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2 **Monte Carlo Simulations to Assess the Uncertainty of Locating and Quantifying CO₂ Leakage Flux**
3 **from Deep Geological or Anthropogenic Sources.**
4

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21 simulations are available from the Zenodo open-source online repository at
22 <https://doi.org/10.5281/zenodo.4573725> (Beaubien and Bigi, 2021b).

23 **Code availability:** The MC-Flux (V1.0) installation package and user's manual are available from the
24 Zenodo open-source online repository at <https://doi.org/10.5281/zenodo.4573575> (Beaubien and Bigi,
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30

31 **Abstract**

32 Accurately locating and quantifying CO₂ leakage to the atmosphere is important for diffuse
33 degassing studies in volcanic / geothermal areas and for safety monitoring of Carbon Capture
34 and Storage (CCS) sites. This is typically conducted by measuring CO₂ flux at numerous points
35 over a large area and applying statistics or geostatistical interpolation. Probability and accuracy
36 of the results will depend on many factors related to survey/data-processing choices and site
37 characteristics, and thus uncertainties can be difficult to quantify. To address this issue, we have
38 developed a Monte Carlo-based program (MC-Flux) that repeatedly subsamples a high-
39 resolution synthetic or real dataset using five different sampling strategies at multiple user-
40 defined sample densities, keeping track of the anomalies found and estimating total flux using
41 four approaches from the literature. This paper describes the use of MC-Flux to assess the
42 potential impact of various sampling and interpretation decisions on the accuracy of the final
43 results. Simulations show that an offset grid sample distribution yields the best results, however
44 relatively dense sampling is required to obtain a high probability of an accurate flux estimate.
45 For the test dataset used, ordinary kriging interpolation produces a range of flux estimates that
46 are centered on the true value while sequential Gaussian simulation tends to slightly overestimate
47 values at intermediate sample spacings and is sensitive to input parameters. These results point to
48 the need for developing new approaches that decrease uncertainty, such as integration with high-
49 resolution co-kriging datasets that complement the more accurate point flux measurements.

50 **Keywords:** Soil diffuse degassing, Carbon Capture and Storage (CCS), CO₂ flux quantification,
51 uncertainty, probability, Monte Carlo

52 **1. Introduction**

53 Accurately finding and quantifying the leakage of deep-origin carbon dioxide (CO₂) to
54 the atmosphere is challenging considering that point flux measurements conducted over large
55 areas are used to characterize spatial anomalies that can be irregular in size, shape, magnitude
56 and distribution and that are superimposed on a background biological flux that is both spatially
57 and temporally variable. In this paper we use a Monte Carlo approach to study the impact of
58 these variables and to quantify some of the related uncertainties.

59 Initial efforts to determine the probability of finding an anomaly using gridded sampling
60 were conducted in the field of mineral exploration. Singer (1972, 1975) used a geometric
61 probability model called ElipGrid to show how triangular and square grids are often equivalent,
62 but that the former can be up to 6% more efficient for certain conditions. The accuracy of this
63 code was proven using a Monte Carlo approach (Davidson 1995), followed by its integration into
64 an environmental software package called Visual Sample Plan (VSP) (Matzke et al. 2014).
65 Instead, the probability of finding anomalies using random sampling, calculated using the
66 formula:

$$P=1- \left[1-x/A\right]^n, \quad (1)$$

where x is the size of the anomaly and A is the size of the survey area, is low and the approach is generally inefficient (Oldenburg et al. 2003).

Uncertainties in quantifying CO_2 leakage can be subdivided into four categories. First, analytical uncertainty is related to site conditions (Bain et al. 2005) and sampling / data processing protocols (Elío et al 2012; Kutzbach et al. 2007). Estimates of the measurement reproducibility range from $\pm 10\%$ (Chiodini et al. 1998) to $\pm 24\%$ for low flux rates (Carapezza and Granieri 2004). Second, the probability of finding anomalies depends on their size and shape versus sample density and distribution. Wong (2018) used a Monte Carlo approach to illustrate that a large number of random samples are needed to obtain a high probability of accurately estimating the average CH_4 flux above a small landfill site.

Third, converting the measured point flux values into an estimated total flux for the entire survey area requires statistical characterization (e.g., arithmetic mean, AM, or minimum variance unbiased estimator, MVUE) or geostatistical extrapolation (e.g., kriging or sequential Gaussian simulation, SGS). Lewicki et al. (2005) compared these four approaches and found them to yield similar results but recommended SGS because this stochastic method honors the histogram and variogram of the original data and provides a measure of the uncertainty. Elío et al. (2016) also found SGS to yield the most robust estimates if the data can be fit with a variogram, otherwise they recommend AM, MVUE or bootstrap resampling for normal, log-normal or mixed populations, respectively. Cardellini et al. (2003) used SGS and data from multiple sites to define an empirical relationship linking flux estimate precision with the number of samples needed within a circular area having a radius equal to the data's variogram range. Schroder et al. (2017) found that AM and MVUE approaches did not accurately estimate leakage rates from a controlled release experiment and thus proposed a new cubic interpolation approach that is not limited by spatial or statistical distribution. At present, SGS is the most commonly used approach, although it too has its limitations (Caers 2000; Gilardi et al. 2002; Emery 2004; Paravarzar et al. 2015).

Fourth, the background biological flux must be estimated and subtracted from the total flux to determine the leakage flux. This can be conducted by making measurements in a separate non-leaking area (Chiodini et al. 2007), but it is much more common to use log-normal

97 probability plots to separate populations of different origins (Chiodini et al. 2020). Elío et al.
98 (2016), however, point out that this approach can be subjective and instead recommend a
99 maximum likelihood procedure. It is extremely difficult to estimate the background uncertainty,
100 given its spatial and temporal variability (Sainju et al. 2008; Leon et al. 2014; Bond-Lamberty et
101 al. 2019), however it can have an important impact on the final leakage uncertainty if
102 background flux represents a significant proportion of the total. Recent efforts to use components
103 associated with the leaking CO₂, such as isotopic signatures or radon (e.g., Bini et al. 2019;
104 Viveiros et al. 2020), have shown how this uncertainty can be significantly reduced. If resources
105 are limited, however, additional analyses may not be feasible or they could lead to a reduction in
106 the total number of flux measurement points (thus reducing spatial resolution).

107 The present work takes a unique approach to quantitatively assess the issues and
108 approaches described above. It better develops and greatly extends the preliminary efforts by
109 Beaubien et al. (2009) and Beaubien (2015) to develop a Monte Carlo – based code (Beaubien
110 and Bigi 2021a) capable of defining the probability of locating leakage flux anomalies and
111 assessing the accuracy of associated quantification. Synthetic and real data (Beaubien and Bigi
112 2021b) are used to explore the effectiveness of different sampling strategies, the impact of
113 anomaly shape and orientation, and to assess the uncertainties linked to the multiple steps
114 (sampling density and strategy decisions, interpolation, background subtraction) required to
115 estimate a final leakage flux. Ideas are presented that could reduce these uncertainties without
116 significantly increasing survey times or costs.

117 **2. Materials and Methods**

118 The MC-Flux program, coded in Visual Basic 6 and run in Windows 10, uses a Monte
119 Carlo approach to study the influence of sampling strategies and sample spacing on the
120 probability of finding a gas leakage anomaly and on the accuracy of leakage flux estimates.

121 Three main functions are available. The first imports or creates high-resolution input
122 data, sub-samples it a user-defined number of times at different spacings using one of five
123 sampling strategies (keeping track of anomalies or calculating statistics to estimate total flux),
124 and saves each sub-sampled equi-probable realization in a simple text file. This function can be
125 used as a stand-alone for determining the probabilities of finding leakage anomalies, or the total
126 flux can be estimated for each sub-sampled realization both statistically and geostatistically using

127 external programs. The second function calls the commercial software Surfer (Golden Software)
128 to perform interpolation using the ordinary kriging, natural neighbor, inverse distance to a power
129 or the radial basis function methods, although only ordinary kriging was applied here. Note that
130 the user must have Surfer installed to access this functionality; development was performed
131 using version 9 however limited testing has shown that MC-Flux also works with version 20.
132 The third function calls the program sGsim from the open source GSLib library (Deutsch and
133 Journal 1997) to conduct sequential Gaussian simulations; this library of DOS executable files
134 can be freely downloaded (<http://www.gslib.com>) and copied onto the user's computer.

135 A flowchart showing the logical structure of the program (Figure S1), the graphical user
136 interface (GUI) developed for selecting options and inputting parameters (Figure S2), and the
137 MC-Flux user manual are given in the Supporting Materials. Note that all GUI input is saved to a
138 configuration file that can be re-loaded to repeat a simulation or make systematic parameter
139 changes.

140 **2.1. Data input**

141 Both synthetic and real data can be used as input for estimating total CO₂ flux while only
142 synthetic data can be used to calculate the probabilities of finding anomalies.

143 Synthetic data is created in two steps. First the background flux field is generated using
144 one of three options: i) all points are assigned a value of 0 (to facilitate recognizing individual
145 anomalies); ii) an internally created normal distribution is randomly placed over the grid (Figure
146 S3a); or iii) an external file is imported into the program. For this last option, we have created a
147 log-normal distribution using Poptools (Hood 2010) and coherently distributed it over a 1km²
148 grid assuming an inverse relationship between topography and biological CO₂ flux (Figure S3b,
149 S4), given that topography influences environmental parameters (e.g., water content, organic
150 matter) that affect soil respiration (Riveros-Iregui and McGlynn 2009). Second, one or multiple,
151 chosen or random, circular or elliptical, gas leakage areas ("vents") are superimposed on the
152 background based on user-defined parameters (number of vents, location, semi-major axis
153 length, semi-major vs semi-minor axis ratio, orientation, and maximum flux rate). This
154 information is saved and can be imported for subsequent simulations. For the "find anomaly"
155 option, vent points are assigned the vent number in a background of zeros, which allows for
156 individual anomalies to be recognized. For the "calculate flux" option, individual vent flux

157 values are calculated using an empirically defined formula based on profiles across multiple gas
158 vents (Figure S5a,b) and plotted on a background flux field (Figure S5c, d).

159 If real data is used, the program imports a text file consisting of a single column of values
160 ordered sequentially for each X point along progressive Y lines. As it is not feasible to collect
161 real data at 1m sample spacing over a large area, a dataset of 548 CO₂ flux measurements made
162 on a regular, 10 m spacing grid in a polygonal area was modified for this purpose. The survey
163 area is located in a well-studied field containing multiple gas vents in the Latera Caldera, Italy
164 (Beaubien et al. 2008; Pettinelli et al. 2010). Flux measurements were made on July 13th and
165 14th, 2014, using three in-house accumulation chamber units that were inter-calibrated prior to
166 the survey. The original grid was extended to a 260 x 375 m area by assigning random
167 background values outside the polygon, and then a high-resolution (HR) grid (1 m node spacing)
168 was created using the average of 10 sequential Gaussian simulations (SGS; Deutsch and Journel
169 1997); SGS was performed to minimize any spatial biasing that may result from gridded
170 sampling (Figure S6a). This data is formed by two main populations (Figure S6b). The lower
171 background population ranges from about 20 to 70 g m⁻² d⁻¹ and has a mean of 51 g m⁻² d⁻¹; these
172 relatively high biogenic fluxes were related to rainfall events shortly before sampling, similar to
173 that observed by Viveiros et al. (2020). This interpretation is supported by the much lower
174 background values observed in the same field (Figure S6c) during a previous campaign in July,
175 2006 (Annunziatellis et al. 2008). The upper, leakage-related population has a maximum value of
176 1645 g m⁻² d⁻¹. The standardized variogram of the HR data consists of a spherical model with a
177 nugget of 0.02, a variance of 0.98 and a range of 37m (Figure S6d).

178 **2.2. Sub-sampling**

179 The program subsamples the high-resolution flux array using one of five possible
180 sampling strategies (Figure S7): i) square grid, where X and Y distances are the same and points
181 are aligned orthogonally; ii) off-set grid, where X and Y distances are the same but every second
182 row is offset horizontally by X/2; iii) triangular grid, where every second row is offset
183 horizontally by X/2 but with all point-to-point distances the same, resulting in Y<X; iv) random
184 grid, where the program steps through a square grid, but at each node a random point is selected
185 within a user-defined radius; and v) purely random sampling. Each subsampling iteration
186 produces a single, equi-probable, sub-sample dataset.

187 Simulations are conducted for different “sample densities”, which are defined by the
 188 sample spacing (grids) or equivalent number of samples (random) chosen by the user. A nested
 189 loop structure produces N unique sub-sampling realizations for M sample densities. Note,
 190 however, that the number of unique subsample realizations N for the non-random grid methods
 191 is limited to (sample spacing)² when the input data has a 1m spacing, meaning that fewer unique
 192 simulations can be performed for closely spaced grids.

193 **2.3. Calculations**

194 For the “find anomaly” simulations, each sub-sampled dataset is queried to determine if
 195 any sample points intersect an anomaly, and if yes which ones. After all realizations for that
 196 sampling density are completed, the probability of finding each individual anomaly and the
 197 average number of anomalies found is calculated.

198 For the “calculate flux” simulations, the first step involves calculating the “true” leakage
 199 flux by subtracting the background flux from the total flux for the original HR dataset; this value
 200 is then used as a benchmark for the subsequent subsampling results. For synthetic data the
 201 background and background-plus-vents flux rates are calculated using:

$$202 \quad \varphi_T = \sum_1^n (A_c * \varphi_c) \quad (2)$$

203 Where φ_T is the total flux being calculated, n is the total number of grid cells, A_c is the
 204 surface area of each cell (m²), and φ_c is the CO₂ flux for each cell (g m⁻² d⁻¹). For real data, the
 205 total flux is calculated using Eqn 2 while the background is calculated by multiplying the grid’s
 206 surface area by the average of the lower flux population in a log-normal probability plot of the
 207 entire raw dataset (Chiodini et al. 2007).

208 Four different approaches are used to calculate the total flux of the sub-sampled
 209 realizations, two statistical (arithmetic mean, AM, and minimum variance unbiased estimator,
 210 MVUE) and two geostatistical (ordinary kriging, OK, and sequential Gaussian simulations,
 211 SGS). For both statistical methods the resultant value is multiplied by the total surface area, and
 212 then the previously calculated “true” background leakage is subtracted from this value to yield
 213 the total leakage flux estimate. The AM, which is most appropriate for normally distributed
 214 datasets, is calculated as $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$. The MVUE, $\hat{\mu}_1$, which better describes log-normal
 215 distributions, is calculated as $\hat{\mu}_1 = [\exp(\bar{y})] \psi_n(t)$ (Gilbert 1987; Elío et al. 2016), where \bar{y} is

216 the sample arithmetic mean calculated using the n log-transformed values, t is half of the log
217 transformed variance s_y^2 , and ψ_n is an infinite series (see Figure S8 for details).

218 Regarding the geostatistical methods, OK is implemented using the commercial software
219 package Surfer 9 (Golden Software) and SGS is implemented using the program sGsim within
220 the open source GSLib library (Deutsch and Journel 1997). The OK approach yields a single
221 array for the grid area, whereas SGS produces a user-defined number of realizations for each
222 sub-sampled dataset that are then averaged to produce a single array. As a compromise between
223 level of detail and processing time, an output cell size of 5 x 5 m was chosen for all simulations.
224 Each individual array produced by both OK and SGS is then processed with Eqn 2 to calculate
225 the total flux, and the previously calculated “true” background flux is subtracted to yield the
226 associated leakage flux estimate for each realization.

227 **3. Results and Discussion**

228 **3.1. Probability of finding an anomaly**

229 *3.1.1. Comparison with literature*

230 The accuracy of the MC-Flux Monte Carlo probability estimates were validated using test
231 data reported in Table 1 in Singer (1972) and Table A3 in Davidson (1995) that consist of a wide
232 range of single ellipse sizes, shapes, and orientations coupled with different node spacings for
233 square and triangular grid strategies. The MC-Flux results, based on 5000 realizations, have a 1:1
234 linear correlation with those generated using the mathematical approach of the ElipGrid
235 program, with $R^2 = 0.9999$ for both datasets and most individual simulations within 0.7% (Figure
236 S9a). The test data were modified to convert grid spacing into an equivalent total number of
237 samples to test the purely random sampling approach. The MC-Flux results match very closely
238 with the theoretical probability calculated using Eqn 1 (Figure S9b), with $R^2 = 0.9998$ and most
239 individual simulations within 1%. These results give confidence in the MC-Flux simulations
240 reported below.

241 *3.1.2. Number of simulations necessary*

242 Tests were performed to determine how many simulations are needed to obtain a stable
243 probability estimate. All five strategies were used to sub-sample two different 1 km² grids, one
244 containing a circular anomaly ($a=56.42$, $x/A=0.01$) that was sampled with a c. 120m spacing and

245 one with an elliptical anomaly ($a=79.79$, $b/a=0.5$, $\text{angle}=22.5^\circ$, $x/A=0.01$) that was sampled with
246 a c. 150m spacing. Each test involved 10,000 realizations, each of which yielded 1 or 0 if the
247 anomaly was encountered or not. A total of “n” random samples were collected from this dataset
248 (where $n = 30, 50, 75, 100, 200, 300, 400, 500, 1000, 2000$, or 3000) a total of 200 times each to
249 calculate the probability of finding the anomaly using different numbers of simulations (i.e.,
250 “n”).

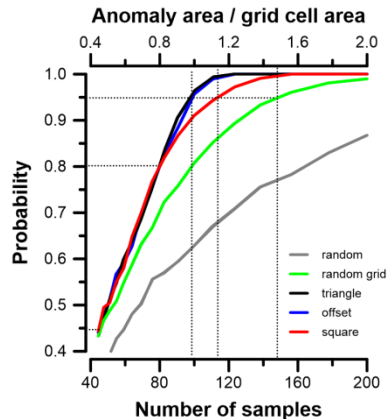
251 All square grid results for the first test yield relatively similar average probabilities but,
252 as expected, the total range of estimated probability narrows significantly with the inclusion of
253 larger numbers of simulations (Figure S10a). The standard deviation of all tests show a linear
254 decrease with increasing number of simulations on a log-log plot (Figure S10b), regardless of
255 anomaly shape, sampling strategy or sample spacing. Based on these results and the fact that the
256 sub-sampling algorithm is fast, all “find anomaly” probability tests reported below were
257 conducted using 5000 simulations ($1\sigma \approx 0.7\%$). In contrast, the OK and SGS flux estimation
258 tests are much slower, and thus 100 ($1\sigma \approx 5\%$) or 500 ($1\sigma \approx 2\%$) simulations were used for these.

259 3.1.3. *Probability of finding a single circular anomaly*

260 The efficiency of the different sampling strategies was first tested for the simple case of a
261 single circular anomaly ($a=56.42$ m; $x=10,000$ m²) in the middle of a 1 km² area ($x/A=0.01$).
262 Simulations were conducted so that each strategy used the same average number of samples, thus
263 spacing was the same for the square, offset and random grids (from 40 to 150 m), 1.07457 times
264 larger for the triangular grid (Singer 1975), and an equivalent number of samples was used for
265 the purely random method.

266 Random sampling gives significantly poorer results across all probability levels (Figure
267 1). All four gridded methods yield similar results up to a probability of 0.45, with the random
268 grid rapidly diverging at higher levels. Its trend is a function of the chosen random search radius
269 (here 50%), such that low values move its trend closer to the square grid results while higher
270 values move it towards the random results. At a probability level above ca 0.8 the square grid
271 method becomes less efficient than the offset and triangular grids, with the latter two being
272 essentially equivalent over all probability levels. This higher efficiency of the triangular versus
273 square grid is in agreement with the maximum improvement of about 6% observed by Singer
274 (1975). As a comparison, the dashed lines in Figure 1 show that to ensure a 95% probability of

275 finding the anomaly, about 100 samples would need to be collected using offset and triangular
 276 grids, 114 using the square grid, 148 using the random grid with a 50% search radius, and 300
 277 using a purely random approach. Sample numbers can also be estimated for other conditions
 278 using the ratio of the anomaly and grid cell areas given in the upper X axis.



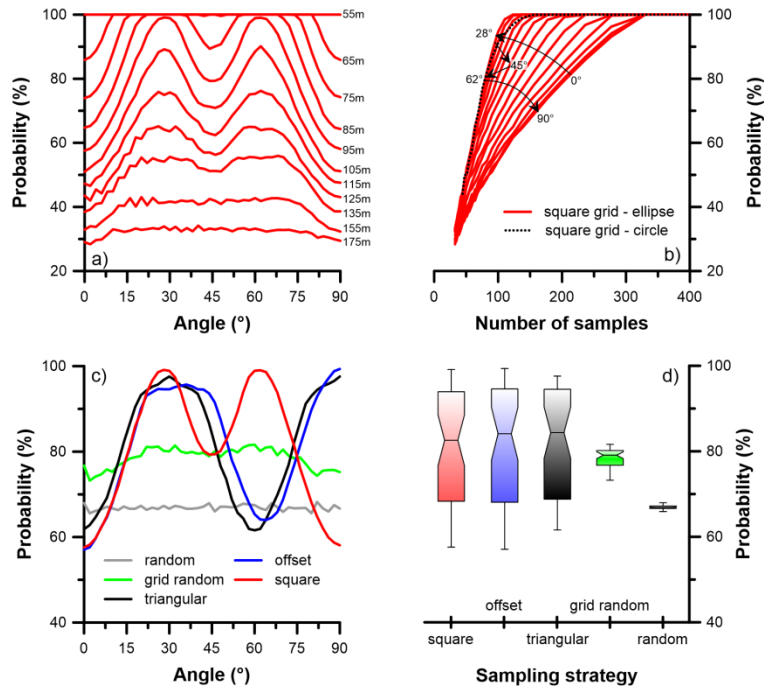
279

280 **Figure 1.** Probability of finding a circular anomaly as a function of sampling strategy and sample
 281 spacing. The lower X axis refers to the modelled conditions of $x=10,000$ and $A=1,000,000$ while
 282 the upper X axis standardizes the trends for all anomaly/cell ratios.

283 3.1.4. Probability at different ellipse angles

284 The probability of finding an ellipse as a function of its orientation was tested using the
 285 five different sampling strategies (spacing from 40 -175 m in 5 m steps, or equivalent). Because
 286 MC-Flux can keep track of individual anomalies and because the location of an anomaly within
 287 the domain has no effect on the Monte Carlo results, 46 ellipses ($x=10,000 \text{ m}^2$, $b/a=0.25$) were
 288 created at angles from $0-90^\circ$ at 2° intervals within a 1 km^2 area (Figure S11).

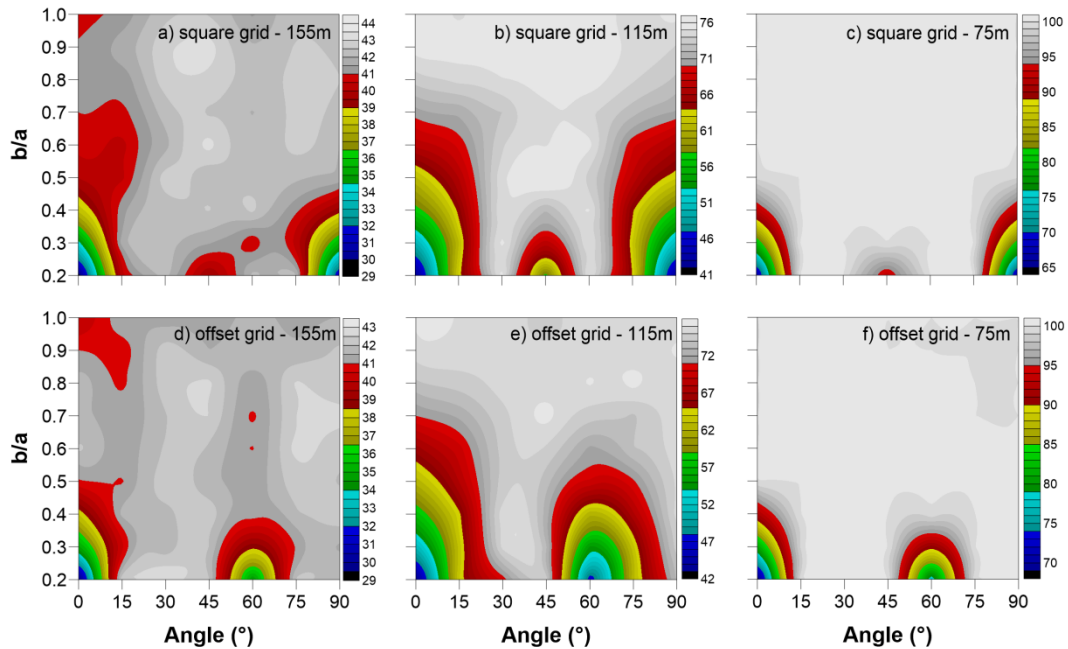
289 Using the square grid results as an example, orientation and sample spacing clearly have
 290 a significant combined effect on the probability of finding a narrow ellipse (Figure 2a). At very
 291 low sampling densities the trends are relatively flat, while at very high densities all directions
 292 tend towards a probability of 100%. Instead, between these extremes maximum probabilities
 293 occur at 28° and 62° , minimum probabilities occur at 0° and 90° , and a lesser minimum occurs at
 294 45° , all due to the geometric positioning of the ellipse within the distributed points. For some
 295 sample spacings the differences are very large; for example, probabilities at a 95 m spacing can
 296 range from 60% at 0° up to almost 100% at 28° . This could mean that between 100 to 300
 297 samples would be needed to attain 95% probability (Figure 2b) depending on orientation. Similar
 298 plots for all sampling strategies are given in Figure S12.



299

300 **Figure 2.** Plots showing the impact of orientation on the probability of finding a $10,000 \text{ m}^2$,
 301 elliptical ($b/a=0.25$) anomaly in a 1 km^2 area: (a) square grids of different spacings (reported
 302 numbers); (b) same data plotted against the number of samples, with the dashed line showing the
 303 circular anomaly trend from Figure 1; (c) probabilities for all five sampling strategies at an
 304 equivalent sampling distance of 95 m; and (d) statistical distribution of the data in (c).

305 Results from a sample spacing of 95 m (or equivalent) are used to compare the relative
 306 trends of all five strategies (Figure 2c). Again the offset and triangular grid results are quite
 307 similar (with a slight shift of $< 5^\circ$), but are quite different from the square grid results. In
 308 particular, maximum and minimum values are opposite at 60° and 90° . The random grid results
 309 are only weakly influenced by anomaly direction and have probability values on the order of
 310 80%, while random sampling, as expected, is not affected by orientation and has a low, flat trend
 311 around 68%. The statistical distribution of these data (Figure 2d) show how the square,
 312 triangular, and offset grid strategies are essentially equivalent, meaning that if the angle is
 313 unknown or variable none of these methods are superior to the others for finding this narrow
 314 anomaly. Instead, the random grid produces a median probability that is only slightly lower than
 315 the other grid methods but with a much narrower range (i.e., more predictable outcome).
 316 Although potentially advantageous, this approach is not necessarily practical for field sampling
 317 campaigns. Similar plots for 75 m and 135 m sample spacing, or equivalent, are given in Figure
 318 S13.



319

320 **Figure 3.** Plots showing the probability of finding a 10,000 m² elliptical anomaly in a 1,000,000
 321 m² area as a function of its shape (b/a) and angle (where horizontal is 0°, moving counter-
 322 clockwise to vertical at 90°) for the labelled grid types and sample spacings.

323

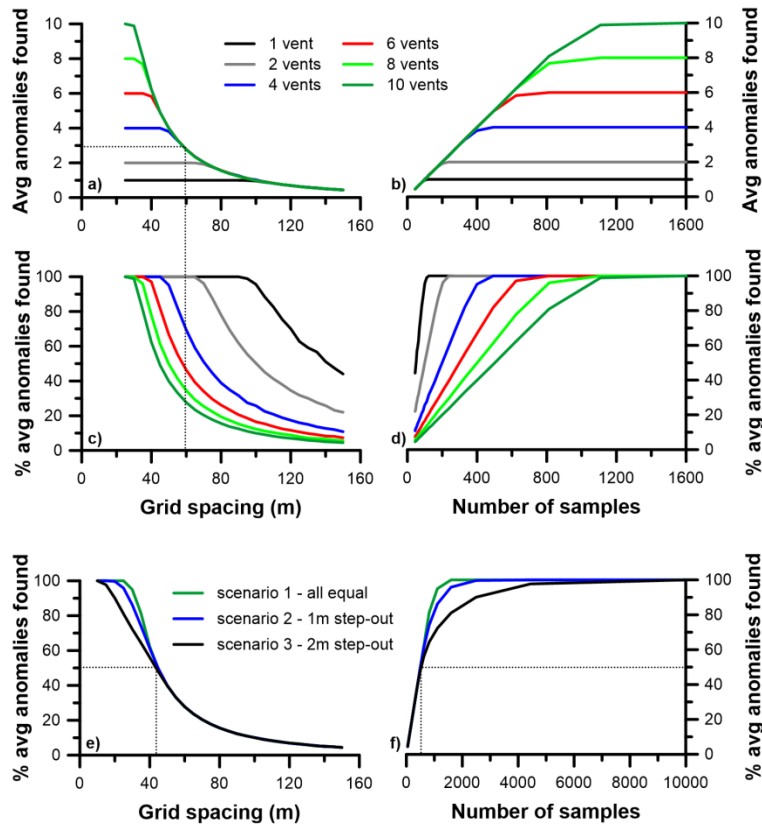
324 Further simulations were conducted to address the combined effect of b/a values from 0.2
 325 to 1.0 (0.1 step size) and angles from 0° to 90° (15° step size). A single domain was defined that
 326 contained all anomalies (Figure S14) and simulations were conducted with the square and offset
 327 grid methods for sample spacings from 40 to 175 m (5 m step size). Trends are similar over a
 328 wide range of sample spacings and the impact of anomaly orientation diminishes at higher b/a
 329 values (Figure 3). Using the square grid results as an example, the total range of probability
 330 values is narrowest for the widest spacing (e.g., 15% difference in Figure 3a) but is much greater
 331 for the other two spacings (e.g., 35% in both Figure 3 b and c). A comparison of the latter two
 332 also shows that the impact is observed at much greater b/a values for the 115 m grid spacing (up
 333 to $b/a=0.7$ for a 5% difference) than for the 75 m spacing ($b/a=0.4$), as progressively narrower
 334 spacing is more likely to intersect wider ellipses that may lie between rows or columns. The
 335 square grid results are highly symmetrical at 0° and 90° (e.g., Figure 3b) whereas the offset grid
 336 results show higher probabilities at higher b/a values for angles at 0° compared 60° (e.g., Figure
 337 3e). That said, the average probability for all b/a values and orientation angles for a given sample
 338 spacing are essentially the same for the square and offset grids.

339 *3.1.5. Average number of anomalies found*

340 Considering that more than one leakage anomaly may occur in a given survey area,
341 simulations were performed to assess how many can be located using different sample spacings.
342 Initial tests involved 6 different scenarios containing 1, 2, 4, 6, 8, or 10 circular anomalies, with
343 all anomalies in each individual scenario having the same size and the sum of their areas being
344 equal to $10,000 \text{ m}^2$ (i.e., $x_{\text{total}}/A=0.01$) (Figure S15). The square and offset grid methods were
345 used for each sample spacing; as results are very similar only the latter results are described here.

346 When plotted in terms of the average number of anomalies found, all scenarios follow the
347 same trends for both sample spacing (Figure 4a) and total number of samples collected (Figure
348 4b). Instead, plotting the same data in terms of the percentage of anomalies found relative to the
349 total number present illustrates how, as expected, the smaller the average anomaly size the
350 smaller the proportion that are found (Figure 4c, d). For example, the dashed lines in Figure 4a
351 and c show how an average of 3 anomalies are found in the 4, 6, 8, and 10 anomaly scenarios
352 when a sample spacing of about 60 m is used, but that this represents 70%, 48%, 35%, and 28%,
353 respectively, of the anomalies actually present. Clearly how a total leakage amount is subdivided
354 and, in turn, how many of these anomalies are found, will have an important impact on eventual
355 quantification estimates.

356 Using the 10 anomaly simulation as the base case, two additional tests were performed to
357 look at the impact of different sized anomalies. Centered on the semi-major axis value from this
358 scenario 1 (i.e., $a=17.8 \text{ m}$), scenario 2 anomalies increase from $a=12.6$ to 22.5 m in ca. 1 m steps
359 while scenario 3 anomalies increase from $a=7.4$ to 26.1 m in ca. 2 m steps (Figure S16). As
360 above, the total surface area of all 10 anomalies in each scenario equals $10,000 \text{ m}^2$. As the
361 distribution of sizes is symmetrical above and below the scenario 1 dimension, trends for all
362 three scenarios are the same up to 50% of the anomalies found (Figure 4e, f). Deviations are
363 observed above this threshold, however, as the smaller anomalies require a closer grid spacing to
364 ensure their discovery. For example, in order to find 9 out of 10 anomalies, 1000 samples are
365 needed for the base case, 1300 for scenario 2 and 2500 for scenario 3, which corresponds to a
366 grid spacing of 32, 27 and 19 m, respectively. While intuitive, these results quantitatively
367 illustrate how challenging it is to locate all leakage areas present, especially if small anomalies
368 occur in a wider size distribution.



369

370

371 **Figure 4.** The absolute number (a, b) and percent (c, d) of anomalies found with square grid
 372 sampling over a range of grid spacings (a, c) and number of samples (b, d) for 6 scenarios having
 373 a different number of equal-sized anomalies whose sum is always $10,000\text{m}^2$. The same for three
 374 scenarios having 10 anomalies of different sizes (e, f); note the expanded X scale in (f).

375 3.2. Quantifying leakage

376 3.2.1. Background flux estimates

377 As stated above, the background biogenic CO_2 flux must be estimated and subtracted
 378 from the total measured flux to calculate the leakage flux rate. Various approaches have been
 379 used in the literature.

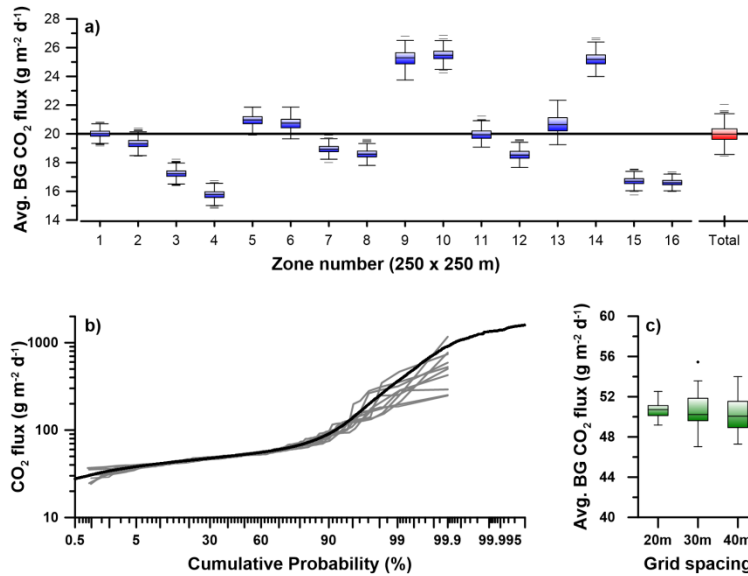
380 For example, the average background value can be calculated using measurements from a
 381 similar, sub-set area where no leakage occurs. To assess this the 1km^2 synthetic background
 382 dataset was subdivided into 16, 250×250 m sub-areas (Figure S3b). MC-Flux was used to create
 383 1000 sub-sampling files for each sub-area (random grid with 35 m spacing) as well as the total
 384 area (random grid with 140m spacing). Each realization yielded about 50 samples which were
 385 used to calculate the average and MVUE flux rates. Although the statistical distribution of the
 386 estimates from each sub-area are relatively narrow, there is a wide range of values (blue boxes in

387 Figure 5a) compared to that for the entire area (red box in Figure 5a) due to the non-random
388 distribution of the synthetic data. Although some sub-areas yield median estimates that are
389 similar to the true value of $20 \text{ g m}^{-2} \text{ d}^{-1}$, others vary by as much as $\pm 4 \text{ g m}^{-2} \text{ d}^{-1}$.

390 Another approach uses a log-normal probability plot of all survey data to separate and
391 characterize the background. To examine this approach the HR Latera dataset (Figure S6) was
392 subsampled 500 times with the offset grid approach using three different spacings (20, 30, 40m).
393 Log-normal probability plots were made for 50 realizations of each of the three datasets, upper
394 background thresholds were estimated graphically based on the inflection point in the data, and
395 the MVUE of the background population was calculated for each realization. Different inflection
396 points (e.g., Figure 5b) and sampled background values lead to background flux estimates
397 (Figure 5c) that vary by up to a maximum of $\pm 3 \text{ g m}^{-2} \text{ d}^{-1}$ for the wider spacings.

398 A third approach, similar to the previous but with background flux calculated using only
399 the area that it is estimated to occupy, has not been assessed because it is impractical for MC
400 simulations. Based on the results above, however, it is likely that this approach would have a
401 similar level of uncertainty. A fourth method, using isotopes or co-migrating gases (e.g., Bini et
402 al., 2020), should instead have significantly lower uncertainties.

403 Although the true background flux uncertainty will be site-specific, and will depend both
404 on the approach used and researcher experience, these results illustrate the potential for errors on
405 the order of a few $\text{g m}^{-2} \text{ d}^{-1}$ for the estimated average. While small, it could have an important
406 impact on the final leakage flux value if the survey area is large and/or the average biogenic flux
407 is proportionally high. As an example, given a $100,000 \text{ m}^2$ area with a total flux of 4 t d^{-1} and a
408 true average background flux of $20 \text{ g m}^{-2} \text{ d}^{-1}$, an estimate that is $\pm 2 \text{ g m}^{-2} \text{ d}^{-1}$ from this value will
409 result in an error of about $\pm 10\%$ in the final leakage estimate. Finally it should be remembered
410 that these examples only consider uncertainty related to statistical sampling, and do not take into
411 account uncertainty caused by temporal variability (given that flux surveys are conducted over
412 many hours during the day and often over multiple days).



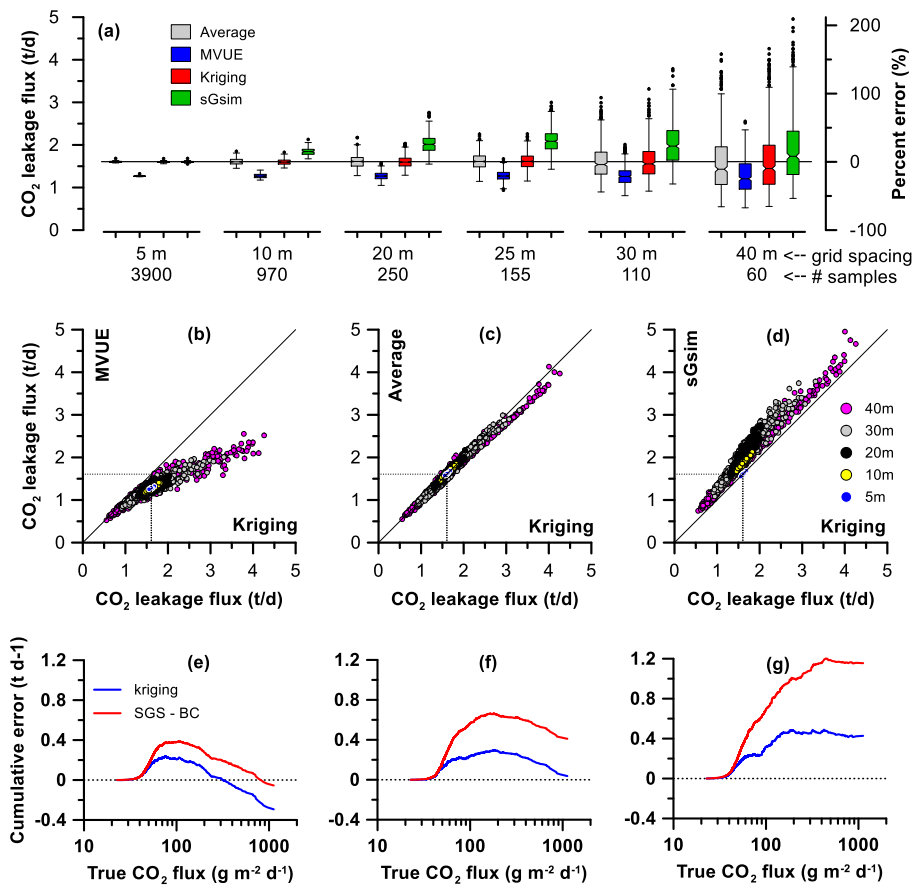
413

414 Figure 5. (a) Range of estimated average background CO₂ flux for the synthetic dataset,
 415 including the total 1km² area (red) and 16 sub-set areas (blue), compared to the true average
 416 (horizontal line). (b) Log-normal probability plots of the high density Latera dataset (black line)
 417 and the first 10 subsampling realizations at 30m sample spacing (grey lines). (c) Range of the
 418 estimated average background flux for the Latera data, based on log-normal probability plot
 419 interpretation of 50 subsample datasets at 20, 30, and 40m offset grid spacing.

420 3.2.2. Leakage flux estimates - calculation approaches

421 All leakage flux simulations discussed in this and the following sections were performed
 422 using the HR Latera data as input (Figure S6). For this section a total of 500 sub-sampling
 423 realizations were performed with MC-Flux using the off-set grid and random strategies at 8
 424 different sample densities each (5, 10, 15, 20, 25, 30, 35, 40m grid spacing, or equivalent number
 425 of samples); these strategies were chosen because they yielded the highest and lowest probability
 426 of finding anomalies, respectively, in Section 3.1. Total flux of the resultant sub-sampled
 427 datasets was calculated using all four methods. Certain parameters were fixed using the “true”
 428 values of the complete HR dataset to standardize the processing of thousands of datasets. For
 429 both kriging and SGS the standardized variogram model was defined as 0.02 Nugget + 0.98 *
 430 Spherical(37) (Figure S6d) and the search radius set to 46m, which is slightly larger than the
 431 variogram range to guarantee sufficient points for the widest grid spacing. Parameters for back-
 432 transforming the normal scores in sGsim were those used to create the original HR dataset (i.e.,
 433 lower tail linear extrapolation to 20 and upper tail hyperbolic extrapolation to 4000 with $\omega = 2$).
 434 Other parameters fixed for the sGsim calculations include: previously simulated nodes to use =

435 16; multiple refinement grids = 0; minimum and maximum data for simulation = 0 and 8; kriging
 436 type = simple. Finally, the average background flux calculated in Section 2.1 ($51 \text{ g m}^{-2} \text{ d}^{-1}$) was
 437 subtracted across the entire grid to convert the calculated total flux to leakage flux. These
 438 simplifications remove any uncertainties related to these parameters, thus highlighting variations
 439 that are primarily related to sub-sampling effects. A comparison of estimates made for 5 datasets
 440 from 3 grid sizes using these standardized parameters versus those calculated individually and
 441 manually for each file (Figures S17, S18) show relatively small changes and indicate that the
 442 approach is valid (Figure S19). Box plots for all data are given in Figure S20, while a subset of
 443 the offset grid simulations is given in Figure 6.



444

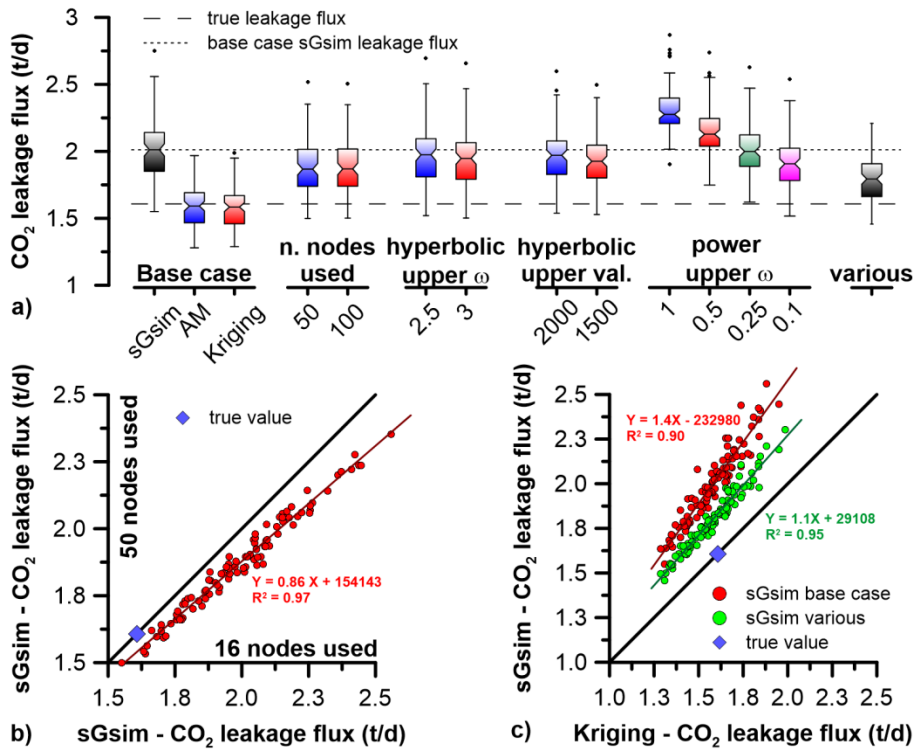
445 Figure 6. CO₂ leakage flux estimates of the Latera HR data using offset grid sub-sampling and
 446 four different calculating methods. (a) Statistical distribution versus the true value (horizontal
 447 line). (b-d) Comparison of kriging estimates with those defined using the average (b), MVUE
 448 (c), and SGS (d) calculation methods; the dashed lines indicate the true value. (e-f) Cumulative
 449 error plots for the 20m grid sub-sampled files that yielded the lowest (e), median (f) and highest
 450 (g) SGS estimates.

451

452 The MVUE approach consistently under-estimates the leakage flux rates (Figure 6a),
453 even at the 5 m grid spacing, likely because this approach is only valid for a single log-normal
454 population (Schroder et al. 2017). The greater scatter of the MVUE data relative to kriging
455 (Figure 6b) is likely due to the variable effect that different leakage samples have on the final
456 calculated value. Although some research has indicated that the MVUE approach may be valid
457 for CO₂ leakage estimates if the data is log-normally distributed (Lewicki et al., 2005; Elio et al.,
458 2016), these results appear to support the affirmation by Schroder et al. (2017) that it is not
459 appropriate. There is a sharp contrast between the MVUE and AM trends, as also shown by a
460 change in their ratio when using only background values versus including progressively more
461 leakage values (Figure S21). The AM data distribution is often centered on the true leakage flux
462 value (Figure 6a) and has a distribution similar to that kriging (Figure 6c), despite the fact that it
463 is also only valid for a single, in this case normal, population.

464 The generally linear relationship between kriging and SGS (Figure 6d) is expected, as the
465 latter has kriging at its core and the more realizations performed the more smoothed and similar
466 to the former the results become (Cardellini et al. 2003). However, for some grid spacings (10 to
467 30 m) SGS on average overestimates CO₂ leakage (Figure 6a) and the slope between kriging and
468 SGS deviates slightly from 1:1 (Figure 6d). Cumulative error plots (i.e., estimated minus true
469 values, summed for increasing true values) for the 20m offset grid subsampled files that gave the
470 lowest (Figure 6e), median (Figure 6f), and highest (Figure 6g) total flux estimates using SGS
471 show how both methods generally overestimate the low values and underestimate the high
472 values, which is a well-known artefact of kriging (Cardellini et al. 2003). However, while their
473 trends are very similar at the low end, SGS continues to over-estimate across a mid-range of
474 values before once again paralleling the kriging trend. These plots show that the final estimate
475 for both methods is a balance between over- and under-estimating across the range of values,
476 rather than a true representation, and how the higher SGS estimates are caused primarily by over-
477 estimation in mid-range values.

478 To better understand the SGS results, sensitivity analyses were performed with MC-Flux
479 using 100 sub-sampled datasets (offset grid, 20 m spacing) as input, systematically changing
480 various SGS parameters. Base case (BC) calculations were made for AM, kriging and SGS
481 (Figure 7a), with SGS BC parameters set to those used to produce the data in Figure 6.



482

483 Figure 7. (a) Statistical distribution of sensitivity analyses of various sGsim parameters; see text
 484 for description. Comparison of sGsim-estimated leakage flux using 16 nodes (base case) and 50
 485 nodes (b) and base case and “various” sGsim-estimated leakage flux versus the corresponding
 486 kriging results (c).

487 The sGsim algorithm uses a large, odd integer as a seed to the pseudorandom number
 488 generator, and each realization can be reproduced exactly by re-running the simulation algorithm
 489 using that seed. In all simulations presented thus far the default value of 69069 has been used.
 490 For this test, the two sub-sample files that yielded the highest and lowest leakage flux estimates
 491 were each processed 100 times with sGsim, each time performing the usual 100 realizations with
 492 the BC parameters but with a unique random seed value. For both input files the statistical
 493 distribution of the leakage estimates were defined by $1\sigma \approx 1.5\%$ and a total (i.e., minimum to
 494 maximum) range of about 8%. This uncertainty, however, is random and does not cause a fixed
 495 upwards or downwards shift in the data, as seen with other parameters.

496 As each new node is simulated, sGsim adds this value to the original dataset used for
 497 kriging of subsequent points, leading to progressively more values and slower computations. To
 498 speed up the algorithm the user can limit the number of previously simulated nodes that are used
 499 at each step. Sensitivity analyses show a drop in the median SGS leakage flux estimate by about

500 0.14 t d⁻¹ (ca. 7%) when the number of nodes is increased from 16 (BC) to 50, but with little
501 subsequent change using 100 nodes (Figure 7a). Both 50 and 100 nodes produce slightly
502 narrower distributions, with greater impacts observed for those sub-sampled files that yield
503 higher leakage flux estimates (Figure 7b). These results are most likely due to the observation
504 that the use of a moving neighborhood of conditioning values can result in sGsim realizations
505 that poorly reproduce second-order statistics, leading to simulated variograms that are biased
506 relative to the theoretical one (Paravarzar et al. 2015). For example, Emery (2004) showed that
507 using 20 nodes resulted in an increase in the apparent variogram range by up to 25%, while
508 results with 100 nodes were better but still produced an increase of 14%. In the context of this
509 work, an overestimated range would extend leakage anomalies over larger areas (compared to
510 kriging) and yield a larger CO₂ leakage estimate.

511 If input data are not normally distributed they must be log-transformed prior to sGsim
512 calculations and then back-transformed for final output, which requires the selection of the
513 extrapolation model type and extreme values for both the lower and upper dataset tails. While
514 lower tail characteristics can be estimated relatively accurately, the upper tail is much more
515 difficult to assess. Estimating the maximum value by extrapolating the CPP is a valid approach
516 (e.g., Cardellini et al. 2003), however the range of potential estimates that could result from
517 probabilistically equivalent sub-sample datasets, like those shown in Figure 5b, indicates the
518 uncertainty in this approach. Initial tests involved changing parameters of the hyperbolic model
519 (Figure 7a). It was found that increasing the ω parameter (related to trend curvature) from 2 (BC)
520 to 2.5 to 3 decreased the median estimate by 1.8% and 3.2%, respectively, while decreasing the
521 maximum value from 4000 g m⁻² d⁻¹ (BC) to 2000 and then 1500 (actual maximum value in the
522 dataset) decreased it by 2.0 and 4.3%, respectively. Modification of the ω parameter for the
523 power model had even larger effects, with a value of 1 (equivalent to the linear model) giving a
524 median value that is 13% above the BC while a value of 0.1 yielded one that was about 5.2%
525 below (Figure 7a).

526 Based on the above results a final test (“various” in Figure 7) was performed that
527 changed three parameters at the same time (50 nodes used and hyperbolic model with a
528 maximum value of 1500 and $\omega = 2.5$). This resulted in a narrowing of the distribution and
529 decrease of the median value by 0.22 t d⁻¹ (c. 11%). Plotted against the kriging results this final

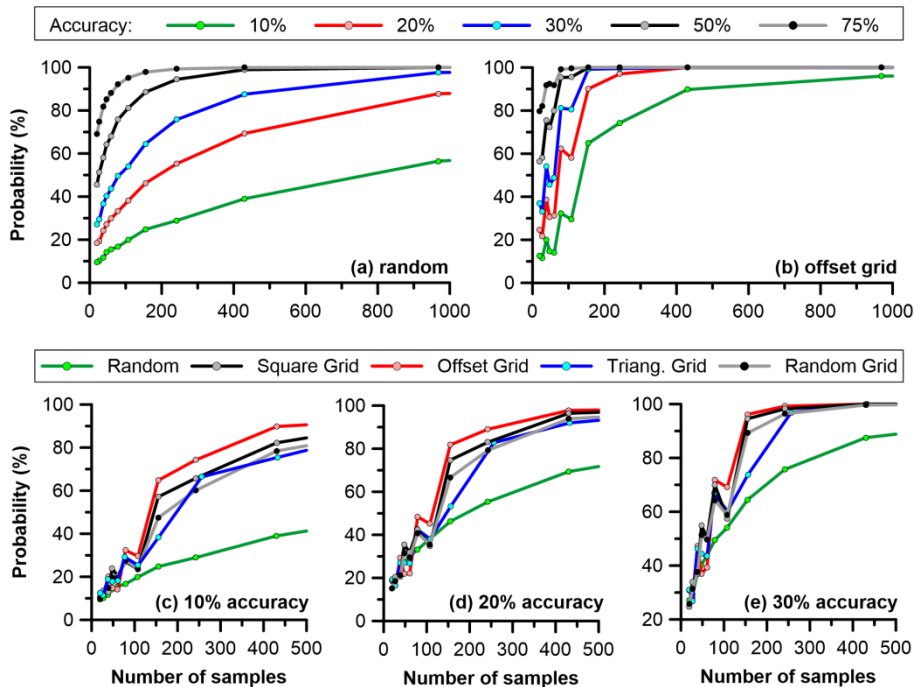
530 test yielded a higher R^2 and a slope closer to 1 compared to the base case (Figure 7c). Despite
531 these improvements, the sGsim still overestimated, on average, the true value by about 0.2 t d^{-1}
532 for these tests on the 20 m offset grid sub-samples. Two possible explanations can be inferred
533 from Emery (2004). First, a continued overestimate of the variogram range is possible despite the
534 increased number of retained nodes. Second, the simulated area may be too small, in that the
535 ratio between variogram range and domain length is 0.14 and 0.1 in the x and y directions,
536 respectively, compared to the recommended value of <0.05 .

537 3.2.3. *Leakage flux estimates - accuracy*

538 Up to 3000 MC-Flux sub-sample realizations were performed using the same input,
539 assumptions and sample densities described above for the results in Figure 6, but applying all
540 five sampling strategies (note that the number of unique square, offset, and triangular grid
541 realizations are limited by $(\text{grid spacing})^2$). Considering the similar behavior of AM and kriging
542 described in the previous section, kriging and sGsim simulations were not performed.

543 The resultant AM data were used to calculate the probability that a given number of
544 samples would yield a certain level of accuracy relative to the true value (Figure 8). Results of
545 the random sampling strategy (Figure 8a) show smooth trends for all considered accuracy levels
546 and low probability of high accuracy even for large number of samples, in agreement with that
547 observed by Wong (2018). For example, there is only a 56% chance that 1000 samples would
548 yield a leakage flux estimate that is within 10% of the true value while the same number of
549 samples have an 88% chance of being within 20%. Unlike Wong (2018), however, we have also
550 examined the response of different gridded sampling strategies, which show markedly improved
551 results. For example, the offset grid data (Figure 8b) show trends that rise much more rapidly
552 with far fewer samples. These trends are more irregular than those for the random sampling,
553 likely due to the effect of sample node spacing combined with the individual size of each
554 anomaly and the average distance between them.

555



556

557 Figure 8. Plots showing the probability that a certain number of samples will produce a given
 558 level of accuracy (lines) for random (a) and offset grid (b) sub sampling, and for all five sub-
 559 sampling strategies at 10% (c), 20% (d), and 30% (e) levels of accuracy.

560 A direct comparison of the results for the different sampling strategies at accuracy levels
 561 of 10% (Figure 8c), 20% (Figure 8d), and 30% (Figure 8e) show that random sampling
 562 consistently provides the poorest results, the different gridded approaches show similar trends
 563 (with triangular grid variations likely due to larger sample spacing for an equivalent number of
 564 samples, see Figure S7), and the offset grid yields the most accurate results (in agreement with
 565 results presented in Section 3.1). From 20 to 100 samples (i.e., 70 to 30 m grid spacing,
 566 equivalent to 200 to 1000 samples km^{-2}) the trends of all 5 strategies are relatively similar, with
 567 a less than 30% chance of 10% accuracy, less than 40% chance of 20% accuracy, and less than
 568 60% chance of 30% accuracy. Instead at the next sample density (i.e., 155 samples, 25 m grid
 569 spacing, 1550 samples km^{-2}) there is a rapid improvement in the grid sampling results, with
 570 probabilities doubling in some cases. To put these results in context, the use of the offset grid
 571 strategy and a desired 90% probability level would require 15m spacing to obtain 10% accuracy,
 572 20m for 15%, 25m for 20% and 30m for 40%. It should be acknowledged that these results are
 573 based on an original dataset having a ca. 10 m sample spacing, and although sGsim processing
 574 was performed to reduce bias, a link cannot be excluded. For this reason similar simulations
 575 using high density datasets from other sites should be conducted in the future.

576 **4. Conclusions**

577 Although it is true that each site is unique and its associated flux data must be interpreted
578 individually to obtain the best results (Elío et al. 2016), the Monte Carlo approach used here
579 (with its necessary standardization of input variables) provides useful information that can help
580 reduce uncertainties and errors in the soil flux method.

581 The offset and triangular grid sample strategies are recommended due to their superior
582 performance under all conditions except low sample densities, where they were equivalent to the
583 other methods. However, because the orientation of elongated anomalies were found to have a
584 large impact on probabilities, due to alignment within grid gaps, any available site information
585 regarding anisotropy, such as air photos, should be taken into consideration for deciding the
586 orientation of the grid itself.

587 Although sequential Gaussian simulation is the most commonly used method for
588 interpolating and quantifying leakage flux data, the results presented here show how its estimate
589 is sensitive to the chosen input parameters and, at least for the dataset used, slightly
590 overestimated for mid-range sample spacings. In contrast, the much simpler and less subjective
591 approaches using the arithmetic mean or kriging yielded probabilistic distributions were centered
592 on the true value and thus may be more appropriate (although kriging accuracy seems to be
593 linked to how well over- and under-estimated values are balanced across the grid). Additional
594 simulations using other high resolution flux datasets from other sites should be performed to
595 confirm these results.

596 The need for large numbers of closely spaced samples to accurately define leakage flux is
597 well known, and the results presented here help to quantify the potential level of uncertainty that
598 can be expected at various sample densities. This raises the question of whether there are other
599 approaches that could yield the same or lower uncertainty levels but with fewer samples, thus
600 freeing resources for other analyses to better separate background and leakage populations (e.g.,
601 Bini et al. 2020). In this regard, both kriging and SGS would benefit from the use of a high
602 resolution dataset of a linked parameter that could be used to both help choose appropriate
603 sample spacing/ locations and for variogram definition and co-kriging. One such possibility is a
604 map of CO₂ concentration anomalies at ground level (Beaubien et al. 2018) or just above it

605 (Barkwith et al. 2020), data which can be collected rapidly at high resolution. Our group is in the
606 process of assessing this approach by combining these two datasets for MC-Flux simulations.

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613 **Conflicts of interest/Competing interests:** The authors have no relevant financial or non-
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615 **Availability of data and material:** Datasets and configuration files used to conduct the reported
616 simulations are available from the Zenodo open-source online repository at
617 <https://doi.org/10.5281/zenodo.4573725> (Beaubien and Bigi, 2021b).

618 **Code availability:** The MC-Flux (V1.0) installation package and user's manual are available
619 from the Zenodo open-source online repository at <https://doi.org/10.5281/zenodo.4573575>
620 (Beaubien and Bigi, 2021a).

621 **Authors' contributions:** Conceptualization [SEB, SL], Methodology, Software, Visualization
622 and Writing – Original Draft [SEB]; Formal Analysis [SEB, GC, MGF]; Validation [GC, MGF];
623 Writing – Review and Editing [GC, MGF, SB]; Supervision [SL]; Project administration and
624 Funding acquisition [SB, SL].

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