

1           **USING DEM TO INVESTIGATE BOUNDARY CONDITIONS FOR**  
2           **ROCKING URM FAÇADES SUBJECTED TO EARTHQUAKE**  
3           **MOTION**

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39 **Abstract:** *Façade overturning is a frequently observed collapse mechanism occurring in*  
40 *unreinforced masonry (URM) buildings during high-intensity earthquake-induced shaking.*  
41 *Following complete separation from a building, the rocking motion of a URM façade and the*  
42 *associated impact against the return walls are the factors that continue to contribute to the*  
43 *façade out-of-plane capacity. Seismic vulnerability studies of URM façades have historically*  
44 *neglected the interaction between building earthquake response and the rocking response of*  
45 *the façade, whereas in the study reported herein this interaction was analysed using the*  
46 *discrete element modelling (DEM) approach, resulting in a façade out-of-plane capacity*  
47 *reduction. The increment in the dynamic rocking capacity caused by the frictional connection*  
48 *between the URM façade and the building was also analysed and is reported.*

49 **Keywords:** Rocking Façade, Unreinforced Masonry, Boundary Conditions, Incremental  
50 Dynamic Analysis, Discrete Element Method

51

## 52 1. INTRODUCTION

53 Façade overturning is a collapse mechanism that is routinely observed after high-intensity  
54 earthquakes (Lagomarsino, 1998, Benito et al., 2007, Hess, 2007, Dizhur et al., 2010), being a  
55 critical life-safety hazard because out-of-plane failure can jeopardise the gravity-load bearing  
56 capacity of the building (Bruneau, 1994) and can result in large quantities of masonry debris  
57 falling outside the building footprint (Abeling and Ingham, 2020). Therefore, details of the  
58 collapse behaviour of unreinforced masonry (URM) façades need to be well understood.  
59 Façades that lose their connection with the adjoining return walls have been observed to behave  
60 as rocking rigid-bodies when disintegration of the masonry did not occur (Shawa et al., 2012,  
61 Lagomarsino, 2015, Giresini et al., 2018). Amongst the different approaches adopted to

62 simulate the free rocking behaviour of rigid bodies, the strategy originally developed by  
63 Housner (1963) is the most widely used where the rocking equations of motion represent a  
64 rigid block rocking on a simply supported rigid base, with the assumption that the coefficient  
65 of friction is large enough to avoid sliding. The cyclic rocking motion of a block dissipates  
66 energy when impacting the ground, which was modelled by Housner (1963) using the  
67 coefficient of restitution, such that a reduction of angular velocity occurs by assuming the  
68 conservation of angular momentum before and after every impact. Several authors have  
69 revisited the Housner (1963) formulation to include sliding and bouncing in the rocking motion  
70 (Aslam et al., 1975, Ishiyama, 1982, Psycharis and Jennings, 1983), to account for different  
71 material stiffnesses and different friction characteristics at the block-base contact (ElGawady  
72 et al., 2011, Lipo and de Felice, 2016, Ther and Kollár, 2016), to reduce discrepancies with  
73 experimental observations (Lipscombe and Pellegrino, 1993, Kalliontzis et al., 2016), or to  
74 reduce the complexity of the rocking motion formulation in order to facilitate the development  
75 of a closed-form solution to the problem (Casapulla and Maione, 2017).

76 The abovementioned research gives a solution to the problem of a free-standing rocking block,  
77 but many rocking elements that concern the earthquake engineering community are one-sided  
78 rocking blocks (e.g. hospital or laboratory equipment generally placed next to walls, or rocking  
79 unreinforced masonry such as URM façades). The motion of the façades when constrained by  
80 impacting with the return walls makes the problem significantly more complex, and therefore  
81 less research has been conducted on this topic. Impact interaction has been accounted for by  
82 restricting the cyclic motion and including an extra coefficient of restitution that changes the  
83 amplitude and the sign of the rotational velocity instantaneously to account for the change in  
84 motion direction and the energy dissipated at the contact (Sorrentino et al., 2008, Bao and  
85 Konstantinidis, 2020). Another adopted approach was to assume that the energy dissipated  
86 between the rocking blocks and the adjacent walls was negligible (Sigurdsson et al., 2019).

87 One-sided rocking models that account for interaction between the rocking façade and steel tie  
88 rods were also reported (Giresini et al., 2015, Giresini and Sassu, 2016, Giresini, 2017,  
89 AlShawa et al., 2019) and simplified approaches were proposed where the interaction between  
90 façades and return walls was represented by a bed of springs (Giresini and Sassu, 2016,  
91 Giresini, 2017). Using the same modelling strategy, more recent models have included two-  
92 directional motion of more complex masonry shapes (e.g. corner failure macroblock)  
93 (Casapulla et al., 2019a, Giresini et al., 2019). Researchers agree that the overturning frequency  
94 of façades is reduced when the damping associated with impact with the return wall is increased  
95 (Sorrentino et al., 2008, Sorrentino et al., 2011, Bao and Konstantinidis, 2020), and when steel  
96 ties are used as a strengthening solution (Giresini, 2017, AlShawa et al., 2019). Bao and  
97 Konstantinidis (2020) considered a sliding-rocking block adjacent to a wall and concluded that  
98 decreasing the friction coefficient and reducing the gap between block and wall are beneficial  
99 strategies to increase the dynamic stability against overturning while accepting sliding.  
100 Casapulla et al. (2017) provides a detailed review of the latest advances on rocking and  
101 kinematic analysis of URM walls subjected to earthquakes.

102 Façades that span over several stories and where the parapet is located at the top of the URM  
103 building experience distinctly different earthquake motion than that occurring at ground level,  
104 because earthquake motion at the ground surface is modified and amplified as it travels through  
105 the structural elements of the building. This amplification phenomenon is acknowledged and  
106 well documented in the literature (Priestley, 1985, Derakhshan et al., 2014, Degli Abbati et al.,  
107 2018) when assessing the seismic performance of URM parapets or upper level URM walls.  
108 The inelastic behaviour of the building also modifies the ground motion up the building height,  
109 dissipating energy as building damage occurs. Depending on the ductility and the stiffness of  
110 the building, the modification of the ground motion may be either beneficial or detrimental to  
111 the seismic demand of the upper level walls (Menon and Magenes, 2011). However in the case



112 of URM façades, studies generally do not account for different input motions at the base and  
113 up the height of the return walls (Sorrentino et al., 2008, Shawa et al., 2012, Giresini et al.,  
114 2019, Sigurdsson et al., 2019). In the study presented herein four topics are addressed using  
115 the discrete element method (DEM): (1) finding a simple alternative to model the amplification  
116 of ground motion occurring in URM buildings, (2) developing and experimentally validating a  
117 suitable strategy to assign a damping parameter for one-sided rocking façades, (3) studying the  
118 seismic vulnerability of URM façades with different boundary conditions, and (4) analysing  
119 the influence when including the seismic behaviour of the building during an assessment of the  
120 vulnerability of rocking façades. Ideally, either the complete building or alternatively part of  
121 the building would be micro-modelled and analysed with thousands of degrees of freedom  
122 (DOF), but such an approach is computationally demanding making such an undertaking  
123 impractical. Instead, an alternative method was investigated and the seismic response for  
124 various configurations of lumped mass-spring models were compared. Once the simpler model  
125 configuration was selected, the resulting motion at multiple levels within the building was  
126 extracted and used as input to the simulated return walls of an experimentally validated 3D  
127 DEM simulation of rocking façades spanning over two building storeys.

## 128 **2. COMPARISON BETWEEN LUMPED MASS-SPRING AND 3D DEM MODELS**

129 A single-unit two-storey URM case study building that was used in the Italian ReLUIIS III  
130 research Project *Masonry Structures* was selected in order to study the modification of ground  
131 motion occurring up the height of a URM building. The selected building was designed with a  
132 relatively simple geometry that was intended to reveal interaction effects between piers and  
133 spandrels. The geometry of the case study building also facilitated a study of the effects of  
134 stress redistribution in the presence of walls with different stiffness, plus a study of torsional  
135 effects. This building was extensively studied by others previously using a multitude of

136 modelling techniques (Cattari et al., 2018, Olcese, 2018). As part of the study reported herein,  
137 the geometry of the building was modelled using the software Rhinoceros® (McNeel and  
138 Associates, 2014), which allows a script to be run that contains an algorithm capable of  
139 exporting a text file of the building geometry compatible with the DEM software 3DEC (Itasca,  
140 2013) (see Figure 1).

(a) Rhinoceros® (Dimensions in m)

(b) DEM

141 Figure 1: Geometrical model of the two-storey case study building

142

143 The DEM software allows modelling of every masonry unit by independent rigid or deformable  
144 elements connected to each other by nonlinear interfaces. The case study building presented  
145 herein was modelled with rigid elements referred to herein as bricks, and linearly elastic  
146 interfaces. This linear behaviour complies with the assumption that façades and parapets will  
147 collapse before the building has suffered enough damage to alter its fundamental frequency,  
148 that will allow a simplification of the problem to focus on the façade boundary condition  
149 problem addressed herein. Rigid diaphragms with elastic connections were modelled as per  
150 Gubana and Melotto (2019). The diaphragm comprised evenly spaced rigid beams oriented  
151 parallel to the shorter walls and placed on top of the masonry walls, whose thickness was  
152 reduced to host the beam support. Elastic contacts between the beam ends and the masonry  
153 were generated and in order to simulate actual diaphragms, a rigid panel was modelled on top  
154 of the beams and connected to them with elastic contacts. This simplified modelling strategy  
155 allowed the cyclic behaviour of a timber floor to be considered in the DEM model.

156 The interfaces between the bricks were defined by three shear and normal material parameters:  
157 normal stiffness ( $j_{kn}$ ); shear stiffness ( $j_{ks}$ ); friction angle ( $^{\circ}$ ). Generally, the estimation of  $j_{kn}$  is

158 calculated as  $E/s$ , where  $E$  is the Young modulus and  $s$  is the joint spacing (see Galvez et al.  
159 (2018) for further details). The value of  $E$  for the case study building was 1,800 MPa, and the  
160 dimensions of the bricks were  $24L \times 6H \times 12W$  cm<sup>3</sup>. However, application of these parameters  
161 to forecast  $j_{kn}$  led to an overestimation of the overall stiffness of the building and a much lower  
162 fundamental period than is typical for a 2-storey URM building. Hence it was decided to reduce  
163  $j_{kn}$  until the building modes of vibrations resulted in values that were similar to previous  
164 observations but respected the relationship with brick dimensions. This tuning strategy to  
165 establish the material properties has been implemented by multiple researchers  
166 (Papantonopoulos et al., 2002, Psycharis et al., 2003, Lemos and Campos Costa, 2017, Meriggi  
167 et al., 2019) who calibrated their models by best fitting stiffness parameters to match  
168 experimental dynamic behaviour or by replicating previous similar research. There are three  
169 types of contact surfaces around a brick, each with its own properties influenced by the brick  
170 dimensions: head joint ( $j_{kn} = 1,280$  MPa/m,  $j_{ks} = 422$  MPa/m, friction angle =  $30^\circ$ ), bed joint  
171 ( $j_{kn} = 5,120$  MPa/m,  $j_{ks} = 1,690$  MPa/m, friction angle =  $30^\circ$ ) and collar joint ( $j_{kn} = 2,560$   
172 MPa/m,  $j_{ks} = 845$  MPa/m, friction angle =  $30^\circ$ ). The aforementioned joint material properties  
173 were selected to ensure that the eigenvalue analysis delivered a period of 0.25 s for the  $Y$   
174 translational mode of vibration, coinciding with the estimate given by equation  $T = 0.0625 \cdot h^{0.75}$   
175 from NZS1170.5 (2004) and AS1170.4 (2007). The same analysis delivered a period of 0.16 s  
176 for the  $Z$  rotational mode (see Figure 2). To these frequencies 5% and 3% Rayleigh mass  
177 proportional (MP) damping ratios were applied, respectively. Rayleigh damping can be derived  
178 using MP or stiffness proportional (SP) damping, with the latter having a strong influence on  
179 the timestep size of the simulation. A time-history simulation of the complete building using  
180 DEM (CBDEM) was considered highly computationally demanding even when using MP  
181 damping. Consequently a simplified structure (SBDEM) without the front and rear façades in

182 the direction of motion,  $Y$ , was dynamically compared to the complete building model via  
 183 eigenvalue analysis (see Figure 2).

(a) 1<sup>st</sup> mode of      (b) 1<sup>st</sup> mode of      (c) 2<sup>nd</sup> mode of      (d) 2<sup>nd</sup> mode of  
 CBDEM (T=0.25 s)    SBDEM (T=0.27 s)    CBDEM (T=0.16 s)    SBDEM (T=0.17 s)

184 Figure 2: First and second modal shapes and periods in the  $Y$  direction of the complete and the simplified  
 185 structure calculated using DEM modal analysis

186

187 Considering the directionality of the input motion, only the modes of vibration with significant  
 188 effective modal mass in the  $Y$  axis were considered, with a difference of 0.01-0.02 s between  
 189 the modal frequencies and similar deformed shapes being observed in Figure 2. Subjecting  
 190 even the SBDEM building to multiple time-history analyses would require a great  
 191 computational effort and would highly restrict the number of earthquakes that could be  
 192 amplified as input to the rocking façade simulations. Hence, developing a simpler easy-to-run  
 193 equivalent model had a high potential for future studies. The simpler model chosen to compute  
 194 the dynamic behaviour of the building was based on the solution of the explicit formulation of  
 195 the equation of motion of a spring-mass model with translational degrees of freedom and  
 196 varying number of springs and lumped masses (SMDOF) performed in the software Matlab®  
 197 (MathWorks, 2008). All the masses of the model were divided equally per lumped mass, and  
 198 the summation of all mass was equal to the mass of the SBDEM model. Similarly, all the  
 199 stiffnesses were equal and adjusted so that the first mode of vibration matched the value of  
 200 0.27 s that was obtained from the SBDEM model (Figure 2b), while damping was fixed at 5%.  
 201 After confirming the directional dynamic equivalence between the CBDEM and the SBDEM  
 202 (see Figure 2), the seismic response of the SBDEM when subjected to two of the ground  
 203 motions in Table 1 (BUCAR0 and RRS228) was compared to the response of the SMDOF.

204

205

(a) Time-history response

(b) Response Spectra

(c) Power Spectra

206 Figure 3: Comparison between the seismic response of the SBDEM and SMDOF for BUCARO earthquake.

207

208 The SMDOF model that provided results most similar to the results from the SBDEM was  
209 identified by comparing the weighted mean error (WME) between models using SBDEM as  
210 the reference. The WME was obtained from the time-history responses and the acceleration  
211 and power spectra (see Figure 3) with the following formulation:

212

$$\text{WME} = \frac{\int_0^X |Y_{DEM}(x) - Y_{DOF}(x)| dx}{\int_0^X |Y_{DEM}(x)| dx} \quad (1)$$

213

214 where  $x$  was time, period or frequency,  $Y$  was acceleration, spectral acceleration or power  
215 amplitude depending on the evaluated case and  $X$  was the total range of  $x$  considered. Observing  
216 Table 2, the errors generated for a 4DOF and 5DOF models were found to have minimal  
217 difference. Consequently a 4DOF substitute structure was chosen to obtain the motion to apply  
218 to the rocking parapets and façade. A 4DOF model is more efficient than a 2DOF with masses  
219 lumped at floors, because most of the mass of typical masonry buildings is located in the walls  
220 rather than in the diaphragms. It was assumed that the primary building structure did not suffer  
221 damage during the earthquake and that even if damage appeared, the parapet and façade would  
222 collapse before the building sustained sufficient damage to change the stiffness and therefore  
223 the response. DEM simulations are known to have high-frequency numerical noise in the  
224 solution, known as the “rattling effect” (Lorig and Hart, 1993, Papantonopoulos et al., 2002,  
225 Jing and Stephansson, 2007, Dimitri et al., 2011). Because of the high-frequency nature of this

226 effect, MP damping is unable to attenuate this effect, which can affect the simulation depending  
227 on the magnitude of the frequencies of phenomena to be modelled. The rattling effect for the  
228 SBDEM can be observed from 0 to 1 s of the response in Figure 3a and in the peak around  
229 0.08 s of the spectra in Figure 3c. In this problem, the rattling error did not affect the main  
230 response of the building, which was approximately 0.27 s.

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232

### 233 3. VALIDATION OF DAMPING PARAMETERS FOR ROCKING FAÇADES

234 Rayleigh damping makes use of the constants  $\alpha$  (MP) and  $\beta$  (SP) to construct the damping  
235 matrix when performing dynamic analysis. The relationship for the damping ratio ( $\zeta_n$ ) for any  
236 circular frequency ( $\omega_n$ ) that results from the application of  $\alpha$  and  $\beta$  can be found in Bathe and  
237 Wilson (1976) as:

238

$$\zeta_n = \frac{\alpha}{2 \cdot \omega_n} + \frac{\beta \cdot \omega_n}{2} \quad (2)$$

239

240 Rocking simulations using SP damping have been recommended by several authors (DeJong,  
241 2009, Tomassetti et al., 2019) because MP damping may introduce overdamped low  
242 frequencies and result in unrealistic results. However, a systematic approach has not yet been  
243 developed to assign the value of SP damping for a 1SR wall. Conversely, a formulation  
244 experimentally developed by Sorrentino et al. (2011) allowed for an estimation of the  
245 restitution coefficient ( $e_{1SR}$ ) caused by multiple impacts of the block into the return walls as:

246

$$e_{1SR} = \left(1 - \frac{3}{2} \cdot \sin^2 \alpha\right)^2 \cdot \left(1 - \frac{3}{2} \cdot \cos^2 \alpha\right) \quad (3)$$

247

248 A parametric analysis was performed using single DEM blocks to represent the cracked  
249 rocking façade and interaction with the return walls to investigate  $\beta$  for 1SR ( $\beta_{1SR}$ ) walls based  
250 on their equivalent  $e_{1SR}$ . The  $j_{kn}$  at the bottom of the walls was calculated as  $E/h$ , where  $h$  is  
251 the height, and  $j_{kn}$  for the joint interacting with the return walls was calculated as  $E/L$ , where  
252  $L$  is the average length of the triangular-shaped URM block assumed to develop in the return  
253 wall once cracking occurs at  $45^\circ$ , according to the observations of Giresini and Sassu (2016).  
254 Assigning the value of  $j_{kn}$  for the return wall is a challenging task with various limitations as  
255 outlined by Giresini (2017). First, it is common practice to assume that the stiffness of the  
256 return walls is constant, even though it is known that the seismic response of URM buildings  
257 is characterised by progressive damage, and the corresponding change in stiffness can have  
258 an influence on the out-of-plane response of URM walls (Menon and Magenes, 2011).  
259 Secondly, the cracks on the return walls are assumed to occur at  $45^\circ$  even though the crack  
260 angle is actually unknown.

261 Four batches of simulations having varying block geometry [height ( $h$ ) and thickness ( $b$ )] and  
262 varying stiffness of the interface between the returning walls and the façade were performed  
263 and the results are summarised in Figure 4. The façade was displaced from the return walls,  
264 released, and allowed to rock and impact against the return walls. A range of  $\beta_{1SR}$  parameters  
265 was applied to the walls where the first three bounces after impacting with the return walls  
266 were studied based on the observations of Sorrentino et al. (2011), who found that the  
267 experimental coefficient of restitution of the third impact calculated using the Housner (1963)  
268 formulation was generally the closest to  $e_{1SR}$ . The formulation for the experimental coefficient

269 can be seen in equation 4, where  $|\theta_n|$  is the maximum absolute rotation at the  $n^{\text{th}}$  impact and

270  $\eta = \tan^{-1}(b/h)$ .

$$e_{exp} = \sqrt[2n]{\frac{1 - \left(1 - \frac{|\theta_n|}{\eta}\right)^2}{1 - \left(1 - \frac{|\theta_0|}{\eta}\right)^2}} \quad (4)$$

271

272 Figure 4a shows the values of  $\beta_{1SR}$  used as part of the parametric study for various values of  $h$   
 273 and a constant value of  $b = 0.11$  m. The values to parametrise  $\beta_{1SR}$  were chosen ranging from  
 274 a maximum expected for a single rocking block as derived from DeJong (2009) and a minimum  
 275 obtained for two blocks vertically stacked obtained using the same principles as DeJong (2009)  
 276 recommended. Assuming from Sorrentino et al. (2011) that  $e_{exp}$  is between 64% and 72% of  
 277  $e_{1SR}$  on the first impact, between 74% and 86% on the second impact, and that on the third  
 278 impact  $e_{exp}$  is similar to  $e_{1SR}$ , the best fitting free rocking results of DEM simulations using  
 279  $\beta_{1SR}$  are highlighted in Figure 4a. To calculate the best fitting results,  $e_{exp}$  (Equation 4) was  
 280 computed from the results of the DEM simulations and compared to the aforementioned  
 281 percentages of  $e_{1SR}$  (Equation 3) on the first, second and third bounce. The best fitting trends  
 282 for different  $h/b$  of façades with  $b = 0.11, 0.3$  m and  $E = 1800, 4050$  MPa, as observed in Figure  
 283 4b, where studied to obtain a dimensional regression for estimating  $\beta_{1SR}$  as follows:

284

$$\beta_{1SR} = \left( \frac{b}{384} - \frac{E}{5.6 \times 10^6} + 0.00074 \right) \cdot e^{\left( \frac{h}{b} \cdot \frac{-E+6572}{1.2 \times 10^5} \right)} \quad (5)$$

285

286 wherein  $b$  has units of m, and  $E$  of MPa.

287

(a) Values of  $\beta_{1SR}$  for walls with  $b = 0.11$  m,  $E = 1800$  MPa and different values of  $h/b$ .

(b) Best fitting  $\beta_{1SR}$  for different values of  $E$  and geometry variations

288

Figure 4: Parametric studies.

289



290 The experimental setup reported by Sorrentino et al. (2011) and by Shawa et al. (2012) were  
 291 used to validate the predictions made using equation 5. Sorrentino et al. (2011) tested a wall of  
 292 0.8 m height, 0.11 m thickness,  $E = 1800$  MPa and density =  $1750$  kg/m<sup>3</sup> built with solid clay  
 293 bricks together with two return walls disconnected with a dry joint (see Figure 5a). The tested  
 294 wall was displaced from the return walls, released and allowed to rock in order to study the  
 295 attenuation caused by the impacts against the return walls. Observing the time-history plots in  
 296 Figure 5b,c, the simulation performed using  $\beta_{1SR} = 0.00025$  was found to be closest to the  
 297 experiments during the first bounce, but from the values of  $e_{exp}$  plotted in Figure 5d the  
 298 simulation with  $\beta_{1SR} = 0.001$  showed similar damped motion as the experiments during the  
 299 second and third bounces. The value of  $\beta_{1SR}$  obtained using equation 5 was 0.00095, which was  
 300 deemed an acceptable result for being only 5% different from  $\beta_{1SR} = 0.001$ . After the first  
 301 bounce of simulations with  $\beta_{1SR} = 0.000125$ , 0.00025, 0.0005 and 0.001 the majority of the  
 302 energy was dissipated and the second and third bounces remained mostly undamped.  
 303 Alternatively, simulations with  $\beta_{1SR} = 0.002$ , 0.004 and 0.0058 were found to be overdamped  
 304 after the first impact.

(a) Experimental setup (Sorrentino et al., 2011)

(b) Complete time-history

(c) Expanded view between 0.9 s and 1.9 s

(d) Values of  $e_{exp}$  for different simulations using  $\beta_{1SR}$  compared to  $e_{1SR}$  and experimental testing

Figure 5: Experimental testing from Sorrentino et al. (2011) and simulations using 3DEC

307 Shawa et al. (2012) subjected a 3 m high and 0.25 m thick tuff masonry wall to four earthquakes  
 308 named BagnirWE, SturWE, R1168EW and CalitWE (more information can be found in Shawa  
 309 et al. (2012)). The wall was constructed with return walls to perform 1SR and a physical gap  
 310 of 4 mm was reported between the return walls and the overturning wall. During repeated  
 311 rocking testing debris accumulated under the wall and prevented the wall from returning to its

312 initial vertical position, such that on repeated shaking the wall had rotated ( $\varphi$ ) initial geometry.  
 313 The shake table experimental campaign performed by Shawa et al. (2012) was simulated using  
 314 SP damping and the reported material properties of  $E = 4050 \text{ MPa}$  ( $j_{kn} = 1350 \text{ MPa/m}$ ) and  
 315 density= $1810 \text{ kg/m}^3$ , with care taken to account for the reported geometrical irregularities due  
 316 to debris lodged beneath the wall (Figure 6a,b). It is worth noting that Shawa et al. (2012) was  
 317 capable of reproducing the 1SR tests with a model based on Housner (1963) approach taking  
 318 into account the multiple impacts energy dissipation for the coefficient of restitution and with  
 319 a 2D DEM model with SP damping. Applying equation 5,  $\beta_{1SR}$  was found to be 0.00086, a very  
 320 similar value as the one found by trial and error by Shawa et al. (2012) ( $\beta_{1SR} = 0.00094$ ) for a  
 321 test of the early stages of the testing sequence. In later stages the value of damping increased  
 322 as a consequence of damage due to mortar deterioration and cracking due to the impact with  
 323 the return walls (Shawa et al., 2012). Using equation 5 as a starting point for the assignation of  
 324  $\beta_{1SR}$ , the damping value was slowly increased until reaching a good agreement between  
 325 experimental observations and 3D DEM as seen in Figure 6. When simulating the four  
 326 earthquakes Shawa et al. (2012) found similar values for  $\beta_{1SR}$ . Although equation 5 appears to  
 327 deliver reasonable estimation of  $\beta_{1SR}$ , additional research is required to achieve a robust  
 328 criterion for selection of a damping value that accounts for diffevre boundary conditions  
 329 including free-standing blocks.

(a) DEM model

(b) Diagram of tilted rocking facade

(c) BagnirWE,  $\varphi = 0.191^\circ$ (d) SturWE,  $\varphi = 0.210^\circ$ (e) R1168EW,  $\varphi = 0.272^\circ$ (f) CalitWE,  $\varphi = 0.348^\circ$ 

330 Figure 6: Comparison of top displacement between experiments and DEM simulation.

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#### 4. DIFFERENT BOUNDARY CONDITIONS FOR ROCKING FAÇADES

332 The rocking interaction between the façade (dimensions in Figure 1a) and the case study URM  
 333 building was simulated using three structural configurations and two variations in material  
 334 characteristics to arrive at five different boundary condition scenarios that utilised springs (see  
 335 Figure 7) calibrated to replicate the deformation of the return wall. First, each return wall was  
 336 modelled using a single cohesionless block behind the façade and subjected to the same ground  
 337 motion as at the façade base (W1, see Figure 7a). Secondly, each return wall was modelled  
 338 using four cohesionless blocks positioned at the back of the façade, with the ground motion  
 339 applied to the base and with the amplified motions extracted from each lumped mass of the  
 340 4DOF model being input to each block positioned above the base (W2, see Figure 7b).  
 341 Considering the minimal differences between 4DOF and 5DOF models (Table 2), the use of  
 342 the former model was considered adequate. The aforementioned models were also simulated  
 343 with 0.1 MPa cohesion and 0.07 MPa tensile strength (Giaretton et al., 2016) on the contact  
 344 interface between the façade and the return walls. Finally, a more detailed model that accounted  
 345 for friction (friction angle =  $30^\circ$ ) with no cohesion between the façade and the return walls,  
 346 and including the 4DOF amplified motions, was modelled (W3, see Figure 7c). All façade  
 347 configurations were modelled using a value of  $\beta_{1SR} = 0.00276$  according to equation 5.

(a) W1

(b) W2

(c) W3

349 Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction  
 350 scheme (Colours represent displacement magnitude without scale).

351

352 Elements representing connection between the façade and the return walls in W3 were carefully  
 353 modelled to avoid increments in the overturning resistance being in excess of a realistic  
 354 contribution of masonry interlocking. There were two crucial phenomena to keep in mind while  
 355 assigning the degrees of freedom to the return wall bricks: (1) friction forces resulting on the

356 brick interfaces are proportional to the weight from the upper bricks while the façade is rotating  
357 with respect to the return wall, (2) the interlocked bricks of the return walls need to rotate in  
358 opposite directions to form the crack between the façade and return walls (see Figure 8b).  
359 Therefore, during the induced motion each brick in the return wall (Figure 8a) had free  
360 translational movement in the  $Z$  (vertical) direction and free rotation around the  $X$  axis, while  
361 having the remaining degrees of freedom restricted and being capable of moving horizontally  
362 with the amplified input motion. Low values of the normal and shear stiffnesses of the  
363 connection between the façade and the return walls were chosen in order to reduce  
364 computational time ( $j_{kn} = 1,100$  MPa/m and  $j_{ks} = 440$  MPa/m). These values did not influence  
365 the final capacity of the wall but only the stiffness of the connection (Mordanova et al., 2017,  
366 Pulatsu et al., 2020), which was suitable for this study. Due to the rigidity of the façade and  
367 return wall elements, the lower wall courses remained connected by their edges when the  
368 rotation was well advanced, resulting in a non-realistic resistance. For this reason, lower  
369 elements of the return walls were modelled with an inclined interface to allow breaking in shear  
370 (see lower courses of Figure 8a). The partial elements from the façade (see Figure 8a) were  
371 connected using an interface with appropriate material properties for fired clay bricks to allow  
372 for deformation and detachment if needed. In order to reduce the computation time, the least  
373 possible number of elements were included by modelling only one column of return wall  
374 bricks, and by increasing and modifying the dimensions of the bricks compared to what are  
375 usually used in construction ( $0.25L \times 0.324H \times 0.375W$  m<sup>3</sup>). Prior to running the dynamic  
376 simulation a pushover test was performed to confirm that the boundary conditions were  
377 realistically modelled. Figure 8c shows good agreement between kinematic capacity curves  
378 developed by Casapulla et al. (2019b) and the DEM simulation.

(a) Elements in W3

(b) Scheme of generic façade and  
return wall separation(c) Comparison between kinematic and  
DEM capacity curve

379

Figure 8: W3 details and capacity curve

380

381 The five different models shown in Figure 7 were subjected to the four earthquakes presented  
382 in Table 1 in the positive (+) and negative (-) directions, with increasing acceleration. The  
383 dynamic capacity curve of each wall configuration in terms of PGA and maximum top  
384 displacement ( $d$ ) of the façade was normalized by the wall width ( $w$ ) and is plotted in Figure  
385 9, showing that W2 tended to be equally vulnerable or more vulnerable than W1. Though by a  
386 very small margin, for BUCAR0-, BUCAR0+, 1ST280-, HVSC1+ and RRS228+ the dynamic  
387 behaviour of the building caused a slightly more vulnerable capacity for W2. Conversely, the  
388 dynamic amplification of the building caused a marked reduction in façade capacity for the  
389 remainder of the cases. When cohesion was added to the W1 model a clear increment in  
390 resisting acceleration was observed, while when cohesion was included in the W2 wall the  
391 motion of the return walls made the rigid façade detach at an early stage, resulting in premature  
392 collapse. The dynamic incremental capacity curve of W3 highlights the importance of  
393 interlocking on out-of-plane response. BUCAR0 and RRS228 records both feature a large pulse  
394 and resulted in IDA curves that matched well with the pushover curve obtained via quasi static  
395 analysis undertaken by Casapulla et al. (2019b). The results from 1ST280 and HVSC1 showed  
396 that the capacity of rocking blocks when subjected to earthquakes can be much higher than  
397 obtained from equivalent-static pushover.

398

BUCAR0

RRS228

1ST280

HVSC1

399 Figure 9: Incremental dynamic analyses (Positive (+) direction – Left; Negative (-) direction – Right.  
400 Pushover curves in grey for reference).

401

## 402 **5. CONCLUSIONS**

403 Nonlinear incremental dynamic analyses were performed using the three dimensional discrete  
404 element method to investigate the vulnerability of a rocking façade as part of a two-storey case  
405 study unreinforced masonry building. Due to computational restrictions and simulation time  
406 practicality, dynamically equivalent strategies were developed to simulate the building  
407 behaviour associated with amplification of the ground motion and interaction between the  
408 rocking façade and the return walls. The adopted strategy consisted of developing a 4 degrees  
409 of freedom spring-mass model of the two-storey unreinforced masonry building that was used  
410 to efficiently simulate several time-history analyses and obtain the amplified motion. The  
411 amplified motion of each lumped mass was applied to a discrete element method model of the  
412 rocking façade with different boundary conditions. The equivalence between a full-scale 3D  
413 discrete element method discretised model and the 4 degrees of freedom substitute structure  
414 was demonstrated by subjecting both models to earthquake ground motions and comparing the  
415 time history performance of the building motion, the spectral acceleration and the frequency  
416 power spectra.

417 In order to assure that the rocking simulations of facades using discrete element method were  
418 performed accurately, a formulation was developed to assign an appropriate value of damping  
419 following a stiffness proportional damping approach. The capability of the developed  
420 formulation to be used for any façade independent of the geometry and material properties was  
421 validated using experimental free rocking and earthquake shake-table testing.

422 Comparisons of the incremental dynamic capacity curves showed that modelling the behaviour  
423 of the building, as opposed to the usual strategy of having non-amplified excitation of the return  
424 walls, can reduce the capacity of the façade by as much as 60% depending on the characteristics  
425 of the earthquake. This observation implies that when nonlinear time-history analyses are used  
426 to assess the overturning vulnerability of one-sided rocking parts, the capacity of such elements  
427 may be overestimated. Furthermore, applying amplified motion to return walls having cohesive  
428 bond resulted in premature collapse of the façade such that this strategy is not advised to be  
429 used in future studies. On the contrary, detailed modelling of the interlocking elements of the  
430 façade with the return walls that were subjected to amplified motion was found to considerably  
431 increase the capacity of the façades, depicting greater capacity than shown via the pushover  
432 analysis. While the capacity value will change depending on the level of connectivity between  
433 the return walls and the façade, the increment in capacity due to rocking behaviour will remain.  
434 Lower bricks of the return walls in the detailed model were designed specifically for the  
435 problem studied herein. Further research of the rocking problem is required to address stress  
436 concentration occurring in lower rigid elements due to the overturning moment, because this  
437 concentration could lead to an overestimation of the rocking capacity.

438

439 **Data Availability Statement.** Some or all data, models, or code that support the findings of  
440 this study are available from the corresponding author upon reasonable request.

441

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448

449 **REFERENCES**

450 Abeling, S. and J. M. Ingham (2020). "Volume loss fatality model for as-built and retrofitted  
451 clay brick unreinforced masonry buildings damaged in the 2010/11 Canterbury earthquakes."  
452 Structures **24**: 940-954.

453 AlShawa, O., D. Liberatore and L. Sorrentino (2019). "Dynamic One-Sided Out-Of-Plane  
454 Behavior of Unreinforced-Masonry Wall Restrained by Elasto-Plastic Tie-Rods." International  
455 Journal of Architectural Heritage **13**(3): 340-357.

456 AS1170.4 (2007). Structural design actions, Part 4: Earthquake actions in Australia. , Standards  
457 Australia.,

458 Aslam, M., W. G. Godden and D. T. Scalise (1975). Sliding Response of Rigid Bodies to  
459 Earthquake Motions. A Report of an Analytical and Experimental Study of the Sliding  
460 Response of Rigid Blocks Under Simultaneous Horizontal and Vertical Earthquake  
461 Accelerations. Contract No. W-7405-eng-48. U. S. E. R. a. D. Administration.,



- 462 Bao, Y. and D. Konstantinidis (2020). "Dynamics of a sliding- rocking block considering  
463 impact with an adjacent wall." Earthquake Engineering & Structural Dynamics **49**(5): 498-523.
- 464 Bathe, K. J. and E. L. Wilson (1976). Numerical methods in finite element analysis. NJ,  
465 Prentice-Hall.
- 466 Benito, B., R. Capote, P. Murphy, J. M. Gaspar-Escribano, J. J. Martinez-Diaz, M. Tsige, D.  
467 Stich, J. Garcia-Mayordomo, M. J. Garcia Rodriguez, M. E. Jimenez, J. M. Insua-Arevalo, J.  
468 A. Alvarez-Gomez and C. Canora (2007). "An Overview of the Damaging and Low Magnitude  
469 Mw 4.8 La Paca Earthquake on 29 January 2005: Context, Seismotectonics, and Seismic Risk  
470 Implications for Southeast Spain." Bulletin of the Seismological Society of America **97**(3):  
471 671-690.
- 472 Bruneau, M. (1994). "State-of-the-Art Report on Seismic Performance of Unreinforced  
473 Masonry Buildings." Journal of Structural Engineering, **120**(1): 230-251.
- 474 Casapulla, C., L. Giresini, L. U. Argiento and A. Maione (2019a). "Nonlinear Static and  
475 Dynamic Analysis of Rocking Masonry Corners Using Rigid Macro-Block Modeling."  
476 International Journal of Structural Stability and Dynamics, 10.1142/s0219455419501372:  
477 1950137.
- 478 Casapulla, C., L. Giresini and P. B. Lourenço (2017). "Rocking and Kinematic Approaches for  
479 Rigid Block Analysis of Masonry Walls: State of the Art and Recent Developments." Buildings  
480 **7**(3): 69.
- 481 Casapulla, C. and A. Maione (2017). "Free Damped Vibrations of Rocking Rigid Blocks as  
482 Uniformly Accelerated Motions." International Journal of Structural Stability and Dynamics  
483 **17**(06).

- 484 Casapulla, C., A. Maione and L. U. Argiento (2019b). "Performance-Based Seismic Analysis  
485 of Rocking Masonry Façades Using Non-Linear Kinematics with Frictional Resistances: A  
486 Case Study." International Journal of Architectural Heritage,  
487 10.1080/15583058.2019.1674944.
- 488 Cattari, S., D. Camilletti, G. Magenes, C. F. Manzini, P. Morandi, E. Spacone, F. Cannizzaro,  
489 B. Calderoni, G. Camata, C. Marano, G. Occhipinti and A. E. S. Cordasco (2018). A  
490 Comparative Study On a 2-Storey Benchmark Case Study Through Nonlinear Seismic  
491 Analysis. 16th European Conference on Earthquake Engineering. Thessaloniki, Greece,
- 492 Degli Abbatì, S., S. Cattari and S. Lagomarsino (2018). "Theoretically-based and practice-  
493 oriented formulations for the floor spectra evaluation." Earthquakes and Structures **15**(5): 565-  
494 581.
- 495 DeJong, M. J. (2009). Seismic Assessment Strategies for Masonry Structures. Doctor of  
496 Philosophy in Architecture: Building Technology. PhD Dissertation, MASSACHUSETTS  
497 INSTITUTE OF TECHNOLOGY.
- 498 Derakhshan, H., D. Y. Dizhur, M. C. Griffith and J. M. Ingham (2014). "Seismic Assessment  
499 of Out-of-plane Loaded Unreinforced Masonry Walls in Multi-Storey Buildings." Bulletin of  
500 the New Zealand Society for Earthquake Engineering **47**(2): 120-138.
- 501 Dimitri, R., L. De Lorenzis and G. Zavarise (2011). "Numerical study on the dynamic behavior  
502 of masonry columns and arches on buttresses with the discrete element method." Engineering  
503 Structures **33**(12): 3172-3188.
- 504 Dizhur, D., N. Ismail, C. Knox, R. Lumantarna and J. M. Ingham (2010). "Performance of  
505 Unreinforced and Retrofitted Masonry Buildings During the 2010 Darfield Earthquake."  
506 Bulletin of the New Zealand Society for Earthquake Engineering **43**(4): 321-339.

- 507 ElGawady, M. A., Q. Ma, J. W. Butterworth and J. Ingham (2011). "Effects of interface  
508 material on the performance of free rocking blocks." Earthquake Engineering & Structural  
509 Dynamics **40**(4): 375-392.
- 510 Eurocode 8 (2004). Design of structures for earthquake resistance. Part 1: General rules,  
511 seismic actions and rules for buildings. Brussels, Belgium: CEN., European Committee for  
512 Standardization,
- 513 Galvez, F., S. Segatta, M. Giaretton, K. Q. Walsh, I. Giongo and D. Dizhur (2018). FE and DE  
514 Modelling of Out-Of-Plane Two-Way Bending Behaviour of Unreinforced Masonry Walls.  
515 16th European Conference on Earthquake Engineering. Thessaloniki, Greece,
- 516 Giaretton, M., D. Dizhur and J. M. Ingham (2016). "Dynamic testing of as-built clay brick  
517 unreinforced masonry parapets." Engineering Structures **127**: 676-685.
- 518 Giresini, L. (2017). Design Strategy for the Rocking Stability of Horizontally Restrained  
519 Masonry Walls. Proceedings of the 6th International Conference on Computational Methods  
520 in Structural Dynamics and Earthquake Engineering (COMPDYN 2015),  
521 10.7712/120117.5620.18188: 2963-2979
- 522 Giresini, L., C. Casapulla, R. Denysiuk, J. Matos and M. Sassu (2018). "Fragility curves for  
523 free and restrained rocking masonry façades in one-sided motion." Engineering Structures **164**:  
524 195-213.
- 525 Giresini, L., M. Fragiaco and P. B. Lourenço (2015). "Comparison between rocking analysis  
526 and kinematic analysis for the dynamic out-of-plane behavior of masonry walls." Earthquake  
527 Engineering & Structural Dynamics **44**(13): 2359-2376.

- 528 Giresini, L. and M. Sassu (2016). "Horizontally restrained rocking blocks: evaluation of the  
529 role of boundary conditions with static and dynamic approaches." Bulletin of Earthquake  
530 Engineering **15**(1): 385-410.
- 531 Giresini, L., F. Solarino, O. Paganelli, D. V. Oliveira and M. Froli (2019). "ONE-SIDED  
532 rocking analysis of corner mechanisms in masonry structures: Influence of geometry, energy  
533 dissipation, boundary conditions." Soil Dynamics and Earthquake Engineering **123**: 357-370.
- 534 Gubana, A. and M. Melotto (2019). "Discrete-element analysis of floor influence on seismic  
535 response of masonry structures." Proceedings of the Institution of Civil Engineers - Structures  
536 and Buildings, 10.1680/jstbu.19.00099: 1-30.
- 537 Hess, R. L. (2007). Impacts of a M7.8 Southern San Andreas Earthquake on Unreinforced  
538 Masonry (URM) Buildings. The ShakeOut Scenario, Supplemental Study - Unreinforced  
539 Masonry (URM) Buildings, United States Geological Survey & California Geological Survey,
- 540 Housner, G. W. (1963). "The Behavior of Inverted Pendulum Structures During Earthquakes."  
541 Bulletin of the Seismological Society of America **53**(2): 403-417s.
- 542 Ishiyama, Y. (1982). "Motions of Rigid Bodies and Criteria For Overturning by Earthquake  
543 Excitations." Earthquake Engineering and Structural Dynamics **10**: 635-650.
- 544 Itasca (2013). 3DEC (3-Dimensional distinct element code). Minneapolis (MN)
- 545 Jing, L. and O. Stephansson (2007). Explicit Discrete Element Method for Block Systems –  
546 the Distinct Element Method. Fundamentals of Discrete Element Methods for Rock  
547 Engineering. **85**: 235-316.
- 548 Kalliontzis, D., S. Sritharan and A. Schultz (2016). "Improved Coefficient of Restitution  
549 Estimation for Free Rocking Members." Journal of Structural Engineering **142**(12): 06016002.

- 550 Lagomarsino, S. (1998). A new methodology for the post-earthquake investigation of ancient  
551 churches. 11th European Conference on Earthquake Engineering. Balkema, Rotterdam,
- 552 Lagomarsino, S. (2015). "Seismic assessment of rocking masonry structures." Bulletin of  
553 Earthquake Engineering **13**(1): 97-128.
- 554 Lemos, J. V. and A. Campos Costa (2017). "Simulation of Shake Table Tests on Out-Of-Plane  
555 Masonry Buildings. Part (V): Discrete Element Approach." International Journal of  
556 Architectural Heritage **11**(1): 117-124.
- 557 Lipo, B. and G. de Felice (2016). Smooth-Rocking Oscillator under Natural Accelerograms.  
558 ECCOMAS Congress 2016 - VII European Congress on Computational Methods in Applied  
559 Sciences and Engineering. V. P. M. Papadrakakis, G. Stefanou, V. Plevris (eds. Crete Island,  
560 Greece,
- 561 Lipscombe, P. R. and S. Pellegrino (1993). "Free Rocking of Prismatic Blocks." Journal of  
562 Engineering Mechanics **119**(7): 1387-1410.
- 563 Lorig, L. J. and R. D. Hart (1993). Block dynamics modeling in support of seismic design.  
564 Dynamic Analysis and Design Considerations for High-Level Nuclear Waste Repositories. H.  
565 Q. (ed.). New York, ASCE:: 228-242
- 566 MathWorks (2008). "MATLAB User's Manual. ."
- 567 McNeel, R. and Associates (2014). Rhinoceros 5,
- 568 Menon, A. and G. Magenes (2011). "Definition of Seismic Input for Out-of-Plane Response of  
569 Masonry Walls: I. Parametric Study." Journal of Earthquake Engineering **15**(2): 165-194.

- 570 Meriggi, P., G. de Felice, S. De Santis, F. Gobbin, A. Mordanova and B. Pantò (2019). "Distinct  
571 Element Modelling of Masonry Walls under Out-Of-Plane Seismic Loading." International  
572 Journal of Architectural Heritage **13**(7): 1110-1123.
- 573 Mordanova, A., G. de Felice and A. Genoese (2017). Assessment of the stability condition of  
574 the claudio aqueduct. 3rd International Conference on Protection of Historical Constructions.  
575 Lisbon, Portugal,
- 576 NZS1170.5 (2004). Structural design actions, Part 5: earthquake actions—New Zealand. NZS  
577 1170.5:2004. Standards New Zealand (NZS), Standards New Zealand (NZS) Technical  
578 Committee BD-006-04-11, Wellington, New Zealand.,
- 579 Olcese, S. (2018). Use of discrete element modelling for the seismic assessment of a benchmark  
580 case study masonry structure. Corso di Laurea in Ingegneria Edile – Architettura. Master  
581 Thesis, Università degli Studi di Genova.
- 582 Papantonopoulos, C., I. N. Psycharis, D. Y. Papastamatiou, J. V. Lemos and H. P. Mouzakis  
583 (2002). "Numerical prediction of the earthquake response of classical columns using the  
584 distinct element method." Earthquake Engineering & Structural Dynamics **31**(9): 1699-1717.
- 585 Priestley, M. J. N. (1985). "Seismic Behaviour of Unreinforced Masonry Walls." Bulletin of  
586 The New Zealand National Society for Earthquake Engineering **18**(2).
- 587 Psycharis, I. N. and P. C. Jennings (1983). "Rocking of Slender Rigid Bodies Allowed to  
588 Uplift." Earthquake Engineering and Structural Dynamics **11**: 57-76.
- 589 Psycharis, I. N., J. V. Lemos, D. Y. Papastamatiou, C. Zambas and C. Papantonopoulos (2003).  
590 "Numerical study of the seismic behaviour of a part of the Parthenon Pronaos." Earthquake  
591 Engineering & Structural Dynamics **32**(13): 2063-2084.

- 592 Pulatsu, B., F. Gencer and E. Erdogmus (2020). "Study of the effect of construction techniques  
593 on the seismic capacity of ancient dry-joint masonry towers through DEM." European Journal  
594 of Environmental and Civil Engineering, 10.1080/19648189.2020.1824823: 1-18.
- 595 Shawa, O. A., G. de Felice, A. Mauro and L. Sorrentino (2012). "Out-of-Plane Seismic  
596 Behaviour of Rocking Masonry Walls." Journal of Earthquake Engineering and Structural  
597 Dynamics **5**(2): 253-271.
- 598 Sigurdsson, G. Ö., R. Rupakhety and S. Ólafsson (2019). "A Study of Rigid Blocks Rocking  
599 Against Rigid Walls." **47**: 323-334.
- 600 Sorrentino, L., O. AlShawa and L. D. Decanini (2011). "The relevance of energy damping in  
601 unreinforced masonry rocking mechanisms. Experimental and analytic investigations."  
602 Bulletin of Earthquake Engineering **9**(5): 1617-1642.
- 603 Sorrentino, L., S. Kunnath, G. Monti and G. Scalora (2008). "Seismically induced one-sided  
604 rocking response of unreinforced masonry façades." Engineering Structures **30**(8): 2140-2153.
- 605 Ther, T. and L. P. Kollár (2016). "Refinement of Housner's model on rocking blocks." Bulletin  
606 of Earthquake Engineering **15**(5): 2305-2319.
- 607 Tomassetti, U., F. Graziotti, L. Sorrentino and A. Penna (2019). "Modelling rocking response  
608 via equivalent viscous damping." Earthquake Engineering & Structural Dynamics,  
609 10.1002/eqe.3182.
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617 **Table 1:** Selected ground motions

Event - Station (Component)	Date	$M_w$	$D$ [km]	Soil Type	PGA [g]	PGV [cm/s]	PGD [cm]	$\Delta t$ [s]
Bucarest, Romania - Building Research Institute (BUCARO)	March/4/1977	7.5	150	D	0.21	73.6	24.4	16.2
Northridge, CA, USA - Rinaldi Receiving Station (RRS228)	January/17/1994	6.7	0.1	C	0.84	166.1	28.8	14.9
Nahanni, Canada - Site 1 (1ST280)	December/23/1985	6.8	0.1	A	1.1	46.1	14.6	20.5
Christchurch, NZ - Heathcote Valley School (HVSC1)	February/22/2011	6.3	4	C	1.58	106.6	21.77	29.0

 $M_w$  = Moment magnitude $D$  = Distance from the surface projection of the source

Soil type = According to Eurocode 8 (2004)

Peak ground: PGA = Acceleration, PGV = Velocity, PGD = Displacement

 $\Delta t$  = Duration

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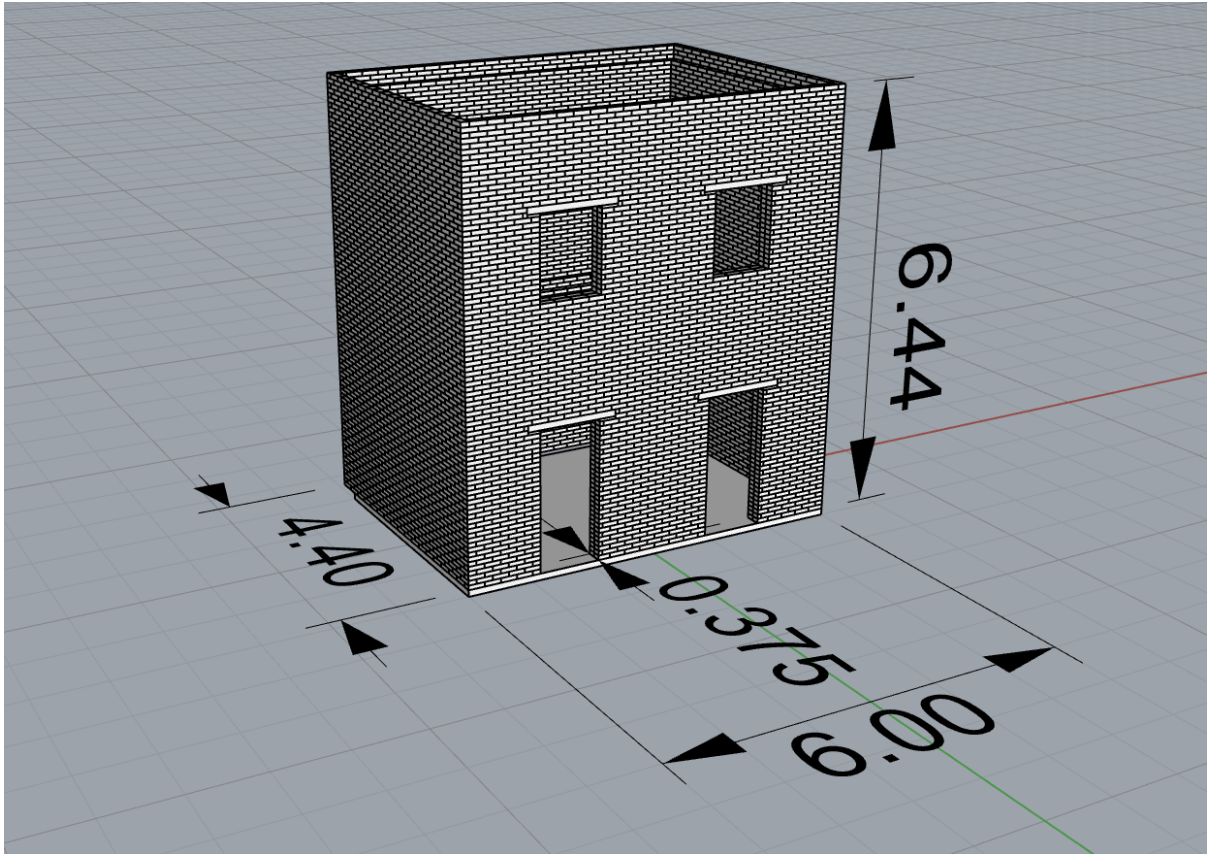


641

642 **Table 2:** Weighted Mean Error as in equation 10.

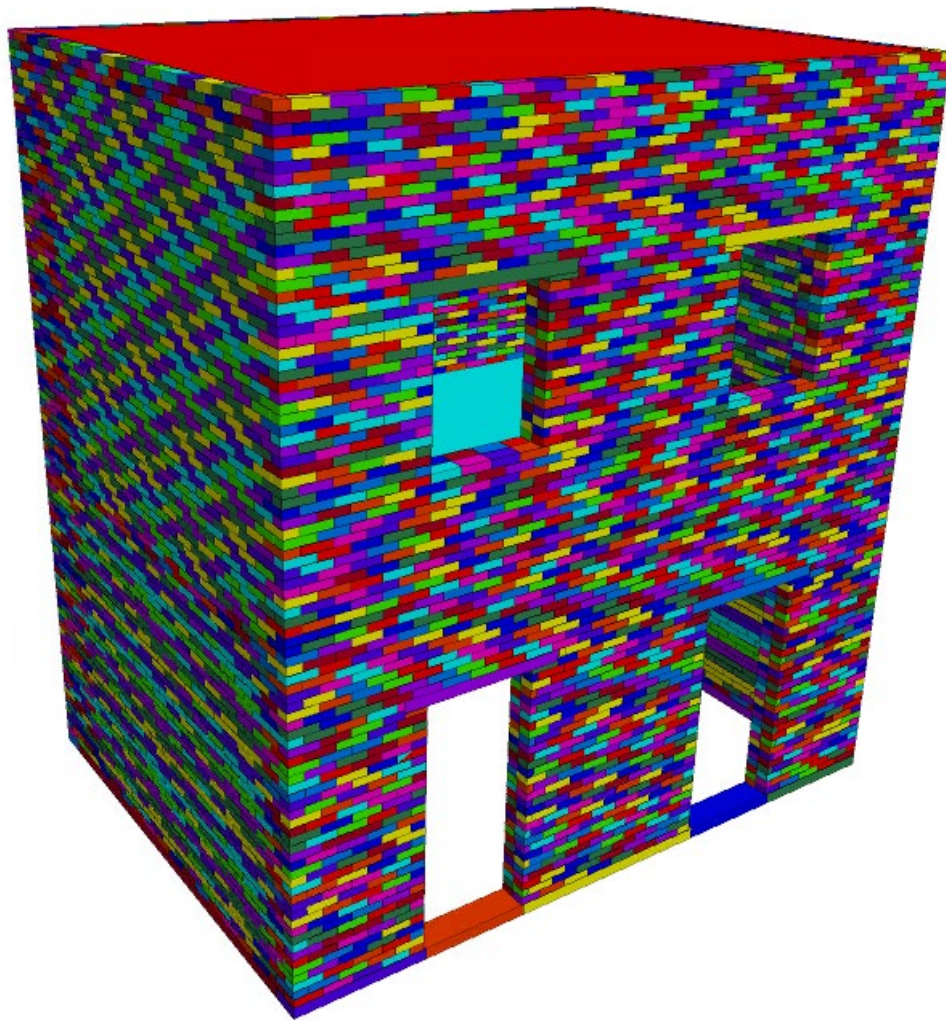
	SDOF		2DOF		3DOF		4DOF		5DOF	
	BUCAR0	RRS228	BUCAR0	RRS228	BUCAR0	RRS228	BUCAR0	RRS228	BUCAR0	RRS228
Acceleration Top Response	0.344	0.407	0.309	0.343	0.305	0.331	0.298	0.326	0.292	0.323
Response Spectra	0.140	0.152	0.082	0.093	0.063	0.074	0.056	0.066	0.062	0.062
Power Spectra	0.273	0.231	0.160	0.141	0.133	0.117	0.119	0.108	0.129	0.105

643



(a) Rhinoceros® (Dimensions in m)

Figure 1: Geometrical model of the two-storey case study building



(b) DEM

Figure 1: Geometrical model of the two-storey case study building

# Displacement [mm]

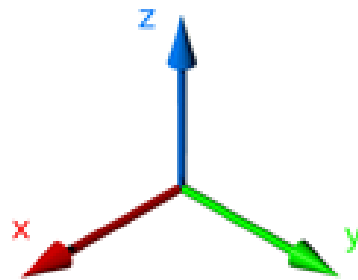
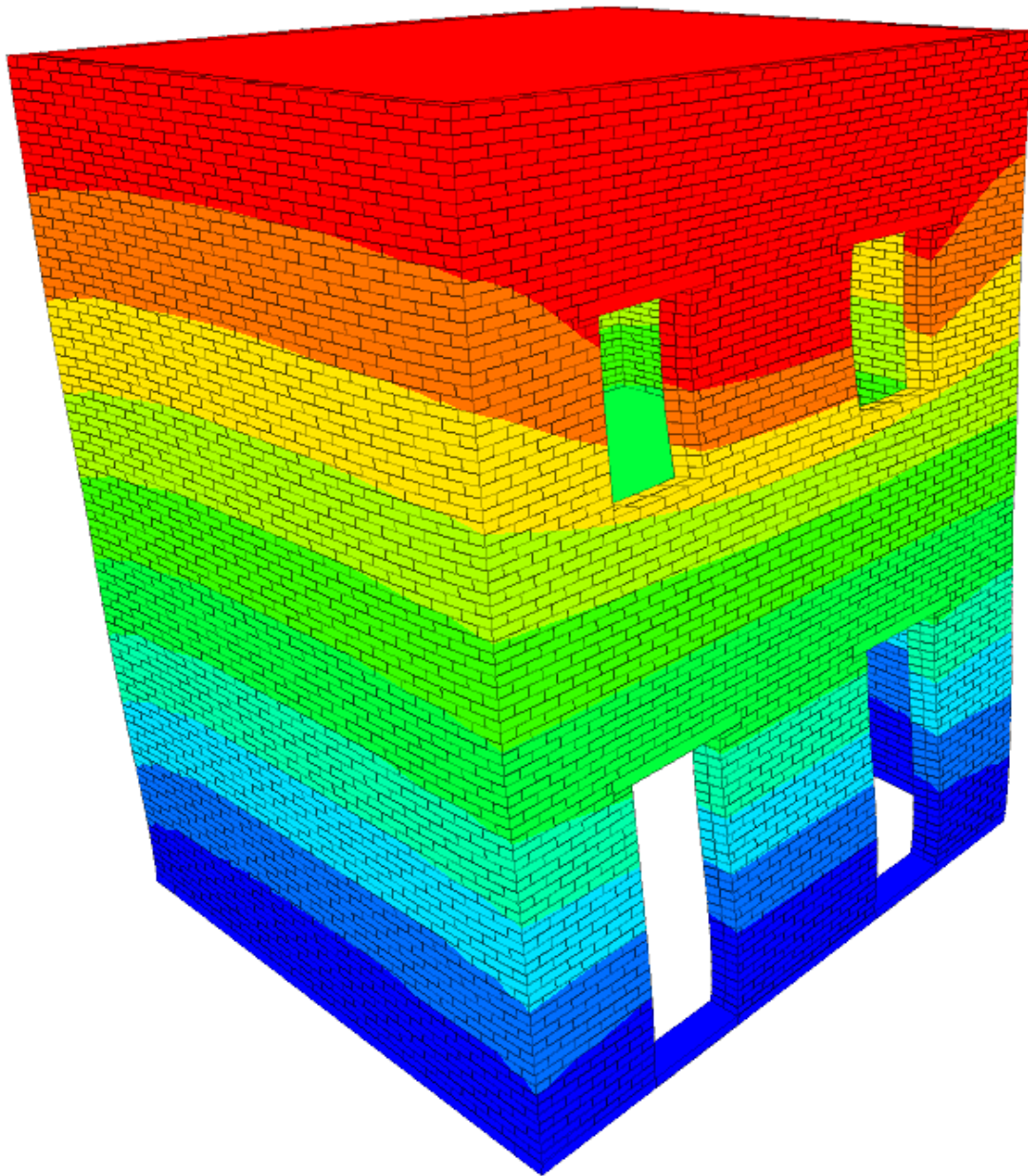


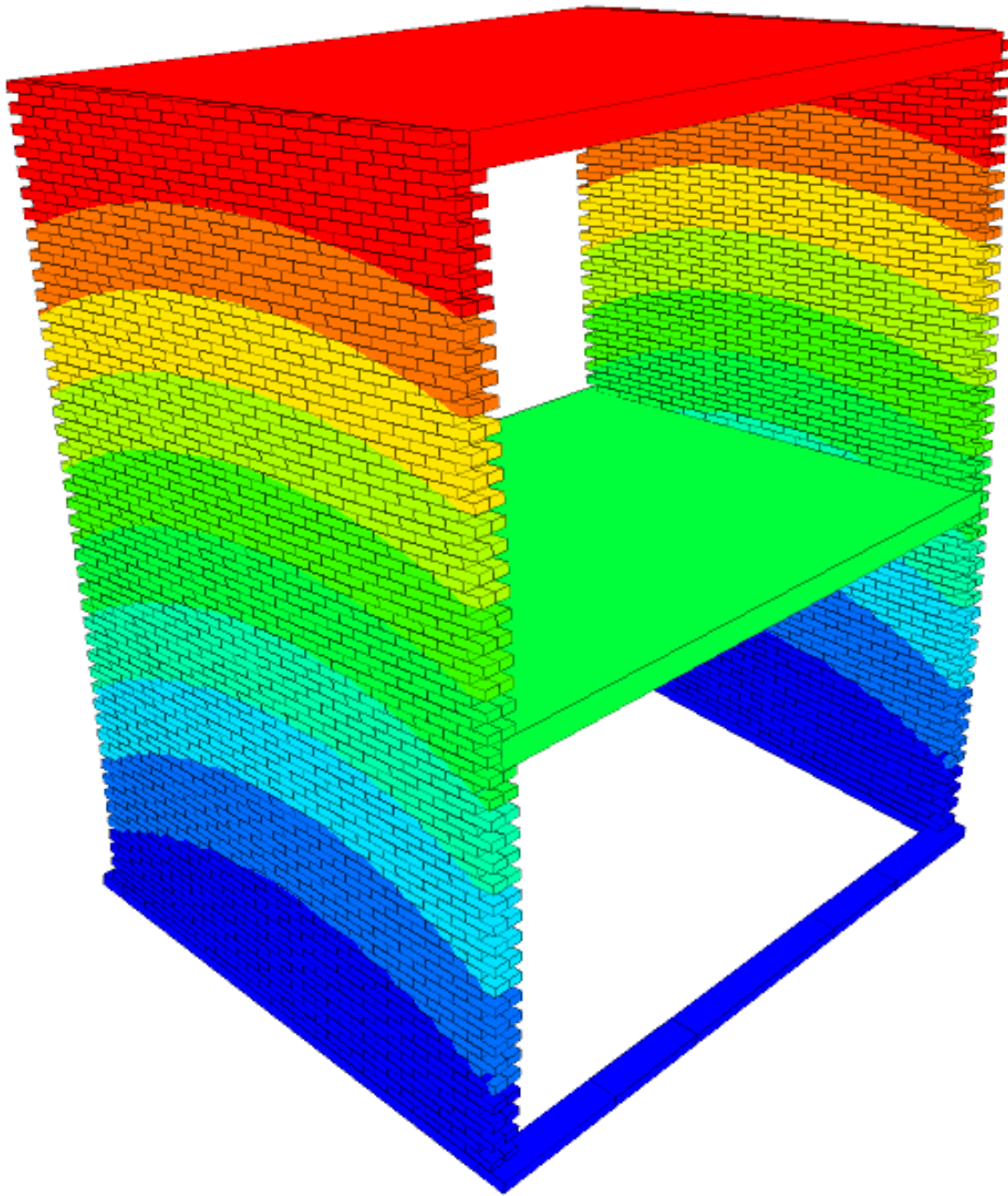
Figure 2: First and second modal shapes and periods in the Y direction of the complete and the simplified structure calculated using DEM modal analysis



(a) 1st mode of CBDEM ( $T=0.25$  s)

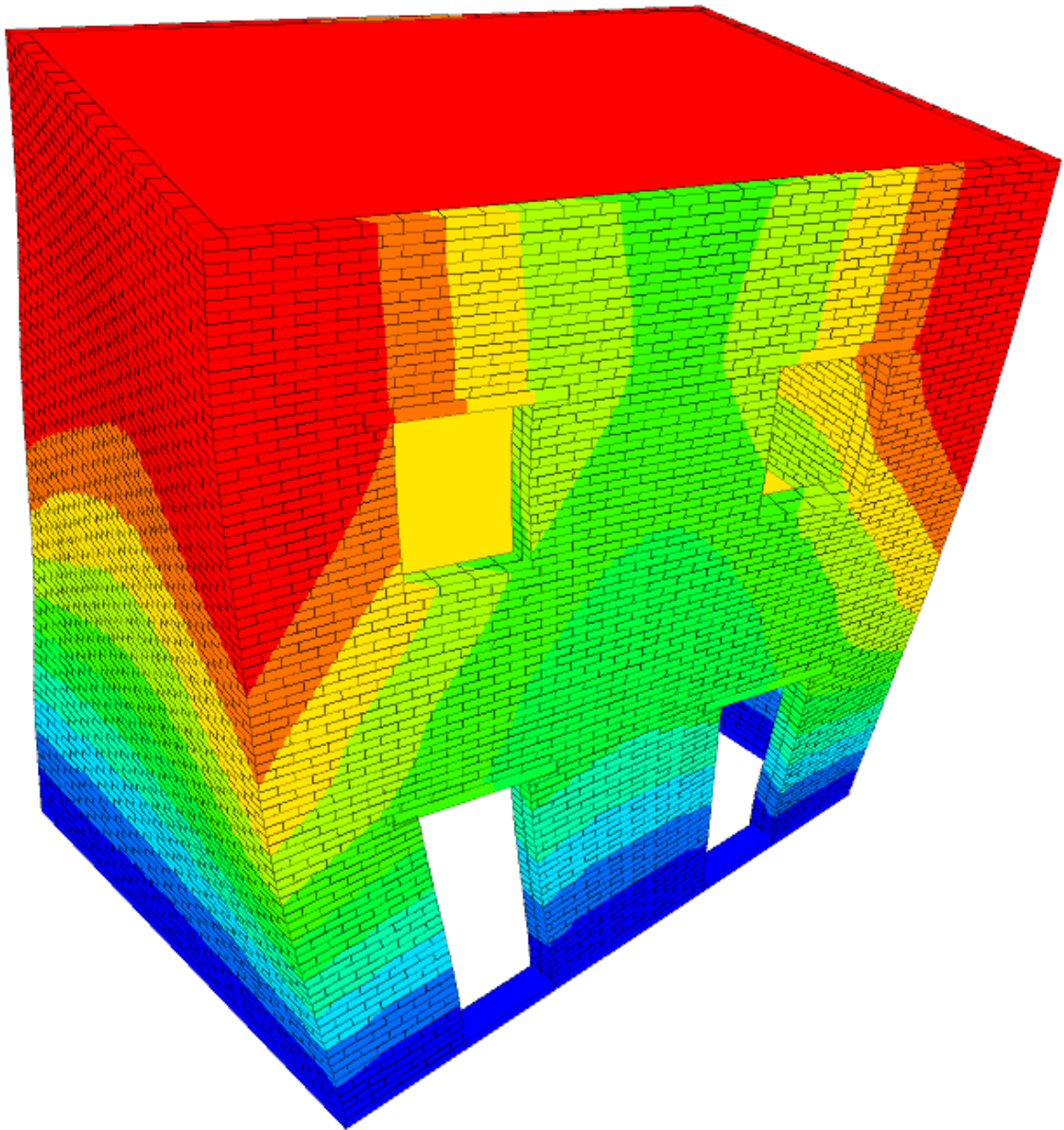
Figure 2: First and second modal shapes and periods in the  $Y$  direction of the complete and the simplified structure calculated using DEM modal analysis





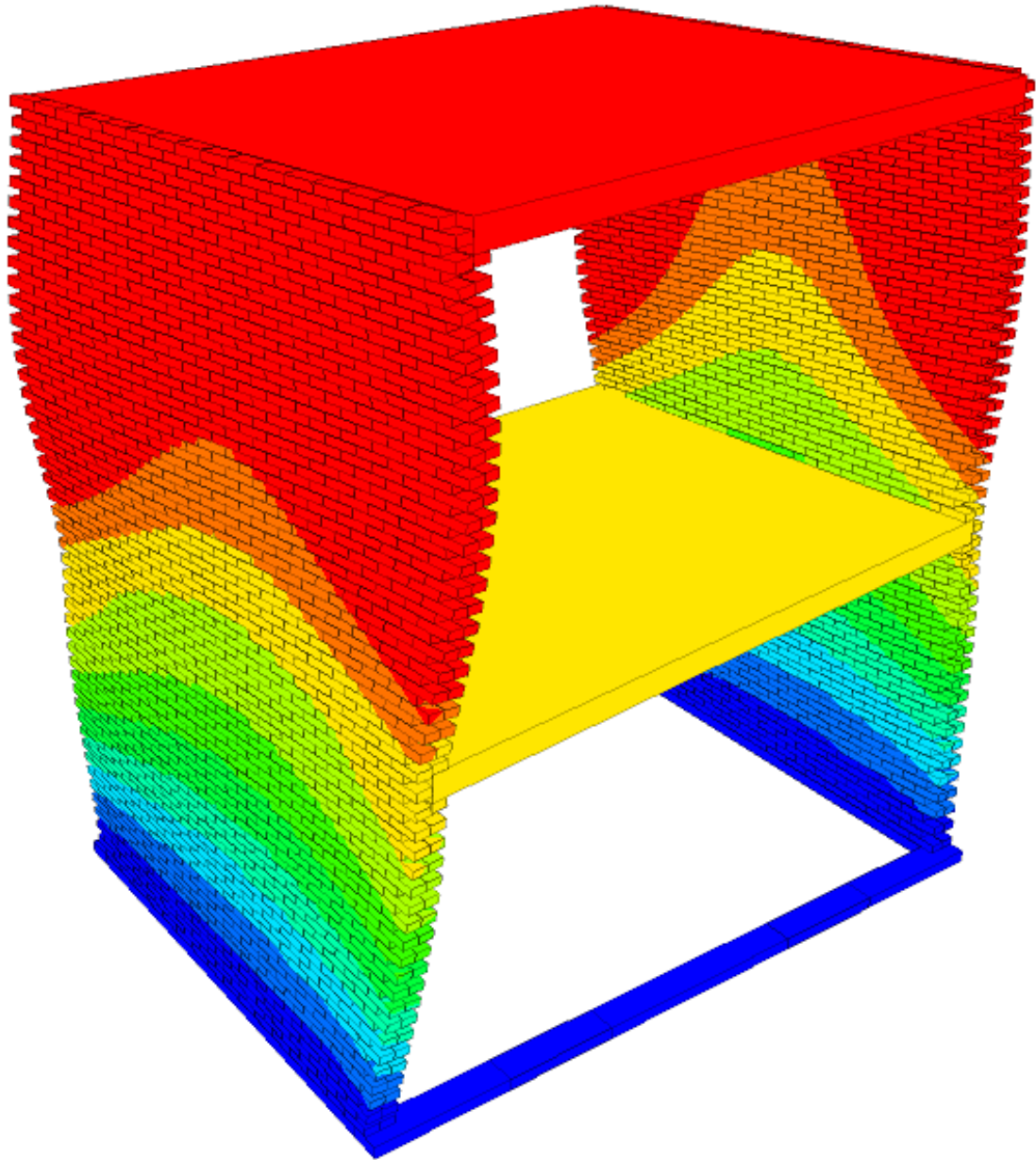
(b) 1st mode of SBDEM ( $T=0.27$  s)

Figure 2: First and second modal shapes and periods in the  $Y$  direction of the complete and the simplified structure calculated using DEM modal analysis



(c) 2nd mode of CBDEM ( $T=0.16$  s)

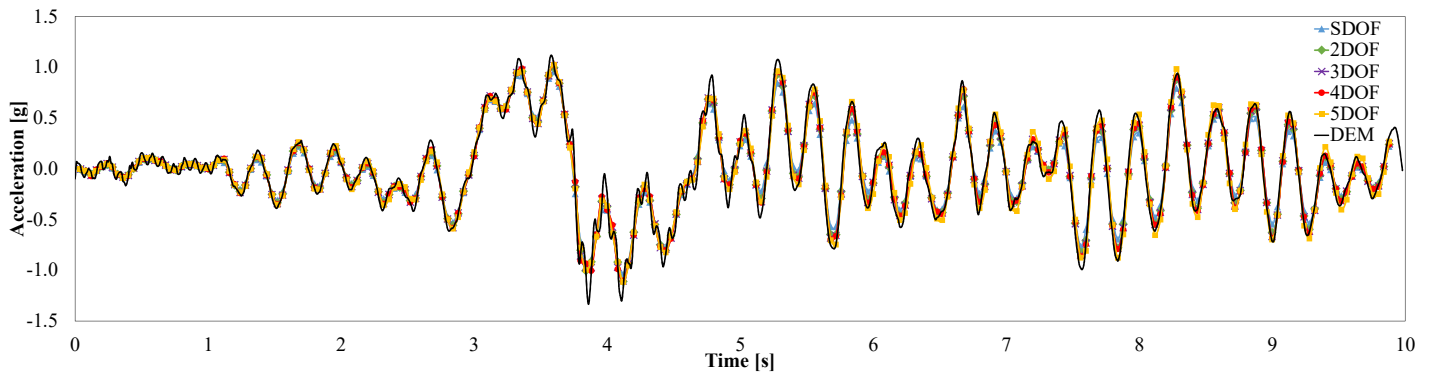
Figure 2: First and second modal shapes and periods in the  $Y$  direction of the complete and the simplified structure calculated using DEM modal analysis



(d) 2nd mode of SBDEM ( $T=0.17$  s)

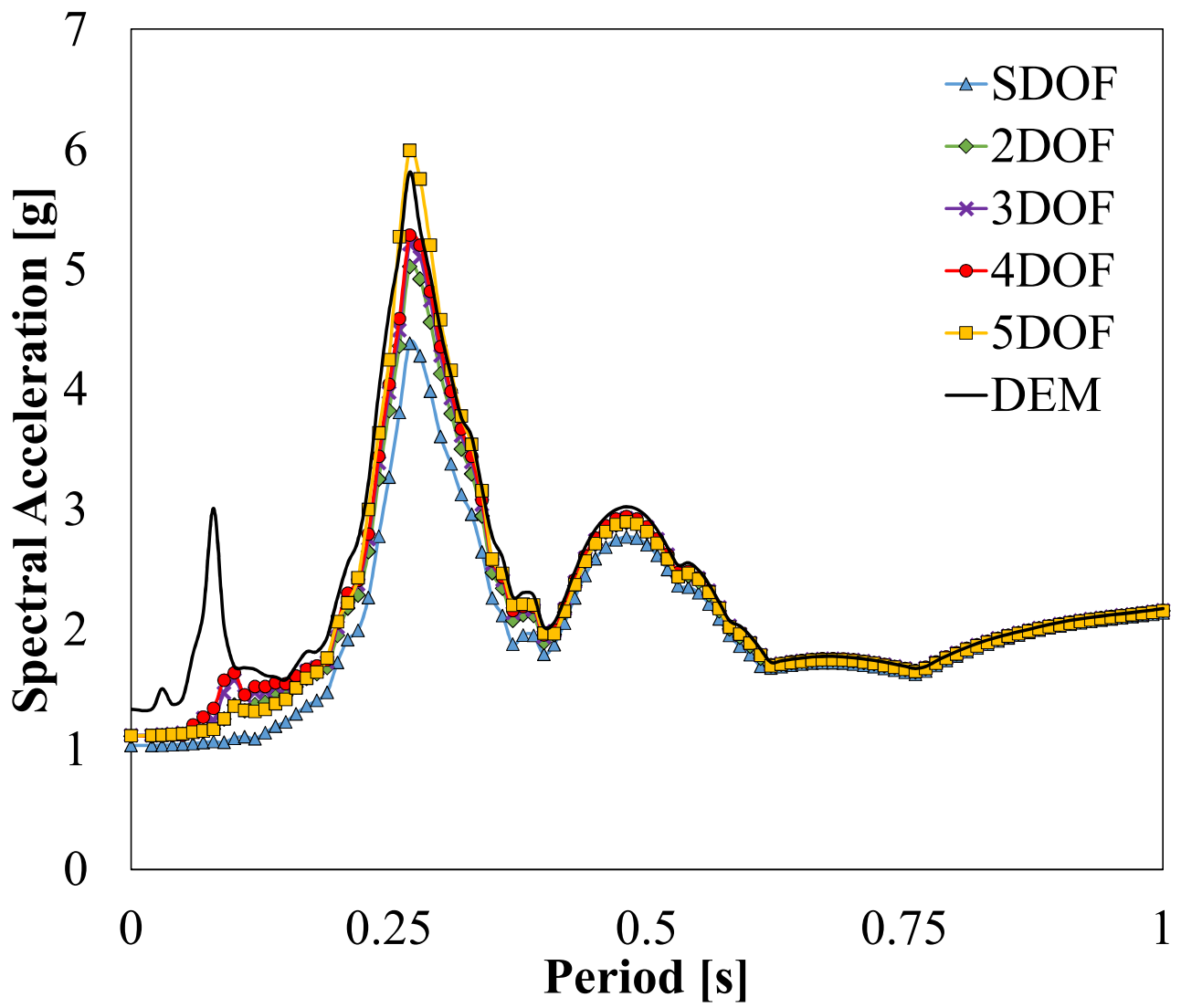
Figure 2: First and second modal shapes and periods in the  $Y$  direction of the complete and the simplified structure calculated using DEM modal analysis





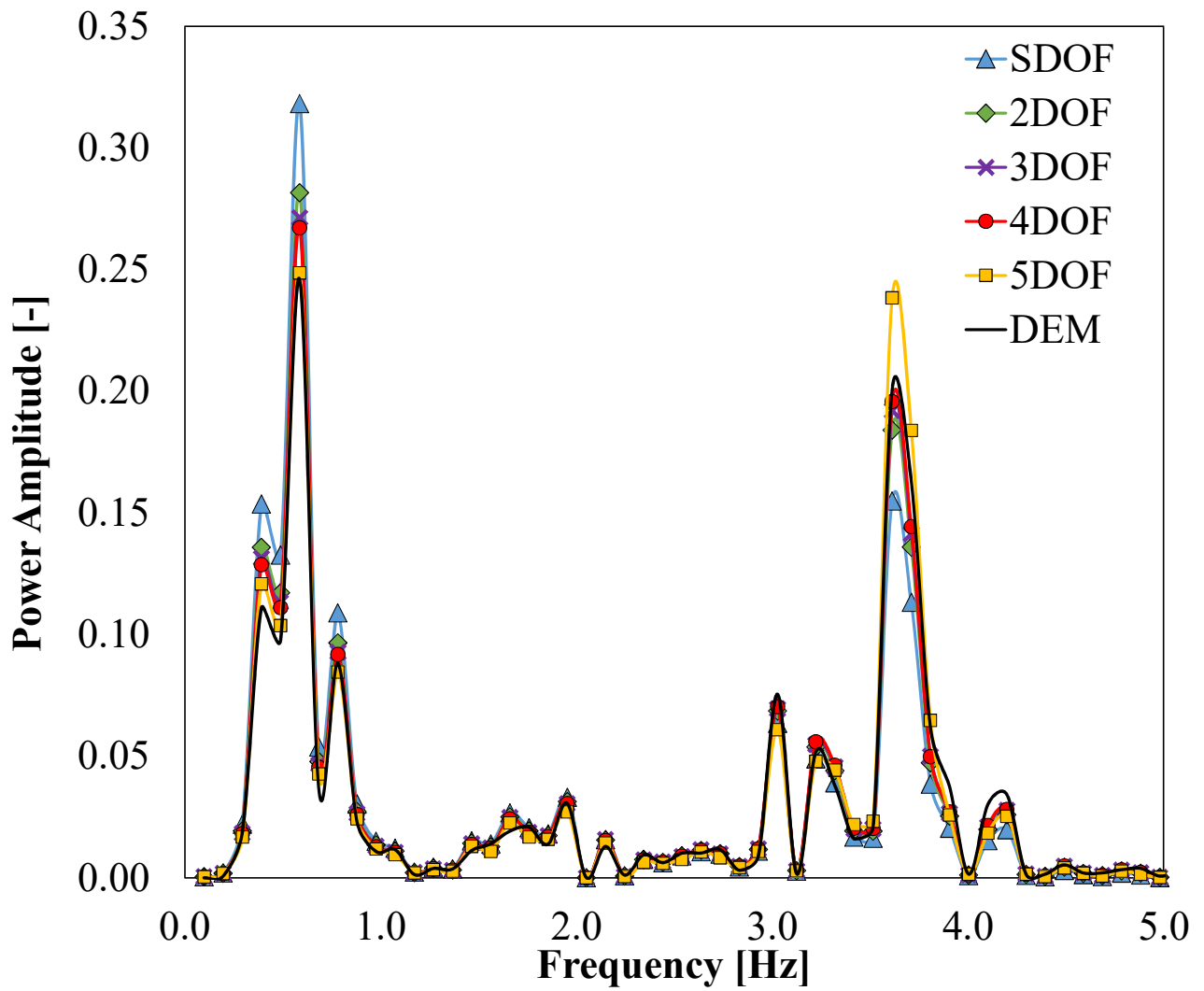
(a) Time-history response

Figure 3: Comparison between the seismic response of the SBDEM and SMDOF for BUCAR0 earthquake.



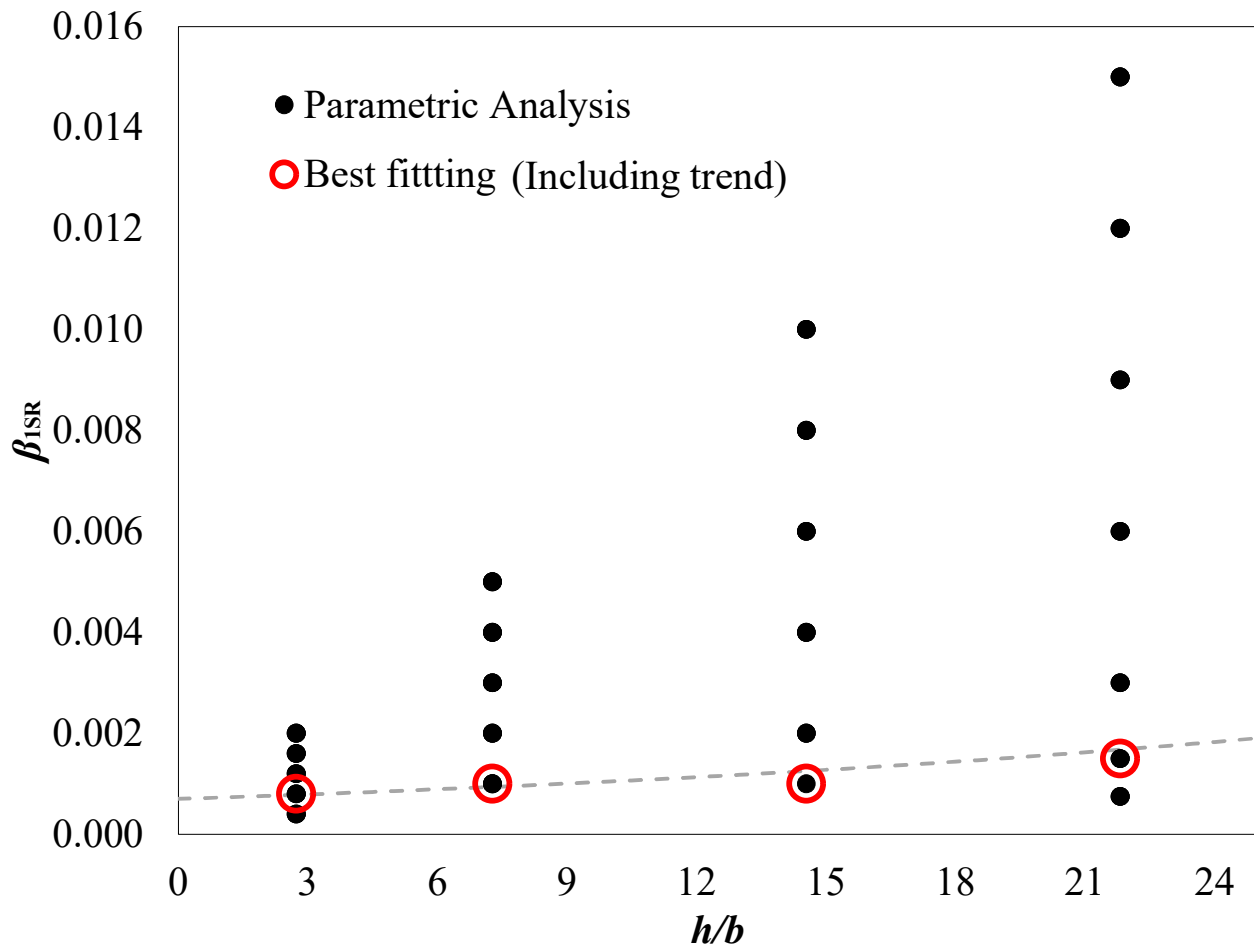
(b) Response Spectra

Figure 3: Comparison between the seismic response of the SBDEM and SMDOF for BUCAR0 earthquake.



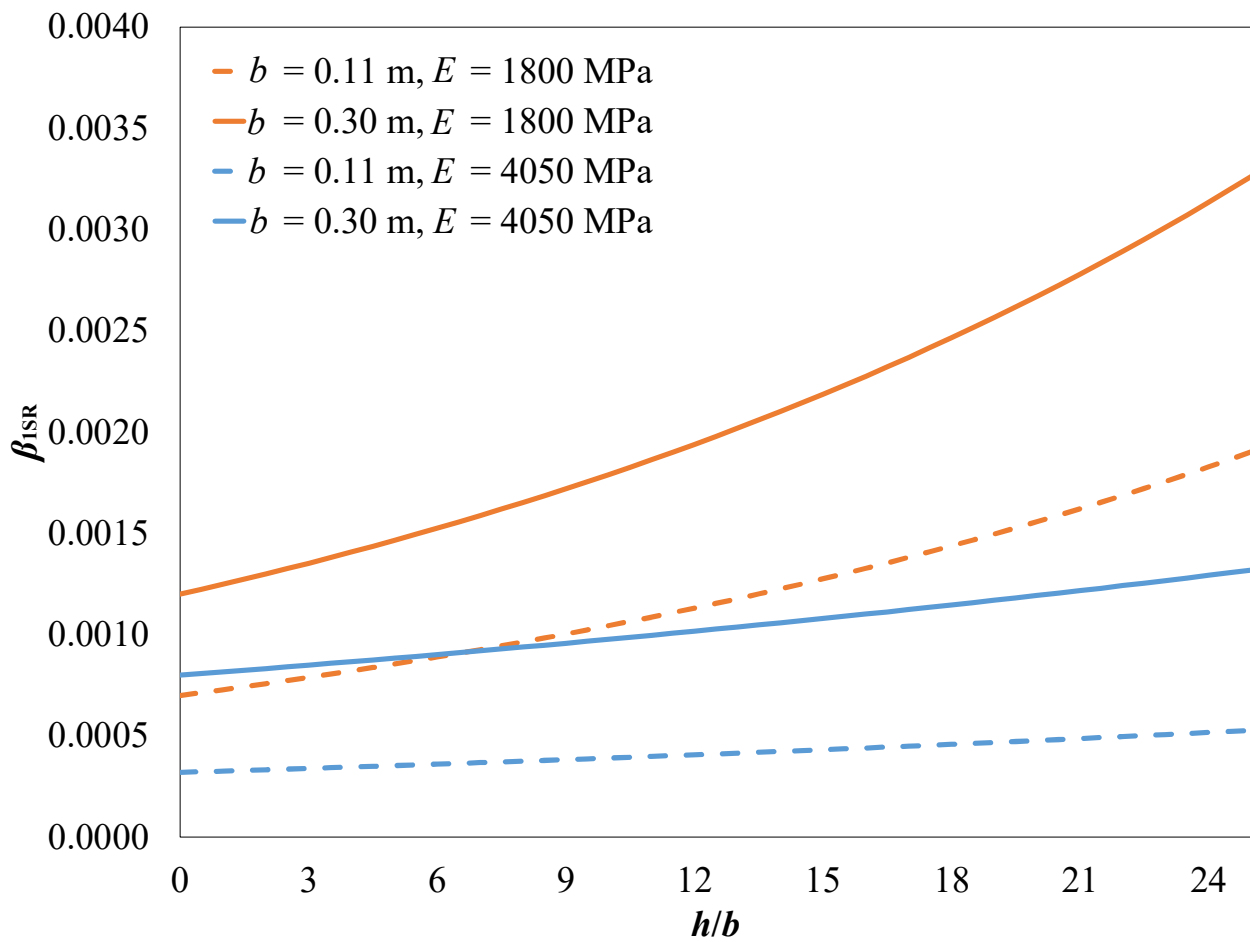
(c) Power Spectra

Figure 3: Comparison between the seismic response of the SBDEM and SMDOF for BUCAR0 earthquake.



(a) Values of  $\beta_{1SR}$  for walls with  $b = 0.11$  m,  $E = 1800$  MPa and different values of  $h/b$ .

Figure 4: Parametric studies.



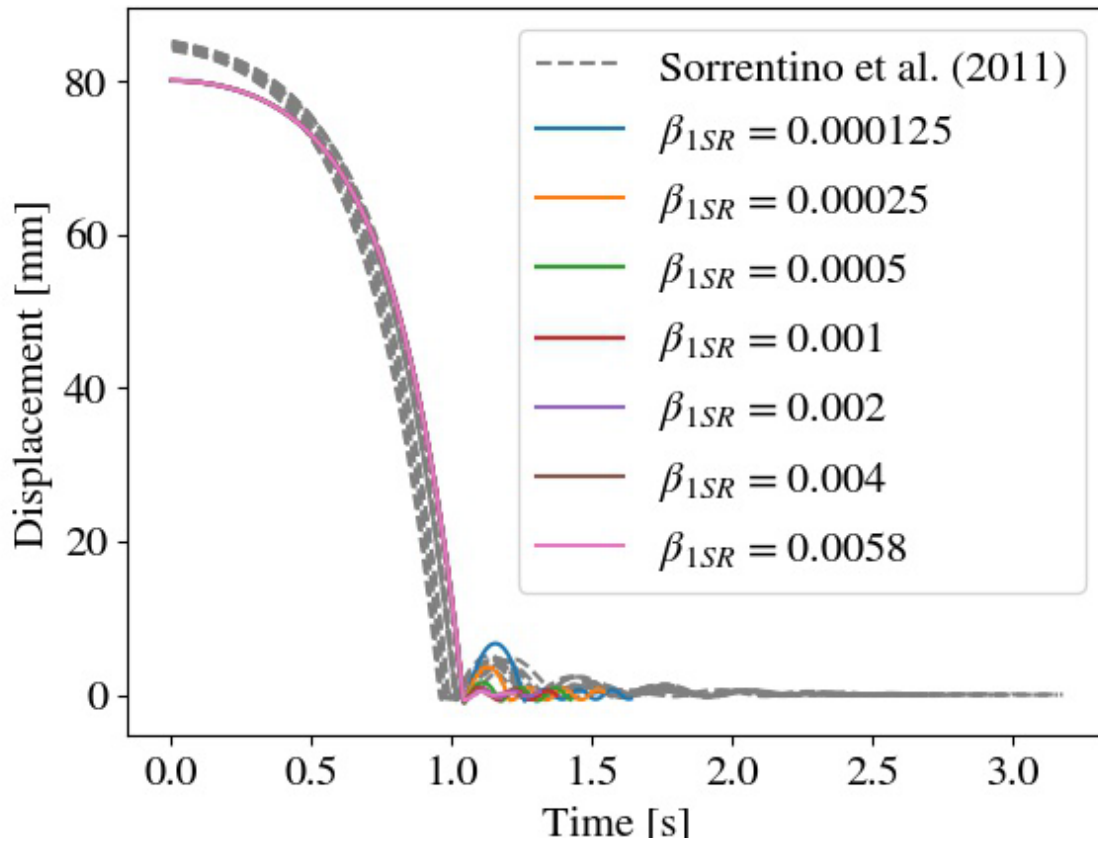
(b) Best fitting  $\beta_{ISR}$  for different values of  $E$  and geometry variations

Figure 4: Parametric studies.



(a) Experimental setup (Sorrentino et al., 2011)

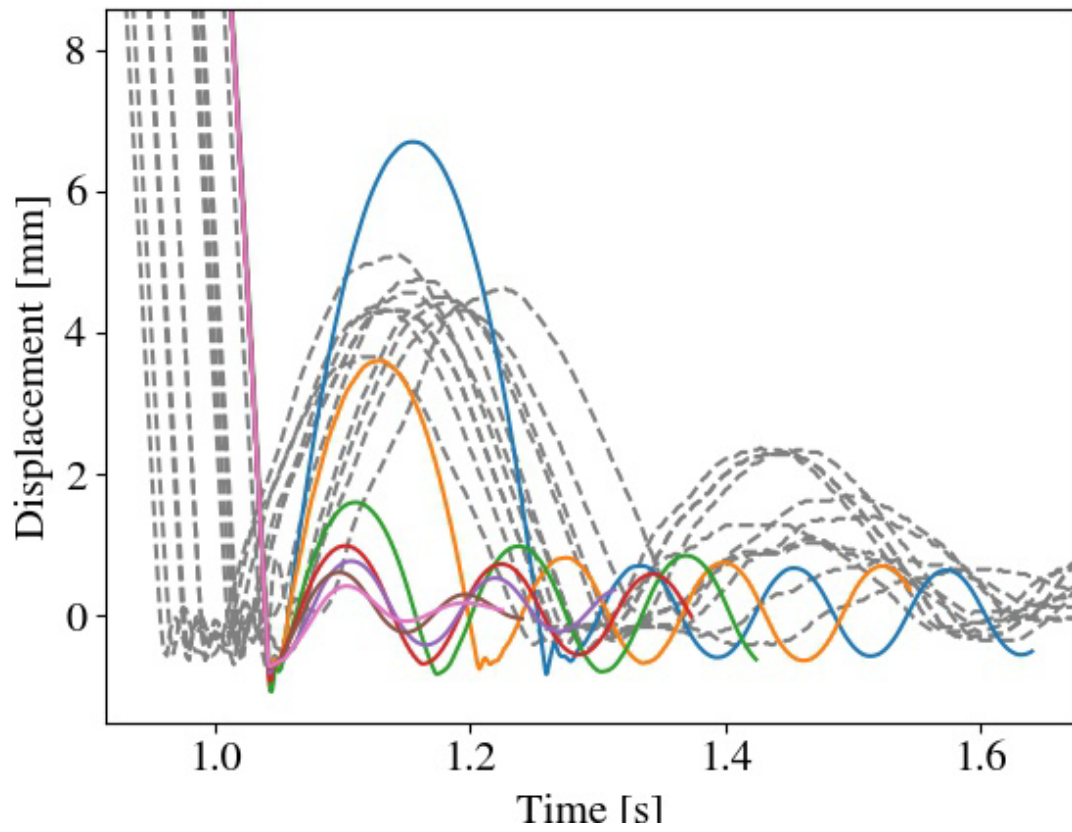
Figure 5: Experimental testing from Sorrentino et al. (2011) and simulations using 3DEC



(b) Complete time-history

Figure 5: Experimental testing from Sorrentino et al. (2011) and simulations using 3DEC

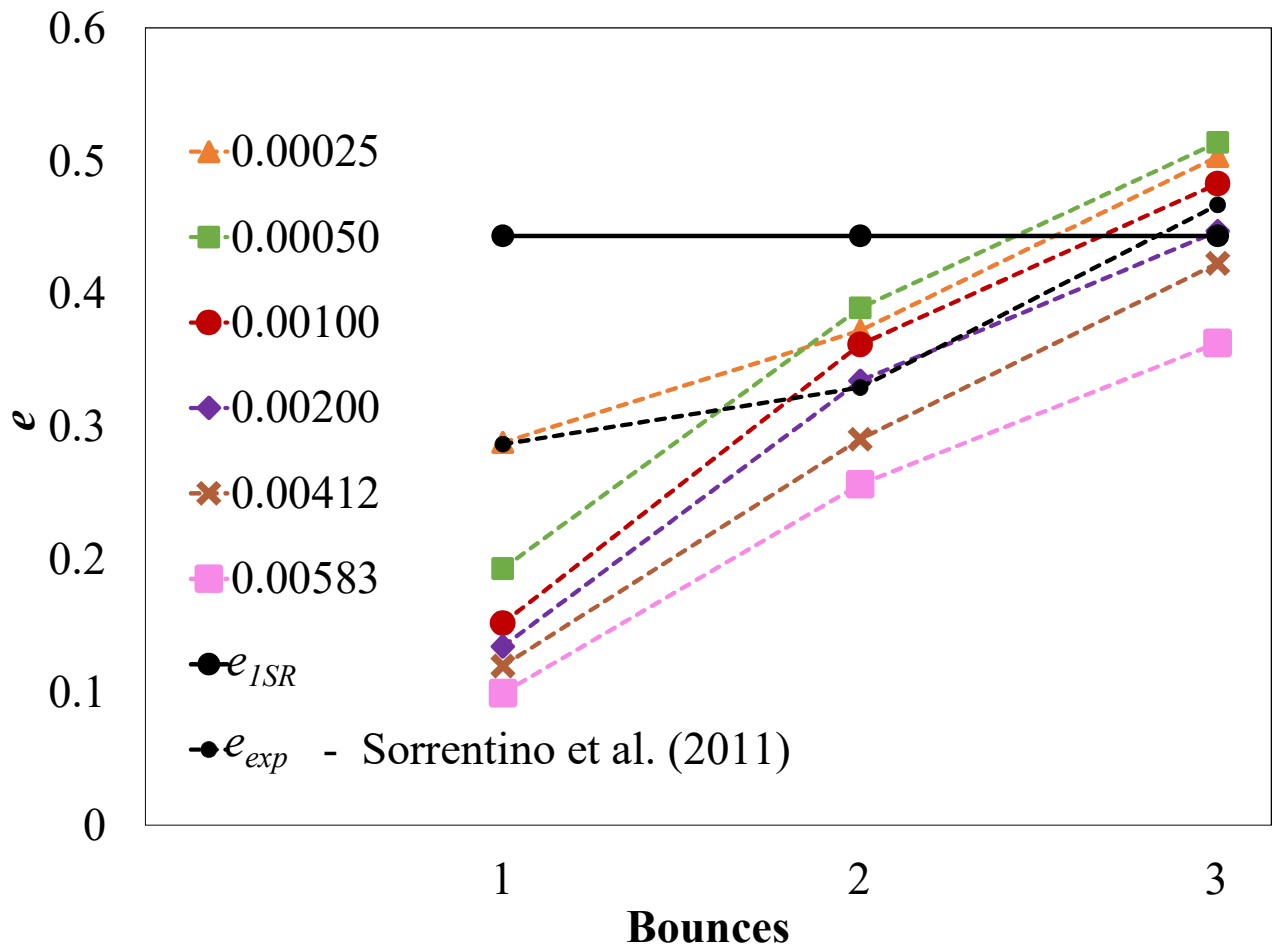




(c) Expanded view between 0.9 s and 1.9 s

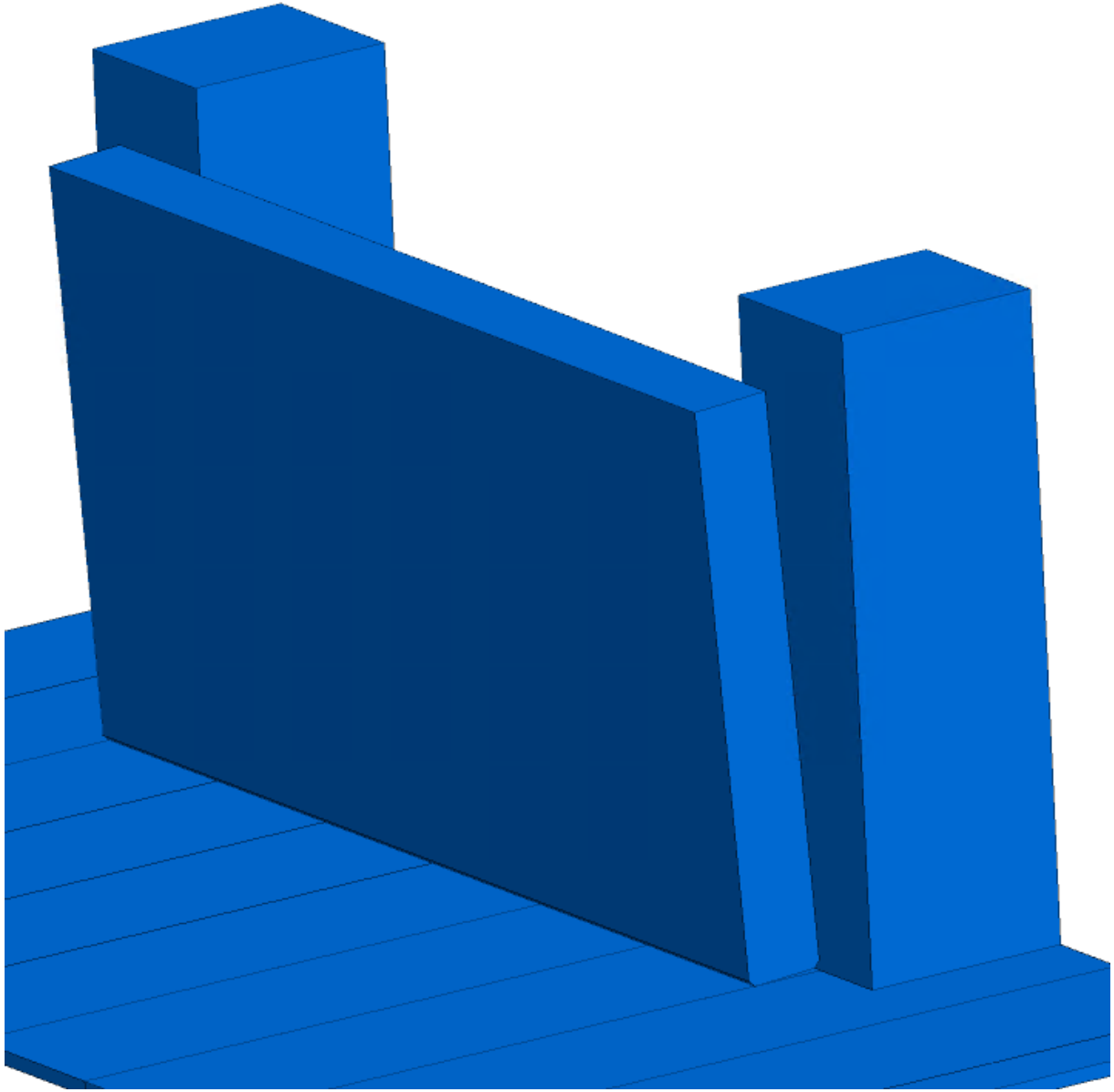
Figure 5: Experimental testing from Sorrentino et al. (2011) and simulations using 3DEC





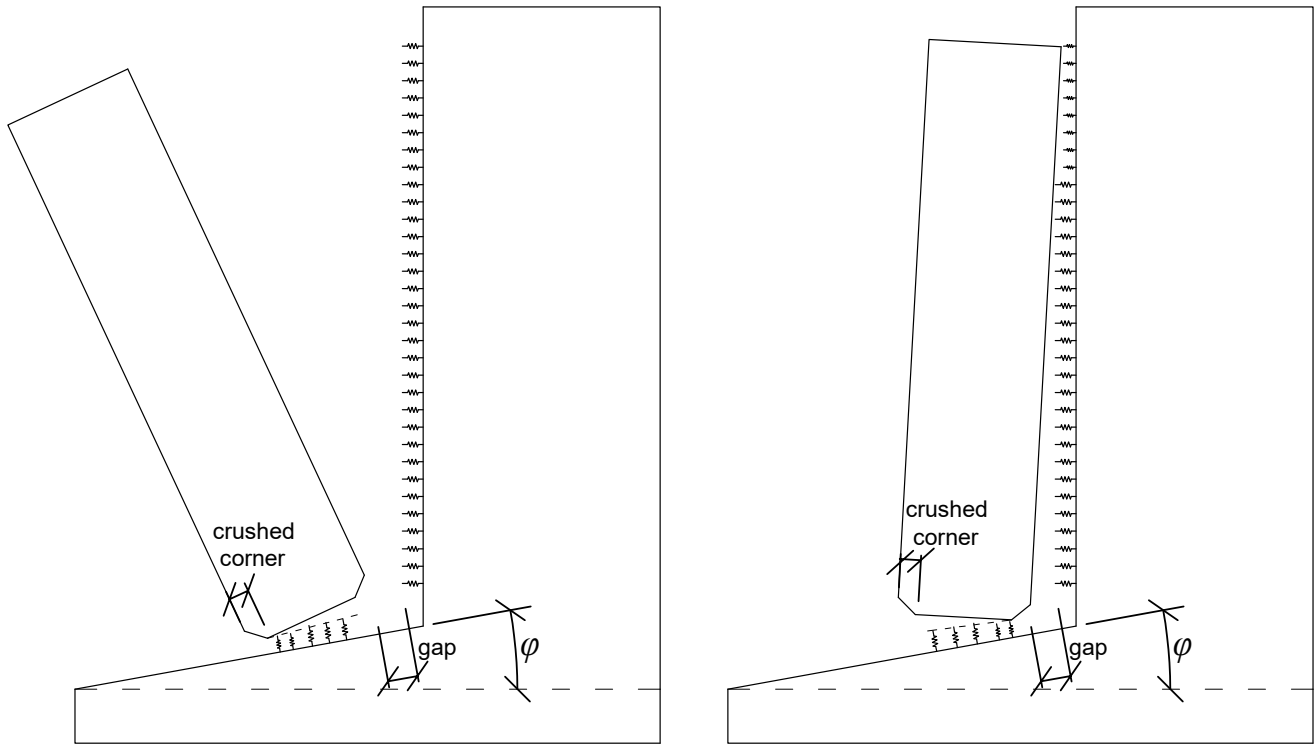
(d) Values of  $e_{exp}$  for different simulations using  $\beta_{ISR}$  compared to  $e_{ISR}$  and experimental testing

Figure 5: Experimental testing from Sorrentino et al. (2011) and simulations using 3DEC



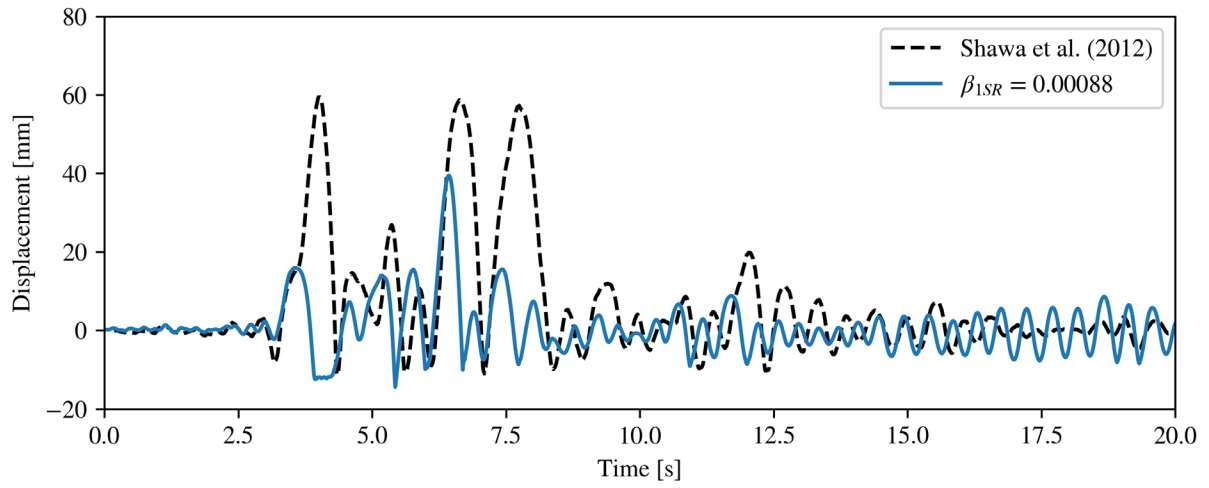
(a) DEM model

Figure 6: Comparison of top displacement between experiments and DEM simulation.



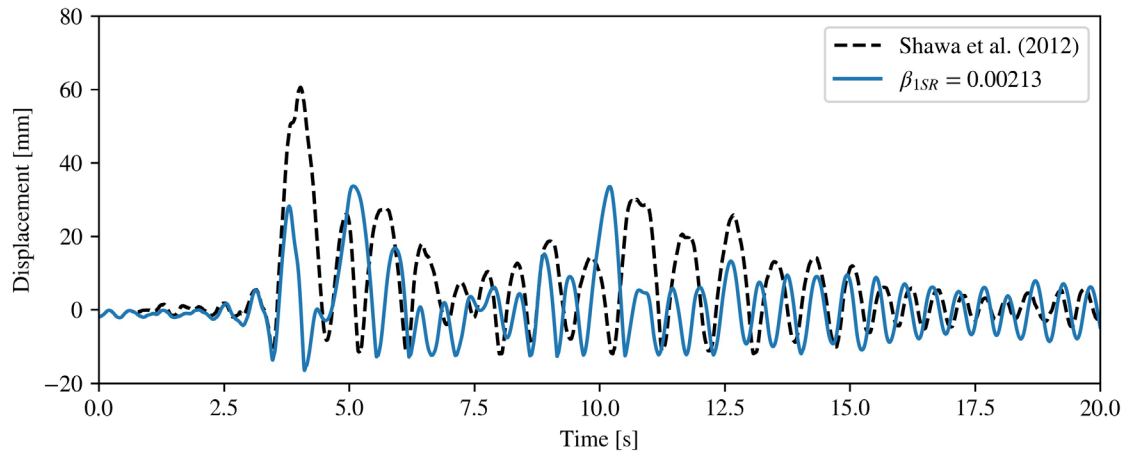
(b) Diagram of tilted rocking facade

Figure 6: Comparison of top displacement between experiments and DEM simulation.



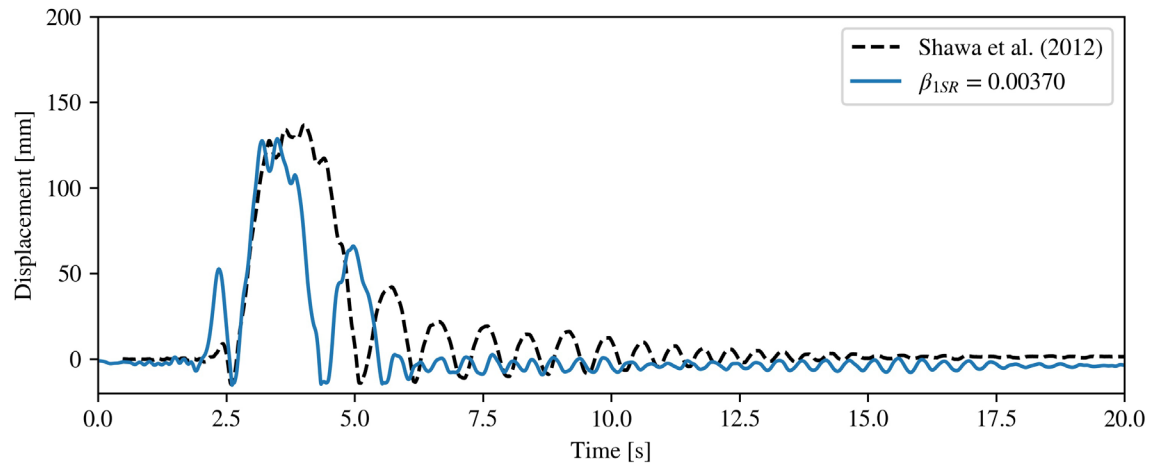
(c) BagnirWE,  $\varphi = 0.191^\circ$

Figure 6: Comparison of top displacement between experiments and DEM simulation.



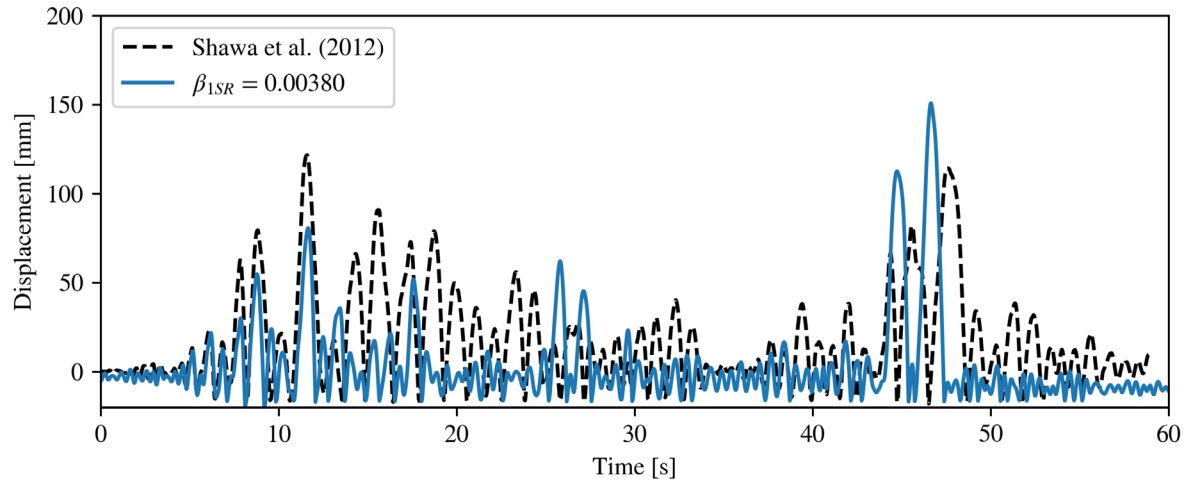
(d) SturWE,  $\varphi = 0.210^\circ$

Figure 6: Comparison of top displacement between experiments and DEM simulation.



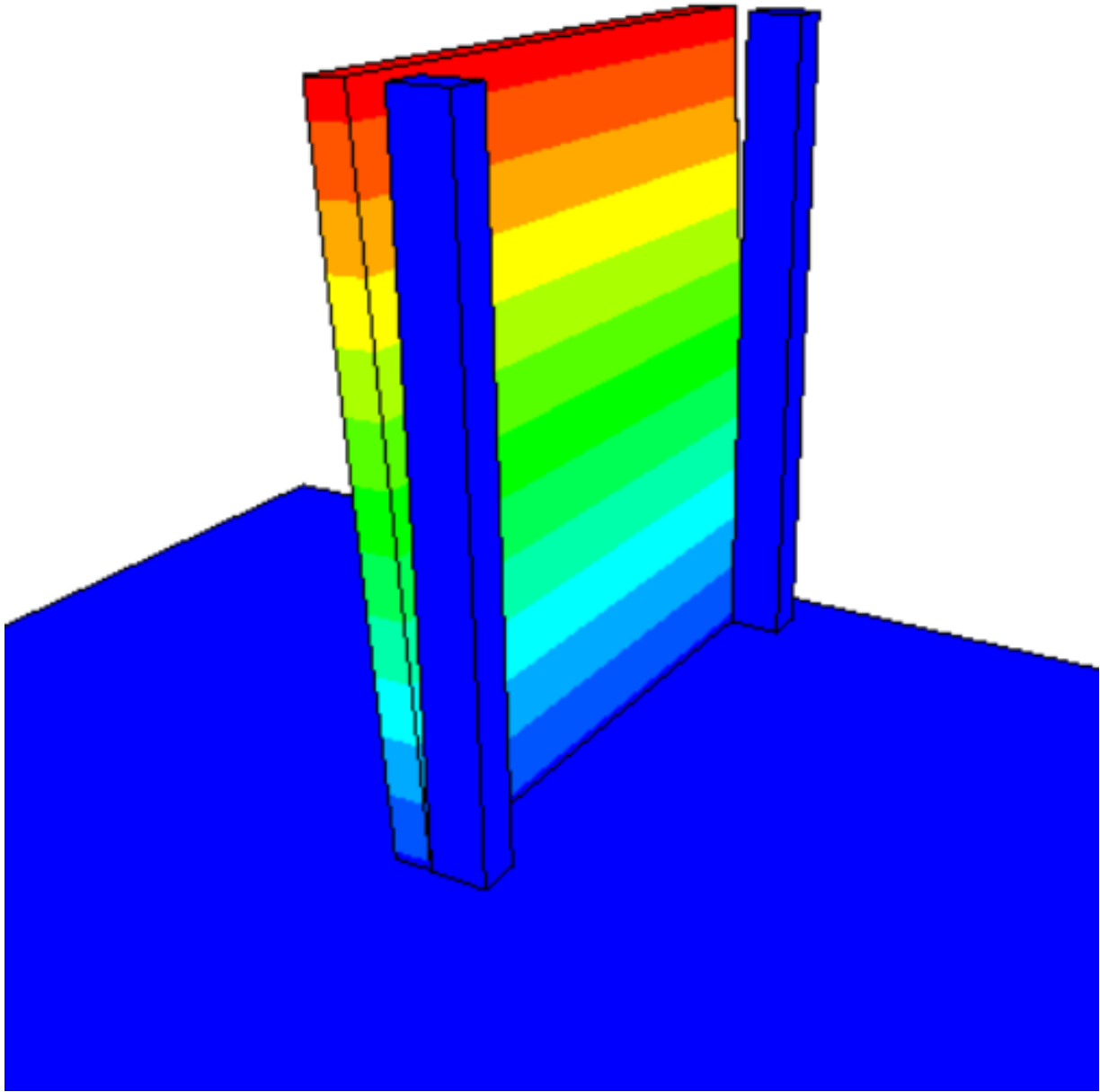
(e) R1168EW,  $\varphi = 0.272^\circ$

Figure 6: Comparison of top displacement between experiments and DEM simulation.



(f) CalitWE,  $\varphi = 0.348^\circ$

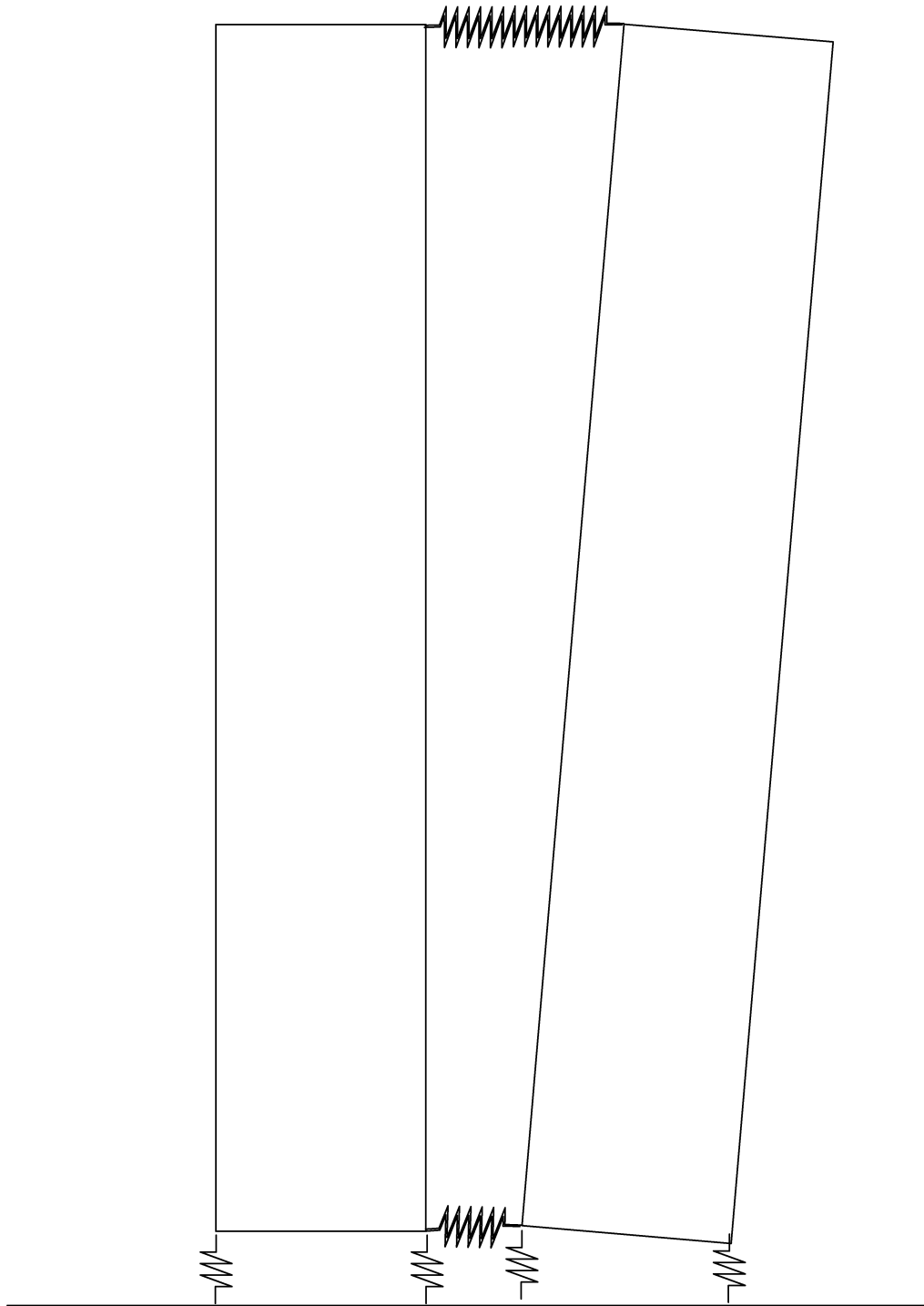
Figure 6: Comparison of top displacement between experiments and DEM simulation.



(a) W1

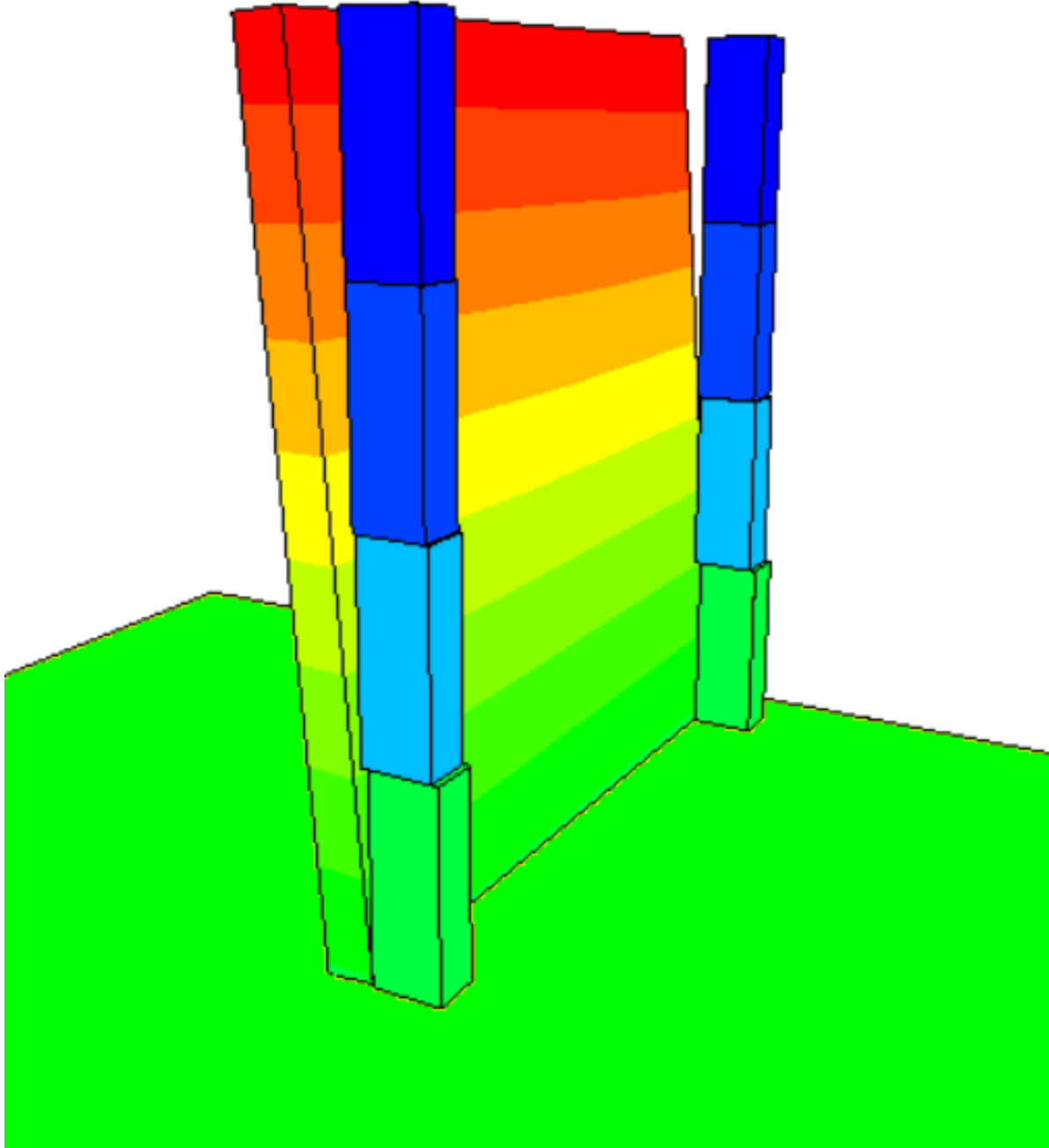
Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).





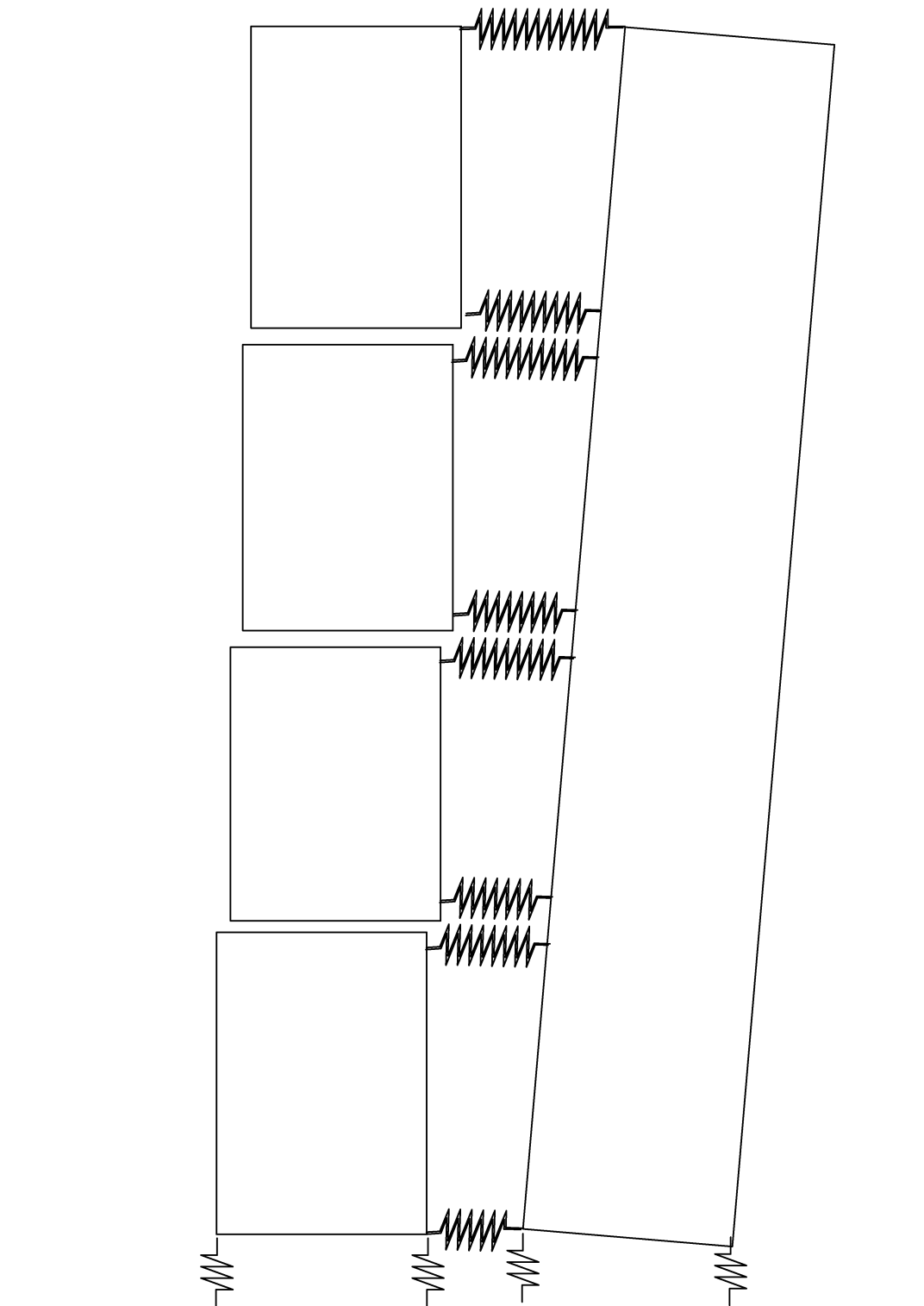
(a) W1

Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).



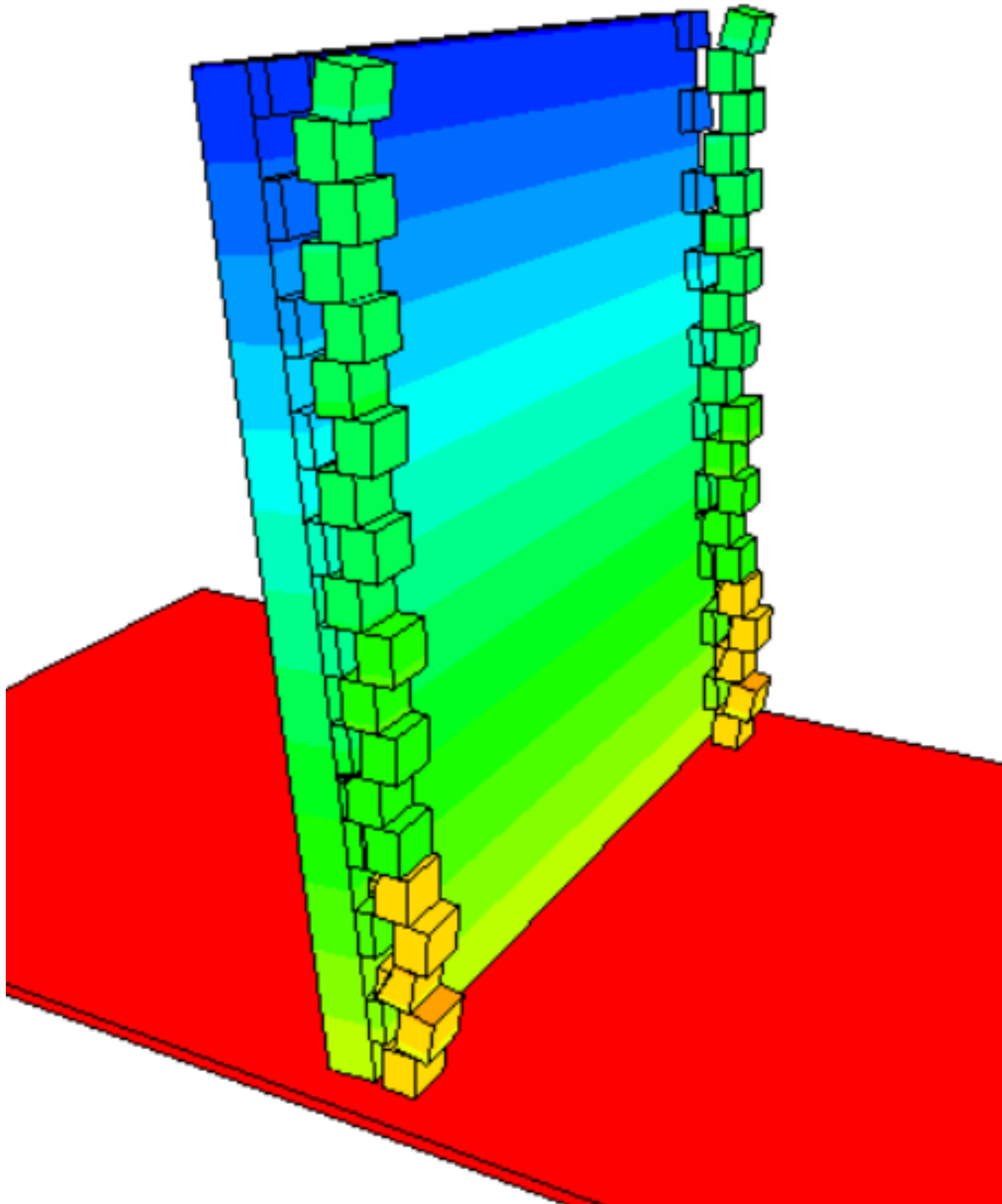
(b) W2

Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).



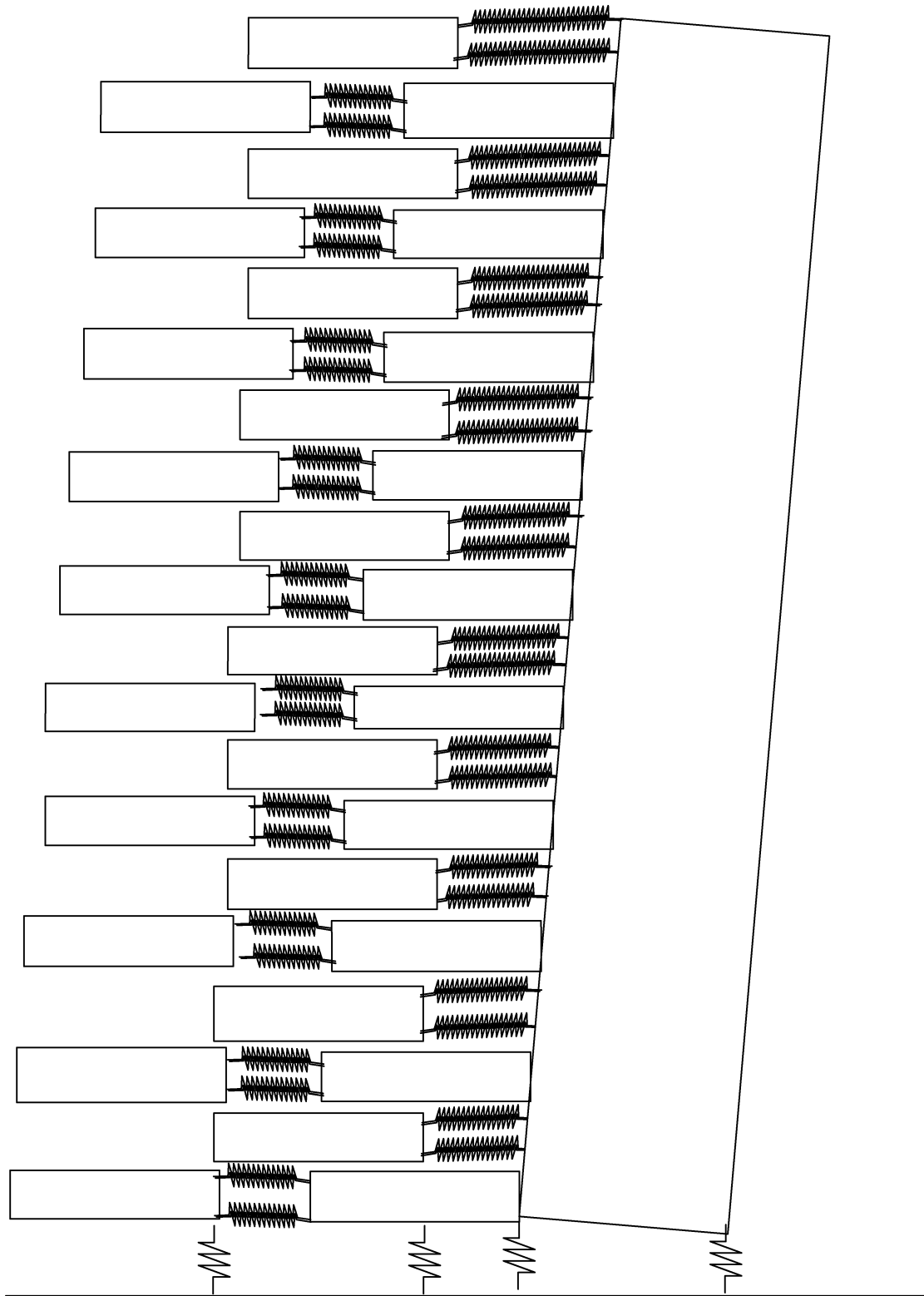
(b) W2

Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).



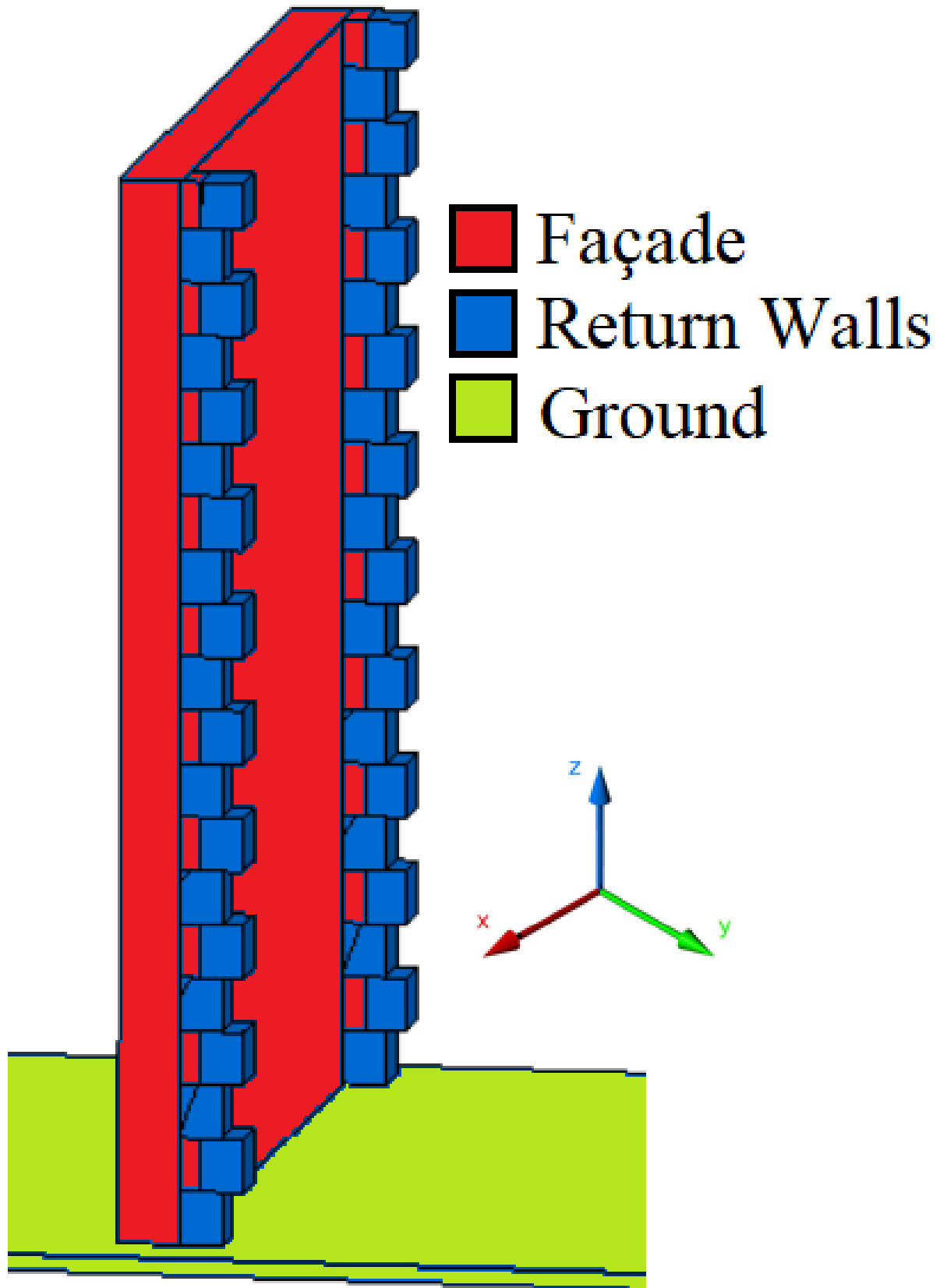
(c) W3

Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).



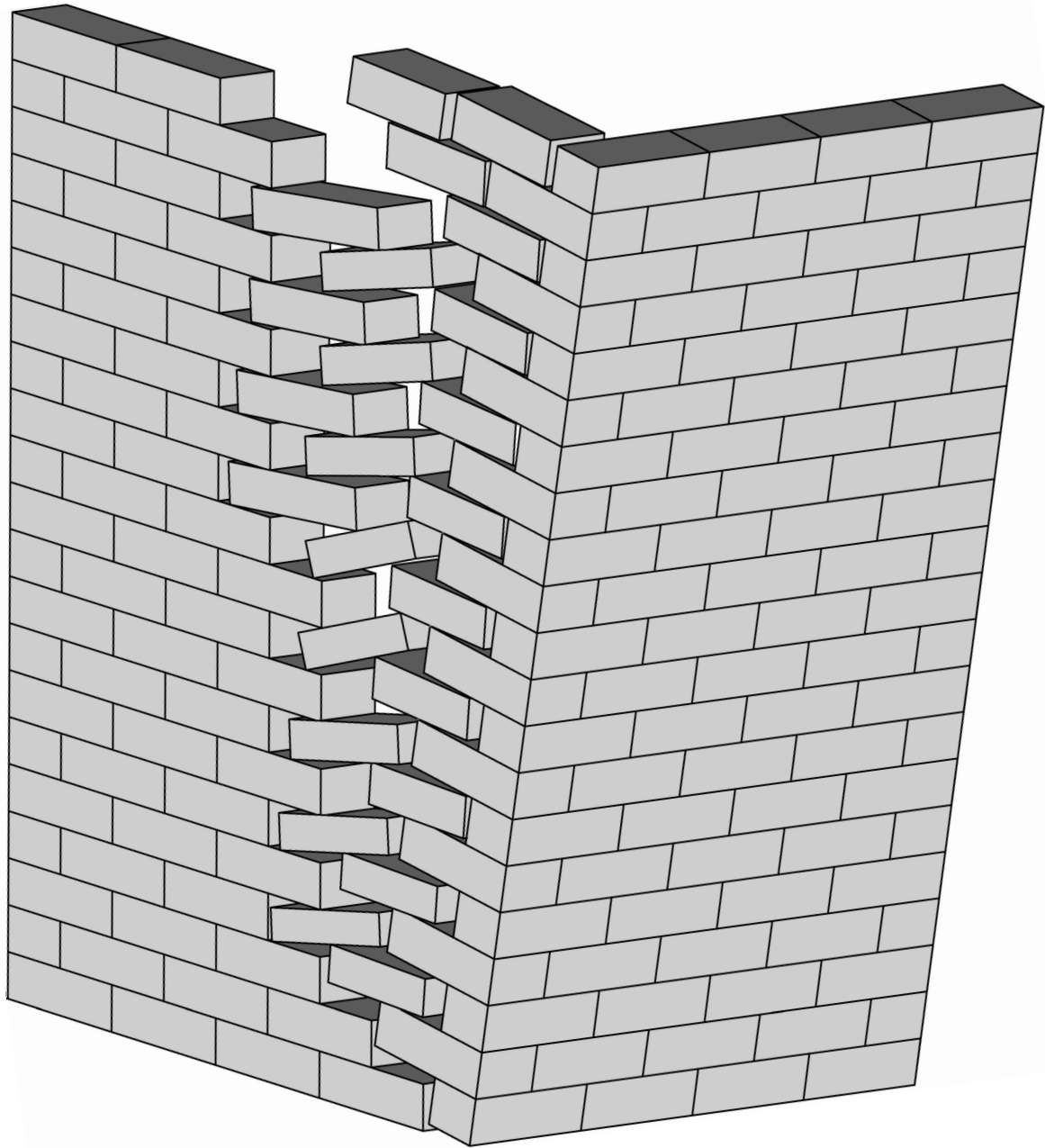
(c) W3

Figure 7: DEM modelling of rocking façade and return walls with associated block and spring interaction scheme (Colours represent displacement magnitude without scale).



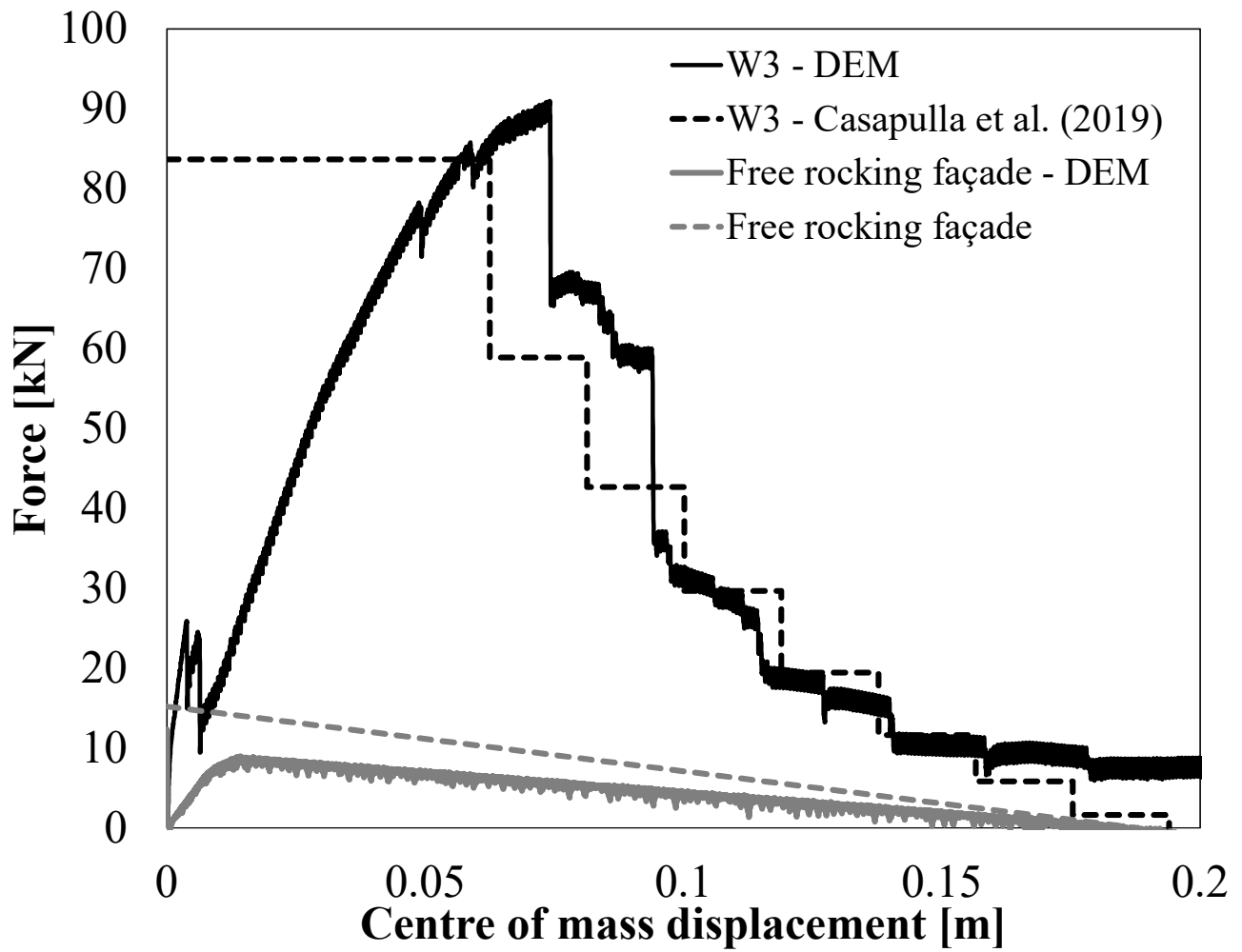
(a) Elements in W3

Figure 8: W3 details and capacity curve



(b) Scheme of generic façade and return wall separation

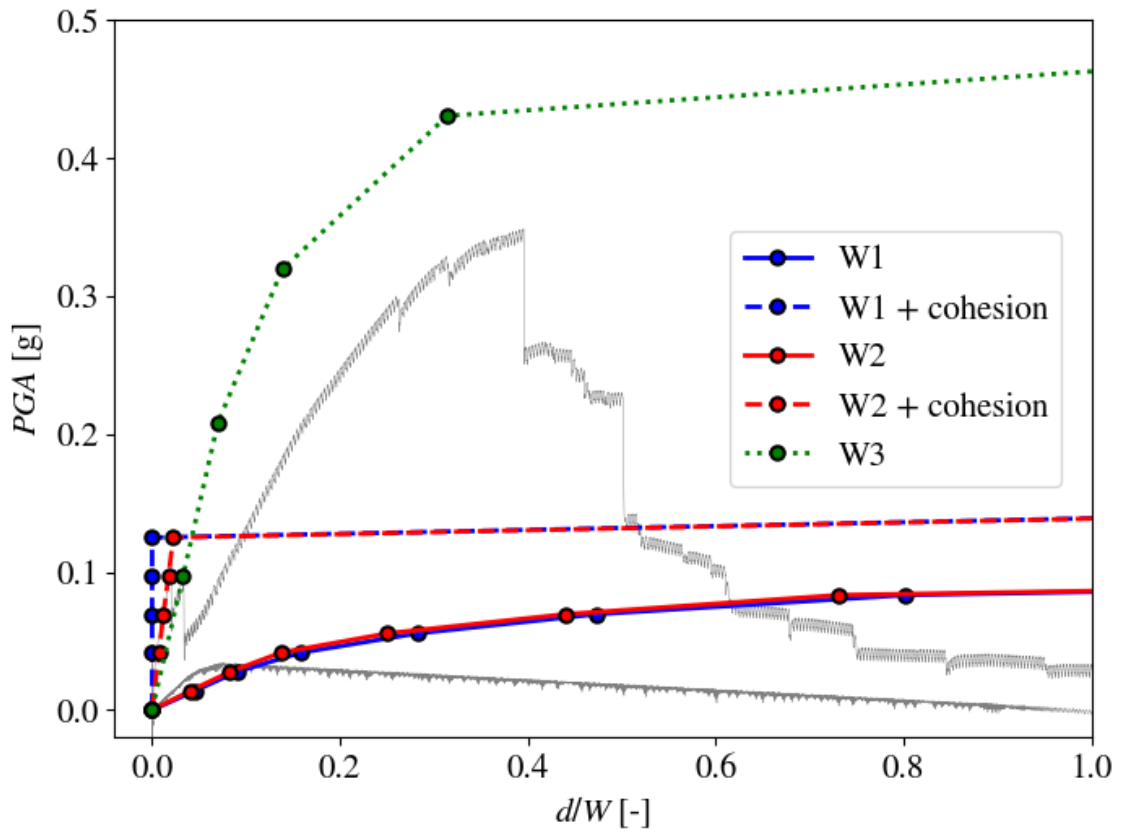
Figure 8: W3 details and capacity curve



(c) Comparison between kinematic and DEM capacity curve

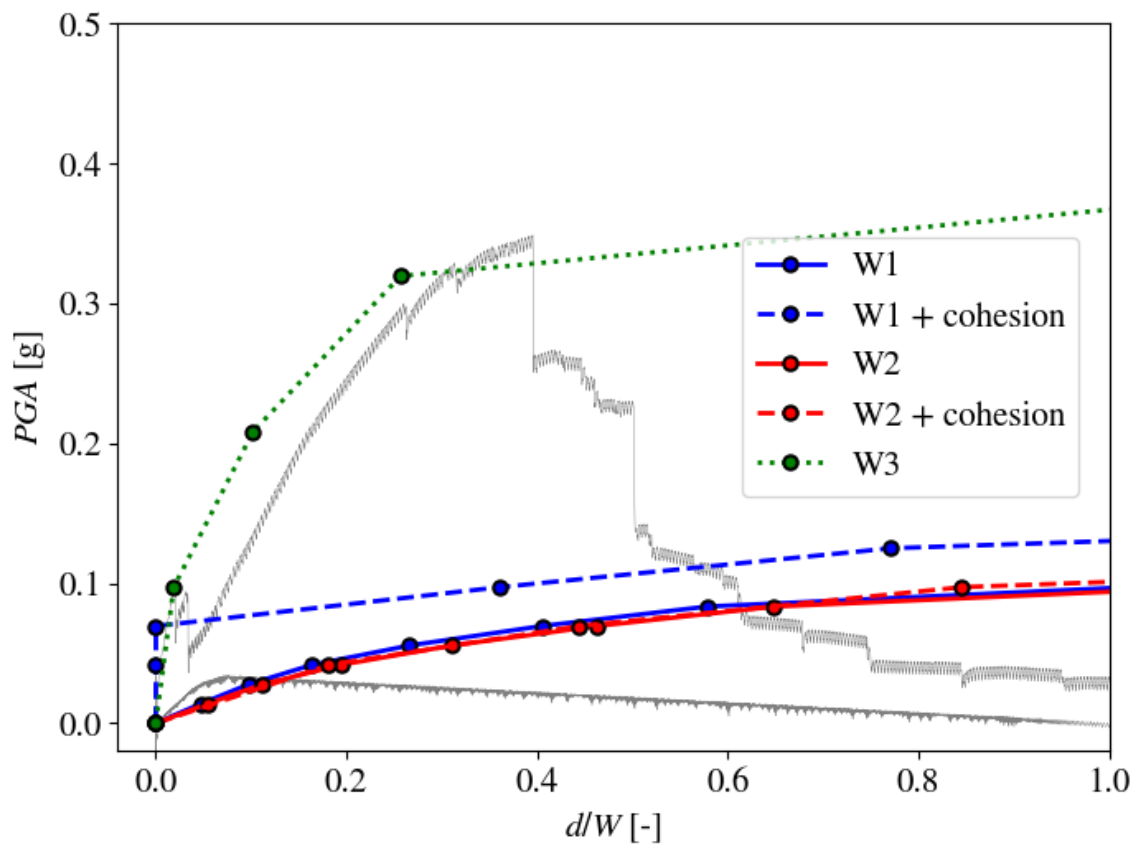
Figure 8: W3 details and capacity curve





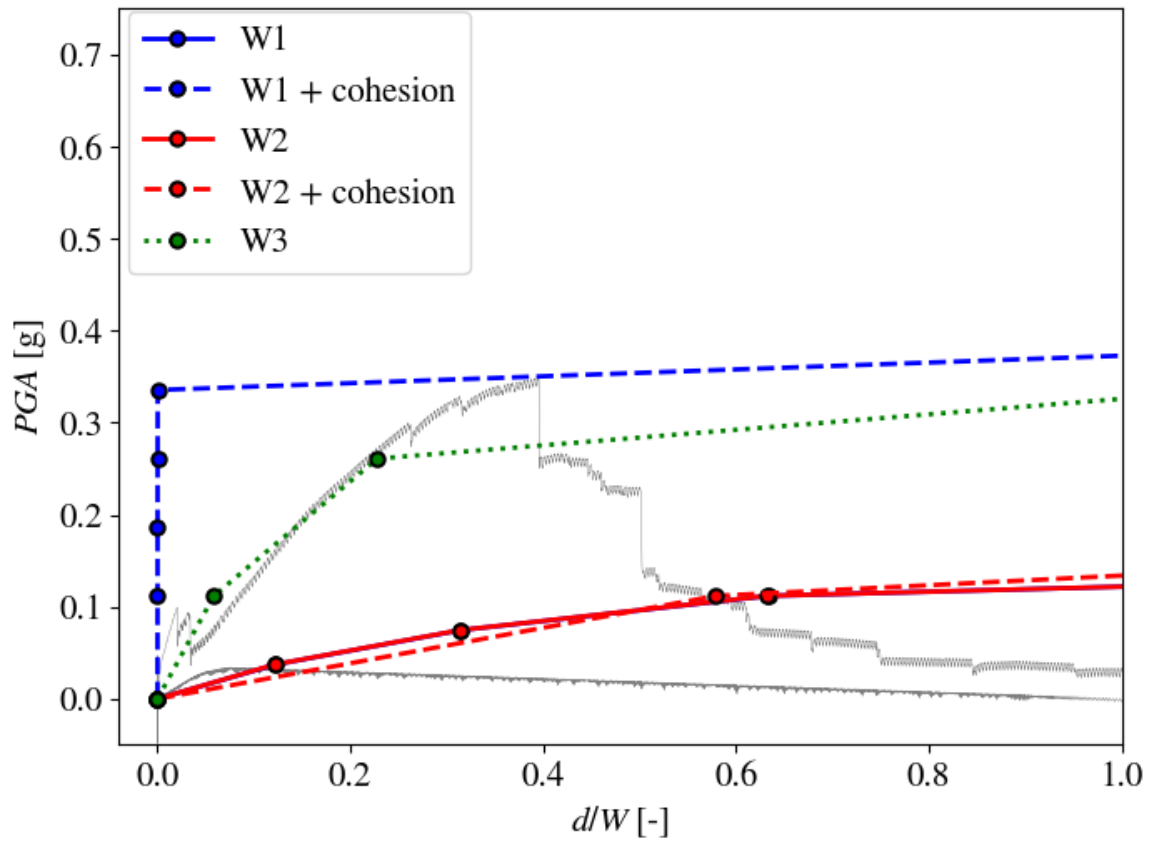
(a) BUCAR0 - Positive (+) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



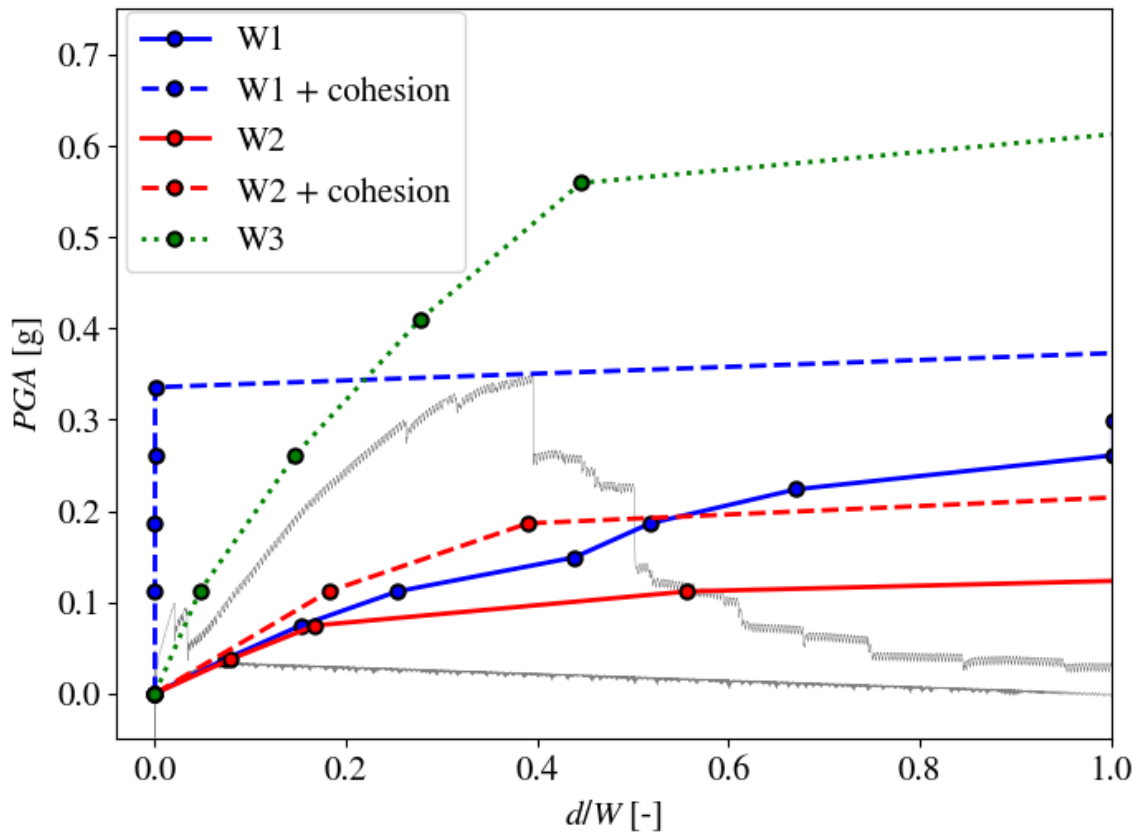
(a) BUCAR0 - Negative (-) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



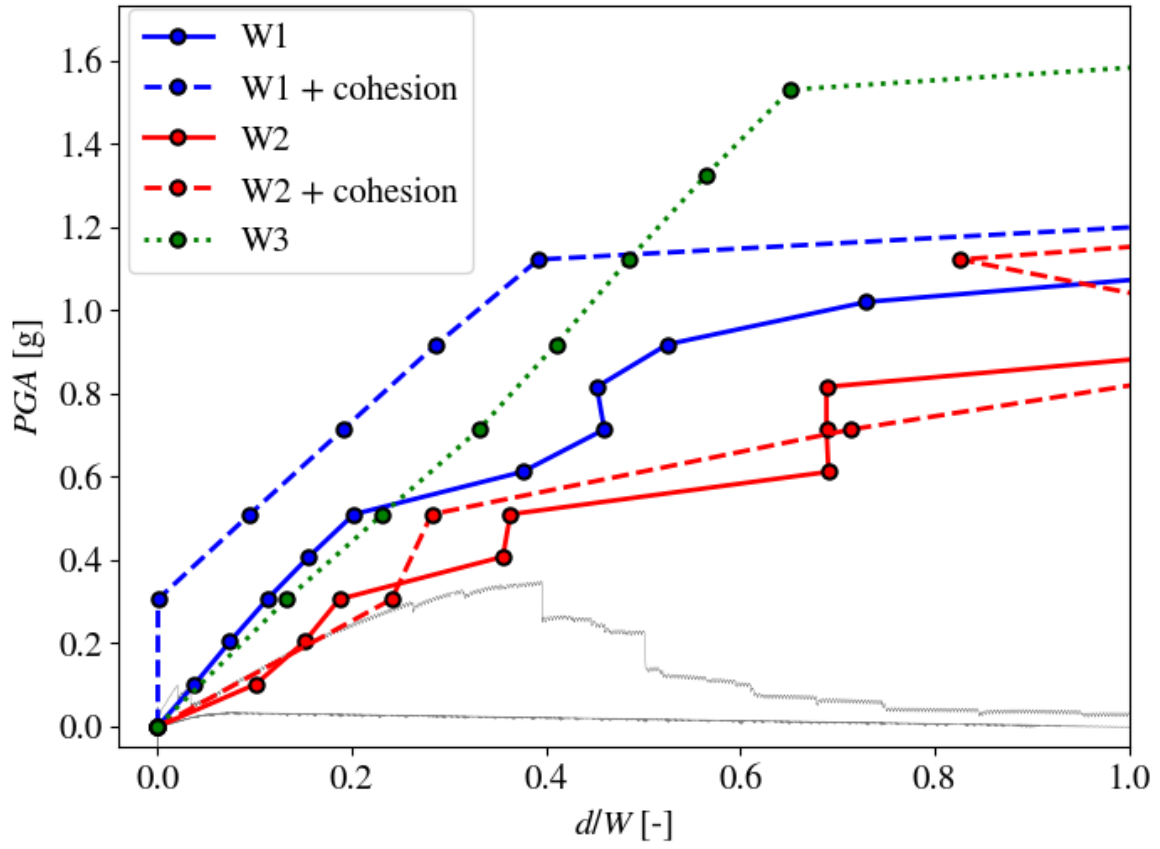
(b) RRS228 - Positive (+) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



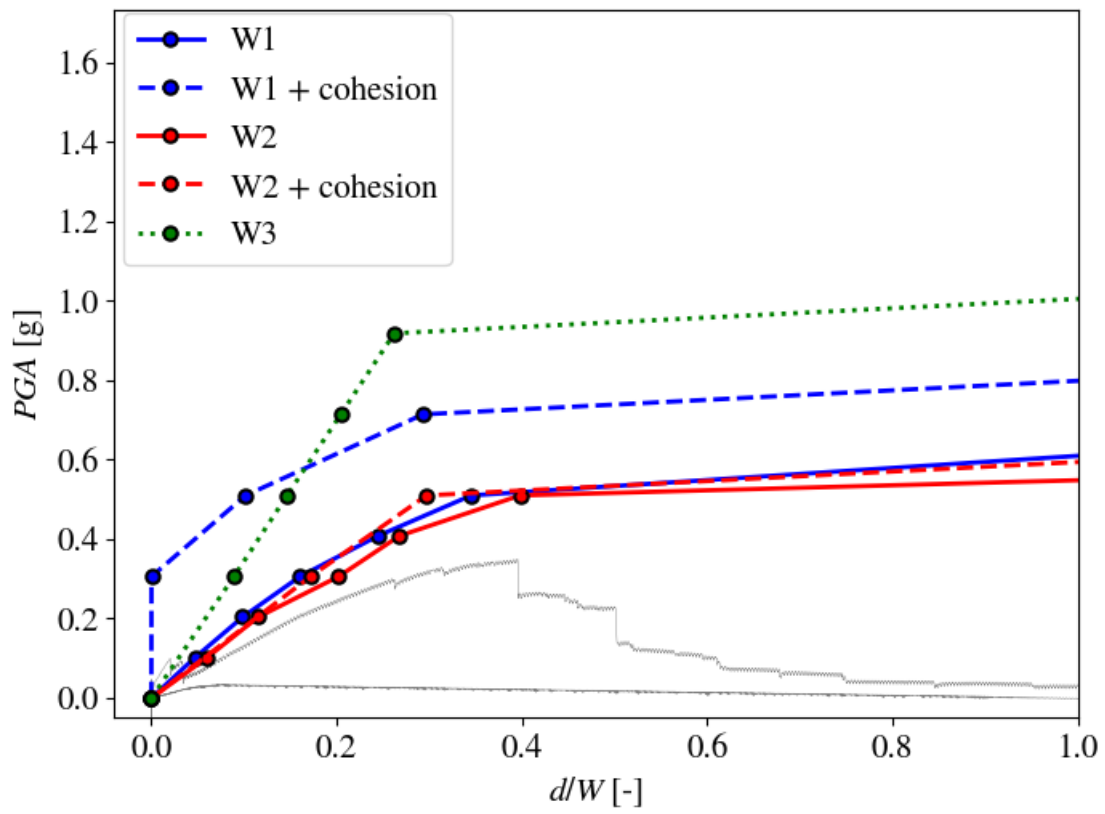
(b) RRS228 - Negative (-) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



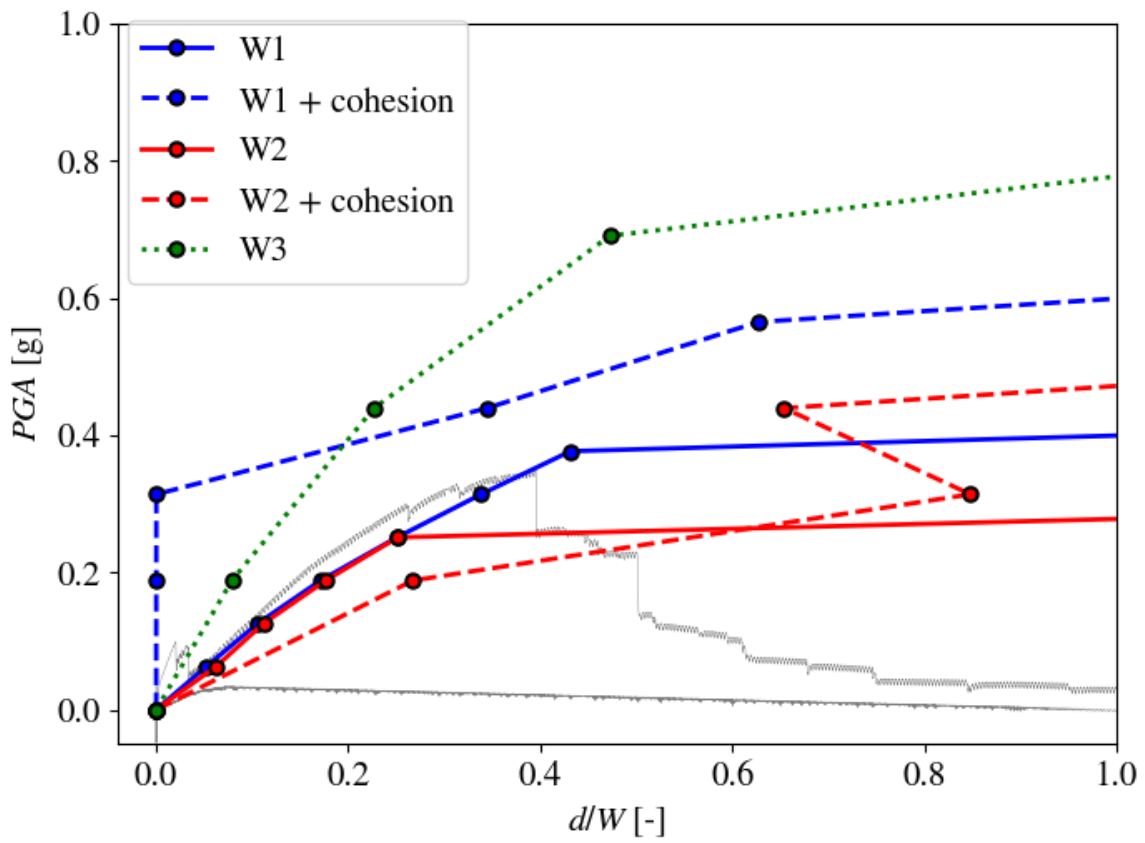
(c) 1ST280 - Positive (+) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



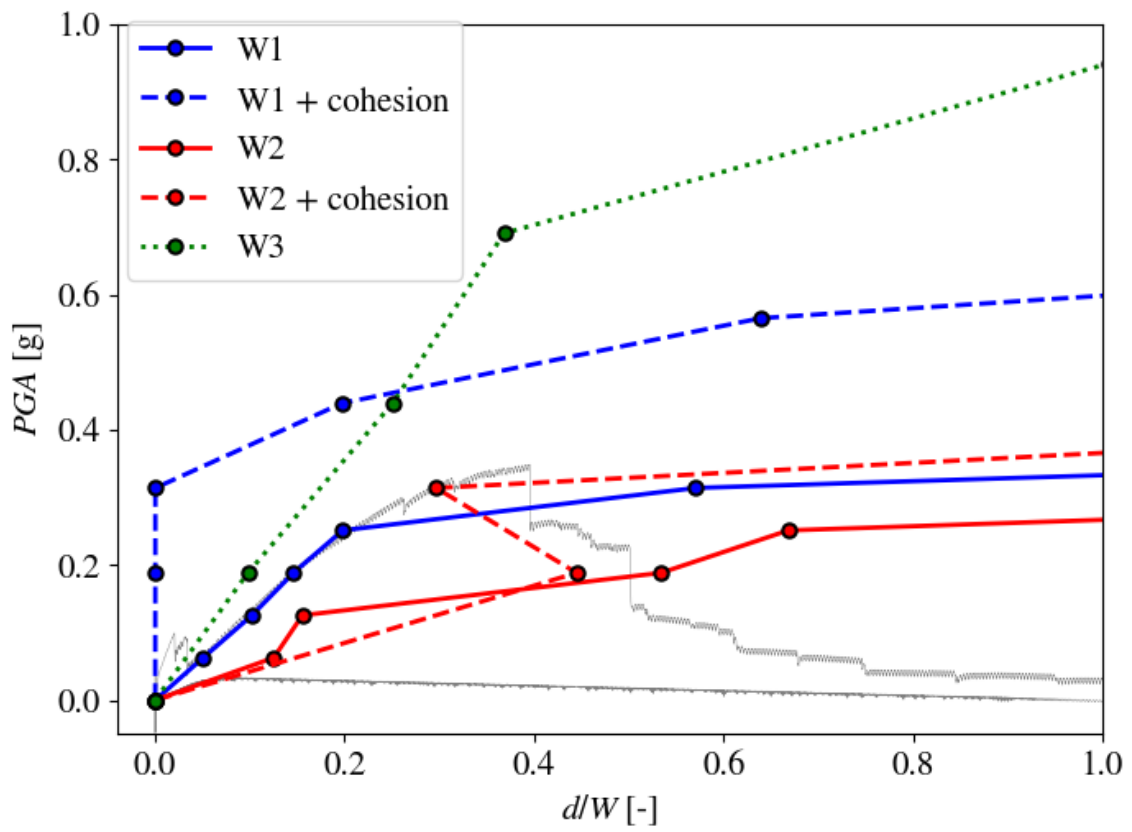
(c) 1ST280 - Negative (-) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



(d) HVSC1 - Positive (+) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.



(d) HVSC1 - Negative (-) direction

Figure 9: Incremental dynamic analyses. Pushover curves in grey for reference.