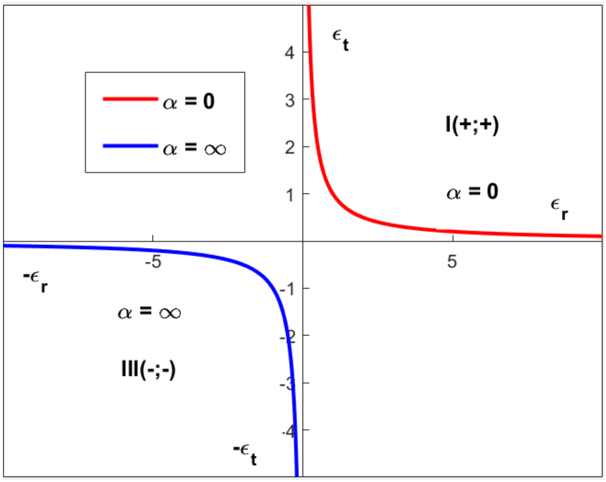

Fig. 3. Interesting behaviour of polarizability in a plane geometry with the corresponding values of ε_r and ε_t.

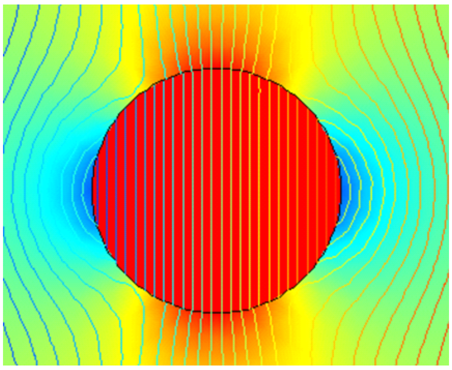

Fig. 4. Electrostatic response of a TRA cylinder using real numerical value of anisotropic ratio γ.

singular with the absence of the losses.

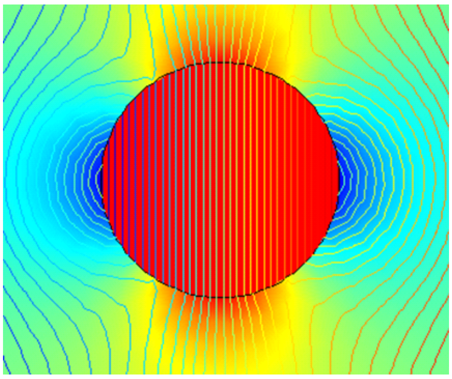

Fig. 5. Electrostatic response of a TRA cylinder using imaginary numerical value of anisotropic ratio γ.

Meanwhile, a layer model with an anisotropic cylindrical shell is used to cloak an inner isotropic core. We have taken some parameters for the outer anisotropic shell such as $\varepsilon_r = 0.1$; $\varepsilon_t = 5.5$. The value of anisotropy is $\gamma = 7.146$. We have visualized the inner core is invisible with a coating of an anisotropic shell illustrated in Fig. 6.

A simple content can be utilized for hiding an inner core of a cylinder. With a large value of ε_r, the field becomes active in the origin presents in Fig. 4. Indeed, when the value of ε_r close to zero, and the large value of ε_t, the field is strong on the outer boundary of the cylindrical shell, but in the origin potential becomes zero has been presented in Fig. 6.

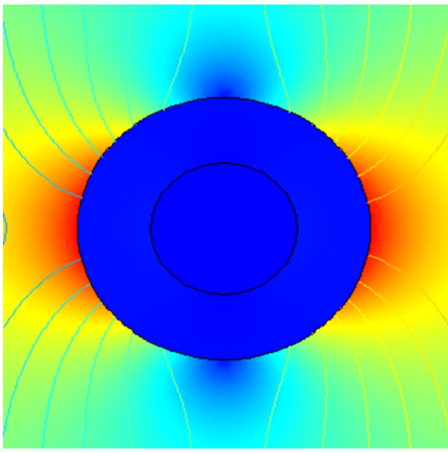

Fig. 6. Invisibility or cloaking of a cylindrical isotropic core using radially anisotropic cylindrical shell.

III. Conclusion

The electrostatic response of a TRA cylinder has been inspected with the limiting case of the absorbance with negligible losses and an invisibility of a cylinder. For this purpose, The cylinder has been performed interesting behavior of TRA cylinder with the distant choice of anisotropy ratio γ by adjusting different values of radial and tangential permittivity of the material. With radial permittivity close to zero and tangential permittivity is large, the cylinder has been performed as a cloak due to the potential inside the core of the cylinder is almost zero.

References

[1] Gao L., Fung T. H., Yu K. W., and Qiu C. W., Electromagnetic transparency by coated spheres with radial anisotropy. Physical Review E, 78.4 (2008), 046609.

[2] Liu H. Z., Li J. L. W., Leong, M. S., Zouhdi, S., Transparent uniaxial anisotropic spherical particles designed using radial anisotropy. Physical Review E, 84 (2011), 016605.

[3] Batool S., Nisar M., Mangini F., Frezza F., Cloaking using anisotropic multilayer circular cylinder. AIP Advances, 10 (2020), 095312.

[4] Batool S., Nisar M., Mangini F., Frezza F., Finding the polarizability of radially anisotropic multilayer circular cylinder, 29 Aug, URSI GASS conference, Rome, (2020).

[5] Batool S., Nisar M., Mangini F., Frezza F., Cloaking and magnifying using radial anisotropy in non-integer dimensional space, IEEE 14th European Conference on Antennas and Propagation (EuCAP), 15 Mar (2020), 1-5.

[6] Pendry J. B., Schurig D., Smith D. R., Controlling electromagnetic fields, science, 23 (2006), 1780-1782.

[7] Alu A, Engheta N., Theory and potentials of multi-layered plasmonic covers for multi-frequency cloaking, New Journal of Physics, 27 (2008) 115036.

[8] Batool S., Nisar M., Frezza F., Mangini F., Cloaking using the anisotropic multilayer sphere. Photonics, 7 (2020) 52-64.

[9] Shen C. L., Electromagnetic scattering by a conducting cylinder coated with metamaterials, Progress In Electromagnetics Research. 2003;42:91-105.

[10] Batool S., Frezza F., Mangini F., Xu YL., Scattering from multiple PEC sphere using Translation Addition Theorems for Spherical Vector Wave Function, Journal of Quantitative Spectroscopy and Radiative Transfer, 248 (2020) 106905.

[11] Batool S., Nisar M., Mangini F., Frezza F., The polarizability of an alternative sequence of isotropic and radially anisotropic multilayer sphere, 29 Aug, URSI GASS conference, Rome, (2020).

[12] Seker S. S., Schneider A., Electromagnetic scattering from a dielectric cylinder of finite length, IEEE Transactions on Antennas and Propagation, 36 (1988) 303-307.

[13] Tarn J. Q., Wang Y. M., Heat conduction in a cylindrically anisotropic tube of a functionally graded material, Journal of Mechanics, 19 (2003) 365-72.

[14] Wallen H., Kettunen H., Sihvola A., Anomalous absorption, plasmonic resonances, and invisibility of radially anisotropic spheres, Radio Science, 50 (2015) 18-28.

[15] Qiu C. W., Luk yanchuk B., Peculiarities in light scattering by spherical particles with radial anisotropy, Journal of the Optical Society of America A, 27 (2008) 1623-1628.

[16] Estakhri N. M., Alu A., Physics of unbounded, broadband absorption/gain efficiency in plasmonic nanoparticles. Physical Review B, 87 (2013) 205418.

[17] Mangini F., Tedeschi N., Frezza F., Sihvola A., Homogenization of a multilayer sphere as a radial uniaxial sphere features and limits, Journal of Electromagnetic Waves and Applications 28 (2014) 916-31.

[18] Mangini F., Tedeschi N., Frezza F., Sihvola A., Realization of a Radial Uniaxial sphere with a multilayer sphere. IEEE URSI General Assembly and Scientific Symposium (URSI GASS), (2014) 1-4.