Dynamic substructuring with time variant coupling conditions for the analysis of friction induced vibrations

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Abstract
Dynamic substructuring methods are originally developed for time invariant systems in order to evaluate the dynamic behavior of a complex structure by coupling the component substructures. Sometimes, the component substructures change their position over time affecting the dynamics of the entire structure. This family of problems can be tackled using substructuring techniques by isolating the time dependency in the coupling conditions among the substructures that are time invariant. Mechanical systems, composed by subsystems in relative motion with a sliding interface, can be analyzed using this approach. In previous works the authors proposed a solution method in time and frequency domain using this approach under the assumption that the relative sliding motion at the contact interfaces is a-priori known, at least approximately. The previous assumption implies that the perturbation generated by the friction induced vibration is neglected. In this work the analysis is extended to account for friction induced vibrations in the formulation of the coupling conditions. This extension allows for the investigation of a wider range of operating conditions including also friction induced instabilities.

1 Introduction

In complex mechanical systems, contact and friction forces arise at the contact interfaces among components in relative motion. These forces can cause the so called friction induced vibrations [1, 2]: a dynamic response of the system in terms of vibration and noise, that in certain conditions can affect structural integrity and comfort [3].

Contact problems could be tackled using the dynamic substructuring approach [4, 5, 6], considering each body in contact as a single substructure. In fact, the dynamic substructuring allows to predict the dynamic behaviour of a complex mechanical system, knowing the dynamic behaviour of its component subsystems [7, 8].

Although the classical substructuring approach assumes that the system is time invariant, time-variant systems composed by time-invariant substructures subjected to time-variant coupling conditions have been tackled with this approach [9, 10, 11]. The numerical analysis of configuration dependent problems in the framework of dynamic substructuring provides interesting results reducing the computational effort. For sliding contact problems with friction, equilibrium conditions apply on the tangential directions at the contact interface, that are not considered in the compatibility conditions because of sliding. This represents a non collocated interface as defined in [12, 6].

The approach presented in [11] assumed that the time-variant compatibility and equilibrium conditions, arising from sliding contact, are a-priori known. The assumption made in [11] does not allow to account for the relative displacement at the contact interface due to the system deformation, that is relevant for dynamic contact problems [13] where it is the main cause of friction induced vibrations phenomena such as dynamic instabilities [14, 15, 16], stick-slip [17, 18] or sprag-slip [19, 20].

For this reason the authors proposed an enhanced substructuring method that provides a more reliable definition of compatibility and equilibrium conditions [21]. In this case, the time-variant coupling conditions due to sliding are estimated accounting for the deformation of the contacting bodies introducing also geometric nonlinearities.

In this paper, substructuring methods are applied to evaluate the response associated with unstable friction induced vibrations of a beam on beam system [22]. In particular, the results obtained using the method proposed in [11] are compared with those of a well assessed numerical method [23] in order to highlight possible limitations. Then, the results obtained using the enhanced method proposed in [21] are also compared with those of the well assessed numerical method [23].

2 Numerical methods for contact dynamic analysis

In this section two different approaches are presented to investigate the dynamic response of system composed by different bodies in frictional contact in which the mutual interaction can give rise to friction induced vibrations. A well known numerical approach [23] is described, to be used as reference for the result comparison with a substructuring based method, developed by the authors and detailed in [11].

2.1 Forward increment Lagrange multiplier method

The method proposed by Carpenter et al. in [23] is a very efficient numerical method to simulate the dynamic response of a mechanical system with a frictional contact interaction among different parts. It employs the explicit β_2 integration scheme proposed by Newmark [24] to evaluate a predictor of displacements at a given step without accounting for contact. When predicted nodal positions show a compenetration, Lagrange multipliers are introduced on contact nodes in order to correct the nodal displacement. The first correction is computed supposing a sticking contact, but if the computed friction forces exceed the sticking limit, relative sliding between parts is allowed and friction forces are those due to sliding. Hence, the correct displacement is evaluated by using a Gauss-Seidel iterative procedure.

2.2 Substructuring with time variant coupling conditions

Contact problems are naturally prone to be investigated using dynamic substructuring. In fact, the system is generally composed by different subsystems and their contact interaction could be defined in the substructuring framework.

A contact problem can be represented as system composed by n interacting subsystems. For the single linear subsystem r, the equation of motion can be expressed as:

$$\mathbf{M}^{(r)}\ddot{\boldsymbol{u}}(t)^{(r)} + \mathbf{C}^{(r)}\dot{\boldsymbol{u}}(t)^{(r)} + \mathbf{K}^{(r)}\boldsymbol{u}(t)^{(r)} = \boldsymbol{f}(t)^{(r)} + \boldsymbol{g}(t)^{(r)} \tag{1}$$

where:

$\mathbf{M}^{(r)}$, $\mathbf{C}^{(r)}$, $\mathbf{K}^{(r)}$ are the mass, damping and stiffness matrices of subsystem r;

$\boldsymbol{u}(t)^{(r)}$ is the vector of displacements of subsystem r;

$\boldsymbol{f}(t)^{(r)}$ is the vector of external forces on subsystem r;

$\boldsymbol{g}(t)^{(r)}$ is the vector of connecting forces with other subsystems (internal constraint forces).

Before considering the contact interaction, the equation of motion of the n independent subsystems can be written as:

$$\mathbf{M}\ddot{\boldsymbol{u}}(t) + \mathbf{C}\dot{\boldsymbol{u}}(t) + \mathbf{K}\boldsymbol{u}(t) = \boldsymbol{f}(t) + \boldsymbol{g}(t) \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are block diagonal matrices.

For contact problems, compatibility and equilibrium do not necessarily apply to the same set of DoFs. In fact, if sliding with friction occurs, the compatibility applies only on the direction normal to the contact interface (no penetration), whilst equilibrium of tangential and normal forces must be imposed. Moreover, due to the relative displacement between subsystems, compatibility and equilibrium conditions change over time.

Compatibility requires that a given pair of matching DoFs at time t, must share the same displacement [11]. This condition can be generally written as:

$$\mathbf{B_C}(t)\boldsymbol{u}(t) = \mathbf{0} \tag{3}$$

where each row of $\mathbf{B_C}(t)$ defines compatibility between a pair of matching DoFs at time t.

Internal constraint forces apply only on connecting DoFs. The equilibrium condition requires that sum of connecting forces at a pair of matching DoFs at time t is zero [11]. The resulting set of equilibrium conditions can be expressed as:

$$\mathbf{L_E}(t)^T\boldsymbol{g}(t) = \mathbf{0} \tag{4}$$

Using the set of the three previous equations (the so called three field formulation):

$$\begin{cases} \mathbf{M}\ddot{\boldsymbol{u}}(t) + \mathbf{C}\dot{\boldsymbol{u}}(t) + \mathbf{K}\boldsymbol{u}(t) = \boldsymbol{f}(t) + \boldsymbol{g}(t) \\ \mathbf{B_C}(t)\boldsymbol{u}(t) = \mathbf{0} \\ \mathbf{L_E}(t)^T\boldsymbol{g}(t) = \mathbf{0} \end{cases} \tag{5}$$

it is possible to include contact problems in the substructuring framework. Note that, when relative displacement occurs at the contact interface $\mathbf{B_C}$ and $\mathbf{L_E}$ depend on time t.

The substructuring problem can be solved by automatically satisfying the equilibrium condition, this can be obtained by defining a unique set of Lagrange multipliers $\boldsymbol{\lambda}$ corresponding to connecting force intensities:

$$\boldsymbol{g}(t) = -\mathbf{B_E}(t)^T\boldsymbol{\lambda}(t) \tag{6}$$

Note that, when friction forces are present at the interface, the matrix $\mathbf{B_E}(t)$ is different from the matrix $\mathbf{B_C}(t)$ previously defined to enforce the compatibility condition [21].

In this approach, known as dual assembly, each interface DoF is considered as many times as there are substructures connected through that DoF.

The interface equilibrium condition (4) is thus written:

$$\mathbf{L_E}(t)^T\boldsymbol{g}(t) = -\mathbf{L_E}(t)^T\mathbf{B_E}(t)^T\boldsymbol{\lambda}(t) = \mathbf{0} \qquad \forall\boldsymbol{\lambda} \tag{7}$$

Since Eq. (7) is always satisfied by any set of connecting force intensities $\boldsymbol{\lambda}$, the system of equations (5) becomes:

$$\begin{cases} \mathbf{M}\ddot{\boldsymbol{u}}(t) + \mathbf{C}\dot{\boldsymbol{u}}(t) + \mathbf{K}\boldsymbol{u}(t) + \mathbf{B_E}(t)^T\boldsymbol{\lambda}(t) = \boldsymbol{f}(t) \\ \mathbf{B_C}(t)\boldsymbol{u}(t) = \mathbf{0} \end{cases} \tag{8}$$

Eq. (8) can be explicitly solved in the time domain using an approach similar to the one presented in [23], i.e. by relating compatibility at time $t_{n+1} = t_n + h$ with the dynamic equilibrium at time t_n, where h is the integration time step. If both compatibility and dynamic equilibrium were considered at the same time instant, the problem would be singular. The equation of motion becomes:

$$\begin{cases} \mathbf{M}\ddot{\boldsymbol{u}}_n + \mathbf{C}\dot{\boldsymbol{u}}_n + \mathbf{K}\boldsymbol{u}_n + \mathbf{B}_{\mathbf{E}n+1}^T \boldsymbol{\lambda}_n = \boldsymbol{f}_n \\ \mathbf{B}_{\mathbf{C}n+1}\boldsymbol{u}_{n+1} = \mathbf{0} \end{cases} \tag{9}$$

In this notation the subscripts n and $n+1$ indicate the functions at time t_n and t_{n+1}. The first equation of (9) can be rewritten as:

$$\ddot{\boldsymbol{u}}_n = \mathbf{M}^{-1}\left[\boldsymbol{f}_n - \mathbf{C}\dot{\boldsymbol{u}}_n - \mathbf{K}\boldsymbol{u}_n - \mathbf{B}_{\mathbf{E}n+1}^T \boldsymbol{\lambda}_n\right] \tag{10}$$

For explicit integration of the equation, the Newmark β_2 scheme can be used [24], that for $\beta_2 = 0.5$ reduces to the central difference scheme:

$$\ddot{\boldsymbol{u}}_n = \frac{1}{h^2}\left(\boldsymbol{u}_{n+1} - 2\boldsymbol{u}_n + \boldsymbol{u}_{n-1}\right) \tag{11}$$

By substituting the central difference expression of $\ddot{\boldsymbol{u}}_n$ in the equation of motion (10), and by using $\dot{\boldsymbol{u}}_n = \left(\boldsymbol{u}_n - \boldsymbol{u}_{n-1}\right)/h$ one obtains the displacement $\boldsymbol{u}$ at time $n+1$. It can be expressed as the sum of a predictor term $\boldsymbol{u}^\star_{n+1}$, that can be directly evaluated since it depends only on the displacements of the system at times t_n and t_{n-1}, and a corrector $\boldsymbol{u}^c_{n+1}$ that represents an incremental displacement associated with the contact forces $\boldsymbol{\lambda}$:

$$\boldsymbol{u}_{n+1} = \boldsymbol{u}^\star_{n+1} + \boldsymbol{u}^c_{n+1} \tag{12}$$

where

$$\boldsymbol{u}^\star_{n+1} = h^2\mathbf{M}^{-1}\left[\boldsymbol{f}_n - \frac{1}{h}\mathbf{C}\left(\boldsymbol{u}_n - \boldsymbol{u}_{n-1}\right) - K\boldsymbol{u}_n\right] + 2\boldsymbol{u}_n - \boldsymbol{u}_{n-1} \tag{13}$$

and

$$\boldsymbol{u}^c_{n+1} = -h^2\mathbf{M}^{-1}\mathbf{B}_{\mathbf{E}n+1}^T \boldsymbol{\lambda}_n \tag{14}$$

The corrector is unknown because contact forces $\boldsymbol{\lambda}_n$ must be estimated and constraints $\mathbf{B}_{\mathbf{E}n+1}$ must be defined. The amplitude of contact forces can be obtained by combining Eqs. (14) and (12) with the second line of Eq. (9):

$$\boldsymbol{\lambda}_n = \left(h^2\mathbf{B}_{\mathbf{C}n+1}\mathbf{M}^{-1}\mathbf{B}_{\mathbf{E}n+1}^T\right)^{-1}\mathbf{B}_{\mathbf{C}n+1}\boldsymbol{u}^\star_{n+1} \tag{15}$$

Contact forces depend on matrices $\mathbf{B}_\mathbf{E}$ and $\mathbf{B}_\mathbf{C}$ at time t_{n+1}. In [11] the authors proposed a definition of matrices $\mathbf{B}_\mathbf{E}$ and $\mathbf{B}_\mathbf{C}$ based on the rigid, a-priori defined, relative motion of component subsystems due to the kinematic boundary conditions.

2.3 Results comparison

In this subsection the results of transient simulation performed with the substructuring method (referred as “Subs” in the following) are compared with the results of the forward increment Lagrange multiplier method (referred as “FiLm” in the following).

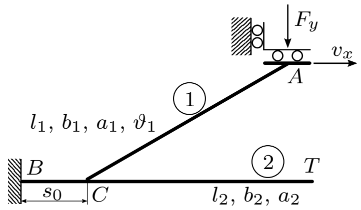

Figure 1: Mechanical system used in the simulations.

Table 1: Geometrical characteristics of the model.

Dimensions		Beam 1	Beam 2	
Length	l	75	100	mm
Thickness	b	1.5	2	mm
Width	a	10	20	mm
Angle	ϑ	30	0	degree

The mechanical system considered in this work is composed by two subsystems connected through a sliding interface with a constant friction coefficient μ. As shown in Figure 1, the two subsystem are: an oblique beam that slides along the upper horizontal edge of a cantilever beam. The oblique beam, of length l_1 and section $a_1 \times b_1$, forms an angle ϑ with the horizontal beam, of length l_2 and section $a_2 \times b_2$. The horizontal beam is fixed at its end B, while time-variant boundary conditions (vertical force F_y and velocity v_x) are applied at the upper end A of the oblique beam. During the simulation the contact point C, initially distant s_0 from the fixed end, moves to the right. Geometrical characteristics, mechanical properties and boundary conditions are reported in Tables 1 and 2. Note that a viscous proportional damping is considered in the model. The vertical load F_y, the velocity v_x and the friction coefficient μ are applied smoothly during the first 0.1 s and then remain constant up to the end of simulation. Moreover, in order to damp the transient vibrations generated by the horizontal acceleration and by the application of the vertical load, high damping coefficients ($\alpha_0 = 40\,\mathrm{s}^{-1}$ and $\beta_0 = 4.0e - 8\,\mathrm{s}$) are used during the first 0.2 s and gradually reduced to the desired values α and β in the next 0.05 s.

Table 2: Mechanical properties and boundary conditions of the model.

Dimensions		Beam 1	Beam 2	
Young mod	E	71	2.38	GPa
Poisson ratio	ν	0.33	0.40	
Density	ρ	2770	1200	kg/m^3
M prop. damping	α	0.2	40	s^{-1}
K prop. damping	β	4.0e-8	4.0e-8	s
Friction coefficient	μ	0.30		
Force	F_y	-0.20		N
Velocity	v_x	30		mm/s

The two beams are modeled separately by plane stress element in ANSYS. To reduce the computational burden, a modal reduction of both the substructures is performed using the Craig Bampton method with 20 fixed interface modes. Besides the modal DoFs of the modal reduction, only the physical DoFs on which the boundary conditions apply and the ones involved in the contact are maintained, for a total of 243 DoFs.

The value of the friction coefficient μ used in the simulation is chosen so that some instabilities due to modal coupling arise, when the oblique beam slides along the horizontal one. Figure 2 shows the locus plot of complex eigenvalues [11] of the coupled system when the oblique beam, starting form position s_0, slides toward the free end of the horizontal cantilever beam. The locus plot is obtained by coupling for every position the two substructures using primal assembly, as proposed in [11]. Note that, the real part of some

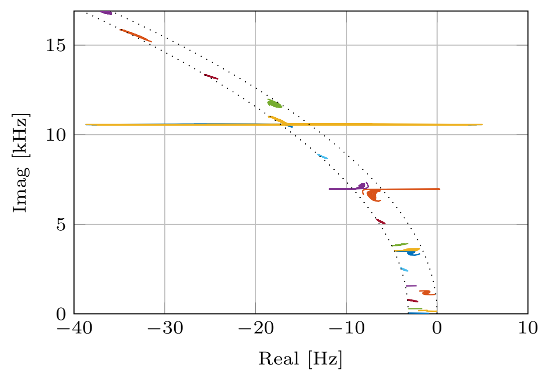

Figure 2: Locus plot of the 20 lowest eigenvalues for a friction coefficient $\mu = 0.3$.

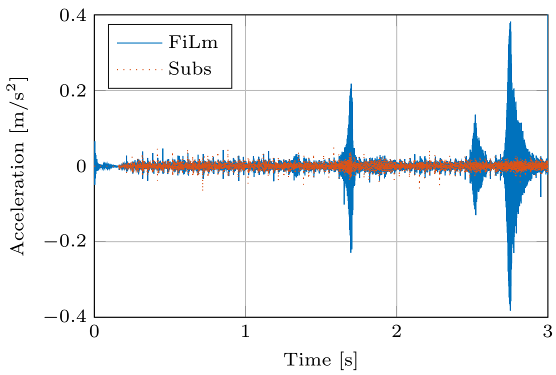

Figure 3: Vertical acceleration of point T: comparison between "Subs" and "FiLm" method.

eigenvalues reaches positive values, i.e. possible dynamic instabilities at some position of the contact point. Specifically, instabilities may occur around 7 kHz and 10.5 kHz.

When the system passes trough an instability region, it is expected that the vibration amplitude increases with an exponential law, whilst it extinguishes when the instability region is overcome. Figure 3 shows the vertical acceleration of the free end T of the horizontal beam evaluated using the two different method previously outlined. Figure 4 shows the spectrograms of the vertical accelerations in Figure 3. Result obtained using the FiLm method show typical bursts of vibration associated with the crossing of instability region. On the other hand, the results obtained with the Subs method highlight that, although the substructuring based method, as it is formulated in [11], is capable of detecting the changes in the global dynamics of a time variant coupled system, it does not provide reliable results in case of contact dynamic instabilities. This limitation was already mentioned by the authors in [11] and it is due to the a-priori definition of coupling conditions that do not account for sliding associated with static and dynamic body deformation. This is in fact a crucial aspect in order to reproduce unstable friction induced vibrations, since the energy absorption of the structural system depends on the phase shift between the oscillations of contact forces and relative displacement [15]. Therefore, a lack in precision in the estimation of these quantities can heavily affect the correctness of results. Thus, an improved time variant substructuring method will be introduced in the following section.

3 Enhanced time-variant substructuring method

In order to get more reliable results also in presence of friction induced vibrations, an improved method was proposed by the authors and detailed in [21]. This enhanced method uses the same general integration scheme shown in section 2.2 but in this case the matrices $\mathbf{B_E}$ and $\mathbf{B_C}$ are defined accounting also for system deformation. The contact algorithm now estimates the deformed position $\boldsymbol{U}^\star$ of the whole system at time t_{n+1} using the predictor of displacements $\boldsymbol{u}^\star$:

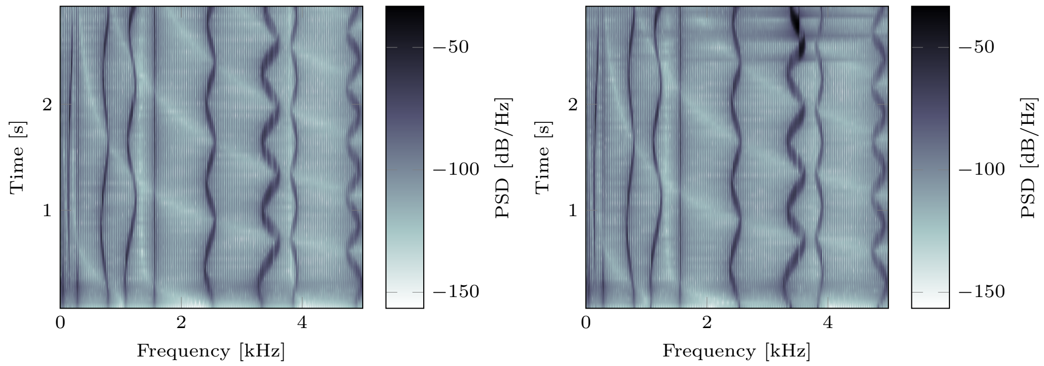

Figure 4: Spectrograms of the vertical acceleration of point T obtained using the “Subs” (left plot) and “FiLm” (right plot) methods.

$$\boldsymbol{U}^{\star}_{n+1} = \boldsymbol{X} + \boldsymbol{u}^{\star}_{n+1} \tag{16}$$

where $\boldsymbol{X}$ represents the undeformed position. Since the system deformation affects the coupling conditions, the coupled system is geometrically nonlinear. Moreover, the variation of the tangential displacement at the contact can be observed, thus allowing to account for friction induced vibrations.

3.1 Numerical validation for unstable FIV

In this subsection the results of transient simulation performed with the enhanced substructuring method (referred as “EnSubs” in the following) are compared with the result of the forward increment Lagrange multiplier method (FiLm). The mechanical system and the boundary conditions are the same described in section 2.3.

Figure 5 shows the vertical acceleration of the free end T of the horizontal beam evaluated using the EnSubs method and the reference FiLm method. Results highlight a substantial superposition of the two responses thus confirming the effectiveness of the enhanced time variant substructuring method in retracing also the friction induced vibrations due to contact induced instabilities.

Figure 6 shows the horizontal velocity of point C and the normal contact force λ during the simulation with the “EnSubs” method. Results confirm the sliding contact assumption for all the simulation time.

In this case, the proposed method is computationally more efficient with respect to the classical FiLm method: the computational time required by EnSubs is 30% lower than that required by FiLm. In fact, although the two subsystems are always in sliding contact, the FiLm method for each time step supposes that the bodies are in sticking contact and, if sticking condition is not verified, it relaxes the constraint along the

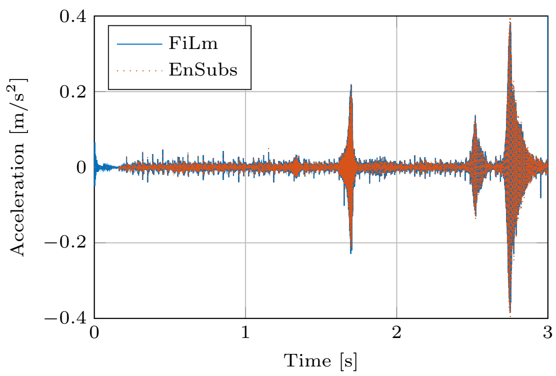

Figure 5: Vertical acceleration of point T: comparison between “EnSubs” and “FiLm” method.

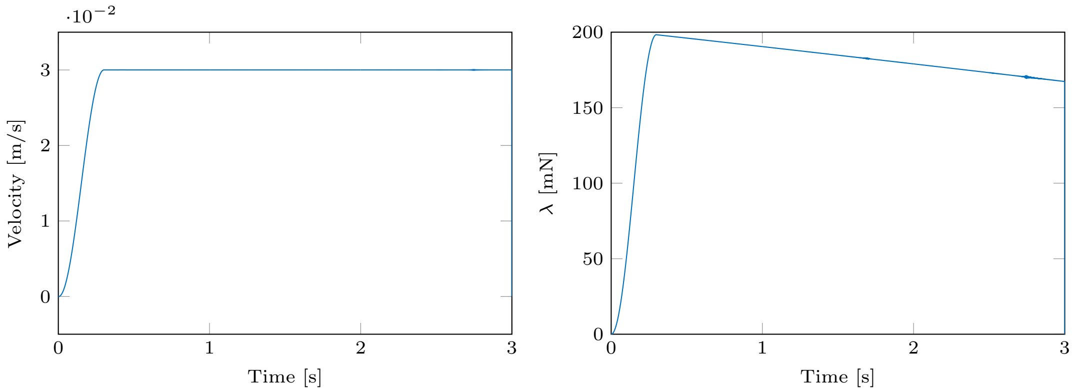

Figure 6: Relative velocity and normal contact force for the "EnSubs" method.

tangential direction and approximates the correct position with an iterative procedure. On the contrary, the EnSubs method provides directly the correct solution at the subsequent time step thus reducing the computational burden.

Conclusions

In this paper, two substructuring based methods, previously proposed by the authors, and a well-established numerical method are adopted to evaluate the time response of a mechanical system, composed by two subsystems connected through a frictional contact interfaces. The first substructuring method assumed that the time-variant compatibility and equilibrium conditions, arising from sliding contact, are a-priori known. The comparison of the results with the reference method highlighted that this approach provides a correct estimation of the variation of the global dynamics of the system due to the relative sliding. However, it is not able to reproduce friction induced vibrations, since it does not properly account for the relative displacement at the contact interface due to the body deformation. The second substructuring method is an enhancement of the previous one, on which the deformation of the mechanical system is accounted for in the definition of the compatibility and equilibrium conditions. Hence, this method, providing a more reliable definition of coupling conditions is able to retrace the dynamic response of the system also in case of unstable friction induced vibration due to mode coupling. The comparison of the computational time highlights a significant reduction of the computational burden with respect to the reference method.

Acknowledgments

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