Y(4260) --> gamma + X(3872) in the diquarkonium picture

$Y(4260) \to \gamma + X(3872)$ in the diquarkonium picture

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Abstract

The observed $Y(4260) \to \gamma + X(3872)$ decay is a natural consequence of the diquark-antidiquark description of Y and X resonances. In this note we attempt an estimate of the transition rate, $\Gamma_{\rm rad}$, by a non-relativistic calculation of the electric dipole term of a diquarkonium bound state. We compute $\Gamma_{\rm rad}$ for generic composition values of the isospin of X and Y. Specializing to $I = 0$ for $X(3872)$, we find $\Gamma_{\rm rad} = 496$ keV for $Y(4260)$ with $I = 0$ and $\Gamma_{\rm rad} = 179$ keV for $I = 1$. Combining with BESIII data, we derive upper bounds to $B(Y \to J/\Psi + \pi + \pi)$ and to $\Gamma(Y \to \mu^+\mu^-)$. We expect to confront these results with forthcoming data from electron-positron and hadron colliders.

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Introduction

Exotic, hidden charm, mesons known as X, Y, Z resonances have been interpreted in [1, 2] as tetraquarks, namely states made by two diquark pairs $[cq][\bar{c}\bar{q}']$ with q, q' light quarks. Each pair is in color $\mathbf{3}$ or $\bar{\mathbf{3}}$ configuration, spin $s, \bar{s} = 1, 0$ and relative orbital momentum $L = 0, 1$. The scheme has met with some degree of success at explaining the rich phenomenology which has emerged from electron-positron and proton-(anti)proton collider experiments. More information is expected in the future data from LHCb, BES III and Belle II.

The long-standing conviction, based on consideration of large-N QCD, that tetraquark states could only materialize in the form of hadronic resonances too broad to be experimentally resolved, has been recently proven incorrect in [3]. Tetraquarks in large-N QCD have been further studied in [4]. The recent discovery of two pentaquark states of opposite parity [5] has reinforced the case for a new spectroscopic series of hadrons, in which diquarks (antidiquarks) replace antiquarks (quarks) in the classical scheme [6].

Recently, a new paradigm for the spin-spin interactions in hidden-charm tetraquarks has been proposed, which assumes the dominance of spin-spin couplings inside the diquark or the antidiquark [7]. This simple ansatz reproduces the mass ordering of the three, well identified, spin 1^+ states, $X(3872)$, $Z(3900)$ and $Z(4020)$ and the pattern of their observed decays. In addition in Ref. [7] the diquark spin assignments of $L = 1$ states is discussed, pointing out that $Y(4260)$ has the same spin distribution as $X(3872)$ namely

$$
\begin{aligned}
X &= |0_{cq}, 1_{\bar{c}\bar{q}}; L = 0\rangle + |1_{cq}, 0_{\bar{c}\bar{q}}; L = 0\rangle \\
Y &= (|0_{cq}, 1_{\bar{c}\bar{q}}; L = 1\rangle + |1_{cq}, 0_{\bar{c}\bar{q}}; L = 1\rangle)_{J=1}
\end{aligned}
\tag{1}
$$

States are in the basis $|s, \bar{s}; L\rangle$ where s ($\bar{s}$) is the diquark (antidiquark) spin and L the relative orbital angular momentum.

A similar scheme has been extended to exotic, hidden beauty mesons [8], and shown to give a consistent picture of the decays of $\Upsilon(10890)$ into $\Upsilon(nS)\pi^+\pi^-$ or $h_b(nP)\pi^+\pi^-$, which occur via the intermediate Z_b, Z_b' states [9].

The suppression of spin-spin interactions between a quark and an antiquark in different diquarks, underlined in [7], suggests that the overlap of the two constituents is very small, *as if* diquark and antidiquark were well separated entities inside the hadron. In the present paper we pursue this idea to the extreme consequences by considering the approximation where a diquark and an antidiquark can be described as pointlike. X, Y, Z would be, in this case, bound particle-antiparticle systems, that we call diquarkonia for brief. We shall see that this extremely simplified picture leads to a reasonable approximation to the mass spectrum of S and P wave tetraquarks.

The diquarkonium picture has been introduced by A. Ali *et al.* [10] to study the production and decay of the $Y(10890)$ considered as the b-tetraquark analog to the $Y(4008)$. The annihilation of a diquarkonium with $s, \bar{s} = 0$ has been treated in [10] as the annihilation of a pair of spinless, pointlike particles. The extension to $Y(4260) \to \mu^+\mu^-$ deserves further consideration, given that the diquark and the antidiquark in the Y have not the same spin and the coupling to the photon is not simply determined by the charges.

We study the diquarkonia mass spectrum in the non-relativistic approximation, using the Cornell potential previously applied to charmonia [11,12] and then, equipped with the corresponding wave functions, we compute the predicted rate of the ED1 allowed transition with $\Delta L = 1, \Delta s = 0$

$$Y(4260) \to \gamma X(3872) \tag{2}$$

which arises naturally from (1).

Using the masses of the identified X, Y, Z states, we find parameters of the potential rather similar to the Cornell parameters and confirm the identification of the $Z(4430)$ as the first radial excitation of $Z(3900)$.

We compute the rates of the radiative transition for isospin $I = 0, 1$ of $X(3872)$ and $Y(4260)$. Assuming $X(3872)$ to be an isospin singlet, we find

$$\begin{aligned} &\Gamma(Y(4260) \to \gamma X(3872)) = \\ &= 496 \text{ keV } (I: \ 0 \to 0) \qquad (3) \\ &= 179 \text{ keV } (I: \ 1 \to 0) \qquad (4) \end{aligned}$$

and compare this result to the available experimental information [14].

The rate of the radiative decay (2) has been computed in Ref. [13] in the molecular scheme, describing $Y(4260)$ and $X(3872)$ as DD_1 and $DD^\star$ bound states respectively. The resulting rate turns out to be considerably smaller than the values indicated in (3) or (4).

Diquark masses

For S-wave states diquarkonia one writes the rest frame Hamiltonian

$$M(S\text{−wave}) = 2M_{cq} + 2\kappa_{cq}(\boldsymbol{s}_c \cdot \boldsymbol{s}_q + \bar{\boldsymbol{s}}_c \cdot \bar{\boldsymbol{s}}_q) \tag{5}$$

where $\boldsymbol{s}$ ($\bar{\boldsymbol{s}}$) denotes the quark (antiquark) spin and M_{cq} is the effective diquark mass. In the $|s, \bar{s}\rangle_J$ basis, S-wave tetraquarks with $J^P = 1^+$ are described [7] by

$$J^P = 1^+ \quad C = + \qquad X_1 = \frac{1}{\sqrt{2}} (|1,0\rangle_1 + |0,1\rangle_1) = X(3872) \tag{6}$$

$$J^P = 1^+ \quad G = + \qquad \begin{cases} Z = \frac{1}{\sqrt{2}} (|1,0\rangle_1 - |0,1\rangle_1) = Z(3900) \\ Z' = |1,1\rangle_1 = Z(4020) \end{cases} \tag{7}$$

M_{cq} can be estimated from the $X(3872)$ and $Z(4020)$ masses, subtracting the spin-spin contributions

$$\begin{aligned} M(X) &= M(Z) = 2M_{cq} - \kappa_{cq} \\ M(Z') &= 2M_{cq} + \kappa_{cq} \\ M_{cq} &= \frac{1}{4} \ (M[Z(3900)] + M[Z(4020)]) \approx 1980 \text{ MeV} \qquad (8) \end{aligned}$$

As a first approximation, we shall use M_{cq} as input mass in the Schrödinger equation that gives the diquarkonia wave functions and masses.

In the case of charmonium, the input charm quark mass in the Schrödinger equation is obtained from the leptonic width $\Gamma(J/\Psi \to e^+e^-)$, see [11]. In our case, the leptonic width of $Y(4260)$ is not available yet and we shall be content to use the value (8) as input. We have verified that the various quantities are little sensitive (only to few percents) to variations of the input diquark mass around this value.

Bound state masses

The simplest description of diquarkonia is in term of a non-relativistic potential, $V(r)$. For this first exploration we take the Cornell potential [11, 12] with one Chromo-Coulombic and one confining term

$$V = -A\,\frac{1}{r} + \nu\,r \tag{9}$$

For charmonia, one finds [12]

$$A = 0.47,\;\; \nu = 0.19\ \text{GeV}^2\ \text{(charmonium spectrum)} \tag{10}$$

For diquarkonia, we leave the parameters as free variables to be determined by comparison of diquarkonia eigenvalues $1S$, $2S$ and $2P$, to the mass differences of the $J = 1$ states, $X(3872)$ or $Z(3900)$, $Y(4260)$ and $Z(4430)$, subtracted of spin dependent terms. The subtraction is straightforward for the S-wave states, but for P-waves it requires the determination of not well known spin-orbit couplings [7], which introduces a non negligible uncertainty.

Let us assume, as in [1, 7], that we can write

$$\begin{aligned} &M(X) = M_0(1S) + \text{spin interaction terms} \\ &M(Y) = M_0(2P) + \text{spin interaction terms}, \\ &\text{etc.} \end{aligned} \tag{11}$$

where in the r.h.s. we have introduced the eigenvalues of the Schrödinger equation, $M_0(1S)$, etc.

Explicitly, spin interaction terms are obtained from a parametrization of the constituent quark

Diquarkonium	X	Z	Z'	Y
$1S$	3871.69 ± 0.17	3888.7 ± 3.4	4023.9 ± 2.4	
$2S$		4485^{+40}_{-25}		
$2P$				4251 ± 9

Table 1: Masses of well identified X, Y, Z states used in the text [15].

Hamiltonian, which generalizes Eq. (5) to include orbital angular momentum excitation [7] [1]

$$M = M_{00} + B_c \frac{\boldsymbol{L}^2}{2} - 2a\ \boldsymbol{L} \cdot \boldsymbol{S} + 2\kappa_{qc}\ [(\boldsymbol{s}_q \cdot \boldsymbol{s}_c) + (\boldsymbol{s}_{\bar{q}} \cdot \boldsymbol{s}_{\bar{c}})] \tag{12}$$

Obvious manipulations lead to

$$\begin{aligned} M = M_{00} + B_c \frac{L(L+1)}{2} + a\ [L(L+1) + S(S+1) - 2] + \\ + \kappa_{qc}\ [s(s+1) + \bar{s}(\bar{s}+1) - 3] \end{aligned} \tag{13}$$

and we read

$$\begin{aligned} M(X(3872)) &= M_{00} - \kappa_{qc} \\ M(Y(4260)) &= M_{00} + B_c + 2a - \kappa_{qc} \\ M(Z(4430)) &= M'_{00} - \kappa_{qc} \end{aligned} \tag{14}$$

(M'_{00} is the analog of M_{00} for the first radial excitation) so that

$$\begin{aligned} M_0(1S) &= M(X(3872)) + \kappa_{qc} \\ M_0(2P) &= M(Y(4260)) - 2a + \kappa_{qc} \\ M_0(2S) &= M(Z(4430)) + \kappa_{qc} \end{aligned} \tag{15}$$

and

$$M_0(2S) - M_0(1S) = M(Z(4430)) - M(Z(3900)) \tag{16}$$

$$M_0(2P) - M_0(1S) = M(Y(4260)) - M(X(3872)) - 2a \tag{17}$$

We use the mass values summarized in Tab. 1 [15] and take the value $a = 73$ MeV from the fit to the masses of Y states in [7][2] to which we attribute a theoretical error estimated to be not less

[1]Signs are chosen so that, for B_c, a, κ_{qc} positive, energy increases for increasing $\boldsymbol{L}^2$ and $\boldsymbol{S}^2$. As remarked in [7], this Hamiltonian is not the most general one as it does not include tensor terms which are known to be important in charmonium. The Hamiltonian describes well the $J = 1$ states but it could not be reliable for states with higher J.

[2]see Eq. (47) there, for the case in which: $Y_3 = |(1_{cq}, 1_{\bar{c}\bar{q}})_{S=0}; L = 1\rangle = Y(4220)$, the narrow structure in the h_c channel [17](S is the total tetraquark spin). Identifying $Y_3 = Y(4290)$, the broad structure in the h_c channel [17], would lead to a result consistent with $A = 0$.

than 50%. We find

$$M_{2S} - M_{1S} = 0.60 \pm 0.03 \text{ GeV}$$
$$M_{2P} - M_{1S} = 0.23 \pm 0.07 \text{ GeV} \tag{18}$$

We solve numerically the Schrödinger equation [16] using the diquark mass in (8).

Results for the mass-differences are reported in Fig. 1, in the plane of the eigenvalue differences $2S - 1S$ and $2P - 1S$. The result for the Cornell potential with charmonium parameters is given by the round dot whereas the squared box with errors corresponds to the eigenvalue differences estimated in (18). Lines indicate the results computed with fixed A while varying ν. Approximate agreement with the mass formula point is obtained for

$$A \sim 0, \;\; \nu = 0.25 \text{ GeV}^2 \text{ (diquarkonium spectrum)} \tag{19}$$

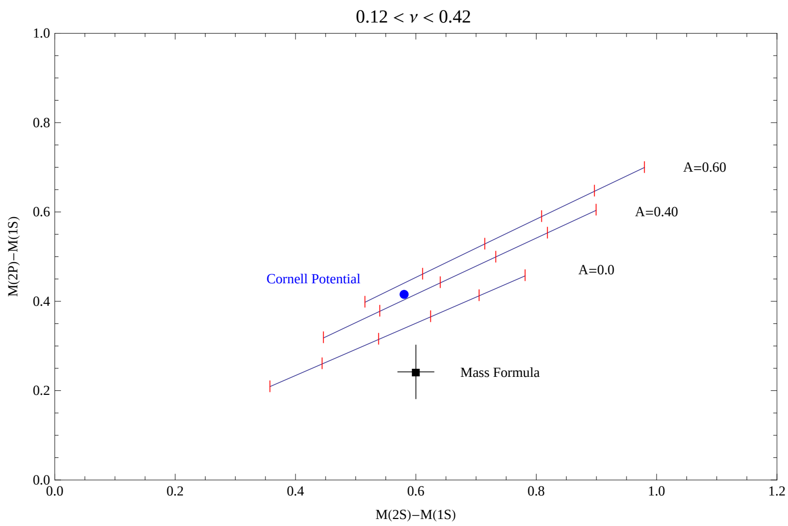

Figure 1: Results for the mass-differences in the plane $M(2S) - M(1S)$ and $M(2P) - M(1S)$ (in GeV). The round dot represents the result for the Cornell potential with charmonium parameters given in Eq. (10) and the squared box with errors corresponds to the eigenvalue differences estimated in (18). Lines indicate the results computed with fixed A and varying ν.

The difference $2S - 1S$ is well reproduced for both sets of parameters, Eqs. (10) and (19), reinforcing the case for $Z(4430)$ to be the first radial excitation of $Z(3900)$ [2, 4]. The difference between the parameters in (10) and (19) may be due to the inaccuracy of the mass formula or to the fact that the diquark is not as pointlike as the c quark, therefore less sensitive to the short distance effects embodied by the Coulomb term.

The ED1 transition

We consider the process

$$Y(4260) \to \gamma + X(3872) \tag{20}$$

as the ED1 transition from a P-wave to a S-wave tetraquark with the same spin structure. Diquarks are taken as pointlike objects of electric charge Q

$$Q = \begin{cases} +\frac{4}{3} \text{ for } [cu] \\ \\ +\frac{1}{3} \text{ for } [cd] \end{cases} \tag{21}$$

The Hamiltonian (radiation gauge) is

$$H = eQ\ \boldsymbol{v} \cdot \boldsymbol{A}(\boldsymbol{x}) \tag{22}$$

where $\boldsymbol{A}$ is the vector potential, $\boldsymbol{x}$ the coordinate and $\boldsymbol{v}$ the relative velocity of the particles in the centre of mass system, with the diquark reduced mass

$$\mu = \frac{1}{2} M_{2q} \tag{23}$$

and M_{2q} given by (8). In the dipole approximation where we set $\boldsymbol{A}(\boldsymbol{x}) \approx \boldsymbol{A}(\boldsymbol{0})$, the matrix element for the decay is

$$\mathcal{M}_{if} \;=\; e\, \frac{1}{\sqrt{2\omega}}\, \langle X, m | Q\, \boldsymbol{v} | Y, k\rangle \ \cdot \boldsymbol{\epsilon}(\boldsymbol{q}) = \tag{24}$$

$$= ie\omega\, \frac{1}{\sqrt{2\omega}}\, \langle X, m | Q\, \boldsymbol{x} | Y, k\rangle \ \cdot \boldsymbol{\epsilon}(\boldsymbol{q}) \tag{25}$$

where $\boldsymbol{\epsilon}$ and $\boldsymbol{q}$ are the polarization vector and momentum of the photon, $\omega = E_f - E_i$ its energy and m and k label the spin states of X and Y respectively.

The total rate is obtained by (25)

$$\Gamma \;=\; e^2 \int \frac{d^3q}{(2\pi)^3 2\omega}\, \omega^2\, (2\pi)\, \delta(E_f - E_i - \omega)\ \ (\delta_{ij} - n_i n_j)\, \frac{1}{3} \sum_{m,k} \langle Q\, x^i\rangle\ \langle Q\, x^j\rangle^\star =$$

$$= \frac{4\,\alpha\,\omega^3}{9} \sum_{m,k,i} |\langle Q\, x^i\rangle|^2 \tag{26}$$

where we used

$$\int d\Omega\, (\delta_{ij} - n_i n_j) = \frac{2}{3}(4\pi)\delta_{ij} \tag{27}$$

with $n^i = q^i/\omega$.

Diquarkonium wave-functions and transition radius

Consider first diquarkonia with a given flavor composition, *e.g.* $Y_u = [cu][\bar{c}\bar{u}]$ or $Y_d = [cd][\bar{c}\bar{d}]$. In the non-relativistic approximation, state vectors corresponding to Y (P-wave) or X (S-wave) are written as

$$N_Y \langle Y, k| = \langle 0| \int d^3x \; R^{2P}(r) \frac{x^i}{r} \epsilon_{ijk} \left[d_a^j \left(\frac{\boldsymbol{x}}{2}\right) (d_c)^a \left(-\frac{\boldsymbol{x}}{2}\right) + d_a \left(\frac{\boldsymbol{x}}{2}\right) (d_c)^{aj} \left(-\frac{\boldsymbol{x}}{2}\right) \right] \quad (28)$$

$$N_X \langle X, m| = \langle 0| \int d^3x \; R^{1S}(r) \left[d_a^m \left(\frac{\boldsymbol{x}}{2}\right) (d_c)^a \left(-\frac{\boldsymbol{x}}{2}\right) + d_a \left(\frac{\boldsymbol{x}}{2}\right) (d_c)^{a,m} \left(-\frac{\boldsymbol{x}}{2}\right) \right] \quad (29)$$

where d and d_c (or d^m and d_c^m) are destruction operators of diquark and antidiquark with spin $S = 0$ ($S = 1$) and $R(r)$ the radial wave functions. We have made explicit the color index $a = 1, 2, 3$. The normalization factors are obtained from (non-relativistic) identities of the form

$$\langle 0| d_a^j(\boldsymbol{x}) [d_b^l(\boldsymbol{y})]^\dagger |0\rangle = \delta_a^b \; \delta^{jl} \; \delta^{(3)}(\boldsymbol{x} - \boldsymbol{y}), \text{ etc.} \quad (30)$$

to wit

$$N_Y^2 = 2^6 (2N) \, \frac{2}{3} \, (4\pi) \, \delta_{kk'} \; \delta^{(3)}(0) \quad (31)$$

$$N_X^2 = 2^6 (2N) \, (4\pi) \, \delta_{mm'} \; \delta^{(3)}(0) \quad (32)$$

where (27) has been used and the number of colors is $N = 3$.

The transition radius is then computed between normalized states to be

$$\langle X, m| x^i |Y, k\rangle = \frac{1}{\sqrt{6}} \; \epsilon_{mik} \; \langle r\rangle \quad (33)$$

$$\langle r\rangle = \langle r\rangle_{2P\to 1S} = \frac{\int_0^\infty r \left[y^{1S}(r) y^{2P}(r) \right] dr}{\sqrt{\int_0^\infty dr \, (y^{1S}(r))^2} \sqrt{\int_0^\infty dr \, (y^{2P}(r))^2}} \quad (34)$$

and we have introduced the reduced radial wave functions of the $1S$ and $2P$ wave-functions $y(r) = rR(r)$ computed numerically [16].

Finally, we consider the general isospin structure of $Y(4260)$ and $X(3872)$, defining

$$X(3872) = \cos\theta \, X_u + \sin\theta \, X_d$$
$$Y(4260) = \cos\phi \, Y_u + \sin\phi \, Y_d \quad (35)$$

and obtain

$$\langle X, m| Q \, x^i |Y, k\rangle = \frac{1}{\sqrt{6}} \; \epsilon_{mik} \; Q_{\text{eff}} \; \langle r\rangle$$

$$Q_{\text{eff}} = \left(\frac{4}{3} \cos\theta \cos\phi + \frac{1}{3} \sin\theta \sin\phi \right) \quad (36)$$

	charm. potential, Eq. (10)	diquark. potential, Eq. (19)	$Q^2_{\rm eff}$
$\langle r\rangle$, GeV^{-1}	1.84	2.15	
$\Gamma(I=0\to I=0)$, keV	361 $(3.0\cdot 10^{-3})$	496 $(4.1\cdot 10^{-3})$	25/36
$\Gamma(I=1\to I=0)$, keV	132 $(1.1\cdot 10^{-3})$	179 $(1.5\cdot 10^{-3})$	1/4

Table 2: Transition radius and corresponding decay widths for $Y\to\gamma X$. In parenthesis the branching ratio, assuming $\Gamma_{Y(4260)}=120$ MeV [15].

Diquarkonium rate

With (26) and (36), we obtain

$$\Gamma(Y(4260)\to\gamma+X(3872))=\frac{4\,\alpha\,\omega^3}{9}\,Q^2_{\rm eff}\,\langle r\rangle^2=154.2\times Q^2_{\rm eff}\left(\frac{\langle r\rangle}{\rm GeV^{-1}}\right)^2\ \text{keV} \tag{37}$$

Note that $0\le Q^2_{\rm eff}\le(4/3)^2$, with zero attained when $Y=Y_u$ and $X=X_d$ or viceversa and the maximum when Y and X have only u-flavor.

As indicated by data, we take $X(3872)$ close to a pure $I=0$ state. For the two sets of parameters of the potential, Eqs. (10) and (19), we summarize in Tab. 2 (i) the numerical values of the transition radius and (ii) the rate for $Y(4260)$ with $I=0,1$.

With the indicated numerical value of the radius, we are at the border of the dipole approximation, since $\omega\langle r\rangle\sim 0.8$, not so much smaller than one. The situation, however, is not so different from the radiative transition $\chi_{c2}\to J/\Psi\gamma$ which has $\omega\langle r\rangle\sim 0.86$, with estimated $\sim 10\%$ corrections, see [19].

The result found in Ref. [14] can be stated as

$$\frac{B(Y\to\gamma X)B(X\to J/\Psi\ \pi\pi)}{B(Y\to J/\Psi\ \pi\pi)}=5\cdot 10^{-3} \tag{38}$$

which, assuming [15]

$$B(X\to J/\Psi\ \pi\pi)\gtrsim 2.6\cdot 10^{-2} \tag{39}$$

becomes

$$\frac{B(Y\to\gamma X)}{B(Y\to J/\Psi\ \pi\pi)}<0.2 \tag{40}$$

Using our result, we predict

$$B(Y\to J/\Psi\ \pi\pi)>\begin{cases}2.1\cdot 10^{-2} & (I:0\to 0)\\ 0.78\cdot 10^{-2} & (I:1\to 0)\end{cases} \tag{41}$$

From the value of $\Gamma(Y \to X\gamma)$ we can also estimate $\Gamma(Y \to e^-e^+)$. We use the well known formula for the peak cross section

$$\sigma(e^-e^+ \to X\,\gamma)_Y = \frac{12\pi}{m_Y^2}\frac{\Gamma(Y \to X\gamma)\Gamma(Y \to e^-e^+)}{\Gamma(Y \to \mathrm{All})^2} \tag{42}$$

with the experimental determination [18]

$$\sigma(e^-e^+ \to Y(4260) \to X\,\gamma) = \frac{0.33\ \mathrm{pb}}{\mathcal{B}(X \to \pi^+\pi^- J/\Psi)} \tag{43}$$

and the input values in Table 1 and 2 (diquarkonium potential)

$$\Gamma(Y \to e^-e^+) \lesssim \frac{226}{(\Gamma(Y \to X\gamma)/\mathrm{keV})}\ \mathrm{keV} = \begin{cases} 0.45 \\ 1.26 \end{cases} \mathrm{keV} \tag{44}$$

and

$$\sigma(e^+e^- \to \mu^+\mu^-)_Y \lesssim \frac{2871}{(\Gamma(Y \to X\gamma)/\mathrm{keV})^2}\ \mathrm{pb} = \begin{cases} 0.01 \\ 0.09 \end{cases} \mathrm{pb} \tag{45}$$

for Y isospin equal to 0 or 1, respectively.

Conclusions

We estimated the transition rates $\Gamma(Y(4260) \to \gamma + X(3872))$ under the assumption that Y is a diquarkonium confined by a Cornell-like potential, either isospin singlet or triplet. Our results reinforce the case for $Z(4430)$ to be the first radial excitation of $Z(3900)$ [2, 4]. Mass differences between states with different orbital excitations computed with a linearly rising potential and no Chromo-Coulombic term approximately agree with the mass formula derived from the constituent quark model in [7]. The results obtained, together with upper bound estimates of $B(Y \to J/\Psi\ \pi\pi)$ and of the Y electronic width, can be confronted with future data, from electron-positron and hadron colliders.

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