

Interpreting Mueller matrix of anisotropic material

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Abstract—The polarization imaging methodology has evolved to accept increasingly complex parametric measurements. The Mueller matrix is now generally calculated using a polarimetry technique. In our manuscript, we study the anisotropic wood sample using polarization imaging techniques. Herein, We attempt to study the difference between wood horizontal and vertical fibers. We calculated 3×3 Mueller matrix elements, which can be used to described an intuitive overview of the anisotropic material characteristics. We interpreted experimental results of Mueller matrix coefficients in terms of a graphical representation.

Index Terms—Scattering, Polarization imaging, Mueller matrix measurements

I. INTRODUCTION

Several scientists have begun investigating the polarized light imaging as a possible diagnostic method during recent years [1], [2], [3]. The basic Linear Degree of Polarization (LDOP) imaging can be convenient to demonstrate the difference between the biological tissues in both ex vivo and in vivo conditions [4]. Since the LDOP imaging of anisotropic tissues may be sensitive to the sample orientation, a Rotating Linear Polarization Imaging (RLPI) has been developed to provide the collection of orientation-insensitive parameters to differentiate between different microstructural characteristics of fiber tissues. Mueller matrix imaging approach has also been tested in different clinical uses [5]. Even though it explains comprehensively, how the polarization states are converted from the incident to the transmitted light, a Mueller matrix provides detailed knowledge about the structure’s optical properties [6]. In this manuscript, we investigated the connection between the parameters of polarization imaging and the morphological characteristics of wood. The experimental study provides new insight into polarization imaging with different imaging techniques for fiber tissues. This paper provides a summary of recent findings of Mueller matrix coefficients of wood as fiber tissue. Measurements in both orientation (horizontal and vertical) have been obtained.

II. METHODOLOGY

A. Experimental setup

Wood samples (horizontally and vertically oriented) have been illuminated by linearly polarized light. The wavelength of illumination is 450 nm from a LED, which is collimated by a

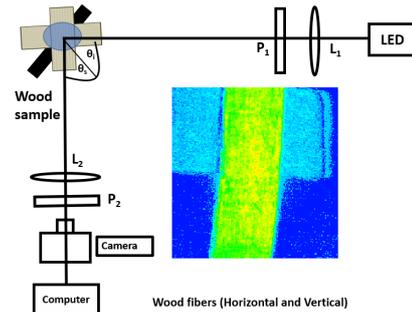


Fig. 1. Experimental setup

lens L_1 and then propagates through a linear polarizer P_1 . The samples were illuminated orthogonally, observed at different large angles in order to see the behavior of light according to the respective angle. Backscattered light in terms of photons from the sample passes through the lens L_2 and second polarizer P_2 . Finally, images were captured by a camera, which is connected to the computer. In our experimental setup, both polarizers P_1 and P_2 can rotate around their optical axis to vary polarization angles for illumination θ_i and detection θ_s .

We selected wood as a sample for anisotropic material, since wood contains both horizontal and vertical fibres. The images were mathematically inspected, to determine the properties of the light diffused towards the observer. In each experiment as illustrated in Fig 1, we captured series of images $I(\theta_i, \theta_s)$. For each incident polarization, we took a pair of images corresponding to the parallel θ_s and perpendicular $\theta_s + \pi/2$ detections. In this way, we recorded 16 images (parallel images=8 and perpendicular images=8) for wood sample (horizontal and vertical) respective to different angles of observation shown in Fig 2.

B. Mueller matrix polarimetry

Polarimetry is the measurement technique, that can be applied for interpretation of the polarization of light. In general, when the light interacts with optical elements that include polarizers, filters, lenses surfaces, scattering media etc, it can

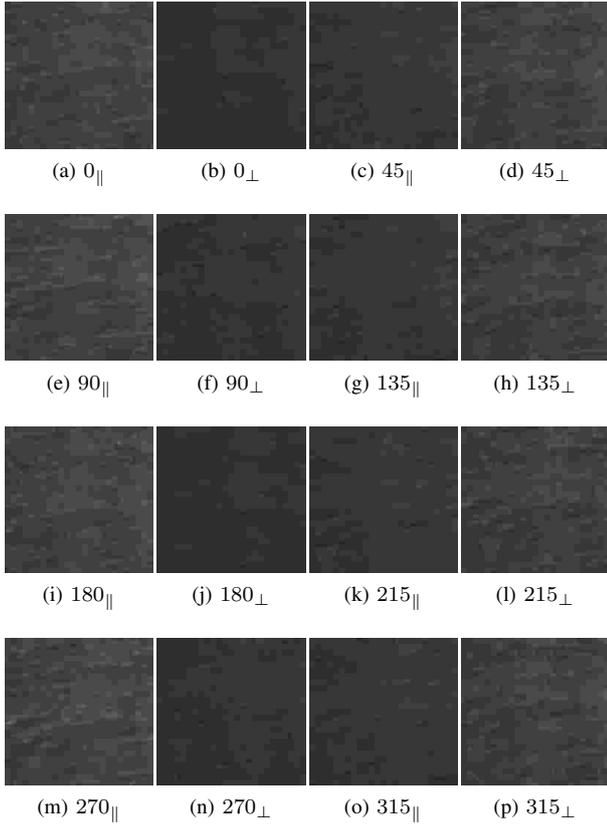


Fig. 2. Images for wood horizontal sample (parallel and perpendicular) direction with respect to the observation angles at 40°

change the state of its polarization. This interaction with any optical element or material can be defined as a multiplication of the Stokes vector with a 4×4 matrix, $S' = MS$. Stokes vector elements demonstrated as the total intensity of light I , the amount of linear horizontal or vertical polarization Q , the amount of linear $+45^\circ$ or -45° polarization U and the amount of right or left circular polarization contained in the light beam V . The stokes matrix is modified in terms of Mueller may be written as:

$$\begin{bmatrix} I_{out} \\ Q_{out} \\ U_{out} \\ V_{out} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I_{inp} \\ Q_{inp} \\ U_{inp} \\ V_{inp} \end{bmatrix} \quad (1)$$

C. Determining the Mueller matrix

If the Mueller matrix all elements are unknown, they can be determined experimentally with different orientations of polarizers. Fig. 3 represents the method of measurements for each matrix element. The element M_{11} gives the information of the unpolarized light input to output intensity, so this factor can be interpreted as a simple transmission. M_{12} is obtained by measuring the total reflected intensity for an incoming beam that is scaled the linear polarization horizontally and subtracting from this the total reflected intensity for a vertically polarized incoming beam. M_{12} can be expressed as the linear

M_{11} (OO)	M_{12} (HO-VO/2)	M_{13} (PO-MO/2)	M_{14} (LO-RO/2)
M_{21} (OH-OV/2)	M_{22} (HH+VV/4) - (HV-VH/4)	M_{23} (PH+MV/4) - (PV+MH/4)	M_{24} (LH+RV/4) - (LV+RH/4)
M_{31} (OP-OM/2)	M_{32} (HP+VM/4) - (HM-VP/4)	M_{33} (PP+MM/4) - (PM-MP/4)	M_{34} (LP+RM/4) - (LM-RP/4)
M_{41} (OL-OR/2)	M_{42} (HL+VR/4) - (HR-VL/4)	M_{43} (PL+MR/4) - (PR-ML/4)	M_{44} (LL+RR/4) - (RL-LR/4)

$o = \star$ $H = \rightarrow$ $P = \swarrow$ $L = \sum$
 $v = \downarrow$ $M = \searrow$ $R = \ominus$

Fig. 3. Formation of Mueller matrix elements

Intrinsic Property	Extrinsic Measurable
Linear Birefringence (LB)	Linear Retardance (LR)
Circular Birefringence (CB)	Circular Retardance (CR)
Linear Dichroism (LD)	Linear Extinction (LE)
Circular Dichroism (CD)	Circular Extinction (CE)
Absorption (A)	Transmission (T)

Fig. 4. Terminologies for polarization

extinction at $0^\circ/90^\circ$. Extinction here denotes the output-to-input light intensity ratio. In our case the polarization terminology may need some clarification, since there are a variety of related terms; transmission/reflectance, absorption, extinction, etc. illustrated in Fig. 4. If the sample under measurement displays only intrinsic or extrinsic optical property, the Mueller matrix can be interpreted as shown in Fig. 5.

III. PHYSICAL INTERPRETATION OF MUELLER MATRIX

A. Anisotropic Mueller matrix

In our experiment, we investigated the 3×3 Mueller matrix, for the characterization of wood (horizontal and vertical) as a sample at observation angle θ_i varies from 10° to 70° . During our numerical tests and elaborations, we have observed some important features of wood [7], [8], [9], [10].

T	-LE	-LE'	CE
-LE	D	CR	-LR'
-LE'	-CR	D'	LR
CE	LR'	-LR	D _c

T=Transmittance/Reflectance
LE=Linear Extinction
CR=Circular Extinction
LR=linear Retardance
CR=Circular Retardance
D=Depolarization

Fig. 5. Mueller Matrix described with single optical properties

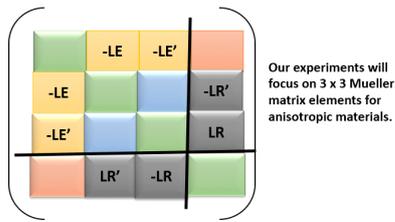


Fig. 6. Anisotropic Mueller Matrix

For the horizontal wood sample, the coefficients of Mueller matrix M_{23} and M_{32} have small values with opposite signs. For the vertical wood sample, values are negligibly small but have the same signs. For the sake of intuitive understanding, it is suggested that the diagonal values can be overlooked to focus the off-diagonal elements as represented in Fig. 6. The first row and first column of matrix tell us the information about linear extinction. The signs and magnitude should overlap across the diagonal for M_{12} and M_{21} as well as for M_{13} and M_{31} for homogeneous anisotropic material/sample. For the Mueller matrix elements M_{22} and M_{32} , the anisotropic sample can show higher values compared to the other elements.

B. Graphical representation of Mueller matrix coefficients

Fig. 7 represents the measured Mueller matrix coefficients for the wood (horizontal and vertical) with the observation angle varying from 10° to 70° . We have taken an average of three repeated measurements and normalized all matrix elements. An interpretation of the anisotropic material refers to the M_{12} and M_{21} are close to zero (see Fig. 7a and 7b), in the entire range of observation angles. In both cases of wood (horizontal and vertical), they have exactly the same slope, just adapted a different constant values. A more realistic interpretation of M_{22} is sensitive for the identification of the wood fibers (see Fig. 7g). It can be noted as: in the case of the wood vertical sample, the value of the matrix element M_{22} is substantially higher than the wood horizontal sample. It can be observed that M_{22} show decreasing numerical values with the increasing observation angle. Taking into account the other matrix elements M_{13} , M_{31} and M_{33} represents much lower numerical values rather than M_{22} (see Fig. 7c, 7d, and 7f). Data trends are similar to M_{22} , but it shows more fluctuation across the observation angle. M_{23} and M_{32} apparently propose that there is a remarkable change in the phase between the components of the electric vector \mathbf{E} of electromagnetic radiation of the optical path traveled across the wood fibers (see Fig. 7h and 7e). This information confess us to conclude that the polarization state of light plays a significant role in the interaction of light with a wood fiber orientations.

IV. CONCLUSION

The aim of this study is to determine the Mueller matrix coefficients. Using these Mueller matrix coefficients, we have

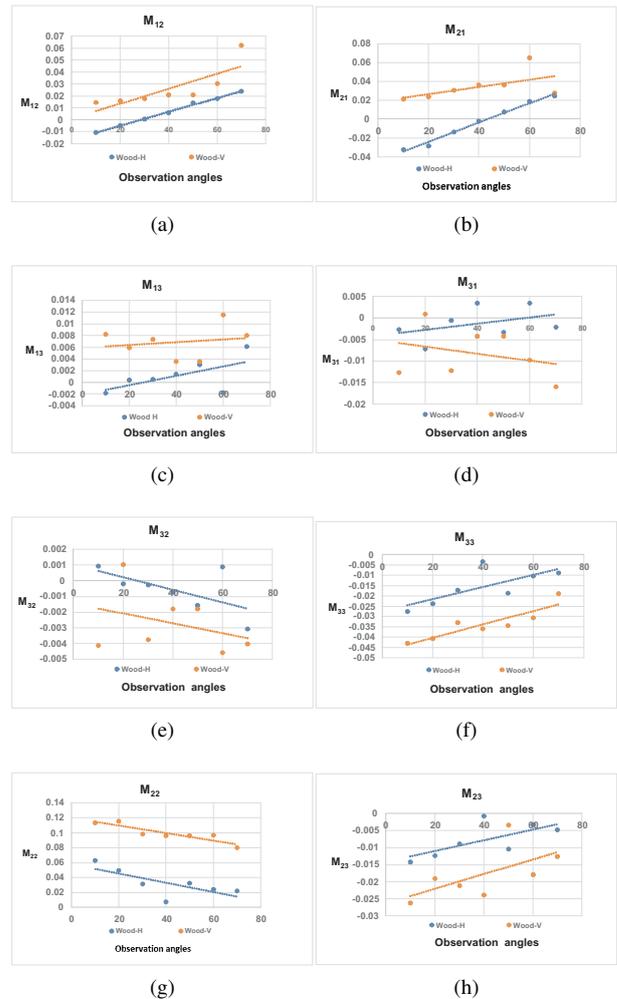


Fig. 7. Mueller Matrix elements M_{ij} for wood fibers of two orientation (horizontal and vertical) as functions of the observation angles.

presented the features of anisotropic materials. The matrix elements M_{22} is the most informative in this context, while the other matrix elements are somehow less sensitive. Thus, the nature of the change in the elements of the Mueller matrices over the entire observation angle reveals the characteristics of the wood fibers using light scattering phenomena.

REFERENCES

- [1] Tuchin V. V., "Polarized light interaction with tissues", Journal of biomedical optics, vol. 21, pp. 071114, 2016.
- [2] Alali S., Vitkin I. A., "Polarized light imaging in biomedicine: emerging Mueller matrix methodologies for bulk tissue assessment", Journal of biomedical optics, vol. 26, pp. 061104, 2015.
- [3] Gurjar R. S., Backman V., Perelman L. T., Georakoudi I., Badizadegan K., Itzkan I., Dasari R. R., Feld M. S., "Imaging human epithelial properties with polarized light-scattering spectroscopy", Nature medicine, vol. 7, pp. 1245-1248, 2001.
- [4] Zaffar M., Pradhan A., "Assessment of anisotropy of collagen structures through spatial frequencies of Mueller matrix images for cervical pre-cancer detection", Applied Optics, vol. 59, pp. 1237-48, 2020.
- [5] Angelsky O. V., Tomka Y. Y., Ushenko A. G., Ushenko Y. G., Ushenko Y. A., "Investigation of 2D Mueller matrix structure of biological tissues for pre-clinical diagnostics of their pathological states", Journal of Physics D: Applied Physics. vol. 38, pp. 4227, 2005.

- [6] Batool S., Nisar M., Mangini F., Frezza F., Fazio E., "Polarization Imaging for Identifying the Microscopical Orientation of Biological Structures", 29 Aug, URSI conference Rome Italy, 2020.
- [7] Savenkov S. N., Muttiah R. S., Oberemok E. A., Priezhev A. V., Kolomiets I. S., Klimov A. S., Measurement and interpretation of Mueller matrices of barley leaves, *Quantum Electronics*, vol. 50 pp. 55-60, 2020.
- [8] Chen X., Jiang H., Zhang C., Liu S., Towards understanding the detection of profile asymmetry from Mueller matrix differential decomposition. *Journal of Applied Physics*, vol. 118, pp. 225308, 2015.
- [9] Heinrich A., Bischoff J., Meiner K., Richter U., Mikolajick T., Dirnstorfer I., Interpretation of azimuthal angle dependence of periodic gratings in Mueller matrix spectroscopic ellipsometry, *JOSA A*, vol. 32 pp. 604-10, 2015.
- [10] Jellison G. E., Griffiths C. O., Holcomb D. E., Rouleau CM., Transmission two-modulator generalized ellipsometry measurements, *Applied optics*, vol. 41 pp. 6555-66, 2002.