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MODEL REFERENCE ADAPTIVE METHOD FOR MICROSATELLITE ACTIVE MAGNETIC CONTROL

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Summary. *The paper deals with the problem of the magnetic attitude control for microsatellite in LEO, with three torquerods along with the principal axes of inertia. The resulting attitude dynamics is nonlinear and the actuation torque is always perpendicular to the local Earth magnetic field. The linearization of the attitude dynamics with respect to a nominal attitude, yields a linear periodic system that is useful to develop a linear control algorithm in order to keep the attitude stable. Several methods have been developed using the theory of the linear periodic equations to obtain stable attitude control algorithms. This work presents a novel strategy based on a method of the Direct Adaptive Control theory. Using the dipole model of the Earth magnetic field, the paper shows that the linearized dynamics is represented by coefficients that are analytically computed. As a result, the attitude equations have coefficients that are bounded by maximum and minimum values. Consequently, we are under the conditions to apply a control algorithm, which performs an adaptive law to track the slow variations of the dynamic parameters. A useful control technique exploits the Model Reference Adaptive Control. The method consists in using a LTI system as a reference system and constructing a control law in order to nullify the output error of a system with unknown-bounded dynamic parameters. The approach shown in the paper uses the average on the attitude linearized equations in an orbital period as the model reference system. Using a Lyapunov approach, it is demonstrated that the resulting adaptive control is stable and is able to keep the attitude within some degrees. Numerical testes are shown to support the theoretical results of the work.*

1. INTRODUCTION

2. MISSION

The possibility to adopt a MRAC to control a spacecraft by means of magnetic control attitude system has been tested on a mission characterized by a sun-synchronous and circular orbit. The orbital parameters are listed in Table (1). The spacecraft chosen is a small spacecraft for Earth observation purpose whose payload needs to a stable Nadir pointing.

Table 1. Orbital Parameter and Moments of Inertia

Height	$600\ Km$
Semi major axis	$6978\ Km$
Eccentricity	0
Inclination	$97.8\ ^\circ$
Argument of Perigee	$-$
Orbital Period	$5800\ s$
I_{xx}	$2.33\ Kg\, m^2$
I_{yy}	$2.36\ Kg\, m^2$
I_{zz}	$3.16\ Kg\, m^2$

In the nominal attitude the X axis of the satellite needs to be directed toward the center of the Earth. The yaw axis of the spacecraft coincides with the X axis, the pitch axis with the Y axis and the roll axis coincides with the Z axis. The mass properties of the spacecraft are listed in table (1). The Magnetic attitude control system (MACS) of the satellite consists of position and angular velocity sensors, a triaxial magnetometer and three magnetorquers installed along the directions of the principal moments of inertia.

3. REFERENCE FRAMES

Let's consider the Earth-centered inertial (ECI) coordinate frames $(X_{ECI}, Y_{ECI}, Z_{ECI})$. The following reference frame can be introduced:

1. The Nodal Reference Frame (NRF), (X_N, Y_N, Z_N), related to the ECI by:

$$\left[\begin{array}{ccc} X_N & Y_N & Z_N \end{array}\right]^T = R_{Z_{ECI}}(\Omega)\left[\begin{array}{ccc} X_{ECI} & Y_{ECI} & Z_{ECI} \end{array}\right]^T \tag{1}$$

2. The Orbital Reference Frame (ORF), (X_O, Y_O, Z_O), related to the NRF by:

$$\left[\begin{array}{ccc} X_O & Y_O & Z_O \end{array}\right]^2 = R_{Z_N}(\nu)\, R_{X_N}(i)\left[\begin{array}{ccc} X_N & Y_N & Z_N \end{array}\right]^T \tag{2}$$

3. The Local Reference Frame (LRF), (X_L, Y_L, Z_L), related to the ORF by:

$$\left[\begin{array}{ccc} X_L & Y_L & Z_L \end{array}\right]^T = R_{Z_O}(\varphi)\left[\begin{array}{ccc} X_O & Y_O & Z_O \end{array}\right]^T \tag{3}$$

   where φ is the true anomaly.

4. The Stabilized Reference Frame (SRF), (X_S, Y_S, Z_S), related to the LRF by:

$$\left[\begin{array}{ccc} X_S & Y_S & Z_S \end{array}\right]^T = R_{Y_L}(\pi)\left[\begin{array}{ccc} X_L & Y_L & Z_L \end{array}\right]^T \tag{4}$$

   in order to have the yaw axis toward the center of the earth.

5. The Body Reference Frame (BRF), (X_B, Y_B, Z_B), with the axis directed along the principale axes of inertia of the satellite. It is related to the SRF by:

$$\left[\begin{array}{ccc} X_B & Y_B & Z_B \end{array}\right]^T = T_A\left(\theta_1, \theta_2, \theta_3\right)\left[\begin{array}{ccc} X_S & Y_S & Z_S \end{array}\right]^T \tag{5}$$

4. MODEL REFERENCE ADAPTIVE CONTROL - MRAC

The aim of this work is to simulate the control by means of the MRAC of a small satellite. The MRAC relies on the very simple idea to create a closed loop controller with parameters that can be updated to change the response of the system. The output of the system is compared to a desired response from a reference model. The control parameters are updated according to this error. The goal is for the parameters to converge to ideal values that cause the plant response to match the response of the reference model [1] .

In the specific case study, the reference model is chosen to be a LTI system describing the satellite linearized attitude dynamics averaged over an orbital period. This approach allows the reference model to be controlled through a PID controller whose time invariant gains are computed by means of the linear-quadratic regulator.

In order to obtain the averaged of the control matrix of the reference model, a simple dipole model has been selected in order to compute the geomagnetic field. The dipole has been considered aligned in the direction of the geographic Nord-Sud. The aim of these assumptions is to have a very simple reference model and is based on the fact that the MRAC generally does not need a very accurate reference model [2].

On the contrary, the model to be controlled with the MRAC consists in the non-linear attitude dynamics of a satellite subject to the gravity-gradient torque. The geomagnetic field is computed by the IGRF11 model.

The non linear model is controlled by the MRAC through the tracking error defined as:

$$e_y\left(t\right) = y_m\left(t\right) - y\left(t\right) \tag{6}$$

where y_m and y are respectively the outputs of the reference model and of the non linear model. The adaptive control is computed taking into account the tracking errors associated with the euler angles and the components of the angular velocity in the BRF:

$$u = K_e\left(t\right)e_y\left(t\right) + K_x\left(t\right)x_m\left(t\right) + K_u\left(t\right)u_m\left(t\right) = K\left(t\right)r\left(t\right) \tag{7}$$

The gains K_e, K_x and K_u are computed as follows:

$$\dot{K}_e\left(t\right) = e_y\left(t\right)e_y^T\left(t\right)\Gamma_e - \gamma\, K_e\left(t\right) \tag{8}$$

$$\dot{K}_x\left(t\right) = e_y\left(t\right)x_m^T\left(t\right)\Gamma_x \tag{9}$$

$$\dot{K}_u\left(t\right) = e_y\left(t\right)u_m^T\left(t\right)\Gamma_u \tag{10}$$

where Γ_e, Γ_x and Γ_u are constant weight matrices and γ is a constant needed to avoid the gains to increase too much.

In order to proof the stability of the system we consider an ideal control as follows:

$$u^*(t) = \tilde{K}_x\, x_m(t) + \tilde{K}_u\, u_m(t) \tag{11}$$

able to set the non linear system to follow the planned trajectory.
The candidate Lyapunov function is:

$$V(t) = e_x^T P e_x + tr\left[W\left(K(t)-\tilde{K}\right)\Gamma^{-1}\left(K(t)-\tilde{K}\right)^T\right] \tag{12}$$

where e_x is the error between the state of the non-linear system and the ideal trajectory:

$$\dot{e}_x = \left(A - B\tilde{K}_e C\right) e_x(t) - B\left(K(t)-\tilde{K}\right) r(t) \tag{13}$$

If there exist three positive definite symmetric matrices P Q and W and a positive gain $\tilde{K}_e$ such that the following equation are satisfied:

$$P\left(A - B\tilde{K}_e C\right) + \left(A - B\tilde{K}_e C\right)^T P = -Q \tag{14}$$

$$PB = C^T W^T \tag{15}$$

one can obtain the negative derivative of the Lyapunov function with respect of time [3]:

$$\dot{V}(t) = e_x^T(t)\left[P\left(A - B\tilde{K}_e C\right) + \left(A - B\tilde{K}_e C\right)^T P\right] e_x(t) = -e_x^T(t)\, Q e_x(t) \tag{16}$$

5. DYNAMICS EQUATIONS

The attitude dynamics of the satellite can be derived by:

$$\dot{\vec{\Gamma}} + (\vec{\omega}_r + \vec{\omega}_t) \times \vec{\Gamma} = \vec{M} \tag{17}$$

where $\vec{\Gamma}$ is the angular momentum of the satellite, $\vec{\omega}_r$ is the relative angular velocity of the satellite with respect to the SRF, $\vec{\omega}_t$ is transport angular velocity of the SRF with respect to ECI and $\vec{M}$ is the resultant of the applied torques.
Considering the effect of J_2 on a circular orbit, the $\vec{\omega}_t$ is the sum of the orbital angular velocity $\vec{\omega}_0^{(SRF)}$ and the precession rate of the orbit $\vec{\omega}_p^{(SRF)}$ projected in the BRF [4]:

$$\vec{\omega}_t^{(BRF)} = T_A(\theta_1, \theta_2, \theta_3)\left[\vec{\omega}_0^{(SRF)} + \vec{\omega}_p^{(SRF)}\right] \tag{18}$$

Orbital angular velocity in the SFR is:

$$\vec{\omega}_0^{(SRF)} = \omega_0 \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T \tag{19}$$

The precession rate of the orbit can be approximate in ECI with:

$$\vec{\omega}_p^{(ECI)} = \begin{bmatrix} 0 \\ 0 \\ -\frac{3}{2}\, J_2\, \omega_0\, \cos(i) \left(\frac{R_E}{a}\right)^2 \end{bmatrix} \tag{20}$$

To obtain ω_t in SRF the following sequence of rotation shall be applied:

$$\vec{\omega}_p^{(SRF)} = R_{Y_L}(\pi)\, R_{Z_N}(\varphi+\nu)\, R_{X_N}(i)\, R_{Z_{ECI}}(\Omega)\, \vec{\omega}_p^{(ECI)} \tag{21}$$

obtaining:

$$\vec{\omega}_p^{(SRF)} = \omega_p \begin{bmatrix} -\sin(i)\sin(\varphi+\nu) \\ \sin(i)\cos(\varphi+\nu) \\ -\cos(i) \end{bmatrix} \tag{22}$$

The ω_t is then:

$$\vec{\omega}_t^{(SRF)} = \omega_0 \begin{bmatrix} c_{\omega_1}\sin(\varphi+\nu) \\ -c_{\omega_1}\cos(\varphi+\nu) \\ c_{\omega_2} \end{bmatrix} \tag{23}$$

where:

$$c_{\omega_1} = \frac{3}{2}\, J_2\, \sin(i)\cos(i) \left(\frac{R_E}{a}\right)^2 \quad c_{\omega_2} = \left[\frac{3}{2}\, J_2\, \cos^2(i) \left(\frac{R_E}{a}\right)^2 - 1\right] \tag{24}$$

The gravity gradient and the control torque are the only torques considered in this analysis.

6. REFERENCE MODEL

The reference model needed to plan the trajectories to be followed by the non-linear system is a LTI dynamic model whose dynamic and control matrices are obtained linearizing the equation of motion of a rigid body under the hypothesis of little Euler angles. Furthermore, in order to control the reference model by means of a PID controller, the gravity gradient and control matrix have been averaged over an orbital period. Finally the reference model has the following shape:

$$\dot{x}_m = (A + A_{GG})\; x_m + G\, u \tag{25}$$

where A is the dynamic matrix, A_{GG} is the contribution of the gravity gradient and G is the control matrix.

6.1 LINEARIZED AND AVERAGED EULER EQUATIONS OF MOTION

The components of the angular velocity of the spacecraft in the BRF are:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & \theta_3 & 0 \\ -\theta_3 & 1 & 0 \\ \theta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} + \omega_0 \begin{bmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 & 1 & \theta_1 \\ \theta_2 & -\theta_1 & 1 \end{bmatrix} \begin{bmatrix} c_{\omega_1} \sin(\omega_0 t) \\ -c_{\omega_1} \cos(\omega_0 t) \\ c_{\omega_2} \end{bmatrix} \tag{26}$$

The angular acceleration in the BRF can be obtained differentiating with respect of time the angular velocity:

$$\begin{aligned} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} &+ \omega_0 \begin{bmatrix} 0 & 0 & \omega_0 c_{\omega_1} \sin(\omega_0 t) \\ 0 & 0 & -\omega_0 c_{\omega_1} \cos(\omega_0 t) \\ -\omega_0 c_{\omega_1} \sin(\omega_0 t) & \omega_0 c_{\omega_1} \cos(\omega_0 t) & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ &+ \omega_0 \begin{bmatrix} 0 & -c_{\omega_2} & -c_{\omega_1} \cos(\omega_0 t) \\ c_{\omega_2} & 0 & -c_{\omega_1} \sin(\omega_0 t) \\ c_{\omega_1} \cos(\omega_0 t) & c_{\omega_1} \sin(\omega_0 t) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \\ &+ \omega_0 \begin{bmatrix} \omega_0 c_{\omega_1} \cos(\omega_0 t) \\ \omega_0 c_{\omega_1} \sin(\omega_0 t) \\ 0 \end{bmatrix} \end{aligned} \tag{27}$$

Substituting the relations found in the Euler rigid body equations of motion and neglecting second order terms as $c_{\omega_1}^2$, $c_{\omega_1}\theta_i$ and $c_{\omega_1}\dot{\theta}_i$ and approximating $c_{\omega_2} = -1$ for a Sun-synchronous orbit, we have:

$$\ddot{\theta}_1 + \omega_0 \left(1 - \frac{I_z - I_y}{I_x}\right) \dot{\theta}_2 + \omega_0^2 \left(\frac{I_z - I_y}{I_x}\right) \theta_1 = \frac{M_x}{I_x} - \omega_0^2 \left(1 + \frac{I_z - I_y}{I_x}\right) c_{\omega_1} \cos(\omega_0 t) \tag{28}$$

$$\ddot{\theta}_2 - \omega_0 \left(1 + \frac{I_x - I_z}{I_y}\right) \dot{\theta}_1 - \omega_0^2 \left(\frac{I_x - I_z}{I_y}\right) \theta_2 = \frac{M_y}{I_y} - \omega_0^2 \left(1 - \frac{I_x - I_z}{I_y}\right) c_{\omega_1} \sin(\omega_0 t) \tag{29}$$

$$\ddot{\theta}_3 = \frac{M_z}{I_z} \tag{30}$$

6.2 AVERAGED GRAVITY GRADIENT TORQUE

The gravity gradient torque in the BRF is:

$$\vec{M}_{GG} = \frac{3\mu_\oplus}{r^3} \begin{bmatrix} (I_z - I_y)\,(\hat{r}^{(BRF)} \cdot \hat{j})(\hat{r}^{(BRF)} \cdot \hat{k})\hat{i} \\ (I_x - I_z)\,(\hat{r}^{(BRF)} \cdot \hat{i})(\hat{r}^{(BRF)} \cdot \hat{k})\hat{j} \\ (I_y - I_x)\,(\hat{r}^{(BRF)} \cdot \hat{i})(\hat{r}^{(BRF)} \cdot \hat{j})\hat{k} \end{bmatrix} \tag{31}$$

The scalar products in brackets represent the projection of the unit vector of the orbital radius in the BRF. In order to obtain its components in BRF we proceed considering the rotations from LRF to SRF and from SRF to BRF [5].

$$r^{(BRF)} = \tilde{T}_A\, R_{Y_L}(\pi)\, r^{(LRF)} \tag{32}$$

where $\tilde{T}_A$ is the rotation matrix from SRF to BRF for small euler angles. We obtain:

$$r^{(BRF)} = \begin{bmatrix} -1 \\ \theta_3 \\ -\theta_2 \end{bmatrix} \tag{33}$$

Neglecting second order terms, the components of the gravity gradient torque become:

$$\vec{M}_{GG} = \frac{3\mu_\oplus}{r^3} \begin{bmatrix} 0 \\ (I_x - I_z)\,\theta_2 \\ (I_x - I_y)\,\theta_3 \end{bmatrix} \tag{34}$$

and the averaged contribution of the gravity gradient is:

$$\overline{A}_{GG} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3\mu_\oplus}{r^3}\frac{(I_x-I_z)}{I_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3\mu_\oplus}{r^3}\frac{(I_x-I_y)}{I_z} & 0 & 0 & 0 \end{bmatrix} \tag{35}$$

6.3 AVERAGED CONTROL MATRIX

The control torque arise from the intection between the on board magnetic dipole and the geomagnetic field:

$$\vec{T} = \vec{m} \times \vec{B}^{(BRF)} \tag{36}$$

where $\vec{m}$ is the on board magnetic dipole and $\vec{B}$ is geomagnetic field in BRF.
The term $\vec{B}^{(BRF)}$ can be obtained from the geomagnetic field in the SRF taking into account the rotation matrix T_A linearized for small euler angles:

$$\vec{B}^{(BRF)} = \tilde{T}_A \vec{B}^{(SRF)} = \left[I_{3\times 3} - \tilde{\theta}\right] \vec{B}^{(SRF)} = \vec{B}^{(SRF)} - \tilde{\theta}\vec{B}^{(SRF)} \tag{37}$$

where:

$$\tilde{\theta} = \begin{bmatrix} 0 & -\theta_3 & \theta_2 \\ \theta_3 & 0 & -\theta_1 \\ -\theta_2 & \theta_1 & 0 \end{bmatrix} \tag{38}$$

As the on-board dipole is a function of the angles between BRF and SRF, we can neglect the term $\tilde{\theta}\vec{B}_{SRF}$ and the required torque can be simplified to the following expression:

$$\vec{T} = -\vec{B}^{(SRF)} \times \vec{m} \tag{39}$$

Once the geomagnetic field is known, we need to find the dipole momentum able to generate the required torque:

$$\vec{B}^{(SRF)} \times \vec{T} = \vec{B}^{(SRF)} \times \vec{m} \times \vec{B}^{(SRF)} \tag{40}$$

By developing the cross product, the previous relationship becomes:

$$\vec{B}^{(SRF)} \times \vec{T} = \left|\vec{B}^{(SRF)}\right|^2 \vec{m} - \vec{B}^{(SRF)} \left(\vec{m} \cdot \vec{B}^{(SRF)}\right) \tag{41}$$

In order to find the minimum dipole, it is necessary to impose $\vec{m} \cdot \vec{B}^{(SRF)} = 0$. Since the control torque is always orthogonal to the on board dipole plane, the best choice to do for the dipole is the following [**?**]:

$$\vec{m} = \frac{1}{\left|\vec{B}^{(SRF)}\right|^2} \vec{B}^{(SRF)} \times \vec{T}_{crtl} \tag{42}$$

Calling $\vec{T}_{crtl} = u$, the torque generated by the control becomes:

$$\vec{T} = -\vec{B}^{(SRF)} \times \left[\frac{1}{\left|\vec{B}^{(SRF)}\right|^2} \vec{B}^{(SRF)} \times \vec{u} \right] = -\frac{\tilde{B}_s \tilde{B}_s}{\left|\vec{B}^{(SRF)}\right|^2} \vec{u} \tag{43}$$

where $\tilde{B}_s$ is:

$$\tilde{B}_s = \begin{bmatrix} 0 & -B_z^{(SRF)} & B_y^{(SRF)} \\ B_z^{(SRF)} & 0 & -B_x^{(SRF)} \\ -B_y^{(SRF)} & B_x^{(SRF)} & 0 \end{bmatrix} \tag{44}$$

We can consider the dipole approximation for the geomagnetic field:

$$\vec{B}^{(SRF)} = \frac{W}{r^3} \left[3\hat{r}_s \hat{r}_s^T - I^{3\times3}\right] \hat{m}_s = \frac{W}{r^3} \vec{B}_s' \tag{45}$$

In order to express the geomagnetic field in the SRF, the geomagnetic field is rotated from the LRF to the SRF. The earth magnetic dipole is then rotated from the ECI reference system where it is known to the LRF.

$$\vec{B}_s' = \left[3\hat{r}_s \hat{r}_s^T - I^{3\times3}\right] \hat{m}_s \tag{46}$$

$$= R_{LRF\to SRF} \left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right] R_{ECI\to LRF}\, \hat{m}_{ECI} \tag{47}$$

$$= R_{Y_L}(\pi) \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} R_{Z_O}(\omega_0 t) R_{X_N}(i)\, \hat{m}_{ECI} \tag{48}$$

Approximating the earth magnetic dipole with a dipole parallel to the Z_{ECI} axis, we obtain:

$$\tilde{B}_s = \frac{W}{r^3} \begin{bmatrix} 0 & -c_i & -s_i \cos(\omega_0 t) \\ c_i & 0 & 2s_i \sin(\omega_0 t) \\ s_i \cos(\omega_0 t) & -2s_i \sin(\omega_0 t) & 0 \end{bmatrix} \tag{49}$$

where $c_i = \cos(i)$ and $s_i = \sin(i)$. The time dependent product $\tilde{B}_s\tilde{B}_s$ is then:

$$\tilde{B}_s\tilde{B}_s = \frac{W^2}{r^6} \begin{bmatrix} -s_i^2 \cos(\omega_0 t)^2 - c_i^2 & 2s_i^2 \cos(\omega_0 t) sin(\omega_0 t) & -2c_i s_i sin(\omega_0 t) \\ 2s_i^2 \cos(\omega_0 t) \sin(\omega_0 t) & -4s_i^2 \sin(\omega_0 t)^2 - c_i^2 & -c_i s_i \cos(\omega_0 t) \\ -2c_i s_i \sin(\omega_0 t) & -c_i s_i \cos(\omega_0 t) & -s_i^2 \cos(\omega_0 t)^2 - 4s_i^2 \sin(\omega_0 t)^2 \end{bmatrix} \tag{50}$$

averaging the previous matrix over an orbital period we obtain:

$$\overline{\tilde{B}_s\tilde{B}_s} = \frac{W^2}{r^6} \begin{bmatrix} \frac{s_i^2}{2} - 1 & 0 & 0 \\ 0 & -1 - s_i^2 & 0 \\ 0 & 0 & -\frac{5}{4}s_i^2 \end{bmatrix} \tag{51}$$

and the control matrix becomes:

$$G = -J^{-1} \frac{\tilde{B}_s\tilde{B}_s}{\left|\vec{B}^{(SRF)}\right|^2} \tag{52}$$

where J is the matrix of the moments of inertia. In order to obtain the value of the absolute value of the Geomagnetic field with the dipole approximation we compute the average value the geomagnetic field over an orbit. The unit vector of the geomagnetic field in the SRF is:

$$\hat{b}_s = 1 + 3\left(\hat{m}_s \cdot \hat{r}_s\right) \tag{53}$$

After rotating $\hat{m}_{ECI}$ from ECI to SRF we obtain:

$$\hat{b}_s = 3s_i^2 sin^2(\omega_0 t) + 1 \tag{54}$$

The average over one orbit of $\hat{b}_s$ is:

$$\bar{\hat{b}}_s = \frac{3s_i^2}{2} + 1 \tag{55}$$

7. NUMERICAL SIMULATIONS

We start showing the free non-linear dynamics of the satellite, Figg.(1)(2). The dynamics of the euler angles and of the component of the angular velocity appear to be mutually coupled for initial attitude angles of the order of $10°$.

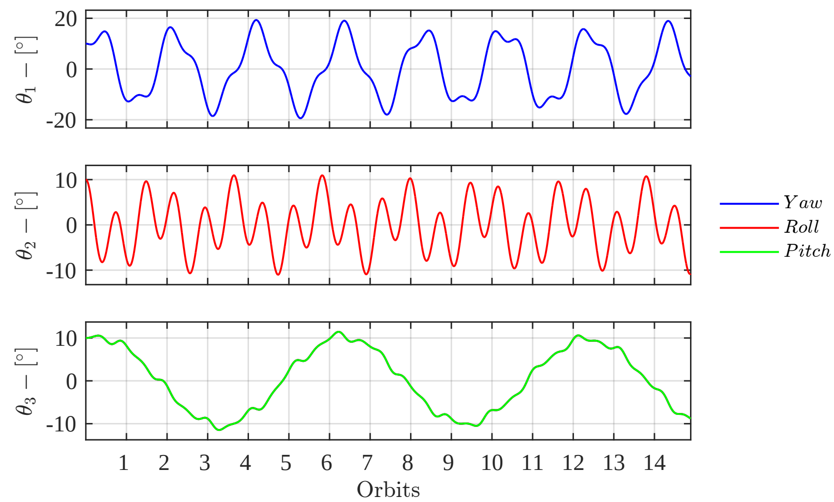

Figure 1. Euler Angles - Free Non Linear Dynamics

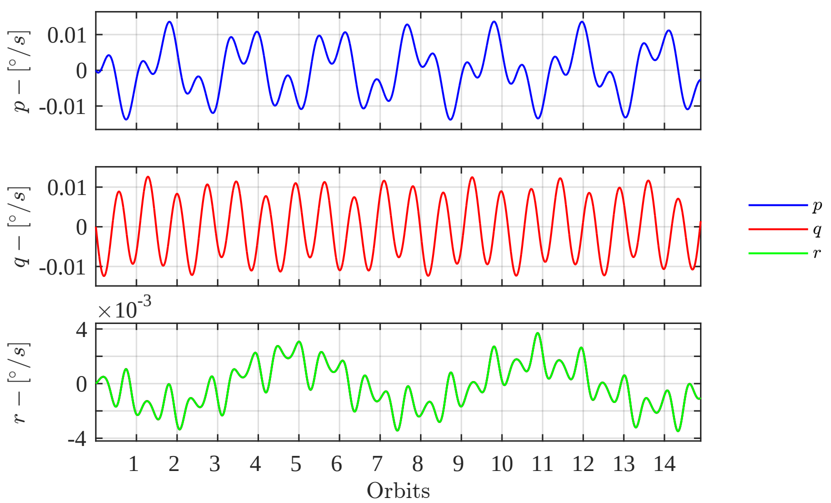

Figure 2. Angular Velocity in BRF - Free Non Linear Dynamics

The first numerical simulations is about the ability of the adaptive control to align the body reference frame (BRF) with the stabilized reference frame (SRF). The initial conditions of the reference model and of the non-linear system have been set to the same values:

$$\begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}_{t=0} = \begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} \tag{56}$$

The same maneuver has been simulated at two different epochs, a month apart from each other, in order to test the adaptive control for different values of the geomagnetic field, Figg.(3)(4)(5). The first epoch (left side in figures) refers to the 21^{st} March 2010, while second epoch (right side in the figures) refers to the 21^{st} April 2010.

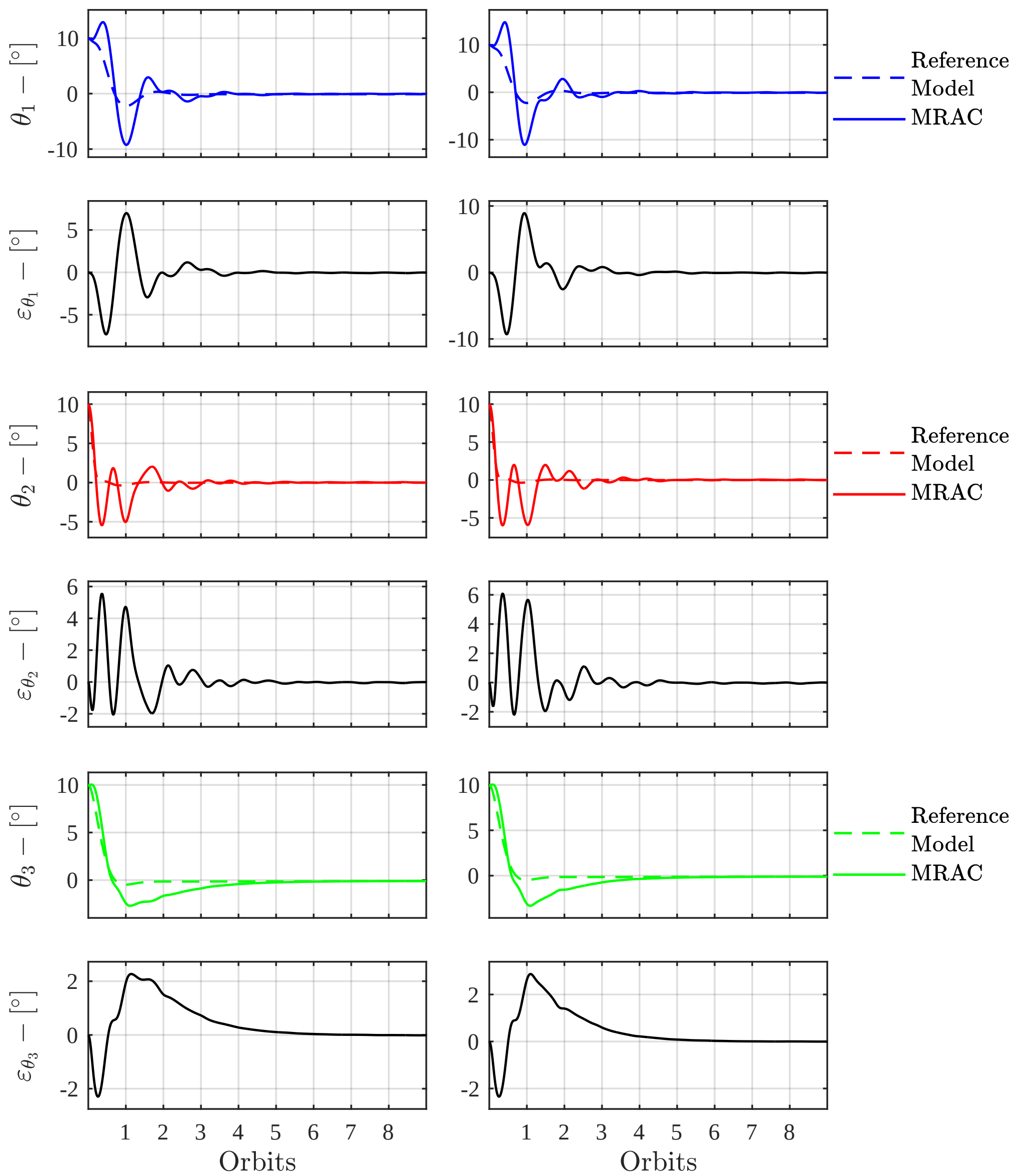

Figure 3. Simulation 1: Euler Angles and errors between Reference Model and Non-Linear Model controlled with MRAC. On the left result for 21^{st} March 2010, on the right for 21^{st} April 2010.

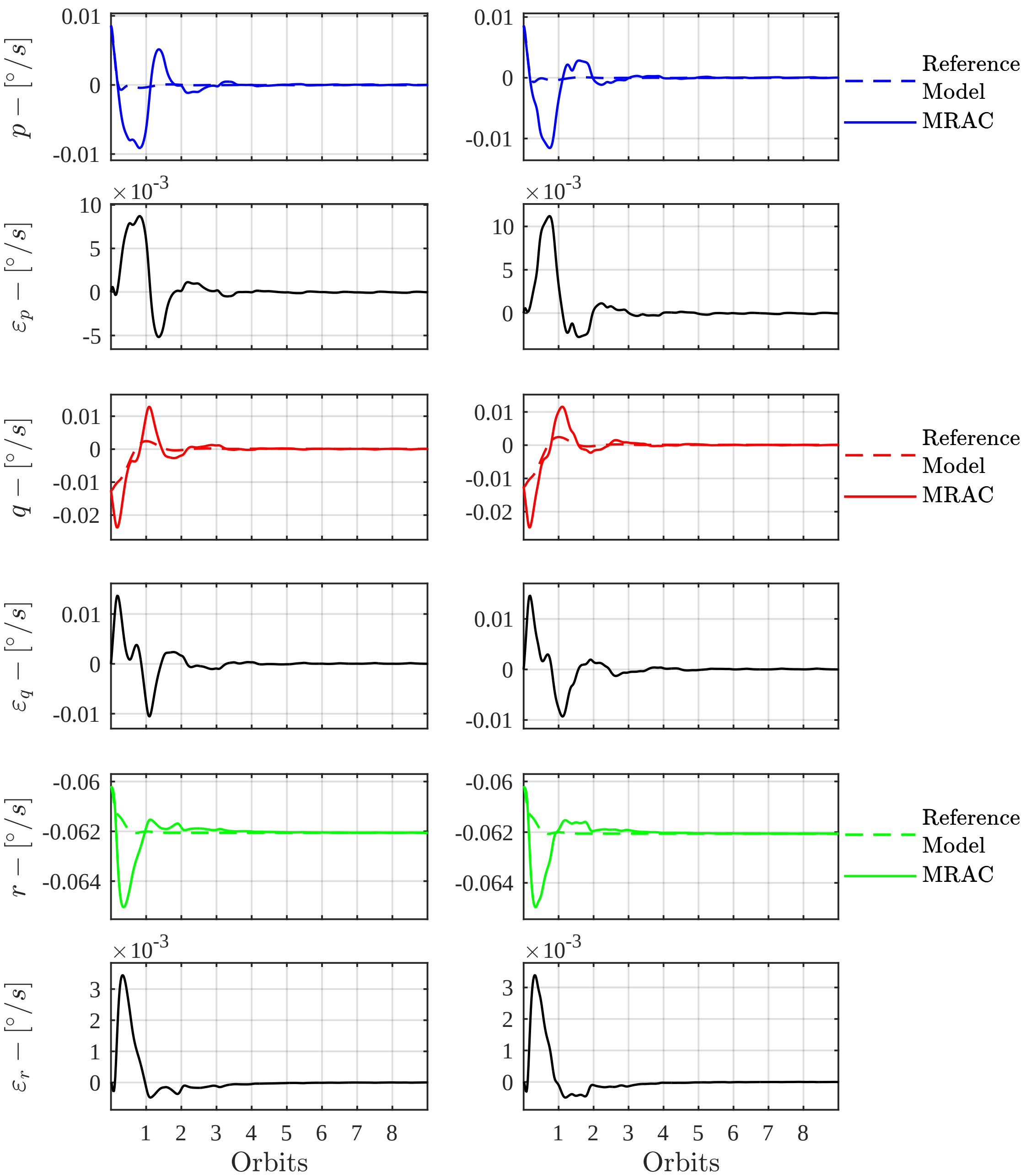

Figure 4. Simulation 1: Components of the angular velocity and errors between Reference Model and Non-Linear Model controlled with MRAC. On the left result for 21^{st} March 2010, on the right for 21^{st} April 2010.

The gains of the adaptive control have been set to null values for $t = 0$. The weight matrices chosen for the first simulation are:

$$
\begin{aligned}
\gamma &= 10^{-9} \\
\Gamma_e &= diag\left[\begin{array}{cccccc} 10 & 10 & 10^3 & 10^{-8} & 10^{-8} & 10^{-8} \end{array}\right] \\
\Gamma_x &= diag\left[\begin{array}{ccccccccc} 10^{-4} & 10^{-4} & 10^{-4} & 10^{-9} & 10^{-9} & 10^{-9} & 10^{-14} & 10^{-14} & 10^{-14} \end{array}\right] \\
\Gamma_u &= diag\left[\begin{array}{cccccc} 10^{-4} & 10^{-4} & 10^{-4} & 10^{-8} & 10^{-8} & 10^{-8} \end{array}\right]
\end{aligned} \tag{57}
$$

The MRAC accomplishes the task of aligning the BRF with the SRF in a time span comparable with three orbits. The controlled non-linear system, in both the epochs taken into account, follows the reference model with maximum errors in angular position comparable with half the initial attitude angles. The controlled system, thanks to the null contribute of the gravity gradient torque in the nominal attitude, is asymptotically stable with null errors associated with the Euler angles and angular velocity.

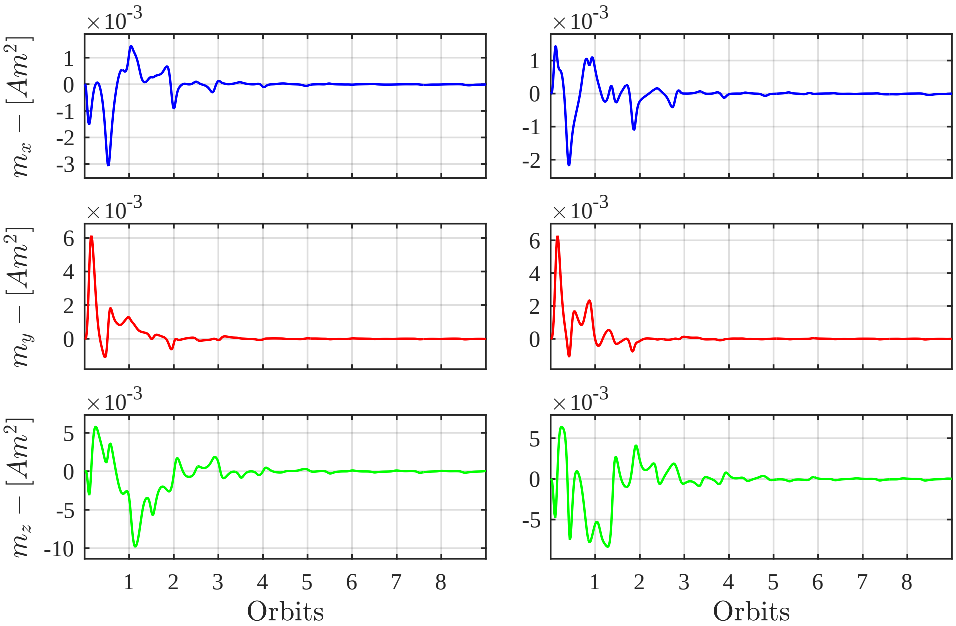

Figure 5. Simulation 1: Magnetic dipole computed with the MRAC. On the left result for 21^{st} March 2010, on the right for 21^{st} April 2010.

In the second simulation, Figg.(6)(7)(8), the reference model has been forced to oscillate around its pitch axis. This behaviour has been obtained adding to the reference model a sinusoidal torque along the Z axis with period equal to two orbital period. The initial conditions of the reference model and of the non-linear system have been set to the same values:

$$
\left[\begin{array}{cccccc} \theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{array}\right]_{t=0} = \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{58}
$$

The gains of the adaptive control have been set to null values for $t = 0$. The weight matrices chosen for the second simulation are:

$$
\begin{aligned}
\gamma &= 10^{-7} \\
\Gamma_e &= diag\left[\begin{array}{cccccc} 10^{2} & 10^{2} & 10^{4} & 10^{-7} & 10^{-7} & 10^{-6} \end{array}\right] \\
\Gamma_x &= diag\left[\begin{array}{ccccccccc} 10^{-5} & 10^{-5} & 10^{-5} & 10^{-10} & 10^{-10} & 10^{-10} & 10^{-15} & 10^{-15} & 10^{-15} \end{array}\right] \\
\Gamma_u &= diag\left[\begin{array}{cccccc} 10^{-5} & 10^{-5} & 10^{-5} & 10^{-9} & 10^{-9} & 10^{-9} \end{array}\right]
\end{aligned}
\tag{59}
$$

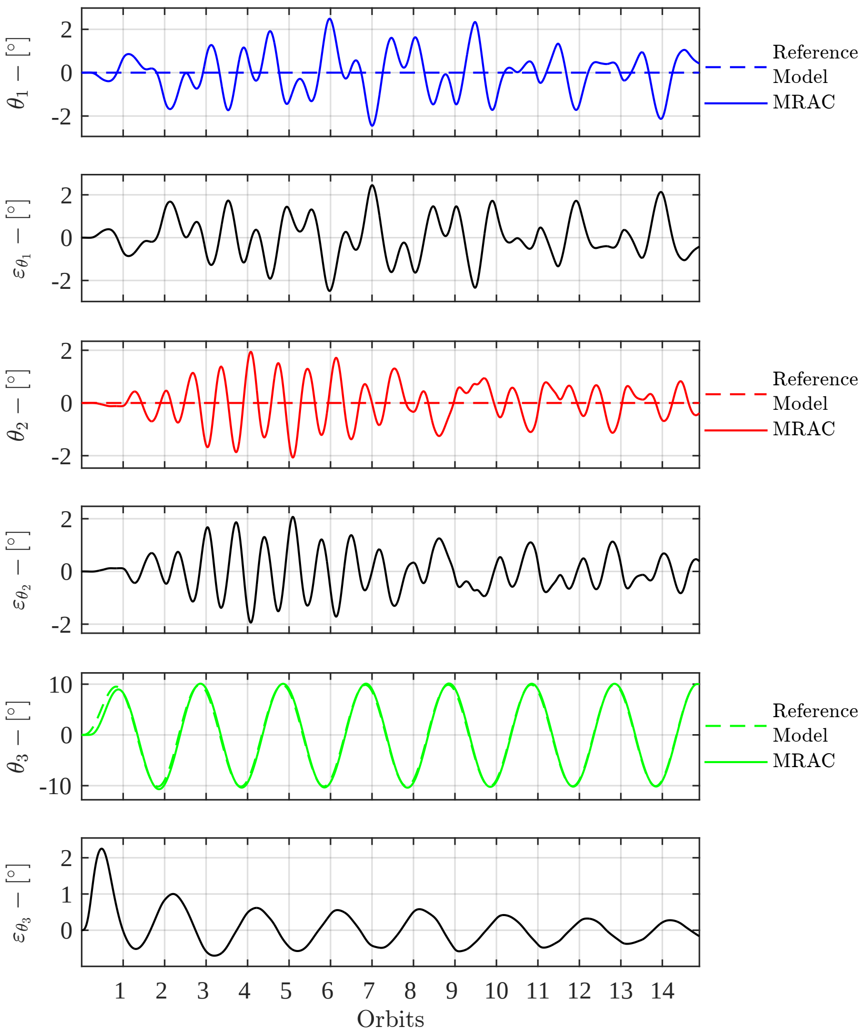

Figure 6. Simulation 2: Euler Angles and errors between Reference Model and Non-Linear Model controlled with MRAC.

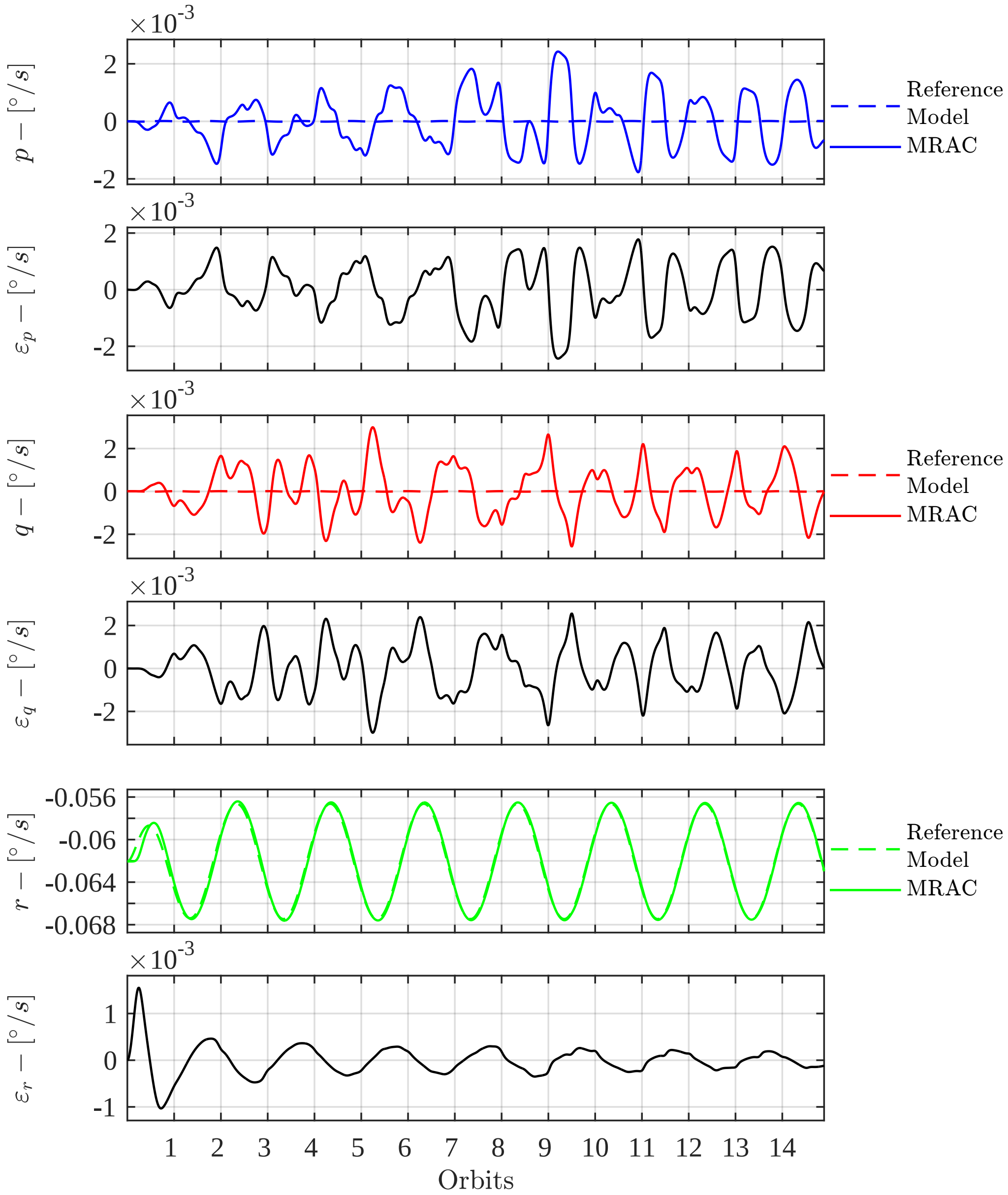

Figure 7. Simulation 2: Components of the angular velocity and errors between Reference Model and Non-Linear Model controlled with MRAC.

The controlled non-linear system follows the reference model along the pitch axis with less than $1°$ of error. The yaw and roll axes oscillate stably around the reference model trajectories with maximum error associated with the angular position of the order of $2°$. This behaviour has to be attributed to the particular nature of the reference model. The reference model is characterized by the fact that the dynamics around the roll and yaw axes are mutually coupled while uncoupled with the dynamics around the pitch axis. The non-linear system has the dynamics around the yaw, roll, pitch axes mutually coupled. The two dynamics can be compared for little Euler angles. With the approach of the MRAC, the non-linear system is forced to follow the reference model but, when the commanded Euler angles increase the non linear system can not follow exactly the reference model because it does not reproduce the real coupling between the dynamics around the three axes.

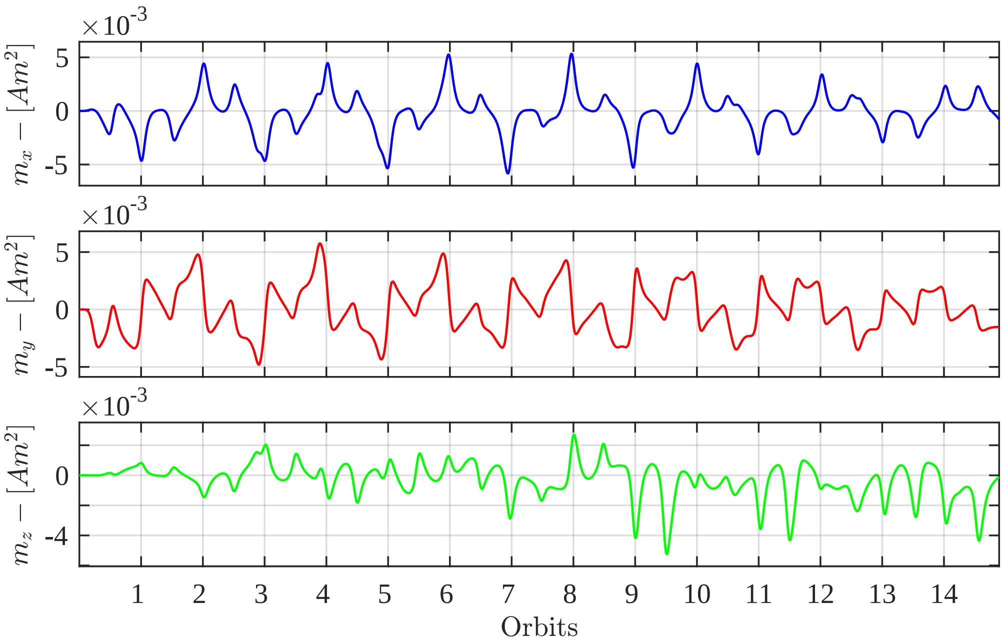

Figure 8. Simulation 2: Magnetic dipole computed with the MRAC.

In the third simulation, Figg.(9)(10)(11), the reference model has been forced to acquire an attitude rotated of $5°$ with respect of Z_{SRF}. The initial conditions of the reference model and of the non-linear system have been set to the same values:

$$\begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}_{t=0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{60}$$

The gains of the adaptive control have been set to null values for $t = 0$. The weight matrices chosen for the third simulation have the following values:

$$\begin{aligned}
\gamma &= 10^{-7} \\
\Gamma_e &= diag \begin{bmatrix} 10^2 & 10^2 & 10^4 & 10^{-6} & 10^{-6} & 10^{-5} \end{bmatrix} \\
\Gamma_x &= diag \begin{bmatrix} 10^{-4} & 10^{-4} & 10^{-4} & 10^{-9} & 10^{-9} & 10^{-9} & 10^{-14} & 10^{-14} & 10^{-14} \end{bmatrix} \\
\Gamma_u &= diag \begin{bmatrix} 10^{-4} & 10^{-4} & 10^{-4} & 10^{-8} & 10^{-8} & 10^{-8} \end{bmatrix}
\end{aligned} \tag{61}$$

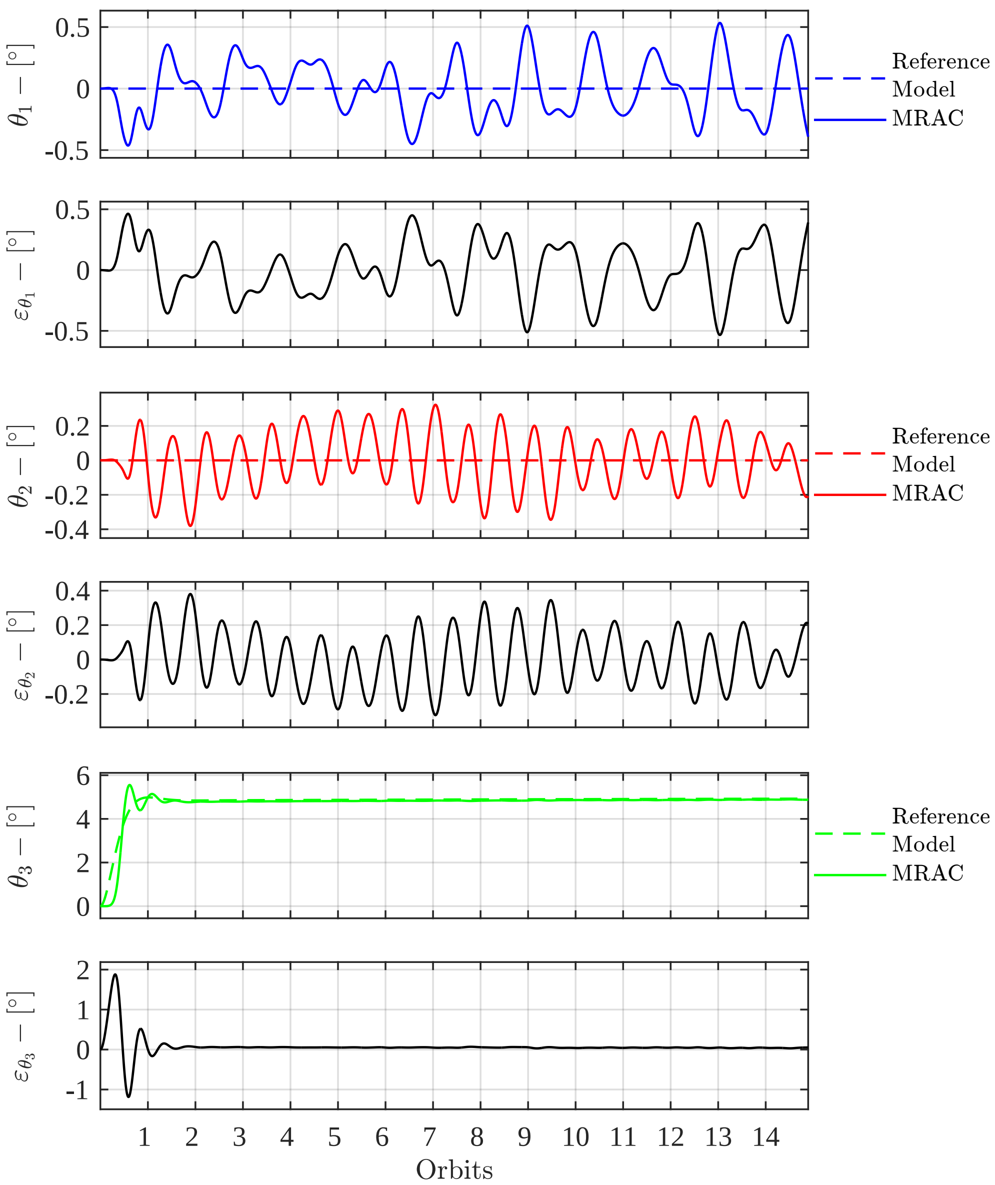

Figure 9. Simulation 3: Euler Angles and errors between Reference Model and Non-Linear Model controlled with MRAC.

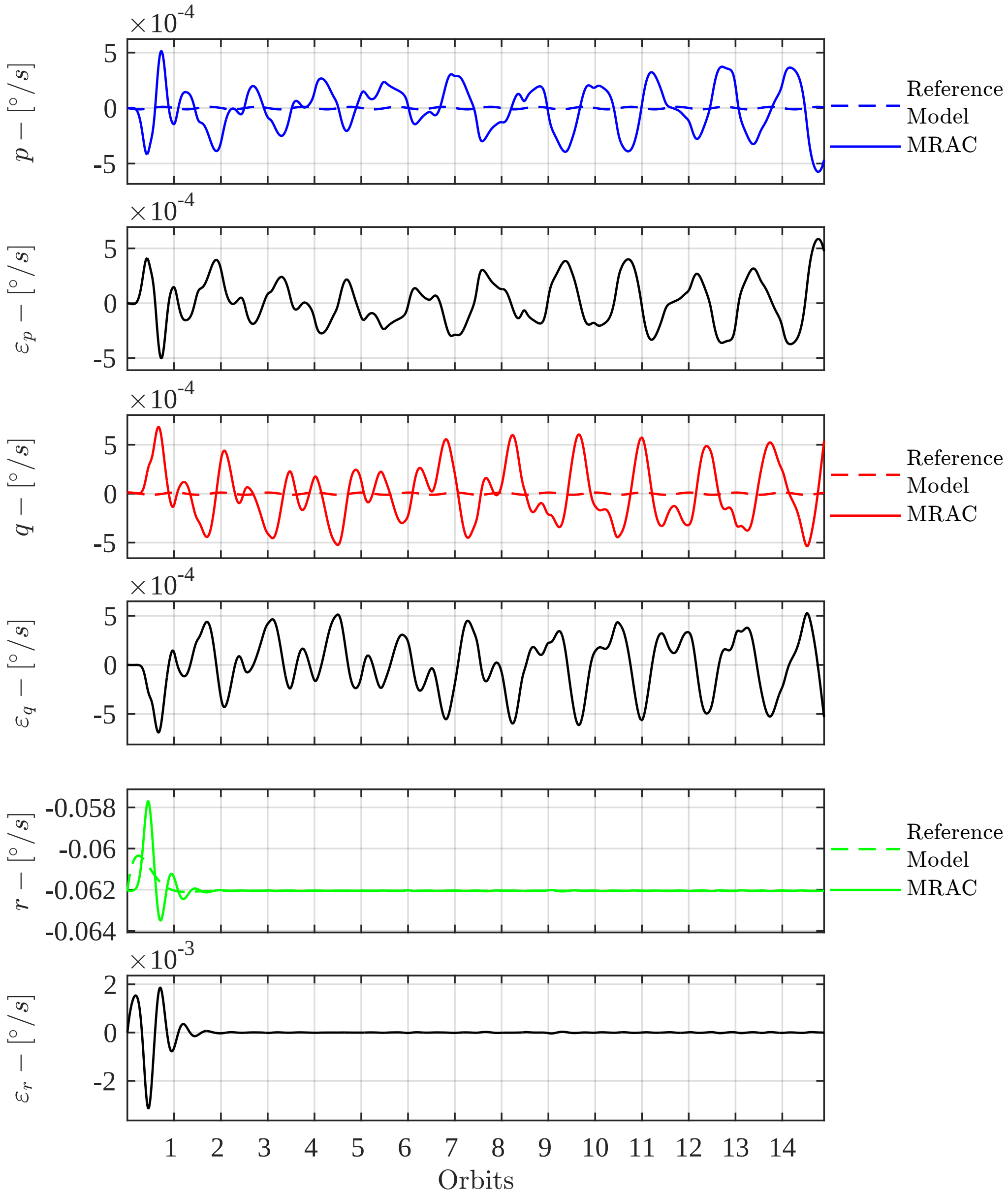

Figure 10. Simulation 3: Components of the angular velocity and errors between Reference Model and Non-Linear Model controlled with MRAC.

In the last simulation the Z of the non-linear system aligns with Z axis of the reference model in a time span comparable with 2 orbits. Due to the value of the commanded pitch angle and due to the same reason exposed for the second simulation, the yaw and roll axes oscillate around the desired trajectory. The maximum amplitude of the oscillation is $0.5°$ for the yaw axis and $0.3°$ for the roll axis. The decreased error with respect of the second simulation is mainly due to the minor value of the commanded angle around the Z_{SRF}.

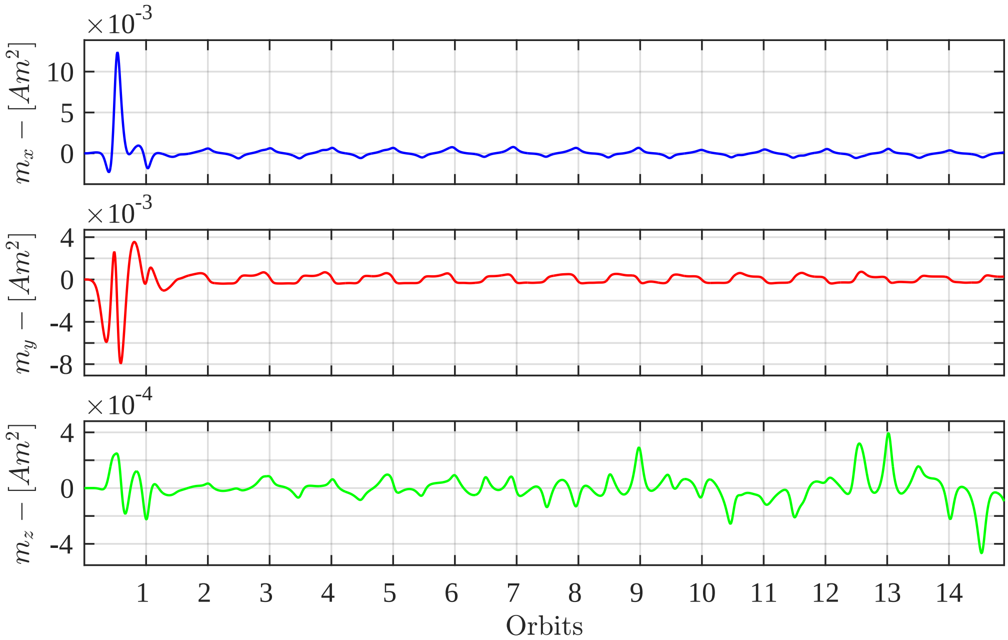

Figure 11. Simulation 3: Magnetic dipole computed with the MRAC.

8. CONCLUSIONS

In this work the MRAC has been applied to the stabilization of a small satellite by means of a magnetic attitude control system. The reference model has been built linearizing and averaging the dynamics and the control matrices, thus obtaining a LTI system. For the control matrix of the reference model the geomagnetic field has been computed according to the magnetic dipole approximation. The non-linear system representing the true dynamics of the spacecraft has been forced to follow the reference model by means of the MRAC. The control accomplishes perfectly its duty when the non-linear system needs to be aligned with the stabilized reference frame representing the nominal attitude. The control, due to the nature of the reference model, is not able to orient exactly the satellite according to an attitude different from the nominal. The MRAC shows an high dependence of its stability from the weight matrices needed for the computation of the adaptive gains.

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