Evaluation of VaR and CVaR for the Makespan in Interval Valued Blocking Job Shops

Carlo Meloni[1]

Dipartimento di Ingegneria Informatica, Automatica e Gestionale
Sapienza Università di Roma,
Via L. Ariosto, 25 - 00185 Roma (Italy)
email: carlo.meloni@uniroma1.it

Marco Pranzo

Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche,
Università di Siena, Siena (Italy)
email: marco.pranzo@unisi.it

Marcella Samà

Dipartimento di Ingegneria,
Università Roma Tre, Roma (Italy)
email: marcella.sama@uniroma3.it

Abstract

The paper deals with the job shop scheduling problem with complex blocking constraints (BJSS) under uncertainties. It proposes a method for the evaluation of the risk that the makespan of a deterministic feasible schedule assumes worse extreme values, considering uncertain activity durations represented by intervals. An interval-valued network approach is proposed to model the feasible solutions characterized by uncertain values for jobs' releases, processing and setup times. The study assumes the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR) as risk measures for the makespan of the feasible solutions, and addresses both modeling and computational issues. They include the implementation and test of a network-based model used with an innovative algorithm for the first time applied to complex BJSS problems to provide an accurate, rapid and viable computation of both risk indices. The impact of dif-

[1]Corresponding author.

Preprint submitted to IJPE *March 5, 2022*

ferent sources of uncertainty (including setups, releases and processing times) on the overall performance of the proposed approach are analyzed. The results of a wide experimental campaign show that the method, for both the computational time and the quality of the evaluations, has broad applicability. It can support the decision-makers for a wide range of practical scheduling cases taking into account their risk sensibility.

Keywords: VaR, CVaR, Scheduling, Risk, Job Shop, Uncertainty

1. Introduction

The *job shop scheduling problem* (JSS) is one of the most studied problems in combinatorial optimization [9, 12, 21]. This interest is keeping momentum due to the technological developments of recent years [60]. With the appearance and diffusion of the Cyber-Physical Systems (CPS) and Industry 4.0 frameworks, new models and methods can be considered to solve industrial problems, such as JSS [61]. In this sense, incorporating shop-floor data and forecasting models in scheduling methodologies is seen as a very promising approach for dealing with real JSSs and their complexity [51, 52].

In JSS the operations of a set of jobs must be processed on a set of machines with an assigned sequence without interruptions or preemptions. Machines can only host one job at a time, and if multiple jobs require the same machine, a system of precedence is prescribed between all conflicting operations. The problem requires the sequencing and scheduling of all operations on the machines so that the overall completion time (*makespan*) is minimized [55, 56]. In its classic version, the JSS includes a buffer of infinite capacity between two consecutive machines. However, in many practical contexts a limitation of this buffer must be introduced for economic or technical reasons. In real-world applications, these situations often occur in addition to other constraints such as release times and setup operations [9, 12, 21].

The *blocking* or *zero-buffer* constraints represent total absence of storage possibilities between a pair of consecutive machines. In this model, a job frees a machine only when the next machine is available for its operation. Otherwise, the job remains on the current machine blocking it even after its processing is completed. Two versions of blocking constrained job shop problem (BJSS) can be distinguished depending on whether the swap between conflicting jobs is allowed or not [23, 38]. This study deals with the no-swap BJSS version, considering only *conflict* (or deadlock) *free* schedules.

Most of the papers in the literature propose deterministic job shop models. However, in some applications the time attributes of the problems may be uncertain. When there are not enough information to fully describe the uncertain time attributes, upper and lower bounds on their values are often available or easier to obtain [9, 28].

Scheduling problems which take uncertainties into account are in general computationally complex and the literature on stochastic and uncertain scheduling is relatively limited compared to the research on deterministic scheduling. These are among the reasons why analysts often use deterministic models that incorporate simplified representations of the most relevant stochastic aspects in order to determine or select the schedule to implement.

However, it is generally difficult to compute a risk measure for a schedule, even when it is given, i.e., the sequencing and allocation of jobs are fixed in advance [53, 59]. Nevertheless, even when optimizing is not the goal of the analysis, providing a risk measure for a schedule computed with deterministic or simplified parameters is very relevant in the operations management practice [25, 39]. In fact, a decision maker may need a quick and reliable support system, capable of adequately measuring the risk of a possible schedule under consideration [26, 53, 59].

The assessment of the risk can be obtained taking advantage of forecasting of time attributes, under the assumption that they are described by means of prediction intervals [2, 37, 62]. To this aim, risk measures often employed are the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR) [11, 24, 54, 59]. We deal with the evaluation of both VaR and CVaR for the makespan of BJSS schedules when the actual time attributes of jobs are uncertain at the scheduling or optimization stage and belong to a given interval provided by an available information/prediction system. The evaluation of these risk measures for a schedule is obtained through a dedicated algorithm on a temporal network model representing the activities of the problem extending the method proposed in [41] to include also the VaR. This method allows the adoption of the considered risk indices as evaluation criteria or outcome functions for different scheduling environments. This study takes into consideration realistic combinations of different sources of uncertainty that can influence the makespan of a given feasible solution for the BJSS. Furthermore, it extends and validates an algorithmic approach enabling the decision-maker to quantify the risk of its deterioration.

The article offers different contributions which can mainly be summarized as follows:

– it proposes an interval-valued network model representing the solutions of complex BJSSs, considering uncertain values of times for jobs' release, processing and setups;

– it introduces and applies an innovative evaluation method for the risk that the makespan of a BJSS schedule deteriorates assuming worse extreme values when time attributes are represented as integer intervals;

– it takes into account realistic combinations of sources of uncertainty affecting a BJSS solution often arising in practice;

– it validates the proposed methodology through an extensive experimental campaign, showing its applicability and its effectiveness in terms of: *i)* accuracy in the evaluation of both VaR and CVaR of the makespan; *ii)* the required computational cost and its suitability for applications; *iii)* the effects of the type and severity of the uncertainties;

– it spurs and motivates to use risk measures such as VaR and CVaR in scheduling applications.

The rest of the paper is structured as follows. Section 2 offers a review of the related literature. In Section 3, the formal description of the considered BJSS model is provided, including different real constraints, the representation of data uncertainties, and an illustrative example. The adopted risk measures on the makespan of BJSS are illustrated in Section 4, where the algorithm for their evaluation is also presented. The description and the discussion of the experimental validation of the proposed method is the theme of Section 5 while Section 6 presents the conclusions and possible directions for further research.

2. Literature review

The job shop scheduling problem shows a wide range of applications characterized by the presence of blocking constraints [12, 21]. They include production [33, 45], logistics [13, 29] and services [35, 46] planning and scheduling. The problem is NP-Hard and its modeling and theoretical basis have been outlined in the literature [23, 38]. The relevance of the BJSS from both an application and an academic point of view has motivated numerous algorithmic contributions in recent decades (e.g., see [3, 30, 40, 43, 47, 48]). In general, these algorithmic approaches are designed for environments in which the timing attributes (i.e., setups, release and processing times) of the problem are assumed to be deterministic. However, in some applications these attributes may be uncertain due to possible changes or due to imprecise information [28, 32, 57]. With the evolution of information technologies and their increasing availability in logistics and production environments nowadays it is often possible to have accurate information for uncertain time values in the form of prediction intervals [60, 61, 62]. This has renewed interest in scheduling problems characterized by interval valued (or bounded) time attributes [4, 7, 8] as cases with incomplete information or imperfect knowledge [14, 15, 18]. Considering that the actual duration of the activities is known only when the activities are performed, the uncertainty on these parameters means that the value of the objective function represented by the makespan is also uncertain [28, 32]. Despite the relevance of the theme, the literature on interval valued scheduling problems is still relatively limited, and to the best of our knowledge no specific studies on BJSS have been proposed so far.

Defining and studying the effects of uncertainties on the makespan of a given feasible schedule of the BJSS and the determination of a schedule with the best performance taking into account the uncertainties are two issues that have considerable practical relevance [25, 36, 52, 53, 60]. For both cases, the study can be approached quantifying the risk that the makespan assumes worse extreme values considering the time intervals of each activity [17, 18, 54]. Calculating a risk measure for a feasible solution is a mean of analysis both for its validation and for an informed decision in case of possible alternative solutions [16, 25, 54, 61]. Furthermore, it may allow to evaluate the effects on the makespan of different sources of uncertainty helping to consider appropriate contrasting actions [26, 39, 58, 59, 60]. Different scalar indicators have been used in the scheduling literature to quantify the risk associated with uncertain performance in terms of $\mathbf{C}_{max}$ (hereinafter, the uncertain quantities are shown in bold type) of a given schedule [6, 17, 36, 53], and often the use of a specific indicator also depends on the personal choices of the decision maker. Having to minimize the $\mathbf{C}_{max}$, two quantile-based measures ordinarily applied in finance and engineering as upside risk indices are receiving increasing attention in scheduling where uncertainty is often associated with some time attributes: VaR and CVaR [32, 34, 54]. These two indicators can also be used in scheduling problems modeled as stochastic activity networks [41, 42, 54]. Although the use of these quantile-based measures to take into account the uncertainty in scheduling seems quite promising, only a few relevant articles appear in the literature. They mainly deal with some specific application [22, 42, 49], or they are limited to some scheduling environment [6, 16, 34, 54]. Compared to other domains, where both VaR and CVaR are used more frequently, their limited diffusion in the scheduling field could be ascribed to the complexity of the adopted methods which often rely in simulation and sampling techniques [10, 20, 34, 53, 54]. The computation of the CVaR of $\mathbf{C}_{max}$ in project scheduling problems modeled through activity networks has been recently addressed in [41], with activity durations considered as integers belonging to known intervals.

3. Blocking Job Shop with Interval-Valued Uncertainties

In BJSS the operations of a set of jobs J must be executed on a set of resources or machines M. Each machine can process only one job at a time. Let *operation* o_{ji} be the processing of a job j on a machine i, which cannot be interrupted or preempted, with p_{ji} indicating its *processing time*. The sequence of operations for each job is given, while the timed sequence of operations for each machine must be determined so that the makespan C_{max} is minimized. In BJSS there is no intermediate storage possibility between two successive machines. Thus, when a job j terminates on machine i its operation o_{ji}, if machine h is available, it moves to h to perform its subsequent operation o_{jh} or, if h is not available, keeps occupying i, thus blocking it. Moreover, no swaps between jobs are allowed.

With respect to the classical BJSS, in this study two sets of realistic additional constraints are considered:

1. each job j has a *release time* $r_j \geq 0$, indicating the earliest time at which job j can start to be processed;

2. given two operations o_{ji} and o_{ki} to be performed on machine i by jobs j and k, the *sequence-dependent setup time* (SDST) $s_{jki} > 0$ ($s_{kji} > 0$) must be spent between the end of operation o_{ji} (o_{ki}) and the start of operation o_{ki} (o_{ji}). The strict positivity of the SDSTs derives from the no-swap hypothesis, while non-negativity is required in the case of blocking with swaps allowed.

An instance of BJSS is said to be deterministic when all its parameters and quantities are considered to be known with certainty. Finding a feasible solution for BJSS translates into assigning to each operation o_{ji} a *starting time*, such that all these constraints are satisfied and no deadlock between jobs exists. Such a solution is a deterministic optimum when the makespan C_{max} is minimized.

This paper investigates the uncertanty influence in terms of the risk of worsening the makespan on a given BJSS deterministic feasible solution. The rationale is to provide, in view of the actual solution implementation, the decision-makers accurate information about the makespan uncertainty which have been neglected or overlooked during the schedule deterministic computation. More specifically, we consider the case in which some attribute a (i.e., processing times, release times and SDST) has an uncertain duration represented as an independent interval of integers time units $\mathbf{T}_a = [\underline{t_a}, \overline{t_a}]$. The values $\underline{t_a}$ and $\overline{t_a}$ are integers, with $\underline{t_a} \geq 0$, and $\underline{t_a} \leq \overline{t_a}$. This assumption also allows to model problems in which the temporal values are not represented by integers by appropriately choosing the time unit.

The width of the intervals depends on the available data and the accuracy of the information/prediction system in use. When a time attribute T_a has a certain value, we have the deterministic case modeled with an interval having coincident extremes $\underline{t_a} = \overline{t_a}$. For sake of simplicity, in the latter case we indicate $T_a = \underline{t_a} = \overline{t_a}$.

To take into account the uncertainty of these time attributes for a feasible solution of BJSS under makespan performance measure, we represent the BJSS solution as a *temporal activity network with integer interval-valued durations* (IIN), extending the method proposed in [41]. A IIN is a discrete network defined by the pair $(G, \mathbf{T})$, where $G = (N, A)$ is a directed graph consisting of a set of nodes N and a set of arcs A. Each node in $N = \{0, 1, \ldots, n-1, n\}$ represents an event. More specifically, each node $m = 1, \ldots, n-1$ models the starting of an activity, while nodes 0 and n are added in N to model the starting and the ending activities of the overall network. A variable v_m is assigned to each node m to model the starting time of the related activity. An arc $a = (m, q) \in A$ represents a time attribute a related to an activity with its associated interval temporal duration $\mathbf{T}_a$.

When modeling a given feasible BJSS solution using a IIN, each node in

$m \in N$ represents an operation o_{ji}, with v_m its starting time, while the arcs in A can be partitioned in four subsets representing the processing (P), SDST (S), job releases (R), and logic constraints (LC):

> P: given a node m associated to operation o_{ji} and a node q associated to its subsequent job operation o_{jh}, an arc $a = (m, q) \in A$ represents the processing time p_{ji} of operation o_{ji} starting at time v_m and having an interval duration $\mathbf{T}_a$;
>
> S: for a node m associated to the end of o_{ji} (i.e., the start of its subsequent job operation) and a node q associated to operation o_{ki} scheduled after it on machine i, an arc $a = (m, q) \in A$ having duration $\mathbf{T}_a$ represents the SDST s_{jki} from the ending of the operation o_{ji} to the beginning of the operation o_{ki};
>
> R: given a node m assigned to the first operation of job j, an arc $a = (0, m) \in A$ having duration $\mathbf{T}_a$ is associated to the release time r_j of job j;
>
> LC: for a node m associated to the end of the last operation of job j, an arc $a = (m, n) \in A$ models a makespan logic constraint linking the end of job j with the objective function C_{max}, computed as v_n.

Given a deterministic feasible solution of BJSS, the corresponding IIN model can be easily constructed assuming the availability of prediction intervals for the time attributes. The resulting directed network is connected and contains single starting and ending nodes. Furthermore, due to the considered no-swap BJSS version, the network represents only conflict or deadlock free schedules, and therefore is acyclic.

Clearly, a single deterministic feasible solution has associated many possible realizations of the values of the uncertain durations. Let the *configuration* $\mathcal{T}$ of the activity durations in a given IIN network Q be the realization for each activity a of a specific integer value t_a from the time interval $\mathbf{T}_a = [\underline{t_a}, \overline{t_a}]$. For each IIN's configuration the corresponding value of C_{max} equals the length of the *critical path* in the corresponding network [9, 12] keeping the same structure as that of the deterministic feasible solution. The dependence of $\mathbf{C}_{max}$ of the network from the specific configuration $\mathcal{T}$ is expressed by $C_{max}(\mathcal{T})$. As an extension we denote with $\mathrm{U}_{\mathcal{T}}(Q)$ the set of all the configurations of a network Q. Its cardinality gives the total number of possible configurations $|\mathrm{U}_{\mathcal{T}}(Q)|$, and can be determined as:

$$|\mathrm{U}_{\mathcal{T}}(Q)| = \prod_{a=1}^{|A|} \Delta_a.$$

where $\Delta_a = \overline{t_a} - \underline{t_a} + 1$ indicates the number of integers from $\underline{t_a}$ to $\overline{t_a}$ in $\mathbf{T}_a$. Activities with certain durations contribute with a unit value factor to the overall product, and the IIN Q of a BJSS solution without uncertainties has only one configuration, resulting $|\mathrm{U}_{\mathcal{T}}(Q)| = 1$.

The objective function $\mathbf{C}_{max}$ in a specific IIN network Q associated a deterministic feasible solution of BJSS belongs to the range $[\underline{C_{max}}, \overline{C_{max}}]$, with $\underline{C_{max}}$ obtained when all time attributes in the network assume their optimistic minimum duration, while for $\overline{C_{max}}$ they take the pessimistic maximum value.

3.1. Illustrative Example

Let us consider a small example for a BJSS problem to better illustrate the concepts just discussed. In this example we have three jobs ($J = \{A, B, C\}$) and three machines ($M = \{1, 2, 3\}$). To complete each job, three blocking operations have to be performed. More specifically, job A has to be processed on machines 1, 2 and 3, in that order, job B on machines 3, 2, 1 and job C on 1, 3, 2.

Table 1: Processing and release times for the illustrative example

j	M1	M2	M3	r_j
A	6	4	6	2
B	7	5	3	1
C	4	2	5	3

Tables 1 and 2 show the input data of this illustrative example. Specifically, Table 1 reports the processing and release times used in the example. Column 1 indicates the job $j \in J$ the row refers to, Columns 2-4 report the processing of

Table 2: SDST s_{jki} for the illustrative example

j \ k	M1			M2			M3		
	A	B	C	A	B	C	A	B	C
A	-	1	1	-	2	1	-	3	4
B	2	-	5	3	-	1	1	-	2
C	2	3	-	3	1	-	1	1	-

job j on, respectively, machines 1, 2 and 3 while Column 5 reports its release time. Table 2 instead presents the SDST s_{jki}. The rows of the table refer to job j, while the columns to job k and machine i the setup relates to.

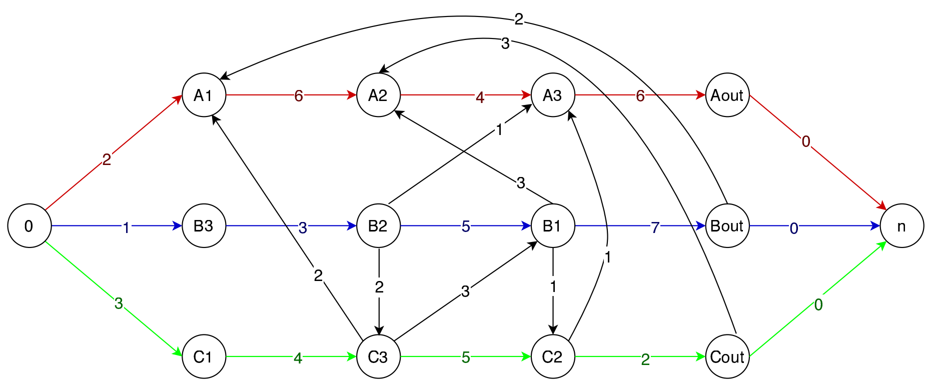

Figure 1: Optimal deterministic solution for the illustrative example

Figure 1 shows graph G of the optimal solution for the given BJSS example. Each node $m \in N$ has a label with a letter indicating the job and a number indicating the machine for the associated operation or the string "out" to signal the end of the job. Colored arcs (red for job A, blue for B, and green for C) are associated with release and processing time constraints, while black arcs are associated with the sequencing decisions taken in the chosen solution. Each arc has the indication of its weight.

However, the given solution could be affected by uncertainty. Graph G of the BJSS solution is used to build the related IIN Q, shown in Figure 2, by including the uncertain arc durations T_a. Specifically, in Figure 2, bold arcs represent uncertain time attributes. All others arcs are deterministic, thus with

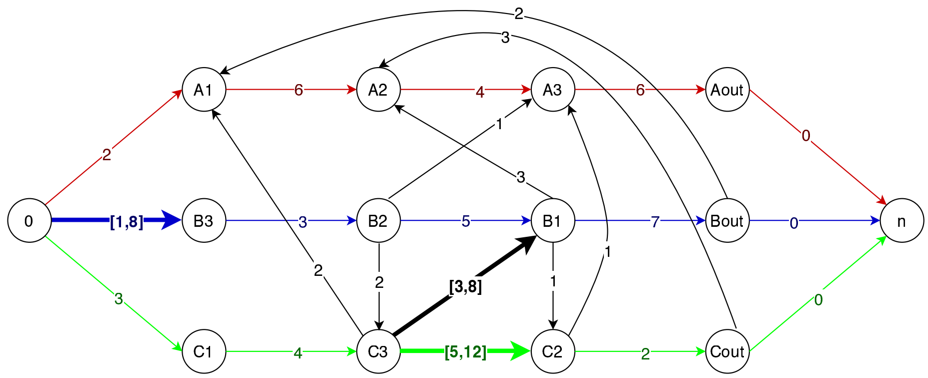

Figure 2: The IIN for the BJSS solution of Figure 1

a duration T_a where $\underline{t_a} = \overline{t_a}$. For example:

- arc $(C3, C2)$ is associated to the processing time of operation $C3$, represented as the interval $[5, 12]$;
- arc $(C3, B1)$ is associated to the SDST between the end of operation $C1$, which correspond to the subsequent beginning on $C3$, and the start of operation $B1$, represented as the interval $[3, 8]$;
- arc $(0, A1)$ is associated to the deterministic release time of job A whose duration is 2;
- arc (C_{out}, n) links the end of job C to the makespan, i.e., the ending node of the IIN. Since this arc is a purely logical constraint, it is not affected by uncertainty, thus its duration is 0.

The IIN Q in this example, considering the uncertainties on the arcs $(0, B3)$, $(C3, B1)$, and $(C3, C2)$, admits $|\mathrm{U}_{\mathcal{T}}(Q)| = \Delta_{(0,B3)} * \Delta_{(C3,B1)} * \Delta_{(C3,C2)} = 384$ configurations.

Figure 3 reports a graph showing the number of configurations for each possible value of the makespan in the illustrative example.

The makespan has an optimistic value $\underline{C_{max}} = 35$ obtained calculating the longest path from node 0 to node n when all the uncertain durations assume

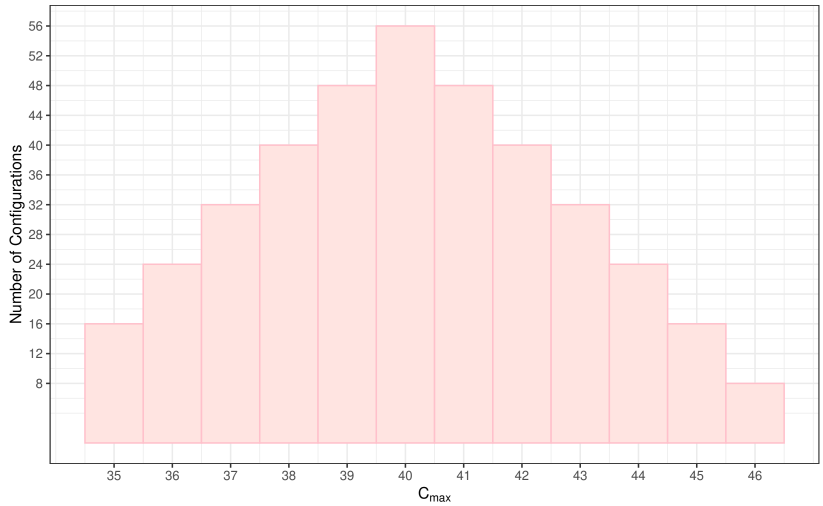

Figure 3: Number of configuration related to each C_{max} value

their minimum integer value. Similarly, at the other extreme, we obtain the pessimistic value $\overline{C_{max}} = 46$.

Figure 4 shows the Gantt charts for the deterministic solution for the (a) optimistic and (b) pessimistic cases, respectively. In the figure, filled blocks represent the processing of a job on a machine, each in its reference color, sketched ones the time a job blocks a machine after its processing is finished while waiting for the next one to be available, and changing colored blocks the SDST between two subsequent jobs on the same machine. Furthermore, dotted lines between Figures 4(a) and 4(b) show the increased time due to the uncertain durations, specifically in blue the release time of job B, in green the increasing processing time of job C on machine 3 and in black the SDST between jobs C and B.

From Figure 4(a) we can see how, in the optimistic case, the longest path between nodes 0 and n involves only operations on M1 plus the ones of job A on the other machines. Instead, in the pessimistic case, the increase on job B release time and on the setup time between jobs C and B on M1 changes not just the length of the longest path, but also the operations it passes through. In this case, operation $B3$ blocks job C on M1. As a consequence, job B has to

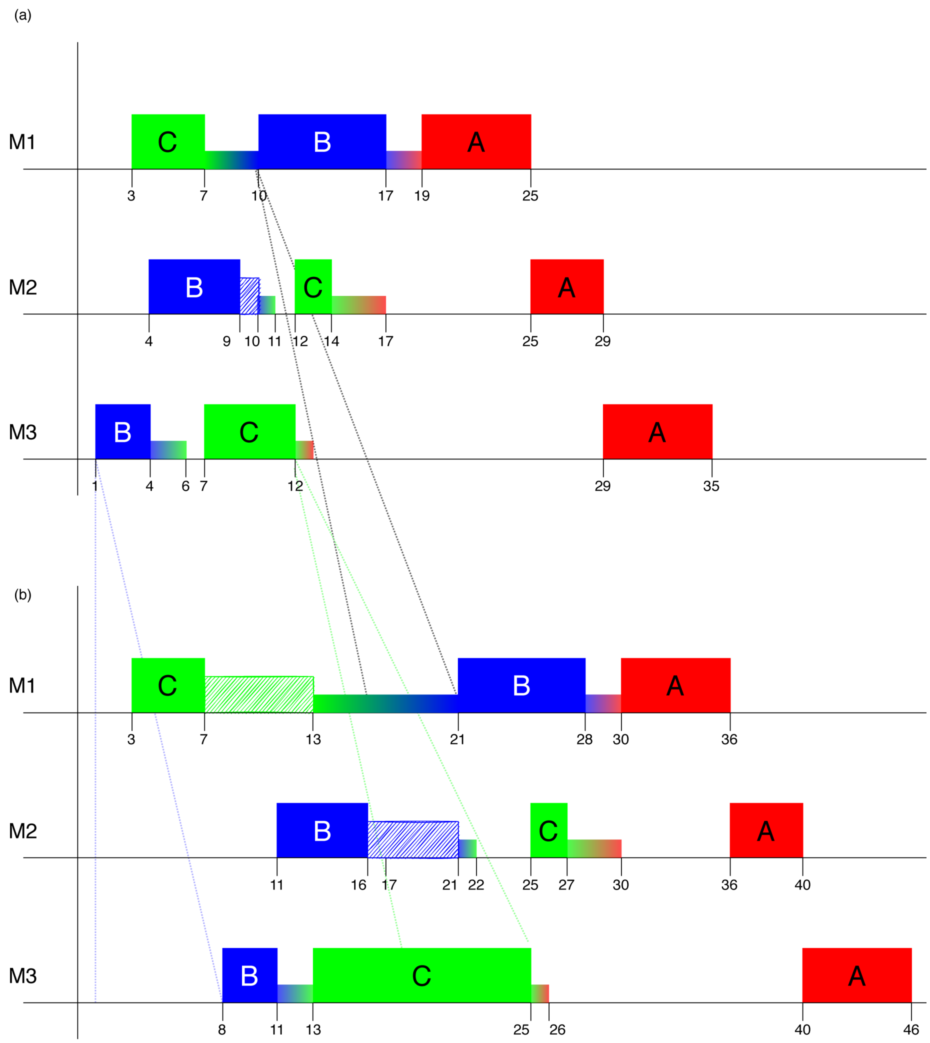

Figure 4: Gantt chart for the (a) optimistic and (b) pessimistic makespan

wait until time unit 21 before it can start its operation on M1 and only after it is done job A can begin to be processed.

Computing the longest path can be useful to compute the makespan in the extreme cases, while for the assessment of the risk associated with a feasible solution, specific indicators and algorithms for their calculation are introduced and discussed in the next section.

4. VaR and CVaR for the Makespan of BJSS

Considering the $\mathbf{C}_{max}$ to be minimized, we focus our analysis on the following upwards quantile-based risk measures at probability level α:

- **Value-at-Risk of $\mathbf{C}_{max}$ ($\mathrm{VaR}_\alpha(\mathbf{C}_{max})$):**

$$\mathrm{VaR}_\alpha(\mathbf{C}_{max}) = inf\{c : prob(\mathbf{C}_{max} \leq c) \geq \alpha\} \tag{1}$$

  It can be defined as the maximum possible loss excluding all $(1-\alpha)100\%$ worse cases. Thus, it does not assess the magnitude of possible losses, but individuates a value exceeded by $(1-\alpha)100\%$ of all possible cases.

- **Conditional Value-at-Risk of $\mathbf{C}_{max}$ ($\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$):**

$$\mathrm{CVaR}_\alpha(\mathbf{C}_{max}) = E(\mathbf{C}_{max} | \mathbf{C}_{max} \geq \mathrm{VaR}_\alpha(\mathbf{C}_{max})) \tag{2}$$

  It is the expected value of all cases above the threshold represented by $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ [11, 50], and can be obtained as:

$$\mathrm{CVaR}_\alpha(\mathbf{C}_{max}) = \frac{1}{1-\alpha} \int_\alpha^1 \mathrm{VaR}_\beta(\mathbf{C}_{max})\, d\beta \tag{3}$$

If the decision maker is interested both by frequency and severity of adverse cases, it is preferable to use CVaR_α instead of VaR_α [11, 54]. The risk neutral choice is represented by a probability level $\alpha = 0$, while higher values of α are chosen by the most risk averse decision maker. As the level α tends to its upper extremum (i.e., $\alpha = 1$), both $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ and $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$ tend to their worst or pessimistic case of value $\overline{C_{max}}$. On the other extremum, as α tends to 0, $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ tends to the best or optimistic completion time $\underline{C_{max}}$, whereas $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$ tends to the expected value of the makespan $E(\mathbf{C}_{max})$.

The finite number of integers Γ contained in the interval $[\underline{C_{max}}, \overline{C_{max}}]$ represents a measure of the amount of uncertainty for a given IIN, and can be obtained as $\Gamma = \overline{C_{max}} - \underline{C_{max}} + 1$. It is useful to note that the two risk measures $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$ and $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$, both have bounded values:

$$\underline{C_{max}} \leq \mathrm{VaR}_\alpha(\mathbf{C}_{max}) \leq \mathrm{CVaR}_\alpha(\mathbf{C}_{max}) \leq \overline{C_{max}}, \forall \alpha \in [0, 1] \tag{4}$$

Furthermore, considering the deterministic counterpart of the expected value of the makespan $C_{max}(E(\mathbf{T}))$, i.e., the completion time that occurs when the duration of each activity $\mathbf{T}_a$ is assigned its expected value $E(\mathbf{T}_a)$, we have (e.g., see [11]):

$$C_{max}(E(\mathbf{T})) \leq E(\mathbf{C}_{max}) \leq \mathrm{CVaR}_\alpha(\mathbf{C}_{max}), \forall \alpha \in [0,1] \tag{5}$$

The definitions of VaR_α and CVaR_α suggest a simple and exact method to determine the $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ and $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$ in IINs [11, 54]. In fact, considering a IIN Q, the configurations in $\mathrm{U}_\mathcal{T}(Q)$ can be sorted in decreasing C_{max} order to find the corresponding values of VaR_α and CVaR_α of $\mathbf{C}_{max}$ individuating the CT_α worst (i.e., highest) configurations in $\mathrm{U}_\mathcal{T}(Q)$, where CT_α denotes the *counting target* for the given value of the probability level α:

$$CT_\alpha = \lceil (1-\alpha)|\mathrm{U}_\mathcal{T}(Q)| \rceil \tag{6}$$

Proceeding in this way for the case of the illustrative example presented in Section 3.1, and using a probability value $\alpha = 0.90$, we obtain the results summarized in Table 3.

Table 3: Analysis of the illustrative example

$\lvert\mathrm{U}_\mathcal{T}(Q)\rvert$	α	CT_α	$\underline{C_{max}}$	$\overline{C_{max}}$	$C_{max}(E(\mathbf{T}))$	$\mathrm{VaR}_\alpha(\mathbf{C}_{max})$	$\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$
384	0.90	10	35	46	39	44	44.820

Unfortunately, this straightforward procedure can only be used for very small instances and is therefore not of practical interest. In the literature, techniques based on *sampling* or the generation and analysis of *scenarios* are widely used to obtain estimates of $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ and $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$ [34, 54]. However, in general these techniques require considerable computational time.

This paper adopts the counting method introduced in [41] to evaluate the CVaR_α for the makespan of a IIN, and extend it to compute $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ and calculate some useful performance indices as illustrated in the following section.

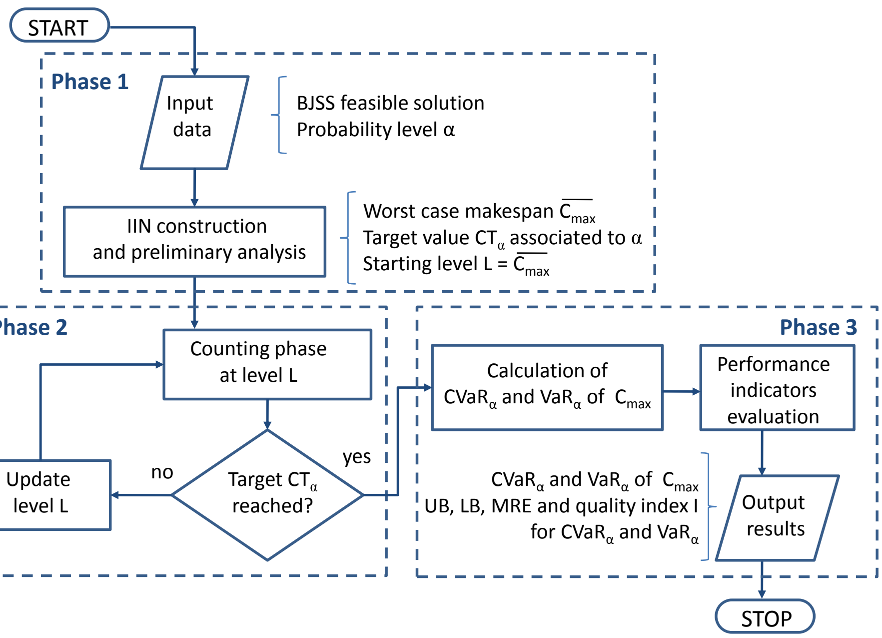

Figure 5: Conceptual scheme of the procedure for $\mathrm{VaR}_\alpha(\mathbf{C}_{max})$ and $\mathrm{CVaR}_\alpha(\mathbf{C}_{max})$.

4.1. Algorithmic scheme

This section illustrates the computational method adopted to evaluate both the VaR_α and CVaR_α of the uncertain $\mathbf{C}_{max}$ in the IIN representing a BJSS solution. To carry out this issue we consider the method proposed in [41]. This method is mainly based on a very fast and accurate procedure which starts at $\overline{C_{max}}$ for the given IIN and counts back all configurations leading to each possible successive C_{max} value. This counting procedure is designed as an iterative process consisting of three phases, as schematically depicted in Figure 5, which continues until enough data is obtained to calculate both CVaR_α and VaR_α.

The first phase of the algorithm, starting from a BJSS solution given as input, constructs the IIN Q and conducts a preliminary analysis calculating $|\mathrm{U}_{\mathcal{T}}(Q)|$, the value of CT_α corresponding to the prescribed probability level α, and the initial value $L = \overline{C_{max}}$ to start the evaluation.

In the second phase the iterative counting process takes place and it refers, at each step, to a specific level L corresponding to a possible value of C_{max}. In the single step at level L the counting is obtained exploiting all possible series-parallel reductions applicable on the *Critical Subgraph* related to the level L [41].

When all the critical subgraphs used in the counting procedure completely reduce to a single arc by series-parallel simplifications, then the proposed method guarantees the exact computation of VaR_α and CVaR_α, differently an heuristic evaluation provides the lower and upper bounds, denoted as $LB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max})$, $UB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max})$ and $LB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max})$, $UB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max})$, respectively.

The third phase of the procedure calculates the results and some additional performance indicators. For both risk measures the results are either exact or computed averaging the related upper and lower bounds. Two indicators are computed to evaluate the quality of the results achieved for CVaR_α. The first of these indicators is the *quality index* I_{CVaR_α}:

$$I_{\mathrm{CVaR}_\alpha} = \begin{cases} 0 & \text{when} \quad \overline{C_{max}} = C_{max}(E(\mathbf{D})) \\ \frac{UB_{\mathrm{CVaR}_\alpha} - LB_{\mathrm{CVaR}_\alpha}}{\overline{C_{max}} - C_{max}(E(\mathbf{D}))} & \text{otherwise} \end{cases} \tag{7}$$

I_{CVaR_α} offers a measure of the progress on the knowledge of CVaR_α achieved applying the algorithm. In fact, I_{CVaR_α} compares the gap between the bounds ($UB_{\mathrm{CVaR}_\alpha}$ and $LB_{\mathrm{CVaR}_\alpha}$) on CVaR_α and the distance between the worst case and the deterministic counterpart of the makespan, assumed as known.

The second of the considered indicators is the *estimated maximum relative error* $MRE_{\mathrm{CVaR}_\alpha}$, determined considering that the approximation adopted when the algorithm has an heuristic behavior can lead at most to an absolute error of $\frac{UB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max}) - LB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max})}{2}$, and therefore it assures a maximum relative error $MRE_{\mathrm{CVaR}_\alpha}$:

$$MRE_{\mathrm{CVaR}_\alpha} = \frac{UB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max}) - LB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max})}{UB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max}) + LB_{\mathrm{CVaR}_\alpha}(\mathbf{C}_{max})} \tag{8}$$

Analogously, the procedure calculates two indicators to assess the quality of the results for VaR_α, namely I_{VaR_α} and $MRE_{\mathrm{VaR}_\alpha}$. These indicators are conceptually similar to those previously described but are adapted to the VaR_α, considering the related bounds and a different reference interval.

$$I_{\mathrm{VaR}_\alpha} = \begin{cases} 0 & \text{when} \quad \overline{C_{max}} = \underline{C_{max}} \\ \frac{UB_{\mathrm{VaR}_\alpha} - LB_{\mathrm{VaR}_\alpha}}{\overline{C_{max}} - \underline{C_{max}}} & \text{otherwise} \end{cases} \tag{9}$$

$$MRE_{\mathrm{VaR}_\alpha} = \frac{UB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max}) - LB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max})}{UB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max}) + LB_{\mathrm{VaR}_\alpha}(\mathbf{C}_{max})} \tag{10}$$

Considering the experimental results reported in [41], we use the counting procedure ES_MinS which due to its rapidity and accuracy is suitable for the application in the validation and final selection phase of the BJSS. The computational complexity of this procedure is shown to be $O(\Gamma^2|A|^2)$, where A is the set of arcs in the IIN, and $\Gamma = \overline{C_{max}} - \underline{C_{max}} + 1$ represents the amount of uncertainty in the network.

5. Experimental Study

This section is dedicated to the description of our experimental campaign which consists of a set of computational tests aimed at investigating the following research aspects:

i) effectiveness of the proposed method in accurately calculating VaR_α and CVaR_α of $\mathbf{C}_{max}$ for the BJSS problem;

ii) computational cost of the risk assessment and its suitability for applications;

iii) effects of the type and severity of the uncertainty in the BJSS instances on both the system performance and the behavior of the risk assessment method;

iv) usefulness of the proposed methodology for decision makers.

Concerning the computational environment, it consists of a machine with a Quad-Core Intel Xeon E5 processor working at 3.7 GHz clock speed, equipped with 32 GB of RAM, and operating under OS X 10.14.6. The risk measures procedure is implemented in plain Python language and run on a single thread.

Section 5.1 illustrates the design of the computational tests. Computational results are presented and discussed in Section 5.2, whereas Section 5.3 offers a discussion on the managerial implications.

5.1. Design of Computational Tests

This study takes into account realistic combinations of sources of uncertainty affecting a BJSS solution often arising in practice, and investigates the validity and applicability of the proposed methodological approach through an extensive experimental campaign. The computational analysis is based on a wide set of instances composed of different sets of complex and realistic cases derived from classical benchmark deterministic job-shop problems. To this aim, we consider 58 instances well established in the literature for the job shop scheduling with blocking constraints. Namely, we consider the blocking versions of the ABZ [1], FT [19], LA [31] and OBZ [5] instance sets, each made up of respectively 5, 3, 40 and 10 instances. Then, we enrich these basic BJSS instances with additional structural constraints and sources of uncertainty, as described in the following to ensure the reproducibility of the analysis.

Regarding the constraints introduced in Section 3, following [30], we introduce for each job $j \in J$ a release time r_j randomly extracted from a uniform distribution as in Equation (11), where p_{ji} is the processing time of operation o_{ji} of job j on the required machine i.

$$r_j = U\left[0,\ 2 \cdot \min_{j \in J}\left\{\sum_i p_{ji}\right\}\right] \tag{11}$$

Moreover, we add SDST constraints by generating for each pair of operations o_{ji} and o_{ki} on a machine i a couple of SDST s_{jki} and s_{kji} from the uniform distribution $U[1,\ 0.25 \cdot p_{ji}^{max}]$, where p_{ji}^{max} is the maximum processing time among all operations of jobs j on machine i in the specific instance under consideration [44].

For the resulting complex scheduling problem there are no viable exact solution methods available. Therefore, each instance has been solved using a state-of-the-art iterated greedy metaheuristic algorithm for BJSS with the makespan objective proposed in [48]. For each of the so built instances we have collected the best 20 feasible solutions (often including the deterministic optimal solutions) obtained by the algorithm for a total of 1160 feasible schedules.

The deterministic solutions so obtained give rise to networks with a structure having on average 161.1 nodes (ranging from 44 to 332) and 1392.1 arcs (ranging form 139 to 4711). As regards the makespan, its coefficient of variation CV (i.e., the ratio of the standard deviation to the mean) is always very low, and on average is equal to 0.01, showing that the solutions collected for each instance have a comparable quality. Table 4 reports in details for each BJSS instance the number of jobs $|J|$ and machines $|M|$, the number of nodes ($|N|$) and arcs ($|A|$) of the corresponding IIN, and the statistics (i.e., average value, standard deviation and CV) of the $\mathbf{C}_{max}$ for the sets of collected deterministic solutions.

Regarding the possible sources of uncertainty affecting the considered instances, we consider seven cases focusing on a specific subset of uncertain attributes $\Theta = \{P, S, R, PS, PR, RS, PRS\}$, keeping all the others deterministic. These are all the possible cases with the categories of time attributes considered in the BJSS model. However, each case also has a practical relevance as it represents situations of possible variability in the time attributes [9, 21, 27, 42]. The latter may depend on the degree of automation and standardization of the activities and processes which are not always under the complete control of the decision maker, e.g., in the BJSS there may be manual procedures in the setups or in the operations, while some activities may be carried out by external personnel or service suppliers.

To better assess the impact of the severity of uncertainties we introduce two different scenarios indicated as Sc10 and Sc25 and characterized as follows. For each case in Θ, the η% of uncertain attributes a are considered. Each of these attributes a is associated to an interval of time values $\mathbf{T}_a = [\underline{t_a}, \overline{t_a}]$, where $\underline{t_a}$ is the basic deterministic value, and $\overline{t_a} = \lfloor \delta \underline{t_a} \rfloor$. Scenario Sc10 adopts the parameters $\eta = 10$ and $\delta = 0.10$, whereas in Sc25 the values are $\eta = 25$ and $\delta = 0.25$, representing considerably more severe situations in terms of uncertainty.

Combining the considered feasible solutions and all cases and scenarios, the

Table 4: Characteristics of the 58 instances and the set of their 20 deterministic solutions

Instance	$\|J\|$	$\|M\|$	$\|N\|$	$\|A\|$	Avg C_{max}	St Dev C_{max}	CV C_{max}
ABZ5	10	10	112	571	2066.6	24.7	0.01
ABZ6	10	10	112	571	1632.6	24.3	0.01
ABZ7	20	15	322	3191	1591.8	11.5	0.01
ABZ8	20	15	322	3191	1603.9	9.3	0.01
ABZ9	20	15	322	3191	1600.3	22.4	0.01
FT06	6	6	44	139	92.1	4.5	0.05
FT10	10	10	112	571	1473.0	22.8	0.02
FT20	20	5	122	1091	1959.8	35.0	0.02
LA01	10	5	62	296	1105.4	28.7	0.03
LA02	10	5	62	296	1103.3	22.2	0.02
LA03	10	5	62	296	995.8	26.0	0.03
LA04	10	5	62	296	1036.9	36.2	0.03
LA05	10	5	62	296	915.6	18.8	0.02
LA06	15	5	92	631	1529.1	22.5	0.01
LA07	15	5	92	631	1416.0	22.6	0.02
LA08	15	5	92	631	1491.4	19.6	0.01
LA09	15	5	92	631	1567.4	28.1	0.02
LA10	15	5	92	631	1555.5	19.3	0.01
LA11	20	5	122	1091	1970.6	34.6	0.02
LA12	20	5	122	1091	1829.7	23.5	0.01
LA13	20	5	122	1091	1949.7	33.4	0.02
LA14	20	5	122	1091	2049.3	19.5	0.01
LA15	20	5	122	1091	2004.9	25.4	0.01
LA16	10	10	112	571	1520.3	21.8	0.01
LA17	10	10	112	571	1328.9	13.0	0.01
LA18	10	10	112	571	1384.1	18.4	0.01
LA19	10	10	112	571	1424.1	18.7	0.01
LA20	10	10	112	571	1530.6	16.2	0.01
LA21	15	10	167	1231	2094.8	39.8	0.02
LA22	15	10	167	1231	1884.0	12.8	0.01
LA23	15	10	167	1231	2101.2	29.8	0.01
LA24	15	10	167	1231	1943.4	16.8	0.01
LA25	15	10	167	1231	1975.4	38.2	0.02
LA26	20	10	222	2141	2710.5	44.5	0.02
LA27	20	10	222	2141	2709.7	27.5	0.01
LA28	20	10	222	2141	2802.6	27.0	0.01
LA29	20	10	222	2141	2492.9	22.2	0.01
LA30	20	10	222	2141	2707.6	14.8	0.01
LA31	30	10	332	4711	3987.1	44.2	0.01
LA32	30	10	332	4711	4215.1	37.7	0.01
LA33	30	10	332	4711	3892.4	40.3	0.01
LA34	30	10	332	4711	4044.7	43.6	0.01
LA35	30	10	332	4711	4183.4	20.5	0.00
LA36	15	15	242	1831	2404.1	24.3	0.01
LA37	15	15	242	1831	2576.3	18.0	0.01
LA38	15	15	242	1831	2326.0	16.9	0.01
LA39	15	15	242	1831	2404.7	15.9	0.01
LA40	15	15	242	1831	2477.4	25.9	0.01
ORB01	10	10	112	571	1643.1	15.1	0.01
ORB02	10	10	112	571	1566.2	16.1	0.01
ORB03	10	10	112	571	1570.3	31.1	0.02
ORB04	10	10	112	571	1554.0	16.4	0.01
ORB05	10	10	112	571	1492.3	33.1	0.02
ORB06	10	10	112	571	1742.7	25.4	0.01
ORB07	10	10	112	571	753.0	17.0	0.02
ORB08	10	10	112	571	1370.2	16.4	0.01
ORB09	10	10	112	571	1450.4	26.0	0.02
ORB10	10	10	112	571	1660.3	21.0	0.01
AVG.	15.4	9.2	161.1	1392.1	1939.0	24.2	0.01

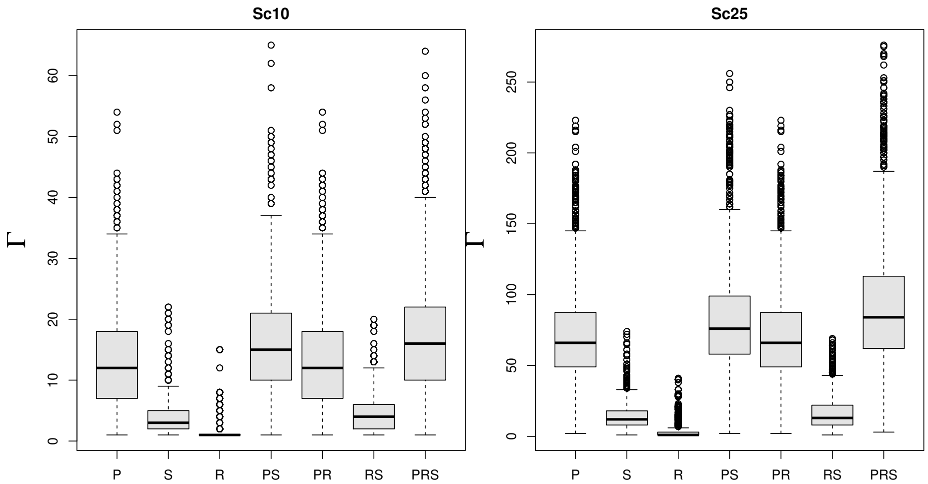

Figure 6: Boxplot for Γ over all the solutions for the cases in Θ under scenarios Sc10 and Sc25

number of schedules for which to calculate the risk measures reaches 16240. This large set of solutions to be evaluated contains IINs which have different spread and extent of uncertainty conducting to different values of $\Gamma = \overline{C_{max}} - \underline{C_{max}} + 1$. These values are summarized in Figure 6 for Sc10 and Sc25, respectively. More precisely, for each scenario is reported a boxplot related to the statistics of Γ over all the evaluations for the seven cases in Θ (reported in the x-axis). Note that, the boxplots in Figure 6 have a different scale for Γ values due to the diverse amount of uncertainty introduced in the IINs in the two scenarios. Each graph in the figure shows the statistics of Γ for the corresponding scenario. Overall, it can be observed that the designed experiments lead to the analysis of a large and diversified case study. Figure 6 shows that, as expected, the cases in which the processing times are uncertain have Γ values tending to be much higher in both scenarios since the processing arcs are relatively more numerous in A. As regards the probability level α of the risk measures CVaR_α and VaR_α, three different values are used ($\alpha \in \{0.99, 0.95, 0.90\}$) which represent different sensitivities of the decision maker regarding the risk of worsening of the makespan and the possibility of delays in the jobs completion. This triples the number of risk assessments to be executed for each indicator reaching 48720.

5.2. Results Analysis

This section reports analyzes the results of the experimental campaign providing answers to the research questions introduced in Section 5. To this aim, Tables 5, 6 and 7 summarize the results obtained for the three probability levels $\alpha = \{0.99, 0.95, 0.90\}$, respectively. Each table contains two sections dedicated to the two scenarios Sc10 and Sc25, and for each of them the results obtained in the 7 cases in $\Theta = \{P, S, R, PS, PR, RS, PRS\}$) are reported by columns. Whereas the rows show the average and the standard deviation over the conducted risk evaluations of the indicators adopted to assess the quality of the results obtained for CVaR_α (reported in the first part of the tables) and VaR_α (in the second part of the tables). The results are analyzed considering the indicators including the relative gap between upper and lower bounds for the risk measures (Gap_{CVaR} and Gap_{VaR}), the *quality index* (I_{CVaR} and I_{VaR}) and the *estimated maximum relative error* (MRE_{CVaR} and MRE_{VaR}) which have been introduced in Section 4.1. Furthermore, we report $\text{Exact}_{\text{CVaR}}$ and $\text{Exact}_{\text{VaR}}$ as indicators of the exact or heuristic behavior of the algorithm. They account for the percentages of cases solved exactly for CVaR_α and VaR_α, i.e., the cases with $LB_{CVaR} = UB_{CVaR}$ and $LB_{VaR} = UB_{VaR}$, respectively.

Overall, the three tables show that high quality results are always reached for both risk measures. In general, as expected from the characteristics of the algorithm used, the quality of the results improves as the α value increases. As regards the impact of uncertainty, with the same α probability level, better results are obtained in the scenario Sc10 characterized by lower levels of uncertainty as shown in Figure 6. While within the same scenario, better results are obtained in cases where the processing times are deterministic and, due to the scenarios structure, the degree of uncertainty remains low.

Regarding the CVaR_α we observe that the average MRE_{CVaR} never exceeds the value 0.01, while both Gap_{CVaR} and the I_{CVaR} are very low even if they are affected by the uncertainty. In fact, their values tend to be higher in scenario Sc25 and, more in general, when processing times are uncertain conducting to more challenging IINs. This influence is also found for the percentage $\text{Exact}_{\text{CVaR}}$

of exact assessments which is greater when the uncertainty is more limited. However, even in cases with greater uncertainty, the detected gap and error remain low, indicating that the algorithm still obtains accurate estimates.

A similar dependence on uncertainties is shown by the algorithm behavior for the VaR_α. Compared to the CVaR_α case, we observe an improvement in the percentages of exact evaluations and a slight worsening of the relative gap, the quality index and MRE_{VaR}. The latter is however always less than 0.02.

These observations show how the method adopted succeeds in effectively and accurately calculating CVaR_α and VaR_α of $\mathbf{C}_{max}$ for complex BJSS problems.

Finally, we analyze the efficiency of the proposed method in terms of the computational effort required. To this aim, the last rows of Tables 5, 6 and 7 report the computation time (indicated as *Time*, and expressed in seconds) of each risk measure, averaged on all the instances and the related standard deviation. The risk assessment algorithm, in general, is rapid and makes slightly increasing computational effort as α values decrease as the algorithm's counting procedures have to process an increasing amount of network configurations. However, this is not really a major flaw as there is usually a greater interest in evaluating CVaR_α and VaR_α for high α values. As expected from the computational complexity analysis discussed in Section 4.1, the computation time is more sensitive to the value of Γ and $|A|$. This explains the longer times required in the more severe Scenario Sc25 and for cases with greater uncertainty and instances with larger networks as analyzed in Table 4 and in Figure 6.

Tables 8 and 9 report for each scenario and for each case considered, the percentage of approximated risk evaluations and the average computation times with respect of the size of the instances. In Table 8 the rows refer to specific values of $|A|$, already reported in Table 4, indicating the number of instances (reported as *#Inst.*) having each size. The results reported in the table averages 120 evaluations for each instance (i.e., considering 3 α values, 20 feasible solutions, and 2 risk measures). As the size of the solutions to be analyzed and the amount of uncertainty increase, the algorithm tends to solve exactly a smaller number of evaluations, however it still provides very accurate approximations.

Moreover, this analysis highlights the impact of the possible sources of uncertainty (i.e., the typology of uncertain arcs) within each scenario. Adopting a similar structure, Table 9 shows the analysis of the averages of the computational times for each scenario and each case considered with respect to the size of the instances, detailing what is shown in an aggregate way in the Tables 5, 6, and 7. The greatest computational effort is required for larger instances with many uncertain arcs and a huge number o configurations. They are identified in particular in the scenario Sc25 for networks with more than 3000 arcs, and in cases with uncertainties on the processing times that are associated with the largest set of arcs. For example, for these solutions in the PRS case (last column in Table 9) the total number of configurations is between 10^{317} and 10^{605}.

The proposed method provides support to the decision maker to quickly calculate a risk measure as an outcome function for the set of feasible schedules available. Therefore, the method can be used for the selection and validation of the solution to be implemented or executed, quantifying the risk of possible deterioration of the makespan to give adequate guarantees on the behavior of the system with regard to extreme cases, according to a level of probability α.

Concerning the BJSS instances, their deterministic solutions found by the metaheuristic algorithm have a very low coefficient of variation, as reported in Table 4. Moreover, the analysis of the experimental results shows that the performances of deterministic solutions are often affected only to a limited extent by uncertainty, confirming that the adopted deterministic approach is quite robust. This aspect is illustrated in aggregate over all the evaluations in Figures 7 and 8 reporting, for each scenario and for each case in Θ, the boxplots of the relative risk measures (indicated in percentage as *Risk(%)*) for CVaR and VaR, respectively. They are computed for each solution with respect to its deterministic makespan C_{max}, i.e., considering Risk $= \frac{\text{CVaR}_\alpha(\mathbf{C}_{max}) - C_{max}}{C_{max}}$ for the case of CVaR, and Risk $= \frac{\text{VaR}_\alpha(\mathbf{C}_{max}) - C_{max}}{C_{max}}$ for the VaR.

Table 5: Summary of results for the risk evaluations when $\alpha = 0.99$

Indicators	Scenario Sc10							Scenario Sc25						
	P	S	R	PS	PR	RS	PRS	P	S	R	PS	PR	RS	PRS
Avg Gap$_{Cvar}$(%)	0.01	<0.01	-	0.02	0.01	0	0.03	0.83	0.03	0	1.29	0.83	0.06	1.57
St Dev Gap$_{Cvar}$(%)	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Exact$_{Cvar}$(%)	94	99	100	86	94	100	84	30	86	100	17	30	80	14
Avg I$_{Cvar}$	0.01	<0.01	0	0.03	0.01	0	0.04	0.38	0.05	0	0.50	0.38	0.06	0.54
St Dev I$_{Cvar}$	0.07	0.01	0	0.13	0.07	0	0.15	0.35	0.16	0	0.34	0.35	0.18	0.33
Avg MRE$_{Cvar}$	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	<0.01	<0.01	0	0.01	<0.01	<0.01	0.01
St Dev MRE$_{Cvar}$	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	<0.01	<0.01	0	<0.01	<0.01	<0.01	0.01
Avg Gap$_{Var}$(%)	0.01	<0.01	0	0.04	0.01	0	0.05	1.65	0.06	0	2.50	1.65	0.10	3.04
St Dev Gap$_{Var}$(%)	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	0.02	<0.01	0	0.02	0.02	<0.01	0.02
Exact$_{Var}$(%)	95	99	100	87	95	100	86	31	87	100	19	31	82	15
Avg I$_{Var}$	0.01	<0.01	0	0.04	0.01	0	0.05	0.44	0.06	0	0.57	0.44	<0.01	0.61
St Dev I$_{Var}$	0.08	0.01	0	0.17	0.08	0	0.17	0.40	0.19	0	0.38	0.40	0.25	0.37
Avg MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
St Dev MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	0	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Avg Time (s)	0.17	0.05	0.05	0.44	0.17	0.05	0.53	54.64	0.75	0.05	84.24	54.61	1.32	129.10
St Dev Time (s)	0.90	0.06	0.03	2.08	0.87	0.07	2.31	137.84	3.64	0.03	206.21	137.66	5.10	293.83

Table 6: Summary of results for the risk evaluations when $\alpha = 0.95$

Indicators	Scenario Sc10							Scenario Sc25						
	P	S	R	PS	PR	RS	PRS	P	S	R	PS	PR	RS	PRS
Avg Gap$_{Cvar}$(%)	0.01	<0.01	0	0.04	0.01	<0.01	0.05	0.97	0.05	0	1.43	0.97	0.09	1.71
St Dev Gap$_{Cvar}$(%)	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Exact$_{Cvar}$(%)	88	99	100	77	88	99	75	18	77	100	10	18	71	8
Avg I$_{Cvar}$	0.03	<0.01	0	0.06	0.03	<0.01	0.07	0.45	0.08	0	0.56	0.45	0.10	0.60
St Dev I$_{Cvar}$	0.11	0.02	0	0.18	0.11	0.03	0.19	0.32	0.19	0	0.29	0.32	0.23	0.28
Avg MRE$_{Cvar}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0	0.01	<0.01	<0.01	0.01
St Dev MRE$_{Cvar}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01
Gap$_{Var}$(%)	0.03	<0.01	0	0.07	0.03	<0.01	0.09	2.00	0.09	0	2.84	2.00	0.16	3.35
St Dev Gap$_{Var}$(%)	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.02	<0.01	0	0.02	0.02	<0.01	0.02
Exact$_{Var}$(%)	90	99	100	80	90	99	77	19	79	100	11	19	74	8
Avg I$_{Var}$	0.03	<0.01	0	0.07	0.03	<0.01	0.09	0.53	0.09	0	0.65	0.53	<0.01	0.69
St Dev I$_{Var}$	0.13	0.03	0	0.21	0.13	0.04	0.23	0.37	0.22	0	0.32	0.37	0.26	0.30
Avg MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.02
St Dev MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Avg Time (s)	0.30	0.07	0.05	0.58	0.29	0.07	0.76	55.56	0.88	0.05	84.77	55.54	1.67	129.52
St Dev Time (s)	1.30	0.10	0.03	2.17	1.26	0.13	2.60	137.45	3.72	0.04	205.76	137.28	5.36	293.13

Table 7: Summary of results for the risk evaluations when $\alpha = 0.90$

Indicators	Scenario Sc10							Scenario Sc25						
	P	S	R	PS	PR	RS	PRS	P	S	R	PS	PR	RS	PRS
Avg Gap$_{Cvar}$(%)	0.02	<0.01	0	0.05	0.02	<0.01	0.06	1.03	0.06	0	1.48	1.03	0.10	1.74
St Dev Gap$_{Cvar}$(%)	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Exact$_{Cvar}$(%)	83	98	100	71	83	98	68	14	73	100	7	14	67	5
Avg I$_{Cvar}$	0.04	<0.01	0	0.08	0.04	<0.01	0.09	0.48	0.09	0	0.58	0.48	0.12	0.61
St Dev I$_{Cvar}$	0.12	0.03	0	0.19	0.12	0.04	0.21	0.30	0.20	0	0.26	0.30	0.24	0.25
Avg MRE$_{Cvar}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
St Dev MRE$_{Cvar}$	0.0	0.0	0	0.0	0.0	0.0	0.0	0.0	0.0	0	0.0	0.0	0.0	0.0
Avg Gap$_{Var}$(%)	0.04	<0.01	0	0.09	0.04	<0.01	0.12	2.17	0.10	0	2.99	2.17	0.18	3.46
St Dev Gap$_{Var}$(%)	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.02
Exact$_{Var}$(%)	85	98	100	73	85	98	71	16	74	100	8	16	68	6
Avg I$_{Var}$	0.05	<0.01	0	0.10	0.05	0.01	0.12	0.57	0.11	0	0.69	0.57	<0.01	0.71
St Dev I$_{Var}$	0.16	0.04	0	0.22	0.16	0.06	0.24	0.34	0.23	0	0.29	0.34	0.27	0.27
Avg MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.02
St Dev MRE$_{Var}$	<0.01	<0.01	0	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0	0.01	0.01	<0.01	0.01
Avg Time (s)	0.39	0.08	0.05	0.66	0.37	0.08	0.87	55.88	0.93	0.06	84.95	55.85	1.75	129.58
St Dev Time (s)	1.43	0.10	0.04	2.22	1.37	0.15	2.68	137.31	3.73	0.05	205.61	137.12	5.37	293.10

Table 8: Percentage of approximated risk evaluations for size of instance

Scenario	$\|A\|$	# Inst.	P	S	R	PS	PR	RS	PRS
Sc10	139	1	0	0	0	3.3	0	1.7	0
	296	5	0.7	0	0	4.0	0.7	0	3.0
	571	18	2.1	0	0	4.5	2.1	0	8.8
	631	5	0.7	0.7	0	4.8	0.7	0	3.7
	1091	6	8.1	0	0	17.6	8.1	0	17.5
	1231	5	16.3	1.7	0	20.0	16.3	0.7	28.3
	1831	5	12.5	0	0	29.0	12.5	0	30.7
	2141	5	10.5	0.3	0	30.7	10.5	1.3	30.2
	3191	3	29.7	4.4	0	63.1	29.7	16.7	58.6
	4711	5	50.3	10.3	0	78.8	50.3	2.3	87.2
Sc25	139	1	1.7	3.3	0	11.7	1.7	0	18.3
	296	5	32.8	0.3	0	52.2	32.8	3.2	57.0
	571	18	65.5	2.8	0	81.8	65.5	5.5	88.2
	631	5	79.0	8.7	0	95.2	79.0	16.5	95.2
	1091	6	90.3	23.5	0	99.2	90.3	24.4	99.4
	1231	5	97.0	20.8	0	99.3	97.0	21.3	99.7
	1831	5	99.3	14.5	0	99.7	99.3	21.0	100
	2141	5	100	29.7	0	100	100	53.3	100
	3191	3	100	62.2	0	100	100	88.1	100
	4711	5	100	88.7	0	100	100	86.8	100

Nevertheless, we observe two types of situations to be further analyzed. The first situation is represented by instances for which the best deterministic solution in the set offers also the best risk profile. The second situation, instead, presents some solution with a better risk profile than that of the best deterministic solution, requiring the decision maker to solve a trade-off. Moreover, as the degree of uncertainty increases (i.e., considering the scenarios and the values of Γ reported in Figure 6), the solutions have a worse risk profile which also tends to have a significant variability on the basis of the α levels and, in these cases, it becomes determining the sensitivity of the decision maker to the risk. When the deterministic solutions remain preferable our analysis provides the scheduler with a certification of its risk related to the uncertainty. Indeed, the proposed methodology provides the decision maker with a tool for easily identifying the presence of schedules with the best risk profiles in the set of available solutions, giving a positive answer to the research question about the usefulness of this method for decision makers. To better highlight this aspect of the results, combining the usefulness of the methodology with the effects of the type and extent of uncertainties, in Tables 10 and 11 we report, for the two

Table 9: Average computation time for size of instance

Scenario	$\|A\|$	# Inst.	P	S	R	PS	PR	RS	PRS
Sc10	139	1	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	296	5	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	571	18	<0.01	<0.01	<0.01	0.01	<0.01	<0.01	0.01
	631	5	0.02	0.01	0.01	0.02	0.02	0.01	0.02
	1091	6	0.03	0.01	0.01	0.05	0.03	0.01	0.05
	1231	5	0.05	0.02	0.01	0.07	0.05	0.02	0.09
	1831	5	0.07	0.02	0.02	0.11	0.06	0.02	0.20
	2141	5	0.08	0.03	0.02	0.12	0.08	0.03	0.15
	3191	3	0.25	0.07	0.06	0.72	0.24	0.14	0.68
	4711	5	0.69	0.09	0.04	1.4	0.67	0.08	1.90
Sc25	139	1	0.02	0.02	0.02	0.03	0.02	0.03	0.04
	296	5	0.09	0.01	0.01	0.18	0.09	0.01	0.19
	571	18	0.19	<0.01	<0.01	0.34	0.19	0.01	0.50
	631	5	0.77	0.03	0.01	1.7	0.77	0.03	2.0
	1091	6	3.3	0.06	0.01	5.4	3.3	0.06	5.3
	1231	5	5.8	0.05	0.01	9.1	5.8	0.05	9.5
	1831	5	10.1	0.06	0.02	15.3	10.1	0.08	27.9
	2141	5	20.4	0.15	0.02	28.7	20.4	0.24	30.3
	3191	3	70.1	0.89	0.07	100.9	70.2	5.8	333.2
	4711	5	144.5	2.5	0.05	223.8	144.4	3.4	296.3

scenarios and for each α value, the percentages of cases in which the preferable risk profile is not associated with the best deterministic solution. The overall percentages for CVaR_α are 7.88% and 17.65% for scenarios Sc10 and Sc25, respectively, whereas for VaR_α are 7.47% in Sc10 and 16.50% in Sc25. The tables report the detailed average results for each of the 7 cases belonging in the range $[0\%, 27.59\%]$ for CVaR_α and $[0\%, 24.14\%]$ for VaR_α. We observe that only for the R case and limited to the Sc10 scenario there are zero cases for all three α values. Furthermore, in Sc10 the cases S and RS have relatively low percentages for each α value. This is explained by the low level of uncertainty present in those cases. In fact, in all other cases, percentages exceeding 10% are obtained. These results confirm the impact of diffusion and the extent of uncertainties in the BJSS model, and highlight it from a more practical point of view.

5.3. Discussion and Managerial Insights

This section discusses the obtained results from a managerial or practical perspective. The latter is often the view that scheduling is a decision to be made rather than a mathematical procedure, and it is the concern of the schedulers. Schedulers face many challenges everyday, and need practical advice on

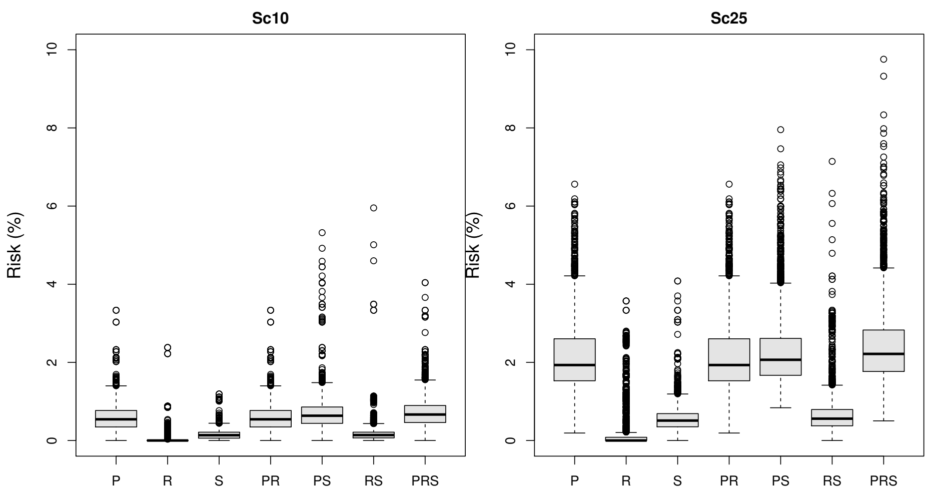

Figure 7: Boxplot for the normalized CVaR risk over all the solutions for the cases in Θ under scenarios Sc10 and Sc25

Table 10: Percentage of cases in which the best deterministic solution does not have the best risk profile under CVaR$_\alpha$

Scenario	α	P	S	R	PS	PR	RS	PRS	Avg
Sc10	0.99	10.34	3.45	0	13.79	10.34	1.72	12.07	7.39
	0.95	10.34	3.45	0	17.24	10.34	3.45	12.07	8.13
	0.90	12.07	3.45	0	15.51	12.07	3.45	10.34	8.13
Sc25	0.99	25.86	10.34	10.34	24.14	25.86	13.79	27.59	19.70
	0.95	20.69	10.34	10.34	22.41	20.69	10.34	27.59	17.49
	0.90	18.97	10.34	10.34	17.24	18.97	10.34	24.14	15.76

how to schedule the shop-floor or a whole factory [39]. In many cases, planners and schedulers see their task as trying to satisfy requirements in an environment affected by variability in information, operations and outcomes. In daily practice, schedulers generally solve one instance at a time with a given characterization of uncertainty in terms of sources and relevance, and with a well defined risk behavior (i.e. in terms of risk index and probability level α to use). The latter is determined on the basis of the decision-makers knowledge, rules and skill settings.

Besides this, we observe that our computational tests contains also very stressful cases for the proposed approach and the experimental results show that the risk assessment methodology is suitable for real contexts. Further

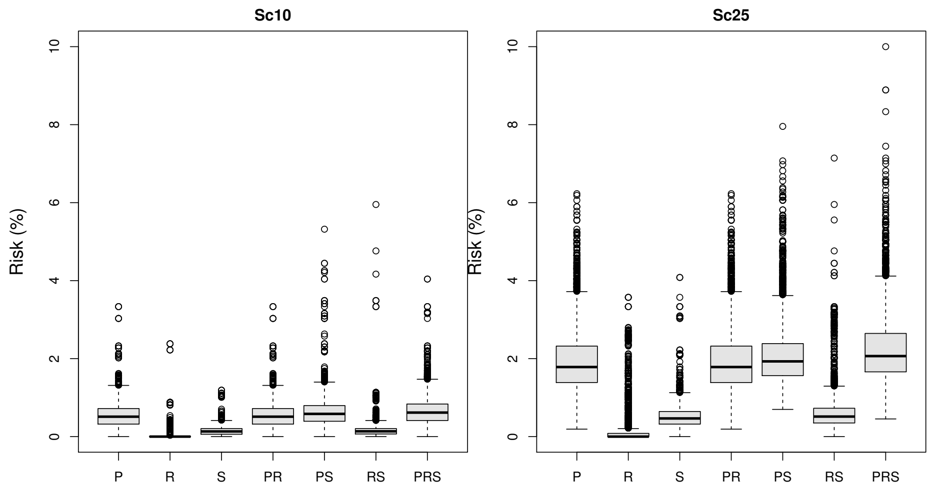

Figure 8: Boxplot for the normalized VaR over all the solutions for the cases in Θ under scenarios Sc10 and Sc25

Table 11: Percentage of cases in which the best deterministic solution does not have the best risk profile under VaR_α

Scenario	α	P	S	R	PS	PR	RS	PRS	Avg
Sc10	0.99	10.34	3.45	0	13.79	10.34	1.72	10.34	7.14
	0.95	12.07	3.45	0	15.52	12.07	1.72	13.79	8.37
	0.90	12.07	1.72	0	10.34	12.07	1.72	10.34	6.90
Sc25	0.99	22.41	10.34	10.34	18.97	22.41	13.79	24.14	17.49
	0.95	22.41	10.34	8.62	24.14	22.41	10.34	20.69	17.00
	0.90	18.97	8.62	5.17	18.97	18.97	12.07	22.41	15.02

managerial insights include both operational and tactical/strategical aspects.

From the operational point of view, the proposed approach allows to evaluate VaR and CVaR as outcome functions for any given feasible solution of BJSS. This allows to validate the solution to be implemented and to quantify the risk giving guarantees on the behavior of the system in the most adverse cases with reference to the prescribed level of α. In addition, in case several schedules are available for implementation, the presented approach helps selecting the best one. A rapid and precise risk assessment procedure can play a fundamental role for the scheduler, even in cases where it is possible to receive updates of the values and quantities affected by uncertainty. It is worth noting that even in situations where the methodology seldom leads to the choice of alternative solutions to the deterministic one, it generally favors the improvement of the quality and safety of both the scheduling system and the decision maker's work, providing valuable support. The experimental results underline that the proposed methodology is very promising both in terms of accuracy of the evaluations and in terms of the required computational times. Thus, this methodology can benefit other types of applications, such as closed-loop scheduling algorithms and monitoring schemes for possible re-scheduling activities. These can be seen as possible future developments requiring further research work.

The characteristics and the behavior of the method make it a useful tactical or strategical tool in the context of Industry 4.0 and Cyber-Physical-Systems architectures. In fact, on the one hand, it can be considered as an alternative or an integration of more complex factory simulation systems or digital twins. On the other hand, this method can help to assess the relevance of the type and extent of the uncertainties within the scheduling system. This supports the management in identifying and evaluating appropriate actions to reduce these uncertainties both from an information and operational point of view. They can include improvements in the estimation of time attributes, the reduction of the variability in the activities execution, and, more in general, the possible sources of uncertainties. Referring to our experimental campaign, these actions may correspond to passing from one scenario to another or, within the same scenario, from one case in Θ to another characterized by a lower amount of uncertainty.

6. Conclusions

This article deals with the BJSS problem and proposes a method for evaluating the VaR and the CVaR as risk measures for the makespan of a feasible schedule when its temporal attributes are uncertain and known only as intervals. More specifically, the paper proposes an interval-valued network approach to model the feasible solutions of complex BJSSs characterized by uncertain values for jobs' releases, processing and setup times. It also introduces an evaluation method for the risk that the makespan of a BJSS schedule assumes worse extreme values, considering the known intervals for the activity durations. Both modeling and computational issues are addressed, implementing and testing a network-based model used with an innovative algorithm for the first time applied to complex job shop problems to provide an accurate, rapid and viable computation of the risk indices considered for the feasible schedules.

The study takes into account realistic combinations of sources of uncertainty affecting a BJSS solution often arising in practice, and highlights the validity and applicability of the proposed methodological approach through an extensive experimental campaign. The latter is based on a wide set of instances composed of different sets of complex and realistic cases derived from classical benchmark deterministic job-shop problems.

The obtained results show the effectiveness of the proposed method in accurately calculating VaR and CVaR of the uncertain makespan for the BJSS problem and the computational effort of the proposed risk assessment and its suitability for real-world applications. Furthermore, the effects of the type and severity of the uncertainty in the BJSS instances are addressed and the usefulness of the proposed methodology for decision makers is highlighted. Moreovoer, the experimental results indicate that the proposed method is compatible with the application in real contexts both for the type and size of the instances and for the computational effort required, allowing an easy use of VaR and CVaR to address cases of BJSS affected by uncertainty taking into account the sensitivity of the scheduler to risk and/or the related industrial policies and possible regulatory requirements.

Regarding a BJSS solution, the proposed approach allows adding important information about the risk of delays or worsening of the makespan. Furthermore, when there are alternative solutions to consider, this approach allows to make an informed choice based on an adequate risk assessment. This contributes to provide a decision support to schedulers leading to improve the overall levels of both quality and safety of the scheduling system.

Overall the framework is widely applicable because it is based on a model that can be easily built for a BJSS solution. The study can be expanded by introducing the use of the proposed method to provide decision support in advanced planning and scheduling systems for multi-stage environments. From an operational point of view, the method can offer a contribution to introduce or improve the risk evaluation functions for the schedule selection, and to implement a risk monitoring for the makespan of a chosen solution in order to adopt specific actions in the cases of risk measures deterioration. At a tactical or strategic decision level, this method can help to assess the relevance of the type and extent of the uncertainties within the scheduling system. This supports the management in identifying and evaluate appropriate ways to reduce relevant uncertainties both from an information (e.g., by improving the estimates of time attributes) and technical (e.g., by reducing the variability of the activity durations) point of view.

Further research directions can include both operational and strategic studies of cases with different or more specific types of uncertainty, and the application of the proposed approach to other scheduling problems characterized by a similar modeling structure. Possible applications could also consider the use of more information and data on the involved uncertainties, and the use of VaR and CVaR as criteria in closed loop scheduling optimization frameworks.

References

[1] Adams J, Balas E, Zawack D, (1988) The shifting bottleneck procedure for job shop scheduling. *Management Science*, 34 (3) 391-401.

[2] Agostino IRS, da Silva WV, Pereira da Veiga C, Souza AM, (2020) Forecasting models in the manufacturing processes and operations management: Systematic literature review. *Journal of Forecasting*, 1-14.

[3] AitZai A, Boudhar M, (2013) Parallel branch-and-bound and parallel PSO

algorithms for job shop scheduling problem with blocking. *International Journal of Operational Research*, 16 (1) 14-37.

[4] Allahverdi A, Aydilek H, Aydilek A, (2014) Single machine scheduling problem with interval processing times to minimize mean weighted completion time. *Computers & Operations Research*, 51, 200-207.

[5] Applegate D, Cook W, (1991) A computational study of job-shop scheduling. *ORSA Journal on Computer*, 3 (2) 149-156.

[6] Atakan S, Bülbül K, Noyan N, (2017) Minimizing value-at-risk in single-machine scheduling. *Annals of Operations Research*, 248, (1-2) 25-73.

[7] Aydilek A, Aydilek H, Allahverdi A, (2013) Increasing the profitability and competitiveness in a production environment with random and bounded setup times. *International Journal of Production Research*, 51, 106-117.

[8] Aydilek A, Aydilek H, Allahverdi A, (2017) Algorithms for minimizing the number of tardy jobs for reducing production cost with uncertain processing times. *Applied Mathematical Modelling*, 45, 982-996.

[9] Baker K, Trietsch D, (2019) *Principles of sequencing and scheduling*, 2nd ed. John Wiley & Sons, New York.

[10] Batur D, Choobineh F, (2010) A quantile-based approach to system selection. *European Journal of Operational Research*, 202, (3) 764-772.

[11] Bertsimas D, Lauprete GJ, Samarov A, (2004) Shortfall as a risk measure: properties, optimization and applications. *Journal of Economic Dynamics & Control*, 28 (7) 1353-1382.

[12] Brucker P, Knust S, (2013) *Complex Scheduling*, 2nd ed. Springer.

[13] Bürgy R, Gröflin H, (2016) The blocking job shop with rail-bound transportation. *Journal of Combinatorial Optimization*, 31, (1) 152-181.

[14] Chanas S, Zieliński P, (2002) The computational complexity of the critical problems in a network with interval activity times. *European Journal of Operational Research*, 136, 541-550.

[15] Chanas S, Dubois D, Zieliński P, (2002) On the Sure Criticality of Tasks in Activity Networks with Imprecise Durations. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 32, (4) 393-407.

[16] Chang Z, Song S, Zhang Y, Ding J-Y, Zhang R, Chiong R. (2017). Distributionally robust single machine scheduling with risk aversion. *European Journal of Operational Research* 256, 261–274.

[17] De P, Ghosh JB, Wells CE, (1992) Expectation-variance analysis of job sequences under processing time uncertainty. *International Journal of Production Economics*, 28 (3) 289-297.

[18] Elmaghraby SE, (2005) On the fallacy of averages in project risk management. *European Journal of Operational Research*, 165 (2) 307-313.

[19] Fisher H, Thompson G, (1963) Probabilistic learning combinations of local job-shop scheduling rules. In: Muth JF, Thompson GL, Industrial scheduling. Prentice-Hall, 225-251.

[20] Fowler JW, Mönch L, Rose O, (2006) Scheduling and Simulation. In Hermann JW, (Ed.), *Handbook of Production Scheduling*, Springer, 109-134.

[21] Framinan JM, Leisten R, Ruiz García R, (2014) *Manufacturing Scheduling Systems. An Integrated View on Models, Methods and Tools.* Springer-Verlag, London.

[22] García-González J, Parrilla E, Mateo A, (2007) Risk-averse profit-based optimal scheduling of a hydro-chain in the day-ahead electricity market. *European Journal of Operational Research*, 181, 1354-1369.

[23] Hall NG, Sriskandarajah C, (1996) A survey of machine scheduling problems with blocking and no-wait in process. *Operations Research*, 44, 510-525.

[24] Heckmann I, Comes T, Nickel S, (2015) A critical review on supply chain risk – Definition, measure and modeling. *Omega*, 52, 119-132.

[25] Herrmann JW, (2006) Decision-Making Systems in Production Scheduling. In Herrmann JW, (Ed.), *Handbook of Production Scheduling*, Springer, 91-108.

[26] Ivanescu CV, Fransoo JC, Bertrand JWM, (2002) Makespan estimation and order acceptance in batch process industries when processing times are uncertain. *OR Spectrum*, 24, 467-495.

[27] Kim SC, Bobrowski PM, (1997) Scheduling jobs with uncertain setup times and sequence dependency. *Omega*, 25 (4) 437-447.

[28] Lai T-C, Sotskov YN, (1999) Sequencing with uncertain numerical data for makespan minimisation. *Journal of the Operational Research Society*, 50, 230-243.

[29] Lange J, Werner F, (2018) Approaches to modeling train scheduling problems as job-shop problems with blocking constraints. *Journal of Scheduling*, 21, (2) 191-207.

[30] Lange J, Werner F, (2019) On Neighborhood Structures and Repair Techniques for Blocking Job Shop Scheduling Problems. *Algorithms*, 12 (11) 242.

[31] Lawrence S, (1984) Resource constrained project scheduling: an experimental investigation of heuristic scheduling techniques (Supplement), *Graduate School of Industrial Administration. Pittsburgh, Pennsylvania, Carnegie-Mellon University.*

[32] Lawrence SR, Sewell EC, (1997) Heuristic, optimal, static, and dynamic schedules when processing times are uncertain *Journal of the Operations Management*, 15, 71-82.

[33] Levner E, Kats VB, Levit VE, (1997) An improved algorithm for a cyclic robotic scheduling problem. *European Journal of Operational Research*, 97, 500-508.

[34] Liao L, Sarin SC, Sherali HD, (2012) A scenario generation-based lower bounding approach for stochastic scheduling problems. *Journal of the Operational Research Society*, 63, 1410-1420.

[35] Liu SQ, Kozan E, (2011) Scheduling trains with priorities: a no-wait blocking parallel-machine job-shop scheduling model. *Transportation Science*, 45, (2) 175-198.

[36] Luh PB, Chen D, Thakur LS, (1999) An Effective Approach for Job-Shop Scheduling with Uncertain Processing Requirements. *IEEE Transactions on Robotics and Automation*, 15, (2) 715-727.

[37] Makridakis S, Wheelwright SC, Hyndman RJ, (2008) Forecasting Methods and Applications, 3rd ed. Wiley.

[38] Mascis A, Pacciarelli D, (2002) Job-shop scheduling with blocking and no-wait constraints. *European Journal of Operational Research*, 143 (3) 498-517.

[39] McKay KN, Wiers VCS, (2006) The Human Factor in Planning and Scheduling. In Herrmann JW, (Ed.), *Handbook of Production Scheduling*, Springer, 23-58.

[40] Meloni C, Pacciarelli D, Pranzo M, (2004). A Rollout Metaheuristic for Job Shop Scheduling Problems. *Annals of Operations Research* 131, (1-4) 215–235.

[41] Meloni C, Pranzo M, (2020) Expected shortfall for the makespan in activity networks under imperfect information. *Flexible Services and Manufacturing Journal*, 32, 668-692.

[42] Meloni C, Pranzo M, Samà M, (2021) Risk of delay evaluation in real-time train scheduling with uncertain dwell times. *Transportation Research Part E: Logistics and Transportation Review*, 152, 102366.

[43] Modali JK, Barbulescu L, Smith SF, (2021) Efficient Primal Heuristic Updates for the Blocking Job Shop Problem. *European Journal of Operational Research*, 295 (1) 82-101.

[44] Naderi B, Zandieh M, Fatemi Ghomi SMT, (2009) Scheduling job shop problems with sequence-dependent setup times. *International Journal of Production Research*, 47 (21) 5959-5976.

[45] Pacciarelli D, Pranzo M, (2004) Production scheduling in a steelmaking-continuous casting plant. *Computers & Chemical Engineering*, 28, (12) 2823-2835.

[46] Pham, D-N, Klinkert, A, (2008) Surgical case scheduling as a generalized job shop scheduling problem. *European Journal of Operational Research*, 185, (3) 1011-1025.

[47] Pranzo M, Meloni C, Pacciarelli D, (2003) A New Class of Greedy Heuristics for Job Shop Scheduling Problems. *Lecture Notes in Computer Science*, 2647, 223-236. Springer.

[48] Pranzo M, Pacciarelli D, (2016) An iterated greedy metaheuristic for the blocking job shop scheduling problem. *Journal of Heuristics*, 22 (4) 587-611.

[49] Quan Y, He L, He M, (2014) Short-term hydropower scheduling with gamma inflows using CVaR and Monte Carlo simulation. *Proceedings of the seventh international symposium on computational intelligence and design (ISCID)*, 108-111.

[50] Rockafeller RT, Royset JO, (2013) Superquantiles and Their Applications to Risk, Random Variables, and Regression. *INFORMS Tutorials in Operations Research*, 151-167.

[51] Romero-Silva R, Hernández-López G, (2020) Shop-floor scheduling as a competitive advantage: A study on the relevance of cyber-physical systems in different manufacturing contexts. *International Journal of Production Economics*, 224, 107555.

[52] Rossit DA, Tohmé F, Frutos M, (2019) Industry 4.0: Smart Scheduling. *International Journal of Production Research*, 57, (12) 3802-3813.

[53] Sarin SC, Nagarajan B, Liao L, (2010) *Stochastic scheduling: Expectation-variance analysis of a schedule*. Cambridge University Press, New York.

[54] Sarin SC, Sherali HD, Liao L, (2014) Minimizing conditional value-at-risk for stochastic scheduling problems. *Journal of Scheduling*, 17, 5-15.

[55] Sterna M, (2011) A survey of scheduling problems with late work criteria. *Omega*, 39 (2) 120-129.

[56] Sterna M, (2021) Late and early work scheduling: A survey. *Omega*, 104, 102453.

[57] Sotskov Y, Sotskova NY, Werner F, (1997) Stability of an optimal schedule in a job shop, *Omega*, 25 (4) 397-414.

[58] Szelke E, Kerr RM, (1994) Knowledge-based reactive scheduling. *Production Planning and Control*, 5 (2), 124-145.

[59] Tao L, Wu DD, Liu S, Dolgui A, (2018) Optimal due date quoting for a risk-averse decision-maker under CVaR. *Interantional Journal of Production Research*, 56, (5) 1934-1959.

[60] Zhang J, Ding G, Zou Y, Qin S, Fu J, (2019) Review of job shop scheduling research and its new perspectives under Industry 4.0. *Journal of Intelligent Manufacturing*, 30, (4) 1809-1830.

[61] Zhang M, Tao F, Nee AYC, (2021) Digital Twin Enhanced Dynamic Job-Shop Scheduling. *Journal of Manufacturing Systems*, 58, 146-156.

[62] Zurita D, Delgado M, Carino JA, Ortega JA, (2018) Multimodal Forecasting Methodology Applied to Industrial Process Monitoring. *IEEE Transactions on Industrial Informatics*, 14, (2) 494-503.