Passive Control Strategy for Wind-induced Parametric Instabilities in Suspension Bridges

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ABSTRACT

A nonlinear reduced-order model of suspension bridges (SB) is presented to study the dynamical response and to investigate aerodynamic stability control strategies coping with vortex-induced vibrations (VIV) leading to principal parametric resonances scenarios. A passive control system consisting of a vibration absorber is incorporated in the model by coupling the dynamics of the bridge with those of an eccentric mass visco-elastically connected to the deck. A direct asymptotic approach is used to investigate the dynamic instabilities induced by the parametric-type forces in the case of a 2:1 ratio between the frequency of the aerodynamic loads and the fundamental torsional frequency of the bridge. It is shown how an optimized passive control system can increase the range of wind speeds for which the bridge does not undergo large-amplitude parametric oscillations signaling the loss of stability of the fundamental equilibrium configuration.

KEYWORDS: *suspension bridges, parametric resonances, passive vibration control*

INTRODUCTION

Suspension bridges (SB) are structures characterized by a relatively low flexural-torsional stiffness which implies remarkably low natural frequencies in the fundamental torsional and vertical bending modes. Moreover, their mechanical behavior is mainly governed by the suspension cables which are structural elements whose geometric nonlinearities play a fundamental role in their static and dynamic response of SB. When subject to severe wind excitations, suspension bridges may be affected by dynamic instability phenomena, such as flutter, arising from the self-excited nature of the aerodynamic loads [1, 2] or parametric instabilities due to vortex-induced vibrations (VIV) [3, 4, 5] in which the aerodynamic loads generated by the airflow separation across the bridge deck section act as a multiplicative forcing term that can induce flexural-torsional parametric instabilities in the bridge at wind speeds lower than those that can induce flutter. In the present work, a nonlinear reduced-order model of SB is coupled with the time-varying aerodynamic loads to obtain the equations of motion for the study of principal parametric resonances. The equations of motion are reduced by the Faedo-Galerkin approach adopting the bridge deck eigenfunctions as trial functions. Nonlinear aerodynamic effects due to flow separation are accounted for by using a quasi-steady aerodynamic theory. The method of multiple scales [6] is adopted to investigate the dynamic instabilities induced by the parametric-type forces in the case of a 2:1 ratio between the frequency of the VIV and the fundamental SB torsional frequency. A passive control system is then studied and optimized so as to increase the range of wind speeds for which the bridge stability with respect to parametric resonance is ensured.

MODEL FORMULATION

The here proposed structural model of suspension bridges is parameterized by one single space coordinate along the bridge span-wise direction and the equations governing the dynamic aeroelastic response are obtained via a total Lagrangian formulation. The cables equilibrium is described by the dimensionless catenary $y_c(x)$ while the kinematic variables are defined as the vertical (in-plane) displacement $v(x,t)$ and the torsional rotation $\phi(x,t)$. All parameters of the mechanical system are cast in nondimensional form. In particular, let $\eta_c = \int_0^1 \sec\theta_c^3 dx$, δ_c and δ represent the nondimensional catenary characteristic length, with $\theta_c = \arctan y_c'$, cables horizontal distance and deck width, respectively; let κ_c, κ_t and β_c be the nondimensional cables stiffness, deck torsional stiffness and horizontal component of the cables prestress tension, respectively, and, finally, let μ_c, $J\mu_d$, c_f and c_t represent the nondimensional mass, mass moment, and damping flexural and torsional coefficients, respectively. Then, the second-order (in time) aeroelastic governing equations can be cast in nondimensional form as

$$\begin{aligned}
&(1+2\mu_c\sec\theta_c)\,\ddot{v} + c_f\dot{v} + v'''' - 2\,\beta_c\,v'' \\
&\qquad - \frac{\kappa_c}{\eta_c}\left(y_c''+v''\right)\int_0^1 \left[2y_c'v' + \cos^2\theta_c\left(v'^2+\delta_c^2\phi'^2\right)\right]dx \\
&\qquad - \frac{2\delta_c^2\kappa_c}{\eta_c}\,\phi''\int_0^1 \left(y_c'\phi' + \cos^2\theta_c\,v'\,\phi'\right)dx = L\,, \\
&\left(J\mu_d + 2\delta_c^2\,\mu_c\sec\theta_c\right)\ddot{\phi} + c_t\dot{\phi} - \left(\kappa_t + 2\delta_c^2\beta_c\right)\phi'' \\
&\qquad - \frac{\delta_c^2\kappa_c}{\eta_c}\,\phi''\int_0^1 \left[2y_c'v' + \cos^2\theta_c\left(v'^2+\delta_c^2\phi'^2\right)\right]dx \\
&\qquad - \frac{2\delta_c^2\kappa_c}{\eta_c}\left(y_c''+v''\right)\int_0^1 \left(y_c'\phi' + \cos^2\theta_c\,v'\,\phi'\right)dx = M
\end{aligned} \tag{1}$$

where the dot and the prime indicate differentiation with respect to the nondimensional time t and space x, respectively, while L and M are the nondimensional aerodynamic lift and moment per reference length, respectively. In the context of a quasi-steady aerodynamic formulation, the dimensionless aerodynamic loads are directly expressed as functions of the static coefficients in terms of the effective dynamic angle of attack $\alpha_{\mathrm{E}} = \phi - \left(\dot{v} + \frac{\delta}{4}\dot{\phi}\right)/\bar{U}$, where $\bar{U}$ is the nondimensional wind speed, as $L = p\,\delta\,C_{\mathrm{L}}\left(\alpha_{\mathrm{E}}\right)\sin\Omega t,\ M = p\,\delta^2\,C_{\mathrm{M}}\left(\alpha_{\mathrm{E}}\right)\sin\Omega t$, being p the nondimensional aerodynamic pressure. Further details of the aerodynamic formulation and the expressions of the static coefficients C_{L} and C_{M} can be found in [7]. Structural and geometrical parameters of the Runyang Suspension Bridge [2] are considering to analyze the case-study here investigated.

ASYMPTOTIC ANALYSIS

The method of multiple scales[6] is here employed to perform the perturbation analysis of the equations of motion governing the dynamics of the suspension bridge. The perturbation analysis will be carried out studying the effects of cubic nonlinearities thus including the terms of the perturbation problems up to order ϵ^3. Thus the parametric forcing term and the damping term are rescaled to appear at third order. Moreover, to make use of complex algebra, let $\sin\Omega t = \frac{1}{2}i(e^{-i\Omega T_0} - e^{i\Omega T_0})$ where $T_0 = t$ is the nondimensional fast time scale and $T_2 = \epsilon^2 t$ is the nondimensional slow time scale in terms of which the nondimensional time derivative is $d/dt = \partial_0 + \epsilon^2\partial_2$. The system exhibits secular terms due to the parametric resonance caused by the 2:1 frequency ratio of the vortex-induced wind excitation with the lowest bridge mode expressed as $\Omega = 2\omega + \epsilon\sigma$. Hence, substituting the external resonance condition into the solvability equation, the following modulation equations for the complex amplitudes A and A^* are obtained:

$$
\begin{aligned}
&i\omega(\partial_2 A + \alpha A) - (i\omega\beta_1\bar{U} + \beta_2\bar{U}^2)e^{i\sigma T_2}A^* + \gamma A^2 A^* = 0 \\
&i\omega(\partial_2 A^* + \alpha A^*) - (i\omega\beta_1\bar{U} - \beta_2\bar{U}^2)e^{-i\sigma T_2}A - \gamma A^{*2} A = 0
\end{aligned}
\tag{2}
$$

By introducing the polar form of the complex-valued modal amplitudes (i.e., $A = \frac{1}{2}a(T_2)e^{\varphi}(T_2)$)

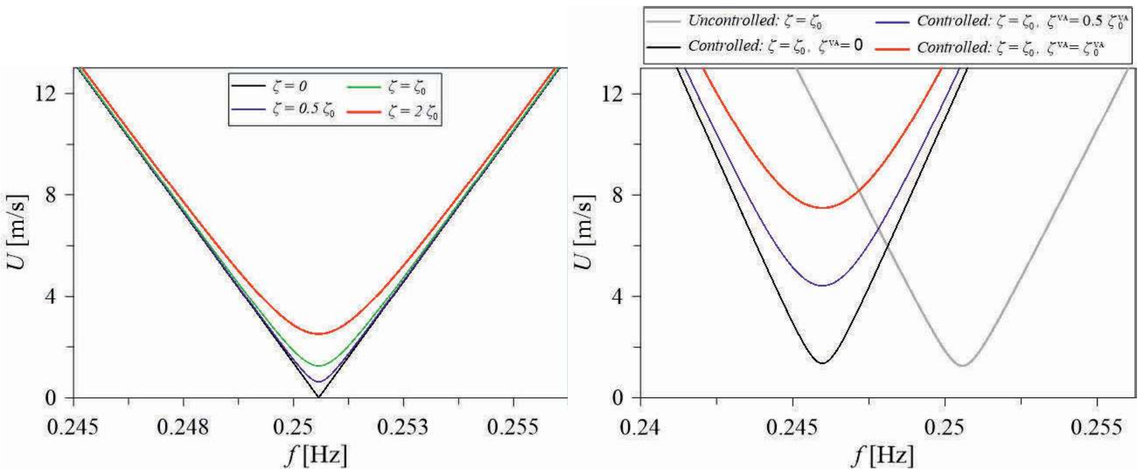

Figure 1: (left) Stability regions showing the sensitivity to structural damping ζ (right) stability regions for the parametric response controlled via VA.

and the relative phase $\Gamma = \sigma T_2 - 2\varphi$ it is possible to obtain the normal form of the modulation equations that governs the dynamics of the real modal amplitude $a(T_2)$. In Fig. 1 (left) (where f is the dimensional frequency of the VIV and U is the dimensional wind speed) are shown the instability regions obtained for the uncontrolled system and their sensitivity with respect to structural damping. Fig. 1 (right) shows the response for the controlled case where the presence of the vibration absorber (VA) lowers the system frequency because of its added mass and increases the critical wind speed (i.e., the speed which leads to the onset of the parametric resonance) by increasing its damping.

CONCLUSIONS

A parametric nonlinear model of SB, including geometric nonlinearities and nonlinear aerodynamics, was proposed to study wind-induced parametric instability and to provide the control of such instability through a passive VA. Investigations via perturbation approach and optimization of the VA mechanical characteristics were carried out to show the effectiveness of the VA system in controlling instability arising from wind-induced parametric aerodynamic loads.

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