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VIBRATION CONTROL OF STEEL LIQUID STORAGE TANKS EQUIPPED WITH INERTER-BASED ISOLATION SYSTEMS

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Abstract. *Base isolation represents a very widely used strategy to mitigate the effects of earthquake excitation on structures. However, it can induce high displacements between the isolation layer and the ground, which may cause serious damage, and even heavy and dangerous consequences in case of industrial components. Among them, big steel tanks for storage of petroleum or other chemical products, should be considered very carefully. Moreover, isolation technique doesn't seem to be effective in the control of the sloshing modes, due to the length of their periods of vibration. This fact can imply severe negative effects on the free surface of the storage tank, where the sloshing wave can exceed the upper limit of the tank, overtopping it, or inducing breaking on the floating roof.*

Moving from the results available in the Literature, in which the introduction in civil applications of a two terminal device, named inerter, able to generate an inertial mass much greater than its gravitational mass, is proposed; the force produced by the inerter is proportional to the difference of acceleration between its terminals. This work concerns the evaluation, through numerical models, of the seismic performance of a passive base isolation system involving a ground inerter system, called IBIS in the following, connecting the isolation layer of a steel liquid storage tank to the ground. The model considered in the numerical analysis consists in a reduced 2DOF linear system. The first degree of freedom is represented by the first sloshing mode; the second is relative to the base isolation system, whose mass includes the basement, the tank and the impulsive component of liquid mass.

The aim is to gain a reduction of the response in terms both of isolation layer displacement and of sloshing height.

The effectiveness of the control strategy proposed has been evaluated considering both a random white noise process and earthquakes (near-fault and far-field) as base input, achieving strong reduction of the response, in terms of sloshing height and isolation displacement.

1 INTRODUCTION

The seismic response of liquid storage tanks has been extensively studied by authors in the past decades [5, 6, 11]. The growth of interest in the engineering field is essentially due to the extremely severe consequences of accidents involving tanks, often occurring in petrochemical plants.

The seismic traditional base isolation has been the topic of both theoretical and experimental works, and its employment in the mitigation of the response of storage tanks has been already proposed, both in passive and semi-active control [7, 16]. Base isolation represents, in fact, a consolidated strategy, commonly used in passive structural control, as it is founded on modifying the structural stiffness of a structure, connecting the basement to the ground by deformable layer of isolators, able to create a flexible level in which the deformation is concentrated, uncoupling the structural displacement from the shaking ground. Therefore, the isolation technique typically acts on the structural stiffness, increasing the fundamental period of the structure, with the result of reducing both the base shear and the overturning moment of the tank, cutting the transfer of the earthquake-induced forces to the superstructure [1].

The employment of this technique presents two main drawbacks. One is related to the onset of the deformable layer, which can experience high displacements. The second one is due to the fact that the isolation period may draw a value comparable to the period of the sloshing modes, implying the incapability of controlling the convective response, or even increasing it.

This paper concerns the optimal design and effectiveness investigation of an isolation system arranged in parallel with a two-terminal device named inerter.

This device has been proposed by Smith and developed in the past years in mechanical engineering with the aim to suppress vibrations in vehicles. In fact, it is able to establish a difference in acceleration between its two terminals, so to produce an amplification mass effect, making the inertial mass much greater than the gravitational mass. This feature makes its use particularly interesting in passive vibration control. The constant of proportionality between the force produced by this device and the relative acceleration takes the name of inertance and has the dimensions of a mass.

The introduction of this device in Civil Engineering has been object of a series of theoretical [3, 4, 9, 10, 12] and experimental [13, 14] research studies. In fact, adding to a conventional TMD -small auxiliary mass- an inerter device- small gravitational mass but large inertial mass- it is possible to obtain a new system, named in literature TMDI, with high inertial mass ratio. This new system manages to combine positive features of a conventional TMD- small mass ratios- with those of an unconventional TMD- high mass ratio: lightness, due to a small auxiliary gravitational mass, and high efficacy and robustness, due to a high inertial mass ratio.

The introduction of inerter-based systems for the mitigation of the response of storage tanks has been already proposed in studies available in the Literature [8, 17]. For instance, Luo et al. proposed to introduce, in parallel with the isolation system, modeled as a linear visco-elastic, a system, called VMD, through which reductions of the sloshing response are achieved. Zhang et al. compared the performances of two different auxiliary control systems, which they plan to place in parallel, with respect to the linear visco-elastic isolation system, respectively a series or a parallel of an inerter and a linear viscous damper.

This work aims at proposing an optimization procedure used for the design of a novel non-conventional isolation system, based on a multi-objective approach, in order to achieve, concurrently, an effective control of the behaviour of both the convective and the impulsive part. Each of them represents a SDOF system, according to the model with lumped masses available in literature. It is worth noticing that the problem the authors dealt with in this paper is similar to a procedure for the optimal tuning of a Tuned Mass Damper system [2, 7, 15], with the difference that the superstructure is here assigned and the aim of the design procedure is the identification of the optimal substructure, whose dissipative capacity, stiffness and inertance have to be found through the multi-objective optimization procedure.

The focus is mainly represented by broad tanks with a large radius, which are widespread throughout the country and typically characterized by a little ratio between the impulsive and the convective masses and by a great first sloshing period.

The paper is organized as follows. The mechanical model is illustrated and the governing equations of motion are derived in the next section. A multi-objective optimization problem is then set in section 3. The main results, in terms of Pareto fronts, frequency response functions and time-history analyses, are discussed in section 4.

2 MECHANICAL MODEL AND EQUATIONS OF MOTION

2.1 Mechanical model

Due to the complexity of the exact mathematical procedure to describe the complete model, an equivalent model, based on lumped masses and stiffnesses, can be introduced, assuming the fluid stored in the tank to be non viscous, incompressible and homogeneous and the flow field to be irrotational. Introducing hypotheses on the distribution of the hydrodynamic pressures, according to the model proposed by Housner for cylindrical tanks and developed by others, the liquid can be considered constituted by three components. The first one, known as impulsive, moves rigidly with the tank walls; the second one represents the interaction between the liquid stored in the tank and the walls; the last one experiences the sloshing motion.

Considering a cylindrical broad tank with circular shape, radius R, filled with a liquid of density ρ_L till a height H, the mass of the $i-^{th}$ convective mode can be expressed as:

$$m_U^i = m\frac{2}{s\lambda_i(\lambda_i^2-1)}tanh(s\lambda_i) \tag{1}$$

where m is the total liquid mass, calculated as $m = \pi\rho_L HR^2$, $s = \frac{H}{R}$ is the aspect ratio and λ_i is the $i-^{th}$ zero of the first derivative of the Bessel function of the first kind. The $i-^{th}$ circular frequency is represented by:

$$\omega_U^i = \sqrt{\frac{\lambda_i g tanh(s\lambda_i)}{R}} \tag{2}$$

The mass and the circular frequency of the first sloshing mode can be obtained substituting λ_1 = 1.8412 in Eqs.1-2. The component related to the liquid-walls interaction has been neglected. Furthermore, only the first sloshing mode is taken into account. The impulsive mass m_L can be derived as:

$$m_L = m - m_U^1 \tag{3}$$

The mechanical model of the proposed isolated reduced order 2DOF system is sketched in Fig.1. A traditional linear base isolation system, called BIS in the following, represented by a SDOF system, whose mass, stiffness and viscous damping coefficients are denoted by m_L, k_L and c_L, is combined with an inerter. Therefore, the connection between the basement and the ground is modeled as a parallel of a Kelvin-Voigt viscoelastic element and a linear inerter.

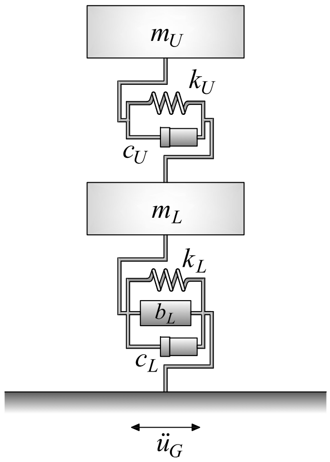


Figure 1: Mechanical model of the 2DOF system.

Table 1: Dimensionless parameters involved in the proposed model.

Description	Symbol	Definition
Isolation damping ratio	ξ_L	$\frac{c_L}{2\sqrt{k_L m_L}}$
Convective damping ratio	ξ_U	$\frac{c_U}{2\sqrt{k_U m_U}}$
Mass ratio	μ	$\frac{m_U}{m_L}$
Frequency ratio	δ	$\frac{\omega_L}{\omega_U}$
Inertial mass ratio	β	$\frac{b_L}{m_L}$

2.2 Equations of motion

The equations of motion of the reduced 2 DOFs for the Inerter Base Isolated System (IBIS), written in terms of the dimensionless parameters listed in Tab.1, result:

$$\ddot{u}_L(t) + 2\mu\xi_U\omega_U(\dot{u}_L(t) - \dot{u}_U(t)) + \mu\omega_U^2(u_L(t) - u_U(t)) + \frac{f_c(t)}{m_L} = -\ddot{u}_g(t) \tag{4}$$
$$\mu\ddot{u}_U(t) + 2\mu\xi_U\omega_U(\dot{u}_U(t) - \dot{u}_L(t)) + \mu\omega_U^2(u_U(t) - u_L(t)) = -\mu\ddot{u}_g(t)$$
$$\frac{f_c(t)}{m_L} = [\beta\ddot{u}_L(t) + 2\xi_L\delta\omega_U\dot{u}_L(t) + \delta^2\omega_U^2 u_L(t)]$$

where $u_L(t)$ and $u_U(t)$, collected in $\mathbf{u}(t)$, respectively represent the displacement of the lower (L) and upper (U) oscillators with respect to the ground, the overdot indicates differentiation with respect to time t and where $\omega_U = \sqrt{\frac{k_U}{m_U}}$ and $\omega_L = \sqrt{\frac{k_L}{m_L}}$. Then, the dimensionless sloshing height, with respect to R, can be calculated as:

$$h(t) = \frac{2(\ddot{u}_U(t) + \ddot{u}_g(t))}{g(\lambda_1^2 - 1)} \tag{5}$$

The Equations which govern the problem of a base isolated system without involving the linear inerter can be obtained from the Eq.4 equaling to zero the inertance contribution. In fact, if the inertance is null, the control system falls back in a base isolation system.

Taking into account the probabilistic nature of the earthquake excitation, neglecting instead the dependence on the excitation frequency, the ground acceleration $\ddot{u}_g(t)$ is firstly modeled as a Gaussian zero mean white noise random process with power spectral density $S_{\ddot{u}_g} = S_0$. Therefore the stochastic properties are assumed to describe the system response. Rewriting the governing equations of motion in the first-order state space form:

$$\dot{z}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}a(t) \tag{6}$$

where

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{u}(t) & \dot{\mathbf{u}}(t) \end{bmatrix}^{\mathbf{T}} \tag{7}$$

is the state vector, $a(t)$ is the applied input process, the matrices **A** and **B** are the state matrix and the input vector, respectively. Since the stationary input process has zero mean and the initial conditions are zero, the response is fully represented by the covariance matrix $\mathbf{G}_{zz}$, that satisfies, by virtue of the stationarity of $\mathbf{z}(t)$, the Lyapunov Equation:

$$\mathbf{A}\mathbf{G}_{zz} + \mathbf{G}_{zz}\mathbf{A}^T + 2\pi S_0 \mathbf{B}\mathbf{B}^T = \mathbf{O} \tag{8}$$

The variances of the displacements of the two oscillators in the controlled configuration, found solving numerically the Eq.8, respectively σ_L^2 and σ_U^2, have been normalized by dividing each of them by σ_{U0}^2, which represents the variance of the displacement of the upper mass with respect to the ground in absence of control, obtaining:

$$I_U = \sqrt{\frac{\sigma_U^2}{\sigma_{U0}^2}} \quad \text{and} \quad I_L = \sqrt{\frac{\sigma_L^2}{\sigma_{U0}^2}} \tag{9}$$

3 MULTI-OBJECTIVE OPTIMIZATION OF THE IBIS

In this section the optimal design of the control system is discussed.

Not all the dimensionless parameters listed in Tab.1 get involved in the optimization procedure as design parameters. In fact, in this work, the sloshing damping factor ξ_U and the mass ratio μ have been supposed known; whereas the three dimensionless parameters referred to the IBIS, i.e. the frequency ratio δ, the damping factor ξ_L and the inertial ratio β, assume the role of design parameters to be optimized. Therefore the proposed procedure aims at designing the optimal connection between the basement of the tank and the ground.

Aiming at achieving the overall protection of both the sub-structures, preserving both the primary and the secondary structures, a multi-objective design criterion has been adopted. Two objective functions have been selected to be minimized: these are the displacement of the upper system relative to the basement- the sloshing displacement- and the lower system displacement relative to the ground- basement displacement. These two functions are in conflict. It is in fact worth noticing that a more effective reduction of the secondary system displacement inevitably corresponds to a worse control of the response of the impulsive displacement. The identification of a solution which corresponds to a minimum- optimal value- for all the objective functions is allowed if they do not conflict with each other.

Therefore, in this case, none of the objective functions can be improved without negative consequences on the other one. Thus, the problem can be stated in the form:

$$\text{find} \qquad min \quad [\mathbf{F}(\mathbf{x})] \qquad \text{subjected to } \mathbf{x}^{min} \leq \mathbf{x} \leq \mathbf{x}^{max} \tag{10}$$

In other terms, the multi-objective optimization procedure consists in finding the design vector **x**, which components vary in a space, capable to minimize the vector of the objective functions **F**(**x**). As a consequence the identification of a unique solution is no longer possible, instead a set of solutions which constitute the Pareto front can be detected.

Table 2: Geometrical and mechanical tank properties [1].

Radius	$R\ [m]$	27.43
Maximum filling level	$H\ [m]$	13.71
Elastic modulus of the tank	$E\ [MPa]$	210000
Steel density	$\rho_s\ [\frac{kg}{m^3}]$	7900
Liquid density	$\rho_L\ [\frac{kg}{m^3}]$	1000
Total liquid mass	$m\ [kg]$	3.24e+07
Convective mass	$m_U\ [kg]$	2.14e+07
Impulsive mass	$m_L\ [kg]$	1.10e+07

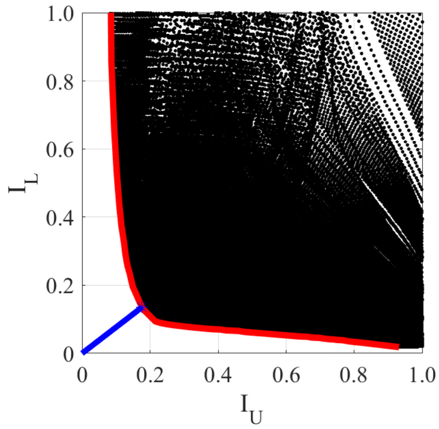


Figure 2: Optimal design of the control system by the Pareto Front.

4 NUMERICAL INVESTIGATIONS

The procedure explained in the previous section has been adopted assuming a case study, whose geometrical and mechanical properties are collected in Tab.2. The results are discussed in this section.

The sloshing damping factor ξ_U = 0.005 and the mass ratio μ = 2.0 have been fixed (see Tab. 1-2) .The three dimensionless design parameters vary in the ranges shown in Tab.3.

4.1 Multi-objective optimization

The problem stated in the previous section can be fitted to the case under consideration substituting in **x** the vector containing the three design parameters:

$$\mathbf{x} = \begin{bmatrix} \xi_L & \delta & \beta \end{bmatrix}^{\mathrm{T}} \tag{11}$$

and considering the following vector of the objective functions:

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} I_U(\mathbf{x}) & I_L(\mathbf{x}) \end{bmatrix}^{\mathrm{T}} \tag{12}$$

The results obtained applying to the system a base motion, modeled as a white noise random input, are illustrated in this section. Both the Pareto front and the performance of the proposed

Table 3: Space of the dimensionless design parameters.

Description	Symbol	$\mathbf{x}^{min}$	$\mathbf{x}^{max}$
Isolation damping ratio	ξ_L	0.01	0.50
Frequency ratio	δ	0.10	10.0
Inertial mass ratio	β	0	10.00

Table 4: Optimal dimensionless parameters found through the multi-objective procedure.

Description	Symbol	Optimal value
Isolation damping ratio	ξ_L	0.50
Frequency ratio	δ	2.24
Inertial mass ratio	β	5.10

IBIS are shown in Fig.2, from which the achievement of a meaningful reduction of the structural response in a large portion of the plan can be deduced.

Moreover the existence of a limit curve, the Pareto front, which delimits a region to whom the access is denied, is highlighted. Among all the points belonging to the Pareto front, the point of minimum distance- blue line in Fig.2- from the origin of the axes has been chosen as the point of the optimality.

At least a triplet of specific values of the dimensionless parameters ξ_L, δ, β can be associated to each point on the plan in Fig.2. Depending on both the ranges of variation of parameters and the number of steps assumed in the analysis, a complete filling of the plan (I_U, I_L) can be achieved, except for the forbidden area.

The triplet of dimensionless parameters associated to the point of optimal performance is listed in Tab.4. Some comments on the optimal values obtained should be done. It is, in fact, worth noticing that ξ_L attains the maximum value ξ_L^{max}, β^{opt} falls in the middle of the range of variation, taking a fairly high value, attesting the inertance beneficial contribution, and δ^{opt} is greater than one, which implies that the period of the control isolation system is lower than the convective period. Instead, values of δ lower than one stand for sloshing mode period stiffer than the isolation system. With the aim of avoiding the onset of crises due to static actions, the adoption of a too flexible isolation system, i.e. the assumption of small values of δ, should be also excluded.

4.2 Sensitivity analyses

Dealing with the issue of establishing the robustness of the proposed seismic mitigation strategy, sensitivity analyses have been performed in order to evaluate the influence of each dimensionless parameter pertinent to the control system on the reduction of the response, paying attention to the system behaviour in a neighborhood of the optimal point.

Denoting with I the ratio between the distance of the current point on the plan (I_U, I_L) from the origin of the axes and the distance of the point of optimal performance, the contour plots obtained fixing $\delta = \delta^{opt}$ and varying the other two parameters around the optimal values are illustrated in Fig.3. The values of the dimensionless parameters divided by the corresponding values in Tab.4 take the superscript N.

The red point in the middle of Fig.3 (B) represents the point of optimality (see Tab.4). It can be noticed that a better control of the response can be achieved increasing ξ_L, suggesting that ξ_L increments induce an advancement of the Pareto Front towards the origin of the axes. Refer, e.g., to the green (D) and red (B) points in Fig.3, which are associated to the same value

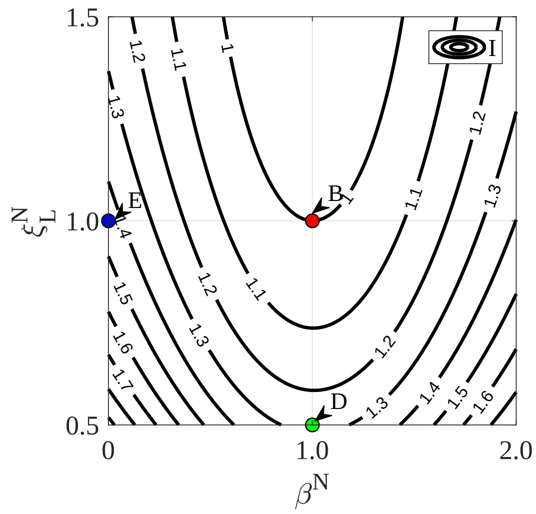


Figure 3: Contour plots of I, assuming δ^{opt}.

Table 5: Optimal dimensionless parameters found through the multi-objective procedure selected for the analyses performed in time and frequency domain.

Description	Symbol	A	B	C
Isolation damping ratio	ξ_L	0.15	0.50	0.50
Frequency ratio	δ	2.56	2.24	1.55
Inertial mass ratio	β	6.8	5.1	0

of β but to different values of ξ_L. Moving on a vertical line, it is also shown in Fig.3 that further increasing ξ_L beyond the optimal value does not lead to a meaningful improvement of the performance. Furthermore, the existence of a minimum for β is well stressed, implying that the selection of a β value should be done carefully.

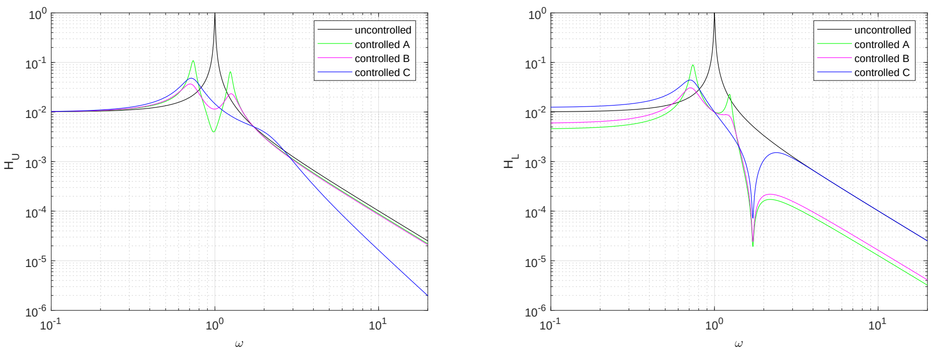
Fig.3 can be used in order to compare the performance of a IBIS - red point- with respect to a BIS- blue point (E), characterized by the same ξ_L, highlighting the achievement of a reduction of I thanks to the employment of the inertance.

4.3 Evaluation of the IBIS seismic performance

The frequency transfer function curves of sloshing displacement and isolation displacement are represented. The values of the parameters are listed in (Tab.5).

In (Fig.4) the effectiveness of the optimally designed IBIS is shown, through the representation of the frequencydomain transfer function curves, from which an effective response mitigation for both sloshing height and isolation displacement can be deduced. For comparative purposes, the responses of an optimized system characterized by a lower xi_L value (A) and of an optimized BIS (C) are shown.

In order to evaluate and assess the seismic effectiveness of the designed control system object of this work, numerical simulations have been carried out, assuming different base motion histories. Time-history analyses have been carried out on the 2DOF system both in the fixed



(a) (b)

Figure 4: Frequency transfer functions of a) sloshing height and b) isolation displacement.

base tank configuration and in the IBIS.

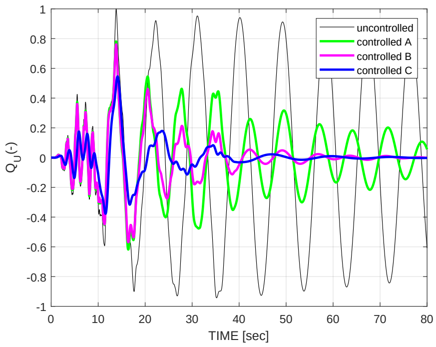
Two quantities have been monitored in order to evaluate the response, i.e. sloshing heights and basement displacements relative to the ground. The responses in time domain in terms of sloshing heights and isolation displacement are illustrated. The values are normalized by dividing the sloshing height and the basement displacement by the peak value of the sloshing height in the uncontrolled configuration. The dimensionless indices obtained make the interpretation of the results easier, in the sense that a value smaller than one means an effective reduction of the response quantities, whereas a value greater than one implies an amplification of the response with respect to the reference configuration. The results shown refer to a system involving the optimal set of parameters.

5 CONCLUSIONS

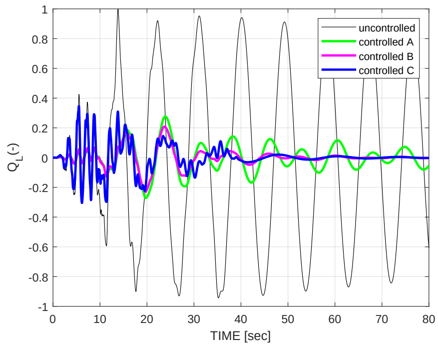
A novel strategy for controlling the sloshing response of seismically excited liquid storage tanks with rigid walls based on an inerter- based isolation system has been developed in this paper. The studies are carried on a reduced 2DOF system. With the aim of controlling at the same time the displacements of the two coupled oscillators- one pertinent to the sloshing part of liquid mass, the second one comprehensive of the impulsive part of the total liquid mass- the design parameters of the inerter- based isolation system, that is, its damping coefficient, its stiffness and its inertial mass, called inertance, are optimized by stating a Pareto multi-objective optimization problem, minimizing the two objective functions related to the displacements of the two oscillators, assuming as a base input a ground acceleration modeled as a Gaussian stochastic process with white noise power spectral density. On the basis of the defined design method an optimized system, able to effectively control both the sloshing and the isolation response, has been obtained.

Sensitivity analyses, in which the influence of the dimensionless parameters selected as representative of the added vibration control system is investigated, have been conducted. Thus, the role of the design parameters in the deployment of points in the plan depicted has been extensively investigated.

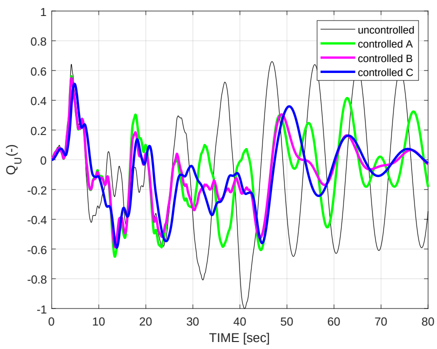
Results obtained by subjecting the system to a series of near fault and far field earthquakes as well as synthetic accelerograms were subsequently reported, showing the gaining of mean-



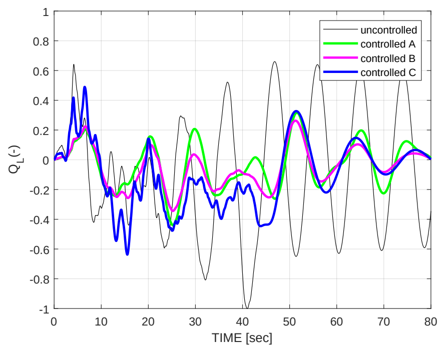
(a)



(b)



(c)



(d)

Figure 5: Time histories of: a) normalized sloshing height, b) normalized isolation displacement for Kobe earthquake; c) normalized sloshing height, d) normalized isolation displacement for Hachinohe earthquake.

ingful reductions of the response.

Further developments will involve a TMDI system in order to study any beneficial effects of its employment in such kind of problems.

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