"Quaderni di Ricerca in Didattica" N.5, 2022
G.R.I.M. (Department of Mathematics and Computer Science, University of Palermo, Italy)

Which strategies do students adopt when reading a text containing a mathematical problem? Initial results of an exploratory study with eye-tracking

Alessandro Gambini
Sapienza University of Rome
Camilla Spagnolo
Free University of Bozen
Roberto Capone
University of Bari
Federica Ferretti
University of Ferrara
Christian Casalvieri
Sapienza University of Rome

E-mail: alessandro.gambini@uniroma1.it

Abstract. Several international studies recognise the central role of understanding in problem-solving within the mathematics teaching-learning process. Interdisciplinary studies have shown how the type of text affects a student's reading and, consequently, performance. It emerged that "selective reading", through which specific attention is paid to certain textual elements, often causes a lack of understanding of the mathematical problem posed. The aim of our research is to understand how some structural and textual aspects influence understanding of a mathematical text. This research, conducted with an eye-tracker tool, shows the initial results of an exploratory study involving university students from the Faculty of Education. In recent years, eye-tracker technology has become an increasingly effective tool for analysing students' learning process. At the same time, a paper- based experiment was conducted via questionnaire, administered to grade 11 high school students. The experimentation is currently underway and initial feedback shows significant results in terms of interpretation of the resolution processes activated by the students.

1. Introduction

Various international studies acknowledge the central role of understanding a text in the teaching- learning process of mathematics (Schommer, 1993; Barton et al., 2004; Arzarello et al., 2011; Les & Les, 2015). Interdisciplinary studies have shown how the type of text affects the student's reading process and, consequently, performance.

It is now well-known how some practices, which can be traced to Brousseau's didactic contract (1988), such as selective reading (whereby the student focuses only on certain textual elements) often lead to a lack of comprehension of the mathematical problem. Our study sets out to understand how certain textual elements and graphic/text layout may influence understanding of a mathematical text and, consequently, the students' performance. Practices like identifying individual phrases or keywords highlight frequent failure to use acquired skills in a transversal context; the linguistic structure and interpretation of texts become merely procedures to carry out rather than tools that help in the representation or communication of information (Ferrari, 2001; Radford, 2000). Some problem- solving processes, especially regarding the understanding of mathematical texts and identification of appropriate strategies, reveal such behavior and require both attention and constant monitoring.

It is in precisely this direction that our research study is headed, with the processes of understanding mathematical texts being analyzed with the aid of eye-tracking technology.

By 'eye-tracking', we mean the technique used in cognitive sciences, psychology, interaction between humans and computers and in many other fields, to capture and register eye movement (i.e. Chen & Yang, 2014, Jian, 2019). Many benefits are offered by this technology: on one hand, it allows the tracking of eye movement, but it also allows the identification of elements which help identify the type of reasoning processes activated by the student in solving a task. Finally, by identifying what the student focuses attention on and the different tracks that guide their eye movement, it is possible to gather information on the process of problem-solving comprehension which is activated (whether correct or not). Eye-tracking is an increasingly popular method, also in mathematics education (Bolden et al., 2015, Cortina et al., 2015, Lin, 2014, Lee & Wu, 2018, Strohmaier et al., 2020).

In this paper, we present the results of the first two stages of a wider experimental study (first finding are highlights in Spagnolo, Capone, Gambini, 2021). The outcomes of a pre-experimental study carried out with university students from the Faculty of Education and using the eye-tracking tool are analyzed, along with a *Paper&Pencil* pilot study performed with grade 11 secondary school students. The gathered data supply information on eye movements and thus the choice and catalyzation of attention by students of different levels during mathematical tasks. The results are therefore significant in interpreting the problem-solving processes which are activated.

2. The Theoretical Framework

2.1. The eye-tracker tool in the context of Mathematics Education

By its very nature, the use of technologies in mathematics learning/teaching processes requires an interdisciplinary approach. In Mathematics Education, various studies have been carried out using the eye-tracker, studies which involved knowledge from other fields of study such as IT, neurology, biology, sociology and cognitive psychology. Cognition is strictly linked to corporeal activity and the position of the body in space and time (Lakoff & Núñez, 2000). Eye movements form part of sensorial experience and, following Radford's approach (2010), their relationship with mathematical representations can shed light on how humans acquire mathematical knowledge (Cortina et al., 2015). Different research studies in the field of eye-tracking have shown that there is a correlation between what is "looked at" and what is "thought" (Rayner, 1998; Just & Carpenter, 1980; Yarbus, 1967). These results are in alignment with other research studies that claim the existence of a correlation between visual fixation and cognitive processing of information (Latour, 1962). As a result, there has been growing interest in eye-tracking within the field of educational research (Scheiter & van Gog, 2019).

As regards the process of mathematical learning, these eye-tracking experiences seem to be in line with Duval's idea that, beginning with the famous statement "there is no noesis without semiosis" (Duval, 2006), highlights how understanding of a concept is born from the relationship between the signifier represented by a sign, a representation, and the meaning or mathematical object. Nemirovsky (2005) claims that all perceptual-motor activity (linked to changes in attention, consciousness and emotive states) contribute to understanding of a mathematical concept. In the field of Mathematics Education research, many studies (e.g. Ferrara & Nemirovsky, 2005, Andrà et al. 2009, 2015; Holmqvist et al., 2011) highlight interesting data on the students' approach to reading mathematical texts, the transformation between different representations (formulae, graphs, words) in order to understand the meaning of a text. It has also been emphasised that there are quantitative and qualitative differences between beginners and experts in the approach to reading a mathematical text and it is precisely because of these reasons that in our trial we used participants from different school years with varying mathematical knowledge.

2.2. Use of the eye-tracker in an enactive perspective

One of the foundations of rational thinking and, particularly, of human attitudes to geometry, reasoning, change of viewpoint, simultaneous treatment of different viewpoints, argumentation, logical ramifications, consists of neural underpinning of mental manipulation of spatial reference (Bertohz, 2000). Our study was designed taking into account recent research within the current of Enactivism (Hutto, 2005), an education model based on the concept that natural cognitive systems participate in generation of meaning, constructing knowledge via sensory-motor interaction (not just informational). In enactive vision, perception "is not conceived as the transmission of information but rather as an exploration of the world in various ways. Cognition is not linked to the functioning of an 'internal mind', a cognitive nucleus, but occurs within the direct interaction between the body and the world in which it lives" (McGann & Torrance, 2005).

All bodily (embodied) processes, including eye movements, contribute to the complex process of knowledge that is strictly integrated (embedded) within the external surroundings. The object to be learnt is enacted not only via neural processes but also via actions, creating an extended cycle of actions in which the subject comes to know reality, modifying and integrating with it.

“Enaction is the idea that organisms create their own experience via their actions. Organisms are not passive receptors of environmental input, but players in their surroundings, in such a way that what they experience is modelled by how they act” (Hutchins, 1996).

Enactivism claims that knowledge is born and develops via dynamic interaction between an organism that acts, and its environment; it is built by an agent via sensory-motor interaction with the environment in which it lives.

"*Organisms do not passively receive information from their environments, which they then translate into internal representations. Natural cognitive systems...participate in the generation of meaning*

...engaging in transformational and not merely informational interactions: they enact a world." (Di Paolo & Thomson, 2014)

Experience of the world, then, is the result of mutual interaction between the organism’s sensory- motor ability and its environment: this is where the term “enaction” arose, coined by Francisco Varela, Evan Thompson and Eleanor Rosch.

Beginning with studies by Maturana and Varela (1980), the ideas of Enactivism have been used in Mathematics Education by radical constructivists who tried to incorporate the concept of consensual domains in an attempt to defeat the criticism that radical constructivism did not tackle learning in social situations. Tom Kieren then introduced an entire range of concepts pertaining to Enactive theory in his research. At the same time, the concept of embodied cognition started to appear in work by numerous other researchers interested in bodily metaphors and gestures in mathematics.

We believe that the phenomena of exploration of a mathematical text via eye-tracking can be applied within the interpretative framework of Enactive Theory in light of the fact that it allows the framing of evolutionary aspects of cognitive development by the student.

3. The Study

International research findings have shown how the number of eye fixations is a reliable and sensitive indicator that can supply valuable insight into the attention flow of participants during mathematical activity (for example, during equation solving, Susac et al., 2014). In particular, the findings of Susac et al (2014) show positive correlations between the number and “positions” of fixations by the student and their efficiency in finding the solution to mathematical problems; this suggests that participants who performed well adopted winning strategies in terms of “knowing where to look for useful information regarding the solution”. It is from this and other findings in Italian and international literature that we have planned our study, structured as follows.

- Stage 1. Pilot study with an eye-tracker conducted among university students on geometry questions.
- Stage 2. Experimental Paper&Pencil study among grade 11 secondary school students tackling geometry questions.

The aim of the research is to understand if, and to what extent, students are aware of strategies that they adopt when faced with a mathematical geometry problem with an image included in the text.

The eye-tracker tool allowed us to investigate students’ eye movements in resolving mathematical tasks and to study the link between these and student performance. To do so, we have analysed both eye movements during problem-solving of mathematical tasks (to understand the order of eye fixations, we used gaze plot videos) and if/to what extent the format of the task influences which part of the page the student focuses attention on (to understand if some elements of the text capture attention more than others, we concentrated on fixations).

During the Paper&Pencil experimentation, we administered the same question in different ways (open-ended and closed formats) and we asked students to state where they looked when trying to solve the problem.

The study was carried out vertically, involving university students and grade 11 secondary school students; for this reason, different questions were set in the two stages. Even though the questions were different, they included the same basic features – both involved geometries, they both included an image in the text, and they were taken from Italian and international standardised assessment tests (the Italian INVALSI and the international OCSE-PISA).

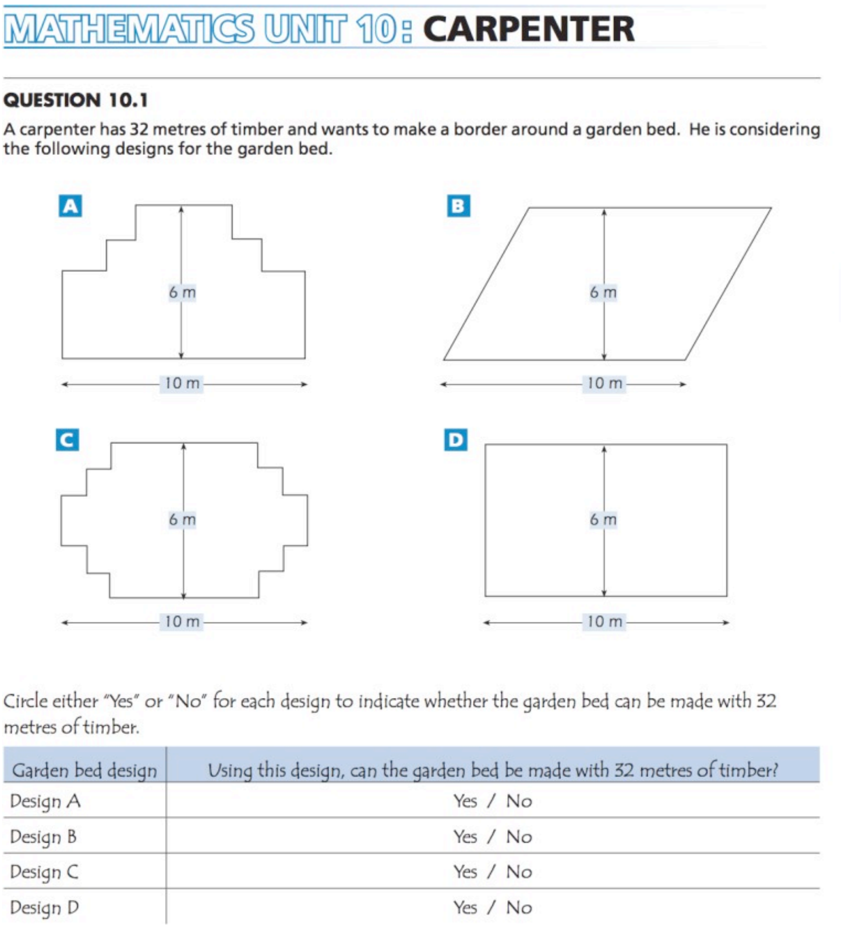
The tasks chosen were built on quantitative results from these assessment tests and focused not only on the extent and manner in which the task structure influences the problem-solving procedure but also on how this actually affects student performance.

3.1. Stage 1: the study conducted with an eye-tracker

The study conducted with an eye-tracker was the result of reflections and findings that emerged during a pilot study conducted in partnership with Free University of Bolzano-Bozen and presented at the 14th International Conference on Technology in Mathematics Teaching.

The administered tasks in this initial stage were based on quantitative results that emerged from the international standardised assessment in mathematics, OCSE PISA 2015 (OCSE-PISA, 2016).

We see in Figure 1:



MATHEMATICS UNIT 10: CARPENTER

QUESTION 10.1

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.

Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

Figure 1. Carpenter problem**.**

Below we highlight the features of the task that allowed us to activate stage 1 of our study with an eye-tracker.

The *difficulty level of the item* is 6 on the overall mathematics literacy scale. The student is asked to model a complex situation, developing a strategy in a non-familiar situation. He/she must display a strong mastery of geometry and apply it to a real context.

The fundamental mathematical concepts and knowledge needed to resolve the problem are plane figures and their properties, while the essential mathematical ability is the following: to respond correctly, it is necessary to be able to deduce (from supplied data) the length of unknown segments. The question is "complex multiple choice"; for each question, the student must select a response from two possible dichotomous options (yes/no). The main difficulty lies in determining the length of the "vertical sides" of the individual figures. It is particularly difficult to determine the perimeter of figure B as the information on the length of the sides is not inferred directly from the stimulus. It is therefore necessary to have good reasoning skills and mastery of basic Euclidean geometry in order to understand that the oblique sides of the parallelogram of figure B are longer than the vertical components of sides of the other figures (where the perimeter is exactly 32m). Only 12.3% of Italian students managed to supply the four correct answers, while 30% managed to identify 3.

This question was chosen from among other geometry tasks with an image in the text, for its level of difficulty as highlighted in international findings.

Percentage of answers from students

MATEMATICA PISA 2003
M266Q01 -
Carpentiere

	Reply 0	Reply1	Reply 2	Reply 3	Reply 4	Invalid	Omitted	Not completed
Area	%	%	%	%	%	%	%	%
North-west	1,1	26,3	20,1	28,2	**17,9**	0,0	4,0	2,3
North-east	1,8	33,4	20,3	27,3	**12,9**	0,0	1,8	2,5
Centre	3,7	27,1	24,4	27,1	**10,5**	0,0	4,4	3,0
South	3,7	27,1	24,4	27,1	**10,5**	0,0	4,4	3,0
Southern islands	3,7	27,1	24,4	27,1	**10,5**	0,0	4,4	3,0
ITALIA	*2,3*	*28,0*	*20,6*	*27,9*	***12,3***	*0,0*	*4,5*	*4,4*
OCSE	*1,5*	*25,2*	*18,9*	*30,0*	***19,5***	*0,0*	*2,4*	*2,4*

* La colonna in grassetto corrisponde a 4/4 risposte corrette.
** La risposta 3 è parzialmente corretta e corrisponde a 3/4 risposte corrette

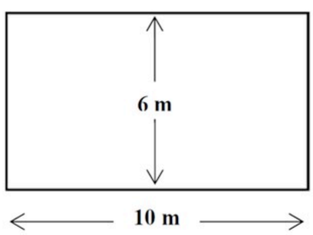
Figure 2. Analysis of OCSE-PISA questions

At a later date, the question was prepared for use with an eye-tracker. Consistent the questions were modulated and administered in an appropriate form. To obtain information about problem-solving processes adopted by students (also with regard to the structure and textual features of the task), the question of Figure 1 was presented in four different ways. This choice allowed an increase in the readability of the task and calibration of the eye-tracker tool. The question remained unvaried, but the designs of the project were shown individually to students (no longer all together).

A carpenter has 32 metres of wooden planks and wants to make

a garden fence. He considers different plans for the fence.

This is the first.



Are there enough planks? Briefly explain your answer.

Figure 3. Example of task from OCSE-PISA

Furthermore, the students were asked to briefly justify their answer. This variation was made in order to observe the eye movement of students while they explained their problem-solving strategy and tracked their problem-solving process.

Students were also told that they could look at the situation as long as necessary in order to supply an answer and that, in responding to a situation, they could make reference also to reflections inherent to previously viewed situations. In this way, it was possible to identify, via recording of eye movements, the focus and fixation on structural and textural elements, and thus, analyze problem- solving strategies.

Eight students in the second year of Primary Education at the Free University of Bolzano-Bozen were involved. Specifically, these were students in education science, as students are teachers-in-training and so they pay attention to any possible educational consequences.

These students were volunteers, and each was involved in the task for two hours. As well as the problem itself, the task involved a non-structured interview during which the student was presented with the results of the eye-tracker investigation. In this way it was possible to compare what the student thought he/she had looked at with what they actually focused on.

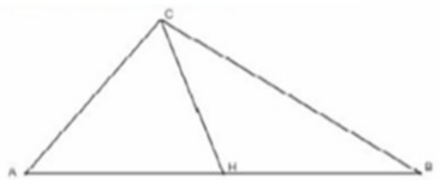
3.2. Stage 2: the Paper&Pencil test

The This stage involved 47 Italian students in grade 11.

The question identified for this stage was chosen from among INVALSI standardized mathematics assessment tests (a national skills assessment test carried out in Italian schools in grades 2, 5, 8 and 13).

The selected question is a multiple-choice format. However, the question was posed in two ways (multiple choice and guided open question). The text was modified to adapt it for the open question format, while it was administered in its original wording for the multiple-choice format.

This was the question chosen:


H is the mid-point of side AB in the ABC triangle.

The triangles AHC and HBC have the same area because

A. ☐ the distance of C from AB is the same in the two triangles and AH=HB

B. ☐ the median CH divides the triangle into two congruent triangles

C. ☐ CH is the height common to both triangles and the relative bases are the same length

D. ☐ the triangles CHA and CHB are both isosceles triangles

Figure 4. INVALSI question for grade 10, school year 2012/13

The question (figure 4) was set for all grade 10 Italian students in the INVALSI mathematics test of school year 2012/13.

The question investigates the use of conjecture and reasoning. These aspects are highlighted through knowledge of fundamental notions in plane geometry.

The national percentages show that 22.7% of students responded correctly (option A), 73.3% replied incorrectly, and 4% did not respond. From the students who made mistakes, 18.8% chose distractor B, 46.6% chose distractor C and 8.1% chose distractor D.

The question was modified for the second (open-ended) format as follows:

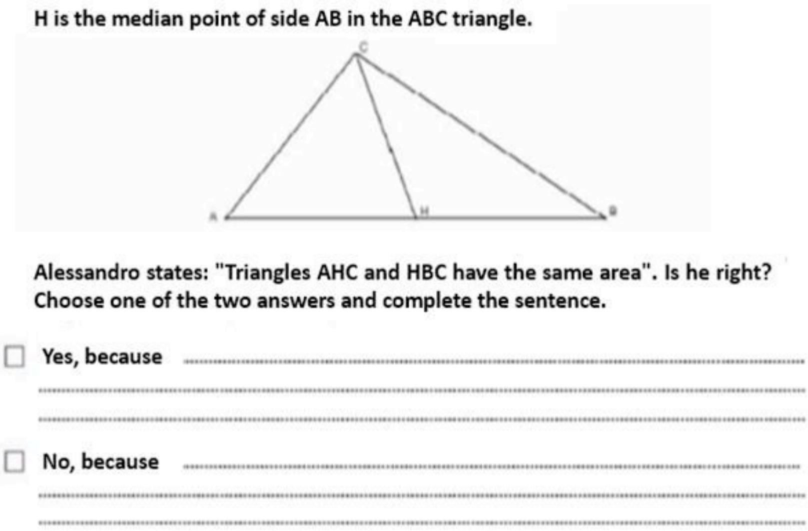


Figure 5. Versions created for the first survey.

The tasks were chosen to highlight also the following aspects:

- Investigate (in the problem-solving that includes an image) which elements the students focus attention on, and which not.
- identify possible differences in eye movements and in the tracking of elements used to resolve problems with the same stimulus when administered in open and closed mode.

To investigate these issues, a questionnaire was planned that was presented to students immediately after tackling the problem-solving task.

The questionnaire included the following questions:

1. What did you look at more – the image or the text?
2. Which elements of the text most captured your attention?
3. Which elements of the image most captured your attention?
4. Which element did you begin with to look for the solution?
5. Which element allowed you to find the solution?
6. Did you read first the text in the question and then look at the image, or vice versa? Why? This questionnaire allowed the students to reflect on which part of the text most significantly captured their attention and why. Each student replied to all six questions in the questionnaire.

4. Results

4.1 Results of Stage 1: study conducted with an eye-tracker

Use The pilot study conducted with the eye-tracker allowed us to clarify the workings of the tool and to focus better on which parts of the text may be classed as "areas of interest" by the Tobi pro-lab software which was used for data analysis of results collected by the tracker.

The interpretative analysis follows an enactive perspective, by which the construction of knowledge occurs through sensory-motor interactions and in which, eye movements therefore also play a fundamental role.

Before proceeding with the analysis, we specify (as illustrated in para.3.1) that in order to allow analysis with the eye-tracker, the task must make reference to only one stimulus at a time and be visible without scanning the page.

The following figure (Fig.6) shows the initial results of analysis referring to stimulus 1.

Figure 6. Initial results of analysis referred to stimulus 1.

In the figure, we can see the parts of the text that the student focused less attention on (colored in green), a little more time/attention (yellow) and the most time/attention (red). It is clear the student was most attracted by the presence of numbers or images, even when these are not relevant in answering the question. Specifically, in both the text and image, the student focused attention on numbers (red part) and the little attention dedicated to text may be an indicator of "selective reading" (Zan, 2012). We can interpret these findings with some of the categories of didactic contract as defined by Brousseau (1988).

We can see similar behaviour in the following situation (Fig.7); in this question, there is a non- standard polygon and again in this task the student focused mainly on the image in the text, returning few times to the text itself.

A carpenter has 32 metres of wooden planks and wants to make a garden fence.

He considers different plans for the fence. This is the third plan.

*Are there enough wooden planks? Briefly explain your answer***.**

Figure 7. Initial results of analysis regarding stimulus 2.

After the test, the students were interviewed, and it emerged that many of them were not aware of where they had focused attention while solving the task.

In this Stage, eye-tracker tool together with interviews allow as to explore the process of the student working. Some recent work shows that this methodology provides effective results (Schindler & Lilienthal, 2019&2020).

4.2 Results of Stage 2: Paper&Pencil

The analysis of stage 2 will be qualitative, but we also supply the quantitative results of the tests given to 47 Italian students in Year 11.
The question was given to 24 students in the closed format (the question text is presented in para. 3.2, Fig 4). Of these, 13 answered correctly and 11 incorrectly; of these 11, 9 chose distractor C while 2 chose distractor B.
The question was given to 23 students in the open-ended format (the question text is presented in para.3.2, Figure 5). Of these, 15 answered correctly, and 8 incorrectly.
Some of the students who responded to the open-ended format question felt obliged to insert elements into the diagram to justify their response. For example, 5 of them (not all from the same class) drew the height on the diagram, renamed it and made reference to this in their reply.

Yes, because AH=HB and CK is the height of both.

Yes, because the area of the triangle is calculated (bxh):2. We ascertained that the bases are congruent because H is the median point of AB, so AH=HB, the height CK is congruent to height TB (see drawing) because CT and AB are parallels and therefore the distance between them is equal at all points of the lines.

The Paper&Pencil experiment revealed some important findings about strategies adopted by students in problem-solving. In particular, some elements were highlighted where students thought they focused their attention and gaze. We will later analyse some illustrative situations.
Most students who replied incorrectly to the closed format question (9 out of 11) identified the CH segment as the height of the triangle ABC (Duval, 1999):
K is the median point of side AB in the triangle ABC. The triangles AHC and HBC have the same area because:

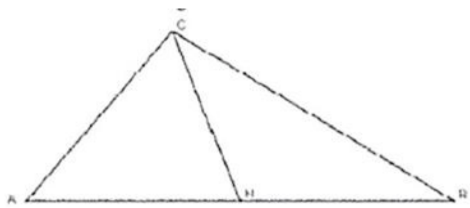


Figure 8. The triangle ABC.

A - The distance from C to AB is the same in the two triangles and AH=HB B - The CH median divides the triangle into two congruent triangles.
C - They share the same height (CH) and the relative bases are the same length D - The triangles CHA and CHB are both isosceles triangles.

In the questionnaire, these students stated that they had focused on the text (particularly on the response options) and had reasoned by exclusion, thus focusing strongly on distractors:
Student 1: “What allowed me to find the answer was the way I acted, by comparing each option with the image in order to identify the solution”,

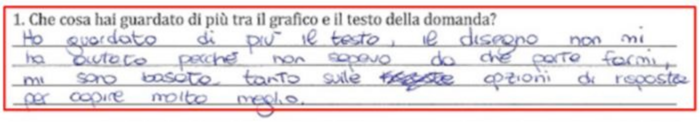
Student 2; “I started with option A and I checked whether it was true or not. I did the same thing with the other three options, and this let me find the solution,” *Student 3; “I looked at the text more because I was checking the various answers provided and whether they were suited to the figure”.*

Student 4: “I focused more on the possible responses, analysing them one by one, and excluding the wrong options”.

On the contrary, students who received the open-ended questions claimed they had focused attention on the diagram, and from cross-checking it emerged that those who did so provided the correct answer.

Student 5: “I looked at the diagram to reason well about the diagram I needed to consider”.

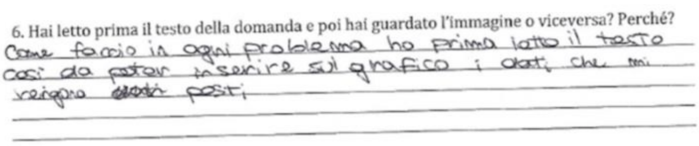
So, the strategies selected to respond to the same question administered in closed-response mode are different: those who responded correctly focused attention mainly on the response options.


1. Che cosa hai guardato di più tra il grafico e il testo della domanda?
Ho guardato di più il testo, il disegno non mi ha aiutato perché non sapevo da che parte farmi, mi sono basato tanto sulle opzioni di risposte per capire molto meglio.

What did you look at more – the graph or the text?
I looked more at the text,the graph did not help me because I did not know which part to look at.
I based my answer a lot on the response options
to understand better.

Figure 9: Student’s answer sheet

Despite this, the analyses regarding the sixth question of the paper questionnaire (Did you read first the text of the question and then look at the diagram, or vice versa? Why?) show that, in most cases, students were convinced they had used the same procedure to resolve all the problems.


6. Hai letto prima il testo della domanda e poi hai guardato l'immagine o viceversa? Perché?
Come faccio in ogni problema ho prima letto il testo così da poter inserire sul grafico i dati che mi vengono posti;

Did you read first the text of the question
and then look at the image, or viceversa?
Why?
As with every task, I first read the text in order to be able to add the data provided to the graph

Figure 10. Student’s answer sheet.

Figure 10 shows the answer sheet of a student who states:

As with every task, I first read the text in order to be able to add the data provided to the diagram

On the other hand, others stated:

I always look at the diagram more, obviously, because it catches the eye more I read the text first because I’ve always done it that way

Generally, I always start by looking at the diagram; in fact, the graphs catch my attention first, then by thinking about them, you practically find the solution to the problem in front of you.

There is also an evident necessity among them to apply the criteria of congruency because in the question there is a comparison between diagrams (Brousseau, 1988). Some of them explained that

“among the triangle formulae, the congruency criteria are recalled” and thus they solve the problem by *“reasoning on topics dealt with in the past”.*

References to criteria of congruency are present in the answer sheets of 6 students:

I thought about whether I knew the rules and formulae regarding triangle areas, and I remembered the criteria of congruency.

The triangles do not respect any of the 3 criteria of congruency, and so they cannot have the same area.

I concentrated on the angles and sides because I was looking for congruency to check one of the 3 criteria.

In answering the second question of the questionnaire (Which parts of the text most captured your attention?), some students stated that they saw the text as a container of data:

The keywords: H, median point, area

The letters H, AB, ABC, because I then looked for these in the diagram.

In this type of reasoning, the student often makes reference to key words to identify the correct operation or formula, they proceed ‘randomly. This happens when students find it difficult to develope the suitable representation of the problem, i.e. to link the information in order to identify a logical problem solving strategy.

One of the most relevant results, which emerged transversally, is that when the same task is administered both in open and close-ended form, the performance and, above all, the problem-solving strategies adopted are very different. In responding to the same question (in different formats), the students claimed that they focused their gaze and attention on different parts of the task. Analysis of the results reveals that the very fact of having focused attention on different parts of the task is one of the biggest factors in success or failure of the problem-solving attempt.

It also emerged that, regarding the close-ended questions, focusing attention mainly on the response options and the decision to proceed by “exclusion” frequently caused students to select the wrong answer. The importance of the strategy chosen emerges above all when faced with performance in reference to the same questions administered in open mode; to respond to open-ended questions, the students focused on other elements (in most cases, the necessary elements) and the number of correct answers was decidedly higher.

In general, the questions in the questionnaire allowed students to recognise their mistakes and, in some cases, to make them aware of how they had focused their attention on irrelevant textual and structural elements to solve the problem.

“Now I understand that the correct answer was not C because CH is not the height”

Finally, 8 students chose to write some general reflections in the margins, caused by reflection on the questionnaire.

In doing this questionnaire, I understood that the important thing is to reason on why certain choices or actions are selected over others, also it’s essential to know the reasons why we do things.

4.3 Conclusions

Please The results of these first stages of experimentation supply rich sources for reflection as regards structural and textual elements of mathematics tests that capture the attention of students, and to what extent these choices affect their mathematical performance. In accordance with the enactive perspective, eye movements (as with all embodied processes) contribute to the complex process of knowledge-building and analysis of these movements supplies precise information on problem-solving processes activated by students. In the first research stage, eye movements were investigated with the use of an eye-tracker during mathematical task completion, and in the second stage, confirmation was achieved of how textual and structural features of the task influence problem-solving choices and, therefore, students’ performance.

The students themselves recognised how their attention is often catalysed by elements that are irrelevant in problem-solving; the data collected, and analysis carried out in these initial stages allowed us to define the planning and implementation of the last step in a wider research project of which this experimental study forms.

References

Andrà, C., Arzarello, F., Ferrara, F., Holmqvist, K., Lindström, P., Robutti, O., & Sabena, C. (2009,January). How students read mathematical representations: An eye tracking study. In *Proceedings ofthe 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol.2, pp. 49-56). PME Thessaloniki, Greece.

Arzarello, F., Bussi, M. G. B., Leung, A. Y. L., Mariotti, M. A., & Stevenson, I. (2011). Experimental approaches to theoretical thinking: Artefacts and proofs. In *Proof and proving in mathematics education* (pp. 97-143). Springer, Dordrecht.

Barton, B., Chan, R., King, C., & Neville-Barton, P. (2004). The mathematical discourse of advancedundergraduate mathematics. In *Proceedings of 27th Mathematics Education Research Group Conference* (pp. 79-86).

Berthoz, A. (2000). *The brain's sense of movement* (Vol. 10). Harvard University Press.

Bolden, D., Barmby, P., Raine, S., & Gardner, M. (2015). How young children view mathematical representations: a study using eye-tracking technology. *Educational research*, *57*(1), 59-79.

Brousseau, G. (1988). Le contrat didactique: le milieu [The didactic contract: the milieu]. *Recherchesen Didactique des Mathématiques, 9* (3), 309-336.

Chen, Y. C., & Yang, F. Y. (2014). Probing the relationship between process of spatial problems solving andscience learning: An eye tracking approach. *International Journal of Science and Mathematics Education*, *12*(3), 579-603.

Cortina, K. S., Miller, K. F., McKenzie, R., & Epstein, A. (2015). Where low and high inference datacon-verge: Validation of CLASS assessment of mathematics instruction using mobile eye tracking with expert and novice teachers. *International Journal of Science and Mathematics Education*, *13*(2),389-403.

Di Paolo, E., & Thompson, E. (2014). The enactive approach. *The Routledge handbook of embodiedcognition*, 68-78.

Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning. In F. Hitt & M. Santos (Eds.), *Proceedings of the 21st annual meeting of the North American Chapter of the International Group for the Psychology of MathematicsEducation* (pp. 3–26). Cuernavaca, Morelos, Mexico.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, *61*(1-2), 103-131.

Ferrara F., & Nemirovsky, R. (2005). Connecting talk, gesture, and eye motion for the microanalysis of mathematics learning. In: H.L. Chick & J.L. Vincent (Eds.), *Proceedings of PME 29* (Research Forum) **1**, 137-142. University of Melbourne, PME: Melbourne, Australia.

Ferrari, P. L. (2001) Understanding Elementary Number Theory at the Undergraduate Level: A Semiotic Approach, in Campbell, S.R.&R.Zazkis (Eds.) *Learning and Teaching Number Theory: Research in Cognition and Instruction*, Westport (CT, USA): Ablex Publishing, pp.97-115

Holmqvist, K., Andrà, C., Lindström, P., Arzarello, F., Ferrara, F., Robutti, O., & Sabena, C. (2011). A method for quantifying focused versus overview behavior in AOI sequences. *Behavior research methods*, *43*(4), 987-998.

Hutto, D. D. (2005). Knowing what? Radical versus conservative enactivism. *Phenomenology and the Cognitive Sciences*, *4*(4), 389-405.

Hutchins, E. (1996). *Cognition in the wild.* Bradford.

Jian, Y. C. (2019). Reading instructions facilitate signaling effect on science text for young readers: an eye-movement study. *International Journal of Science and Mathematics Education*, *17*(3), 503- 522.

Just, M. A., & Carpenter, P. A. (1980). A theory of reading: From eye fixations tocomprehension. *Psychological review*, *87*(4), 329.

Lakoff, G., & Núñez, R. E. (2000). Where mathematics comes from: How the embodied mind brings mathematics into being. *AMC*, *10*(12), 720-733.

Latour, P. L. (1962). Visual threshold during eye movements. *Vision Research, 2*, 261–262.

Les, Z., & Les, M. (2015). Understanding Text. In *Shape Understanding System* (pp. 197-228). Springer, Cham.

Lee, W. K., & Wu, C. J. (2018). Eye movements in integrating geometric text and figure: Scanpaths and given-new effects. *International Journal of Science and Mathematics Education*, *16*(4), 699-714.

Lin, S. S. (2014). Cognitive load for configuration comprehension in computer-supported geometry problem solving: An eye movement perspective. *International Journal of Science and Mathematics Education*, *12*(3), 605-627.

Maturana HR, Varela FJ (1980) Autopoiesis and cognition: the realization of the living. Kluwer, Dordrecht

McGann, M. & Torrance, S. (2005). "Doing It and Meaning It: And the relation between the two". In Ralph D. Ellis;

Natika Newton (eds.). *Consciousness & Emotion: Agency, conscious choice, and selective perception*. John Benjamins Publishing

OECD. (2016). PISA 2015 Technical report. Paris, FR: OECD Publishing.

Radford, L. (2000) *Signs and meanings in students emergent algebraic thinking: a semiotic analysis*, Educational Studies in Mathematics, 42: 237-268.

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the learning of mathematics*, *30*(2), 2-7.

Rayner, K. (1998). Eye movements in reading and information processing: 20 years of research. *Psychological Bulletin, 124*(3), 372–422.

Schindler, M., & Lilienthal, A. J. (2019). Domain-specific interpretation of eye tracking data: towards a refined use of the eye-mind hypothesis for the field of geometry. *Educational Studies in Mathematics*, *101*(1), 123-139.

Schommer, M. (1993). Epistemological development and academic performance among secondary students. *Journal of educational psychology*, *85*(3), 406.

Spagnolo, C., Capone, R., & Gambini, A. (2021). Where do students focus their attention on solving mathematical tasks? An eye tracker explorative study. Proceeding of PME 44, *Khon Kaen, Thailand 19-22July 2021*, 84-92.

Strohmaier, A. R., MacKay, K. J., Obersteiner, A., & Reiss, K. M. (2020). Eye-tracking methodology in mathematics education research: A systematic literature review. *Educational Studies in Mathematics*, *104*, 147-200.

Susac, A. N., Bubic, A., Kaponja, J., Planinic, M., & Palmovic, M. (2014). EYE MOVEMENTS REVEAL STUDENTS’STRATEGIES IN SIMPLE EQUATION SOLVING. *International Journal of Science and Mathematics Education*, *12*(3), 555-577.

Yarbus, A. L. 1967. *Eye movements and vision (B. Haigh, Trans.)*, New York: Plenum Press.

Zan, R. (2012). La dimensione narrativa di un problema: il modello C&D per l'analisi e la (ri) formulazione del testo. *L’insegnamento della matematica e delle scienze integrate*, *35*(2), 107-126.