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of the 44th Conference of the International Group for the Psychology of Mathematics Education

VOLUME 4

Research Reports (S-Z)

Editors:

Maitree Inprasitha, Narumon Changsri and Nisakorn Boonsena

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WHERE DO STUDENTS FOCUS THEIR ATTENTION ON SOLVING MATHEMATICAL TASKS? AN EYE TRACKER EXPLORATIVE STUDY

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Several international studies recognize the central role of the understanding in problem solving in the mathematics teaching-learning process. Interdisciplinary studies have shown how the type of text affects student's reading and, consequently, its performance. It emerged that "selective- reading", through which specific attention is paid to certain textual elements, often involves a lack of understanding of the problematic situation. The aim of our research is to understand how some structural and textual aspects influence the understanding of a mathematical text. This research, conducted using the eye-tracker tool, shows the results of the first phase of a larger study.

RATIONALE

Researches on mathematics education highlight the central role of the mathematics texts' understanding in the undergraduate students' learning of mathematics (Barton et al., 2004). A discussion of the different approaches to the study of the impact of the formulation of a task on the performance of the students can be found in Bolondi, Branchetti & Giberti (2018). It is recognized how attitudes related to the didactic contract in the sense of Brousseau (1988), such as the "selective readings" (Zan, 2012), in which the student focuses attention only on certain textual elements, often lead to a lack of understanding of the problematic situation. The aim of our investigation is to understand how some textual aspects and the graphic and textual arrangement influence the understanding of a mathematical text and, therefore, the students' performance. Attitudes such as the identification of isolated sentences or key words highlight widespread inability to use skills acquired in transversal areas; language training and text interpretation, rather than tools that help in the representation or communication of information, are transformed into indications of procedures to be performed (Ferrari, 2001; Radford, 2000). Some of the problem-solving processes activated in problem solving, especially in reference to the comprehension of the mathematical text and the identification of the resolution strategy, highlight these behaviors and therefore require constant attention and monitoring. And it is precisely in this direction that our research is moving, in which the processes of understanding mathematical texts are analyzed with the support of the eye-tracker tool.

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In this paper we show the results of the first phase of a larger experimental study. This phase involving 8 university students from the Faculty of Education of the University of Bozen-Bolzano. In recent years, eye-tracker technologies have become an increasingly effective tool for analysing students' learning process. The results obtained provide information on eye movement and, therefore, on the choice and catalyzation of the attention of students of different school levels during the mathematical activities and are therefore significant from an interpretative point of view of the activated resolution processes.

THEORETICAL PERSPECTIVE

The eye-tracker in the panorama of Mathematics Education

By its nature, the use of technology in the learning and teaching processes of mathematics requires an interdisciplinary approach. In Mathematics education, several studies have been conducted with the eye-tracker, studies that have also involved knowledge from other fields of study, such as computer science, neurology, biology, sociology and cognitive psychology. Cognition is closely related to body actions and the position of the body in space and time (Lakoff & Núñez, 2000). Eye movements are part of sensory experience and, following the Radford approach (2010), Their relationship with mathematical representation can shed light on how humans access mathematical knowledge. Several studies in the field of eye tracking have shown that there is a correlation between what one "looks" at and what one "thinks" (Rayner, 1998; Yarbus, 1967). These results also agree with other research which support the existence of a correlation between ocular fixations and cognitive information processing (Latour, 1962). Consequently, there has been a growing interest in eye tracking in educational research (Scheiter & van Gog, 2009). As far as the path of mathematical learning is concerned, these eye-tracking experiences seem to be in line with Duval's idea, which, starting from the famous statement "there is no noesis without semiosis" (Duval, 2006), highlights how the understanding of a concept is born from the relationship between the signifier represented by a sign, a representation and the meaning or the mathematical object. Ferrara and Nemirovsky (2005) argue that all perceptual-motor activities, related to changes in attention, consciousness and emotional states, contribute to the understanding of a mathematical concept. In the research in mathematics education, many studies (e. g. Ferrara & Nemirovsky, 2005, Andrà et al. 2009, 2015; Holmqvist et al., 2011) highlight interesting data on students' approach to the reading of mathematical text, on the transformations between different representations (formulas, graphs, words) to understand the meaning of a text. It was also pointed out that there are quantitative and qualitative differences between beginners and experts in the approach to reading of a mathematical text and precisely for these reasons in our experimentation were involved participants of different school grades with different mathematical skills.

Duval frame

The contribution of the Duval ideas to the experimentation conducted is fundamental, in particular for the fundamental role attributed to representation and visualization in understanding mathematics.

Representation refers to a large range of meaning activities: steady and holistic beliefs about something, various ways to evoke and to denote objects, how information is coded. In contrast, visualization seems to emphasize images and empirical intuition of physical objects and actions. (p.3, Duval, 1999). These two processes play a fundamental role in the process of learning mathematics and, even more, as regards the cognitive architecture concerning the apprehension of geometric concepts. Thus, in geometry it is necessary to combine the use of at least two representation systems, one for verbal expression of properties or for numerical expression of magnitude and the other for visualization. What is called a "geometrical figure" always associates both discursive and visual representations, even if only one of them can be explicitly highlighted according to the mathematical activity that is required. Then, students are expected to go to and from between the kind of representation that is explicitly put forward and the other that is left in the background of this discursive/visual association that forms any geometrical figure." (p. 108, Duval, 2006).

The discursive/visual association is complex by a cognitively point of view: oftentimes a contrast between this association and the common association between words and shapes and because its use goes against the perceptual obviousness (Duval, 1998). Each activity in geometry involves the use of at least three registers: the natural language register, the symbolic language register and the figurative register. Figures therefore play an important role, because they represent a concept or situation extensively. They are often much more rapidly receivable than verbal representations and various "gestaltic" mechanisms cause figure to convey information both analytically and synthetically. But not every design can function effectively as a "geometric figure". There are usually four levels of understanding of a geometric figure: perceptual, sequential, discursive and operational (Duval, 1995; 1999). A drawing acts as a geometric figure when it activates the level of perceptual understanding and at least one of the others. The perceptual level involves the ability to recognize figures (for example, distinguish shapes) and to identify components in a figure (recognize sides or other elements). The epistemological function of the perceptual level is identification. Therefore, as regards the understanding of the geometric figures, the activation of the perceptual level is fundamental. As we will see in the analysis of the experiment carried out, the observation of eye movements with eye tracking provides useful information to investigate the activation, or not, of the perceptual level of the geometric figures within the resolution of geometric problems.

THE STUDY

The results of international research have shown that the number of fixations is a reliable and sensitive measure that can provide valuable information on the attention flow of participants during mathematical activities (e.g., during the resolution of equations, Susac et al., 2014). In particular, the data from Susac et al. (2014) show positive correlations between the number and "positions" of the student of the "fixations" and the efficiency of the participants in finding the solution of the mathematical activities, suggesting that the participants who behaved well adopted winning strategies in terms of "knowing where to look for information useful for the resolution".

It is from these and other evidence from national and international literature that we have designed our study. Experimentation was conducted with the eye-tracker tool on geometric task that involved 8 university students at the Free University of Bozen-Bolzano.

The aim of the research is to understand if and how much students are aware of the strategies they put in place when faced with a mathematical question in the geometric field with an image present in the text. The eye-tracker tool has allowed us to investigate what are the students' eye movements in the resolution of mathematical tasks and to study the link between them and the students' performance. To do this we analyzed both the movements of the eyes during the resolution of the mathematical tasks (to understand the order of the fixations we used gaze plot videos), both if and to what extent the structure of the tasks affects the place where the students focus their attention (to understand if some elements of the text captured the attention more than others we dwelt on the fixations).

The questions all have the same basic characteristics: they are geometric, they have an image in the text and they have been taken from international standardized survey OECD-PISA.

The chosen tasks have been built on the quantitative results of this survey and are focused not only on the investigation of how much and how the structure of the task affects the resolution procedures, but also on how much it actually affects the students' performance.

The design and implementation of this experimentation are the result of reflections and evidence highlighted by a first pilot study conducted in collaboration with the University of Bozen-Bolzano and presented at the 14th International Conference on Technology in Mathematics Teaching (Bolondi & Spagnolo, 2019).

The items administered in this phase were constructed from the quantitative results that emerged from the international standardized administration of mathematics of the OECD PISA 2015 (OECD-PISA, 2016).

Let us observe Figure 1:

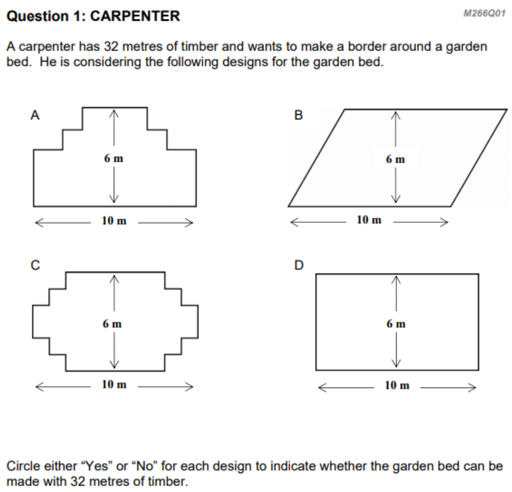

Figure 1. Question 1, OECD-PISA 2015.

Below we highlight the characteristics of the question that allowed us to implement Phase 1 of our study conducted with the eye-tracker tool. The difficulty level of the item is 6 on the overall literacy scale in mathematics. The student, in fact, is required to model a complex situation, developing a strategy in an unfamiliar context. He must show a good mastery of geometry and apply it in a real context. The concepts and mathematical knowledge fundamental to the resolution of the question are flat figures and their properties, while the fundamental mathematical skill is the following: to answer correctly it is necessary to be able to deduce, from the data provided, the lengths of the unknown segments. The question is of the type "complex multiple choice", because for each question you must select an answer between two possible (yes/no). The main difficulties are encountered in determining the overall length of the "vertical sides" of the individual figures. It is particularly difficult to determine the perimeter of Figure B, because the information on the lengths of the sides is not directly inferred from the stimulus. It is therefore necessary a good reasoning ability and a good mastery of basic Euclidean geometry to understand that the oblique sides of the parallelogram of Figure B are longer than the vertical components of the sides of the other figures (whose perimeter is exactly 32 m). Only 12.3% of Italian students were able to provide four correct answers, while 30% were able to identify three. Among the questions of geometric scope with an image in the text, this question was chosen because of the difficulties highlighted internationally. Afterwards the question was prepared for administration with the eye-tracker. In order to obtain information about the resolution processes implemented by the students, also in relation to the structure and textual characteristics of the task, the question in Figure 1 was presented in four different stimuli. This also increased the readability of the task and the calibration of the eye-tracker. The application is unchanged, but the drawings of the projects were shown to the students individually and no longer all together.

In addition, the student was asked to briefly motivate his answer. This variation was made in order to be able to observe the students' eye movements while they explain their solving strategy and retrace the process of solving the problem. It was specified to the students that they could look at the situation for as long as necessary before responding and that, in responding to a situation, they could also refer to reflections inherent to the previous situations displayed. In this way, it was possible to detect, through the recording of eye movements, the focus and permanence on certain structural and textual elements and therefore, to investigate the resolution strategies.

Eight students from the 2nd year of Primary Education at the University of Bozen-Bolzano were involved. In particular, the choice fell on students of educational science as they are not only students but also teachers in training and for this reason, they also pay attention to any educational consequences. These students were identified on a voluntary basis and each of them was involved in the task for two hours. In addition to the resolution of the question, the task included an unstructured and in-depth interview during which the student was confronted with the results of the eye-tracker tool. In this way it was possible to compare what the student thought he had looked at and what he had actually looked at.

THE RESULTS

The data was collected by the Tobi pro-lab software. Before proceeding with the analysis, we specify that to allow an analysis with the eye-tracker, the task must refer to only one stimulus at a time and be visible without scrolling the page.

The following figure (Figure 3) shows the first results of the analysis referring to stimulus 1.

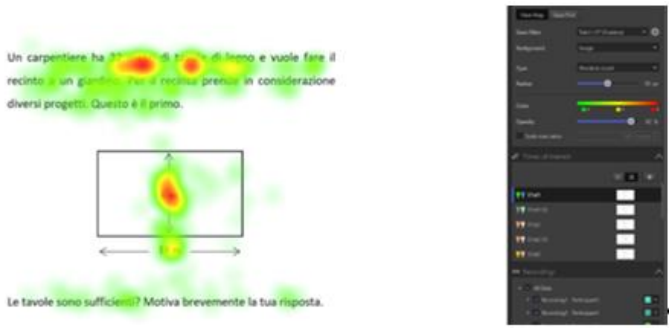

Figure 3: First results of the analysis related to stimulus 1.

In the figure we can see the part of the text where the student lingered for less time coloured in green, the part where the student lingered for a little longer coloured in yellow, and the part where the student lingered for a little longer coloured in red. The student, both in the text and in the image, focuses the attention on the numbers (red part) and the little attention given to the text can be an indication of "selective reading" (Zan, 2012). We can interpret this evidence with some of the categories of the didactic contract in the sense of Brousseau (1988).

We find a similar behaviour also in other situations. The focus on the movements of the eye when a student is reading a mathematical text provided us with insights on the solution processes and provide us outcomes framed within the structural and functional approach to semiotic (Duval, 2006). These results highlight a meaningless: students confuse the graphic elements and number representations (signifiers) with the mathematical object (signified) and they are not able to establish the correct semiotic reference to the mathematical object. The incorrect relation between graphs, numbers and sense confirms the Duval's (1995) cognitive paradox that impels students to identify semiotic representations with the mathematical object.

REMARKS.

The results of this first experimental phase provide rich insights into the structural and textual elements of mathematical tasks that capture students' attention and how much these choices affect their mathematical performance. We investigated which textual and structural characteristics influence -at least in part- the solving procedures and therefore, the students' performance. The eye-tracker tool was used to confirm the students' eye movements while solving mathematical tasks.

The students themselves recognised how their attention is often catalysed by elements that are useless for the solving procedure; the data collected, and the analyses carried out in these first phases made it possible to outline the design and implementation of a new step of the broader research in which this experimental study is inserted.

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