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Snowmass2021 - Letter of interest cosmology intertwined IV: The age of the universe and its curvature

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ABSTRACT

A precise measurement of the curvature of the Universe is of prime importance for cosmology since it could not only confirm the paradigm of primordial inflation but also help in discriminating between different early-Universe scenarios. Recent observations, while broadly consistent with a spatially flat standard Λ Cold Dark Matter (ΛCDM) model, show tensions that still allow (and, in some cases, even suggest) a few percent deviations from a flat universe. In particular, the Planck Cosmic Microwave Background power spectra, assuming the nominal likelihood, prefer a closed universe at more than 99% confidence level. While new physics could be at play, this anomaly may be the result of an unresolved systematic error or just a statistical fluctuation. However, since positive curvature allows a larger age of the Universe, an accurate determination of the age of the oldest objects provides a smoking gun in confirming or falsifying the current flat ΛCDM model.

The curvature of the Universe – The flat Λ Cold Dark Matter (ΛCDM) cosmological model fits current cosmological observations supremely well. Yet, in addition to the well-publicized *Hubble constant* (H_0) tension [1], and tension in the amplitude of mass fluctuations σ_8 (or related parameter S_8) [2], there are additional oddities in the Planck 2018 results that deserve further investigation. Statistically the most significant among them is the 3.4σ preference for a closed universe [3–5]. Moreover, Planck also favors Modified Gravity at $> 2\sigma$ [4,6,7]. This overall tension with the standard flat ΛCDM model can be recast as an anomalous lensing contribution to the Cosmic Microwave Background (CMB) power spectra, described by an unusually high A_L parameter [4,8], which is strongly degenerate with curvature Ω_k (see Fig. 1). A closed universe also alleviates the $\gtrsim 2\sigma$ tension between the low and high multipoles of the CMB angular power spectrum [3,9,10]. This *prima facie* evidence for nonzero curvature can be due to unforeseen systematics in the Planck 2018 data, or could simply be a statistical fluctuation.

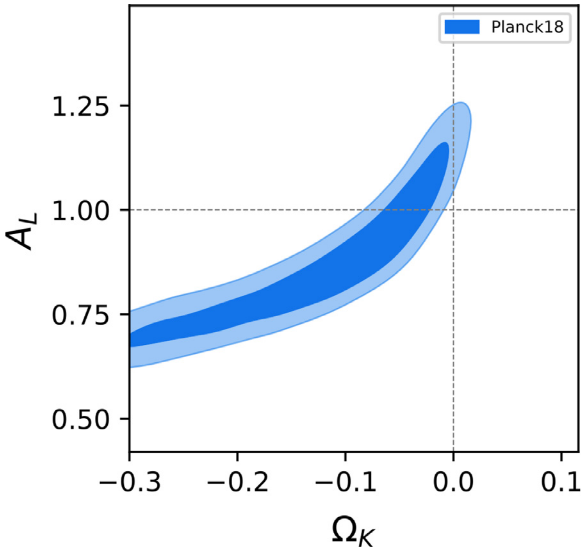

Fig. 1. 68% CL and 95% CL contour plots for Ω_k and A_L (from Ref. [3]).

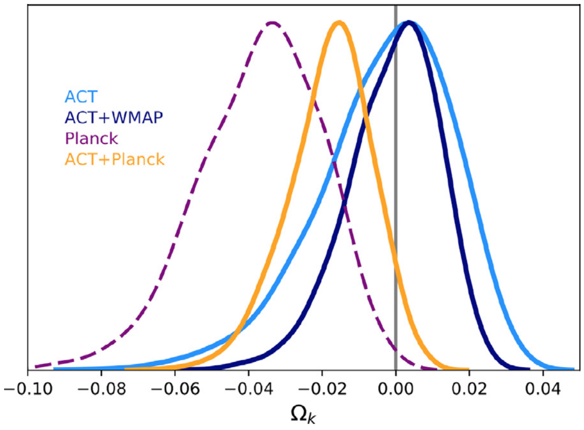

Fig. 2. 1D posterior distributions on Ω_k (from Ref. [14]).

Indeed, while Planck 2018 [4] finds $\Omega_k = -0.044^{+0.018}_{-0.015}$[1], i.e. $\Omega_k < 0$ at about 3.4σ ($\Delta\chi^2 \sim -11$) using the official baseline Plik likelihood [11]. The evidence is reduced when considering the alternative CamSpec [12] likelihood (see discussion in [13]), although the marginalized constraint still favoring $\Omega_K < 0$ at greater than 99% CL ($\Omega_k = -0.035^{+0.018}_{-0.013}$). Moreover, the recent results from the ground-based experiment ACT, in combination with data from the WMAP experiment, are fully compatible with a flat universe with $\Omega_k = -0.001^{+0.014}_{-0.010}$, while slightly preferring a closed universe when combined with a portion of the Planck dataset with $\Omega_k = -0.018^{+0.013}_{-0.010}$ [14](see Fig. 2). A closed universe is also preferred by a combination of non-CMB data, by combining Baryon Acoustic Oscillation (BAO) measurements [15–17], supernovae (SNe) distances from the recent Pantheon catalog [18], baryon density derived from measurements of primordial deuterium [19] assuming Big Bang Nucleosynthesis (BBN). This combination has a much higher H_0 [3] completely in agreement with the SH0ES collaboration value R19 [20]. Letting the curvature free to vary means to increase both the H_0 and the S_8 tensions [3]. Therefore, at the moment there are not theoretical models that can explain at the same time all the tensions and anomalies we see in the data. On the other hand, a flat universe is preferred also by Planck + BAO, or + CMB lensing [21] or + Pantheon data. However these dataset combinations are in disagreement at more than 3σ when the curvature is free to vary [3,5]. In addition, though the error bars are so large that cannot discriminate between the models, a flat Universe is also in agreement with the analysis made by [22] using the $H(z)$ sample from the cosmic chronometers (CC) and the luminosity distance $D_L(z)$ from the 1598 quasars ($\Omega_k = 0.08 \pm 0.31$) or the Pantheon sample ($\Omega_k = -0.02 \pm 0.14$), in agreement with the previous [23]. Finally, in [24] a combination of BAO+BBN+H0LiCOW provides $\Omega_k = -0.07^{+0.14}_{-0.26}$ with H_0 in agreement with R19, while BAO+BBN+CC gives a positive $\Omega_k = 0.28^{+0.17}_{-0.28}$. In [13] it has been pointed out that is hard to believe in a cosmological data conspiracy giving $\Omega_k = 0$. However, a full agreement of the luminosity distance measurements, like Pantheon or R19, with Planck can

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[1] All the bounds are reported at 68% confidence level in the text.

be reached also ruling out both, a flat universe and a cosmological constant [25]. **The Age of the Universe –** The age of the universe is an important piece of the puzzle because it connects H_0 and Ω_m, both of which can be measured in the early and the late universe. The age is not just a prediction of the ΛCDM model, which for Planck 2018 is $t_U = 13.800 \pm 0.024$ Gyr, but can also be measured using very old objects. For example, [26] obtains $t_U = 13.35 \pm 0.16(\text{stat.}) \pm 0.5(\text{sys.})$ Gyr using populations of stars in globular clusters. Nevertheless, while robustness and accuracy tests have been done very extensively for CMB, BAO and SNe; the age method has not been validated to the same level of confidence. For example, one finds the ages of the oldest stars 2MASS J180820025104378 B equal to $t_* = 13.535 \pm 0.002$ Gyr [27], but if the scatter among different models to fit for the age is taken into account the age becomes $t_* = 13.0 \pm 0.6$ Gyr [28]. Also the age of HD 140283, nominally $t_* = 14.46 \pm 0.8$ Gyr [29], becomes $t_* = 13.5 \pm 0.7$ Gyr [28] using the new Gaia parallaxes instead of original HST parallaxes. Therefore, even if at present there is not real tension between the different t_u determinations, most of the error in the age estimates comes from the fact that different stellar models do not agree with each other at the required level of precision to be able to help with the tensions in cosmology. Nevertheless, stellar models can/are expected to improve reducing this error significantly, and this could potentially unveil a tension in the age of the Universe. Trying to alleviate it by changing the Planck model assumptions, would interestingly have an effect on the cosmological tensions. One way to increase the predicted age of the Universe, to be larger than the age of oldest stars, is to lower the Hubble constant because $t_0 \simeq 1/H_0$ [30]. For example, a positive curvature for the Universe, as suggested by Planck 2018, prefers a lower H_0 and thus worsens the Hubble tension, but predicts an older Universe $t_U = 15.31 \pm 0.47$ Gyr. Therefore, it seems that the way to address the H_0 crisis if R19 is correct is to introduce an extremely recent (after $z \sim 0.1$) departure from ΛCDM, requiring a great deal of fine-tuning.

Future – Detecting a nonzero curvature Ω_k could be due to a local inhomogeneity biasing our bounds [31], and in this case CMB spectral distortions such as the KSZ effect and Compton-y distortions, present a viable method to constrain the curvature at a level potentially detectable by next-generation experiments. If nonzero curvature is evidence for a truly superhorizon departure from flatness, this will have profound implications for a broad class of inflationary scenarios. While an open universe is easier to obtain in inflationary models [32–36], with a fine-tuning at the level of about one percent one can obtain also a semi-realistic model of a closed inflationary universe [37,38]. In [39], it has been shown that forthcoming surveys, when combined, are likely to place constraints on the spatial curvature of $\sim 10^{-3}$ at 95% CL, enough for solving the current anomaly in the Planck data. Experiments like Euclid and SKA, instead, may further produce tighter measurements of Ω_k by helping to break parameter degeneracies [40,41].

Summary – In these four LoIs [1,2,42] we presented a snapshot, at the beginning of the SNOWMASS process, of the concordance ΛCDM model and its connections with the experiment. This is a cutting-edge field in the area of cosmology, with much growth over the last decade. On the experimental side, we have learned that it is important to have multiple precise and robust measurements of the same observable, with blinded analysis of experimental data. This provides a unique opportunity to study the same physics from different points of view. On the theory side, on the other hand, it is really important having robust and testable predictions for the proposed physical models that can be probed with the data. With the synergy between these two sides, significant progress can be made to answer fundamental physics questions. During the SNOWMASS process we plan to monitor the new advances in the field to come out with a clear roadmap for the coming decades.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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