Gait Optimization Method for Quadruped Locomotion

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Abstract. The scope of the paper is to develop a methodology for finding optimal gaits of a quadruped robot using genetic algorithm, comparing the results to the ones resulting from natural evolution. The optimization is performed over preimposed contact forces to find the best shapes that guarantees the minimum energy consumption during a single stride cycle. The dynamic formulation of the four-dimensional model is developed without involving any specific kinematic mechanism for the legs, considering the entire gait spectrum a quadruped can exhibit. The optimization model consists of a set of constraints that ensure the feasibility and stability of the gaits. Results are presented for an optimization requiring a constant speed of $1.35\ m/s$. The optimal gait was found to be consistent to nature, suggesting that energy consumption is one of the key factors contributing to the evolution of gaiting patterns in quadrupeds. Eventually, a comparison between different existing gait patterns is carried out in terms of foot contact time and energy consumption.

Keywords: Gait Optimization, Quadruped robot, Genetic Algorithm.

1 Introduction

Nature has always been a source of inspiration for engineers and scientists who have always tried to replicate or at least develop systems that resemble it. This is because nature, in its many forms, is a system that in millions of years of evolution has selected and perfected beings that excel in specific tasks. However, often nature operates a selection characterized by constraints that modern technology can partially overcome, and possible innovative solutions emerge even looking in the groove of nature bio-inspired mechanisms.

In this context, the present paper follows a twofold inspiration in the investigation of quadrupedal robots. On one hand, it is scientifically interesting to explore how quadrupedal locomotion, suitably modelled, can produce gaits that resemble results to which nature arrived over a long evolution. On the other hand, the investigation tries to go beyond the solutions proposed by nature: is it possible to disclose different kind of gaits for a quadrupedal mechanism as the product of a strict optimization process, making these gaits solutions for the best desirable performances?

To obtain a natural and efficient gait for legged robots, two kinds of strategies for sequencing or coordination of the leg movements can be followed. The first strategy assumes that the gaits of animals are optimal, as otherwise they would not have been able to survive the competition and natural selection proposed by Darwin's Theory of Evolution [1]. However, biological locomotion data cannot be used directly for a legged robot due to kinematic and dynamic inconsistencies between animals and legged robots. Today's mechanisms are heavy and have large energy consumption, since they need large number of actuators to move multiple degree-of-freedom legs [2]. The second strategy formulates the gait generation problem of the legged robot as an optimal optimization problem with multiples constraints [3-7]. It generates the optimal gait cycle by minimizing some performance indexes, like motion speed, stability criteria, actuating forces, energy consumption, etc. Evolutionary computation, including the Genetic Algorithm (GA), is a natural choice for the gait optimization of legged robots, since it uses optimization methods based on Darwin's Theory of Evolution [8-10]. The resulting optimal trajectories are then tracked through feedback controllers. Typical controls are Hybrid Zero Dynamics or the simplest PD [11-13]. Non-linear variational optimal controls have been analyzed and studied to include non-quadratic penalty function and non-linear affine systems [14-19] to better follow the trajectories.

The main idea of this paper is based on an optimal optimization process where only the forces transmitted to the ground are considered, without imposing any leg elements or kinematic constraints. The force profiles are morphologically modified through a parametric optimization to assure the body attitude, moving at a certain speed. The gait stability is instead guaranteed by satisfying periodic conditions on a single locomotion cycle. The optimization is performed by GA, and the optimum is determined, over a time period, to minimize the Cost of Transport (COT), i.e. the amount of energy used over time.

The paper is organized as follows: Section 2 describes the dynamic model and definition of the ground reaction forces. Section 3 defines the optimization variables and discusses the objective functions and stability constraints of the optimization. Section 4 provides the optimization results and a comparison between different natural gaits.

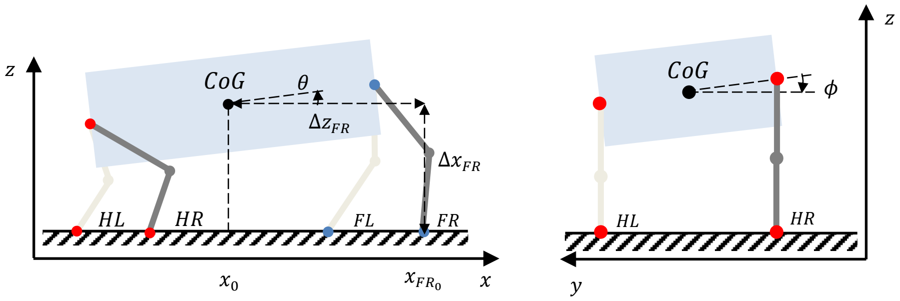
2 Mathematical model for the gait optimization

The optimization model, proposed in this paper, consists on identifying an optimal gait capable of moving a suspended body through the succession of four alternating thrusts generated by legs. The innovation of this approach lies in the absence of a specific kinematic configuration of the legs, thus leaving room for possible free solutions that maybe nature hasn't found. It will then be a later problem identifying the best kinematic configurations that best approximate the optimal solution found. The quadrupedal model is illustrated in Fig. 1, where the body is suspended on four legs, transmitting forces and moments thanks to the interaction with the ground.

The legs are labelled as FL (front left), FR (front right), HL (hind left), HR (hind right) and the period in which the legs are in contact with the ground is called $conctact\ phase\ (CP)$, while the one in which the legs are in the air is called $swing\ phase\ (SP)$.

Newton-Euler equations that govern the rigid body dynamics can be expressed in the fixed reference frame for the translational components and in the body reference frame for the rotational components:

$$m\,\ddot{\boldsymbol{r}} = \boldsymbol{f}_e$$
$$\boldsymbol{I}\dot{\boldsymbol{\omega}} = \boldsymbol{m}_e - \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) \tag{1}$$



a)   ***b***)

Fig. 1. Lateral a) and rear b) view of the model.

where $\boldsymbol{r}$ identify the Center of Gravity (CoG) of mass m, $\boldsymbol{f}_e$ the external forces, with respect to the fixed reference frame; while inertia matrix $\boldsymbol{I}$, angular velocity $\boldsymbol{\omega}$ and external moments $\boldsymbol{m}_e$ are computed in the body reference frame. Assuming $\boldsymbol{I}$ as principal inertia matrix and small pitch and roll angles, we can approximate the equations of motion in the fixed reference frame for a 4 dof rigid body in longitudinal motion x, vertical motion z, roll ϕ and pitch θ (Fig. 1):

$$\begin{aligned} m\ddot{x} &= F_x \quad I_x\ddot{\phi} = M_x \\ m\ddot{z} &= F_z \quad I_y\ddot{\theta} = M_y \end{aligned} \tag{2}$$

where I_x, I_y are the inertias along the two axis; F_x, F_z, M_x, M_y are the total forces and moments in the fixed frame coming from the four legs FR, FL, HR and HL.

From Fig. 2, the duration of the entire locomotion cycle, in which the four legs follow one another, is defined by T and leg contact duration by T_i. The time at which each leg touches the ground is $t_i \in [0,\mathrm{T}]$, where $i = FR, FL, HR, HL$.

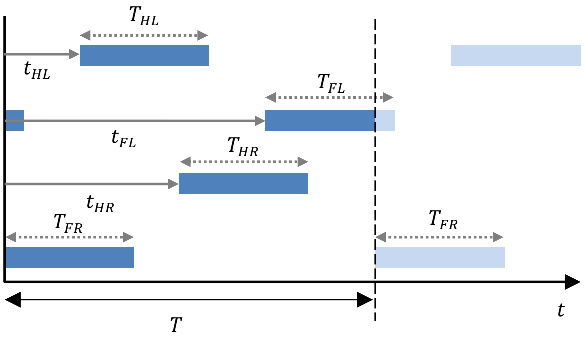
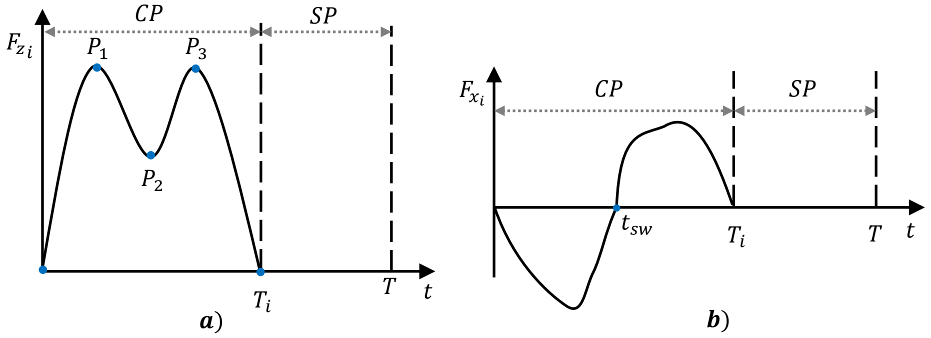


Fig. 2. Temporal diagram of the entire period of locomotion of the quadruped.

When an animal moves on the ground, its limb contact and pushes against the ground and so expressing a ground reaction force. The vertical component F_z of the ground reaction force serves to support the animal's weigh, while the horizontal component F_y allows the animal to accelerate and decelerate. Even when an animal moves at a steady speed, its limbs exert decelerating and accelerating horizontal forces to control the balance. In this paper, the focus of the work is to find compatible contact forces for each leg that allows the body to move at a certain speed and keeping a stable attitude. Considering a single leg, the vertical force F_{z_i} is bound to act only during the CP and it must start and end with zero value to represent the arriving and leaving phases [20]. The shape is defined through a spline function passing across three unknown points $[P_1, P_2, P_3]$ which have to be optimized (Fig. 3a). The longitudinal ground reaction for each leg F_{x_i} is instead identified by the Coulomb expression (Fig. 3b). The force is defined as the product of the grip coefficient μ and the vertical force F_{z_i} with the hyperbolic tangent that allows to consider the acceleration and deceleration grip phase of the foot.

$$F_{x_i} = -\mu F_{z_i} \tanh\big(\gamma(t_i - t_{sw})\big) \tag{3}$$

The $tanh$ function is translated with a parameter $t_{sw} \in [t_i, t_i + T_i]$ that identifies the time at which the foot switch from decelerating to accelerating grip. This model excludes the possibility of foot slipping, as it is a dissipative action that lower the efficiency of the motion.



a)   ***b***)

Fig. 3. Vertical a) and longitudinal b) ground force for one leg during the gait period.

Moreover, the transition phase is governed by the parameter γ that considers the characteristics of the actuators. Eventually, moments M_x and M_y on CoG can be computed by identifying the foot force arm. From Fig. 1a, let's consider the FR foot at the beginning of the contact phase $t_{FR} = 0$ with a position x_{FR_0}. Given the lack of slipping, the horizontal arm between CoG and the foot contact point is $\Delta x_{FR}(t) = x_{FR_0} - x$. In the same way, the vertical arm component ca be evaluated as $\Delta z_{FR}(t) = z_{FR_0} - z$, giving the opportunity to express the two moments like:

$$\begin{aligned} M_{y_i} &= F_{x_i}\Delta z_i - F_{z_i}\Delta x_i \\ M_{x_i} &= F_{z_i}\Delta y_i \end{aligned} \tag{4}$$

where Δy_i, in our case, is a constant parameter stating the mounting spacing of the legs along the y axis. Clearly, the two moments $M_{x_i}(t)$ and $M_{y_i}(t)$ are non-null only during the contact phase where $t \in [t_i, t_i + T_i]$.

3 Optimization model

Usually, in the quadruped legged locomotion, the stability of a gait is guaranteed using criteria such as Zero-Moment Point [21, 22]. Other studies face the stability problem with Poincare map [23, 24] or ground reference points [25]. In this paper, unlike the classical approaches, the gait stability is guaranteed by satisfying periodic limit cycle conditions.

The optimization is performed over a single period which begins and ends when the FR leg meets the ground. The algorithm selected is the well-known genetic algorithm (GA) which has been used to find not only the stride length, frequencies and velocities but also the quantity and form of the forces exchanged on the ground compatibly with the constraints. The parameters $\boldsymbol{p}_{GA}$ serve as inputs for the optimization algorithm to minimize a specific objective function. The optimization is designed to find the optimal gait to maintain constant the initial speed and height, selecting the relative phases of each legs t_i, the time duration of the contact phase, which is here imposed the same for all the legs $T_i = T_{CP}$, and the shape of the normal force through P_j points (see Fig. 3).

$$\boldsymbol{p}_{GA} = [t_i, T_{CP}, P_j] \tag{5}$$

The entire cycle duration T is an imposed parameter as it depends directly from the actuator's technology and hardware available. Sure enough, reducing the time T will raise the cost of technology to be used. Higher computational costs and power density of the actuators are required, along with high accuracy and sensitivity of sensors.

Once the $\boldsymbol{p}_{GA}$ parameters have been assigned, the optimization consists of an iterative resolution scheme that resolves first the vertical dynamics then the horizontal dynamics and ends with the rotational dynamics.

Over the single stride, the average vertical force exerted on the ground by all the limbs must equal the body weight m multiplying the gravity force g:

$$\frac{1}{T}\int_0^T \sum_i F_{z_i}(P_j)\,\alpha\ dt = mg \tag{6}$$

The force shape is randomly defined by the GA with a spline interpolation through P_j points. The parameter α rescales the normal force to satisfy the constraint equation (6) for the entire cycle. The condition (6) involves that the initial velocity along the z axis will be the same at the time T.

$$\dot{z}(0) = \dot{z}(T) \tag{7}$$

However, another condition is needed to assure that the cyclic motion along z axis doesn't diverge. At the end of the cycle, the body needs to return to the initial height:

$$z(0) = z(T) \tag{8}$$

Identifying the initial vertical speed as the double integral of the vertical acceleration, allows to satisfy the requirement (8):

$$\dot{z}(0) = -\frac{1}{mT}\int_0^T \int_0^\tau \left(\sum_i F_{z_i}(P_j)\,\alpha - mg\right) dt\,d\tau \tag{9}$$

The same strategy can be used to maintain a periodic longitudinal speed, as long as the following integral is satisfied:

$$\int_0^T \sum_i F_{x_i}(t_{sw})\ dt = 0 \tag{10}$$

which depends on the parameter t_{sw}, the time at which the transition between acceleration and deceleration grip phase is realized. The equation (10) is solved by a non-linear numerical solver to guarantee the periodicity of the speed:

$$\dot{x}(0) = \dot{x}(T) \tag{11}$$

This doesn't assure that the average speed is the desired one, but at least we guarantee a certain stability of the velocity.

On the other hand, also the resulting moments must assure a stable attitude during the stride cycle, similarly to what has been stated for vertical and longitudinal motion. The following constraints are consequently required:

$$\begin{gathered} \int_0^T M_y(x_{FR_0})\ dt = 0 \\ \int_0^T M_x(\Delta_y)\ dt = 0 \end{gathered} \tag{12}$$

Solving the equations (12), through the identification of the maximum stride span x_{FR_0} and the mounting spacing Δ_y, allows to find the initial and final velocity of roll and pitch that guarantees a periodic motion:

$$\begin{gathered} \dot{\theta}(0) = \dot{\theta}(T) = -\frac{1}{I_y T}\int_0^T \int_0^\tau M_y(x_{FR_0})\ dt\,d\tau \\ \dot{\phi}(0) = \dot{\phi}(T) = -\frac{1}{I_x(\Delta_y)T}\int_0^T \int_0^\tau M_x(\Delta_y)\ dt\,d\tau \end{gathered} \tag{13}$$

Eventually, the objective function, that has to be minimized for identifying the most efficient gait, can be expressed. The energy cost E should be defined as the integral of the absolute value of the power, given the periodicity of the motion and the absence of non-conservative forces:

$$E = \frac{1}{T}\int_0^T |P_{ower}|\ dt \tag{14}$$

Equation (14) must consider kinetic and potential power of the CoG during the entire cycle. Computing the integral of the power absolute value allows a correlation with possible dissipative forces of real applications. In fact, it is reasonable considering dissipative power dependent from quadratic speed and so proportional to $|P_{ower}|$.

4 Results

The mass properties of the body are selected considering the characteristic parameters of quadrupeds in nature, in particular horses. Consequently, the optimization is performed with the purpose of finding the energy efficient gait that moves the body at an average speed of $1.35\ m/s$, with a gait period T of $1s$, which are common parameters of horses walking gait. In nature, walking gaits involve overlapping contact phase such that $\frac{T_{CP}}{T} > 0.5$, i.e. each leg remains in contact more time than in the air, providing a stable base of support. The results show that the optimal gait found by GA follows the characteristics of a classical walking gait for a quadruped animal. In fact, the sequence of the legs is shown to be: $FR - HL - FL - HR$, with a $t_i = [0.25; 0.5; 0.75; 1]$ multiples of ¼ and the time of contact phase of $\frac{T_{CP}}{T} = 0.58$. In Fig. 4 the numerical results of the optimization problem are presented. It can be seen how the body keeps a steady longitudinal speed $\dot{x}$, around the target speed of $1.35\ m/s$, maintaining a stable and restrained attitude and assuring the periodical constraints. What we are showing is one of the possible solutions, as the genetic algorithm can also find points of local optimum, depending on the weights of the objective function.

Consequently, we decided to investigate other gaits, existing in nature, and find out how much energy they consume to make the body move at the same target speed. In a quadrupedal trot, for example, the diagonal forelimb and hindlimb move in phase, while, in pacing, the forelimb and hindlimb on the same side of the body are in phase. Instead, when the forelimbs and hindlimbs each move together in phase, the gait is considered a “full-bound”. To assure the selection of these gaits, we imposed only the time t_i between the legs as constraints:

$$\begin{aligned} &Trot: \quad \begin{matrix} t_{FR} = t_{HL} \\ t_{FL} = t_{HR} \end{matrix} \qquad\qquad Pace: \quad \begin{matrix} t_{FR} = t_{HR} \\ t_{FL} = t_{HL} \end{matrix} \\ &\qquad\qquad Full - bound: \quad \begin{matrix} t_{FR} = t_{FL} \\ t_{HL} = t_{HR} \end{matrix} \end{aligned} \tag{15}$$

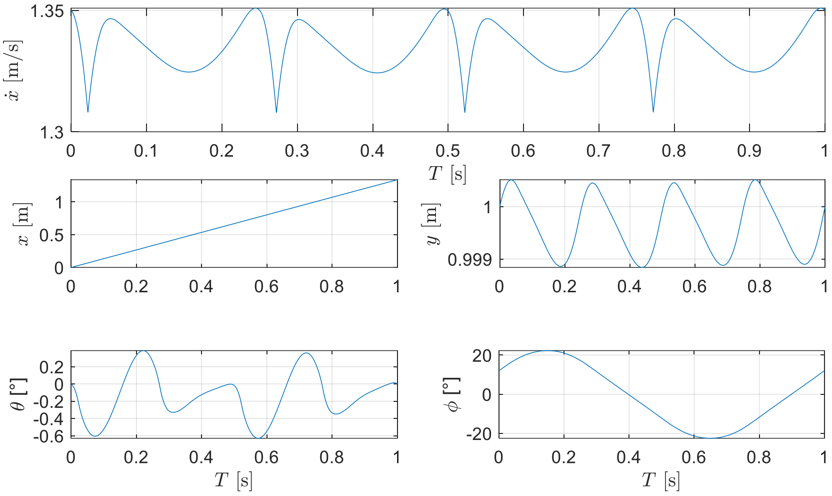
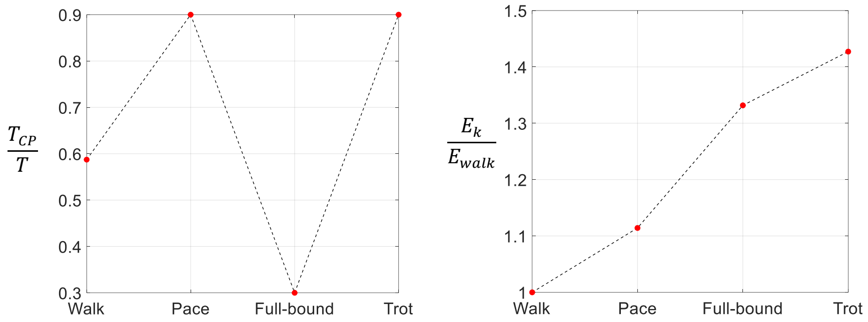


Fig. 4. Motion and attitude of the body during the single stride

In Fig. 5a are shown the resulting ratios $\frac{T_{CP}}{T}$ for each of the analyzed gaits. Trotting and running gaits are typically characterized by $\frac{T_{CP}}{T} \leq 0.5$, however we can see that the optimization isn't consistent to what happens in nature. That's because animals change gait from a walk to a trot or a run only to move at increasing speed, instead we imposed the same target velocity to the body.



a)

b)

Fig. 5. Ratios of the contact time ***a***) and energy consumption ***b***) over cycle time for imposed gaits

Eventually, comparing the energy consumption of these analyzed gaits with the optimal one, we can confirm how the walking gait is the most efficient one for a quadruped at the target speed of 1.35 m/s and a gait period of 1 s (Fig. 5b).

5 Conclusions

In this work, a legged optimization model is proposed to identify different gaits of a quadruped system regardless of the physical characteristics of the system itself. The advantage is to be able to find optimal features of the gait without the constrain of any kinematic mechanism for the legs, body etc., allowing a free search of the optimum. Pre-imposed forces profiles are shaped to guarantee stability of the motion, at a constant speed. The scope is to find optimal gaits that move a body in the most efficient way, then comparing them to the ones resulting from natural evolution to see if there exist more efficient ones.

The optimal gait was found to be consistent to that found in nature, suggesting that energy consumption is one of the factors contributing to the evolution of gaiting patterns in quadrupeds. Future works will consist on finding a kinematic mechanism compatible with the optimal gait and using non-linear feedback controls to control the quadruped robot.

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