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Microwave synthesis of Bessel, Bessel–Gauss, and Gaussian beams: a fully vectorial electromagnetic approach

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Abstract

Bessel, Bessel-Gauss, and Gaussian beams have widely been investigated in optics in the paraxial approximation, under the frame of a scalar wave theory. Such approximations can hardly be applied in the microwave/millimeter-wave range, where the vectorial nature of the electromagnetic fields cannot be neglected, and experimental realizations for some of these beams appeared only recently. In this work, we discuss the generation of Bessel, Bessel-Gauss, and Gaussian beams through a fully vectorial electromagnetic approach. The field derivation of all these beams is first illustrated and numerical evaluations are then reported to compare their different propagation and diffractive behaviors. Finally, an innovative approach for realizing such solutions with planar microwave devices exploiting leaky waves is demonstrated through accurate numerical simulations.

Introduction

More than 30 years ago, Durnin first demonstrated that it was possible to generate electromagnetic waves that do not undergo diffraction over large distances through finite-size apertures [1, 2]. These limited-diffraction solutions are commonly known as *Bessel beams* (BBs). Successively, Durnin and his coauthors compared the power transport efficiency of BBs and *Gaussian beams* (GBs) with the same initial beam waist, showing that the apparent higher localization of GBs is entirely washed at a short distance from the aperture, due to their inherent diffractive spreading [3]. This aspect raised some debates in the optical community (see, e.g. [4–6]), but it is now commonly accepted that both solutions have different propagation and diffractive properties that can be in some way attractive for focusing applications.

Few months after the first experimental demonstration of BBs [2], a new type of solutions, the so-called *Bessel-Gauss beams* (BGBs), was proposed in [7] to derive an analytical description of a limited-diffraction beam, under the paraxial approximation. It turned out that the use of a Gaussian apodization function on the Bessel aperture distribution not only allowed for deriving an analytical expression for a BGB, but also considerably reduced the on-axis oscillations that typically affected BB propagation features due to diffraction from edges. In particular, as opposed to BBs and GBs that only have one degree of freedom (i.e., the axicon angle in BBs and the beam waist in GBs), BGBs offer two degrees of freedom (i.e., both the axicon angle and the beam waist) to control the diffractive, focusing, and power transport properties of the beam, thus allowing for more flexibility in application-oriented designs.

More physical insights about the propagation and diffractive features of BBs, BGBs, and GBs were finally provided in [8], where analytical and numerical results were obtained under the frame of a *scalar theory*. Consequently, these findings do not generally apply at microwave frequencies where the *fully vectorial nature* of the electromagnetic fields has to be taken into account. This difficulty initially hindered the microwave generation of Bessel beams that only appeared for the first time in the last decade [9]. Nonetheless, while BBs are now widespread at microwaves (see, e.g. [9] and refs. therein) BGBs have been discussed only recently on a rigorous theoretical basis [10] (although the benefits arising from the application of amplitude tapering functions on the aperture distribution were yet highlighted in [11]). On the other hand, GBs are commonly generated at microwaves, but mostly by means of bulky devices such as corrugated horn feeders for reflector antennas [12, 13]. Moreover, the beam features of GBs at microwaves are often studied in the far-field region [12, 13], whereas here we are interested in the field behavior within the Fresnel region, and in particular within the *Rayleigh range*. (We should also note that the field generated by a corrugated horn is only an approximate version of a GB [14], and the *gaussicity* figure of merit is introduced for this purpose [15].)

In this paper, we address the microwave generation of BBs, BGBs, and GBs using a fully vectorial approach applied to practical solutions of radiating elements exploiting suitable classes of leaky waves. We first derive in section "Field derivation" analytical representations for the relevant tangential field components required to generate BBs, BGBs, and GBs in the near-field region above the radiating aperture. In section "Leaky-wave approach", an efficient technique to practically realize all these kinds of beams at microwave frequencies by means of a low-cost, low-profile planar device is described. Specifically, the generation of BBs, BGBs, and GBs through radially periodic *tapered* leaky-wave structures is demonstrated. This method is corroborated in section "Numerical results" through accurate numerical simulations. Conclusive comments are drawn in section "Conclusion".

Field derivation

In the following, we will always assume an azimuthally symmetric TM-polarized field (where TM stands for transverse magnetic with respect to the propagation z-axis according to the coordinate reference frame reported in Fig. 1, left) and derive the tangential electric and magnetic field components, assuming that we want to generate an ideal BB/BGB/GB distribution for the vertical z component. As a result of the previous hypotheses (viz., $H_z = 0$ and $\partial/\partial\phi = 0$, ϕ being the azimuthal coordinate of a cylindrical reference frame $\{\rho, \phi, z\}$), E_ρ, E_z, H_ϕ are the only non-zero field components. (A time-harmonic dependence $e^{j\omega t}$ of the fields is tacitly assumed and suppressed throughout the paper.)

From Maxwell's equations in a cylindrical reference frame, the tangential electromagnetic field components can be expressed in terms of E_z through:

$$E_\rho = -\frac{1}{\rho}\frac{\partial}{\partial z}\int_0^\rho E_z \rho' d\rho' , \qquad H_\phi = -j\frac{k_0}{\eta\rho}\int_0^\rho E_z \rho' d\rho' , \tag{1}$$

where k_0 is the free-space wavenumber, η is the characteristic impedance of the medium and ρ' is a dummy variable of integration along the radial coordinate.

It is worth to stress here that BBs arise as *ideal* solutions of Helmholtz equation in a cylindrical reference frame. As a result, the field derivation here proposed for BBs leads to a *full-wave* electromagnetic solution. On the other hand, BGBs and GBs arise as *asymptotic* solutions of Helmholtz equation in the paraxial limit, namely the *paraxial wave equation*. Therefore, a *full-wave* electromagnetic solution for such kinds of beams does not strictly exist over the whole space. In this light, the following section only aims at showing conditions for correctly synthesizing the aperture fields to obtain along the longitudinal component of the electric field either a BB, or a BGB, or a GB, in the near-field region. This approach is demonstrated to be effective within a limited region located in the near field of the radiating aperture (criteria to determine this region are provided in [10]).

Bessel beam

An *ideal*, axially symmetric *scalar* BB is represented through the expression $J_0(k_\rho\rho)e^{-jk_z z}$ where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, k_ρ and k_z are the radial and vertical wavenumbers, respectively, related to k_0 through the separation relation $k_z^2 + k_\rho^2 = k_0^2$. This expression shows that the intensity of an *ideal* BB is invariant along the z-axis, thus revealing its *nondiffracting* nature [1]. In practice, a BB has to be created through a finite aperture; a ray interpretation (see Fig. 1, left) reveals that a *truncated* BB does not undergo diffraction up to an axial distance known as *nondiffractive range* $z_{ndr} = \rho_{ap}\cot\theta_0$, $\theta_0 = \arcsin(k_\rho/k_0)$ being the axicon angle.

As commented at the beginning of this section, we derive the tangential field components for having an *ideal* BB on E_z, i.e., $E_z \propto J_0(k_\rho\rho)e^{-jk_z z}$. As is well known [16, 17], in order to have $E_z \propto J_0(k_\rho\rho)e^{-jk_z z}$, the aperture field, i.e., the electromagnetic field at $z = 0$, is not required to have a stationary Bessel-like character. Indeed, a ray interpretation [18] of radiation from a cylindrical aperture (see Fig. 1, left) reveals that if we assume an inward traveling-wave aperture distribution (represented in cylindrical coordinates by a first-kind zeroth-order Hankel function $H_0^{(1)}(\cdot)$), constructive interference between the emitting rays is expected to occur in a diamond-shaped region whose vertex is determined by the aperture radius ρ_{ap} and yields a nondiffractive range at z_{ndr}. It is worth to recall here that the inward ray bundle turns into an outward ray bundle upon crossing its caustic, i.e., the z-axis (as expressed by the properties of Hankel functions, $H_0^{(1)}(-x) = -H_0^{(2)}(x)$, $x \in \mathbb{R}$), and that $2J_0(x) = H_0^{(1)}(x) + H_0^{(2)}(x)$ [19].

Therefore, if we assume the vertical component of the electric aperture field to be $E_{z,ap} \propto H_0^{(1)}(k_\rho\rho)$, a TM-polarized BB will be

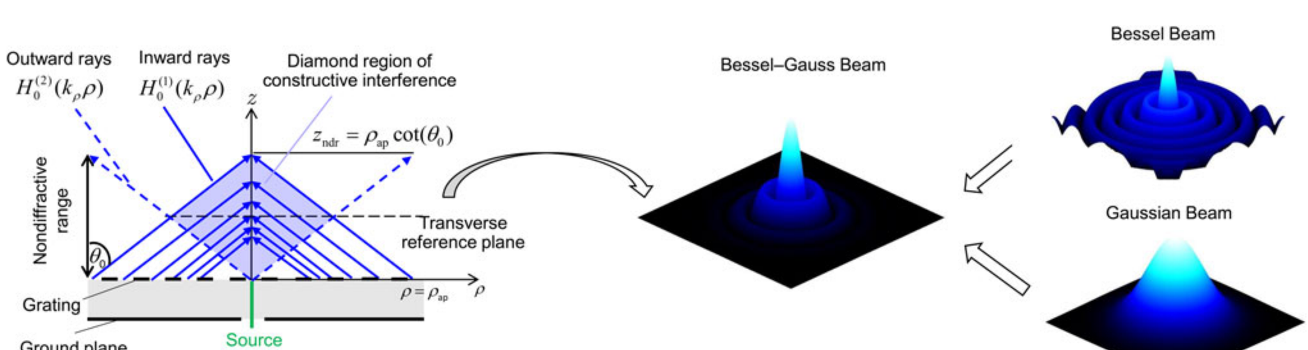

Fig. 1. On the left, a ray interpretation for the generation of a BGB from an inward cylindrical aperture field is represented. In contrast with *truncated* BBs, BGBs exhibit a Gaussian amplitude modulation (the density of the rays decreases according to the Gaussian modulation). In the middle, a perspective view of the transverse profile of an ideal BGB is reported. On the right, it is shown that the BGB transverse profile can be obtained through the product between an ideal BB and an ideal GB.

generated from a radiating aperture, whose tangential components take the form [20]:

$$E_{\rho,ap}(\rho) \propto j\frac{k_z}{k_\rho}H_1^{(1)}(k_\rho\rho)\,,$$
$$H_{\phi,ap}(\rho) \propto \frac{1}{k_\rho}H_1^{(1)}(k_\rho\rho)\,, \qquad (2)$$

where $H_1^{(1)}(\cdot)$ is the first-kind first-order Hankel function.

Bessel-Gauss beam

Under the ray-optics, paraxial approximation, the expression of an ideal BGB is given in [7, equation (2.7)], and is not reported here for brevity. The aperture field required for generating a BGB is easily obtained from [7, equation (2.7)] for $z = 0$ and it is basically an apodized version of a BB, characterized by a Gaussian apodization function. The conditions for obtaining a BGB then follow straightforwardly. Indeed, we wish to have an axially symmetric *vector* BGB having a Gaussian-modulated vertical component over the aperture which leads to $E_{z,ap} \propto J_0(k_\rho\rho)e^{-\rho^2/2w_0^2}$, where w_0 is the beam waist parameter, i.e. the radial distance at which the modulus squared of a Gaussian aperture distribution has decayed $1/e$ with respect to its maximum amplitude (reached at $\rho = 0$). (Note that the definition of w_0 may differ by a $\sqrt{2}$ factor when compared with the relevant literature.)

In order to generate a BGB distribution for the longitudinal component of the electric field, an additional Gaussian taper has to be accounted for in the expression of the aperture fields (see Fig. 1, middle, right). Thus, the resulting equations for the tangential field components are [10]:

$$E_{\rho,ap}(\rho) \propto j\frac{k_z}{k_\rho}H_1^{(1)}(k_\rho\rho)e^{-1/2(\rho/w_0)^2}\,,$$
$$H_{\phi,ap}(\rho) \propto \frac{1}{k_\rho}H_1^{(1)}(k_\rho\rho)e^{-1/2(\rho/w_0)^2}\,. \qquad (3)$$

Equation (3) represents the tangential aperture distribution that is required to obtain a TM-polarized BGB for E_z and within the nondiffractive range.

Gaussian beam

An *ideal* GB is commonly defined through the expression $(-jz_R/\zeta)e^{(-jk_0z-jk_0\rho^2/2\zeta)}$, where $\zeta = z + jz_R$ is the *complex beam parameter* [21] and $z_R = k_0w_0^2$ is the *Rayleigh range*.

The expressions of the tangential electromagnetic field components when E_z takes the form of an ideal GB are obtained in closed form from (1) and read:

$$E_\rho = \frac{-jz_Re^{-jk_0z}}{\rho}\left[1 - e^{-jk_0\rho^2/2\zeta}\left(1 - \frac{\rho^2}{2\zeta^2}\right)\right],$$
$$H_\phi = -jz_{\frac{Re^{-jk_0z}}{\eta\rho}}\left(1 - e^{-jk_0\rho^2/2\zeta}\right), \qquad (4)$$

thus, on the aperture plane $z = 0$ we simply have:

$$E_{\rho,ap}(\rho) \propto \frac{1}{\rho}\left[1 - e^{-\rho^2/2w_0^2}\left(1 + \frac{\rho^2}{2k_0^2w_0^4}\right)\right],$$
$$H_{\phi,ap}(\rho) \propto \frac{1}{\eta\rho}\left(1 - e^{-\rho^2/2w_0^2}\right). \qquad (5)$$

As is known, an *ideal* GB is only defined under the paraxial limit (i.e., in the limit $k_\rho \to 0$ and $k_z \to k_0$). Conversely, the leaky-wave approach proposed here only allows for having a non-zero, although relatively small value of k_ρ (the reasons for this limitation will appear clear in the next section "Leaky-wave approach"). Nevertheless, the synthesis of a GB beyond the paraxial limit will allow for a significant comparison with the synthesis of a BB and a BGB that would be otherwise less interesting if investigated in the paraxial limit. Indeed, paraxial BBs and BGBs, although propagating over large depth of fields, retain larger beam waists [10], thus being less attractive for focusing applications.

Leaky-wave approach

It has recently been shown that radially periodic leaky-wave antennas (LWAs) provide for a simple means to generate either BBs [22, 23] or BGBs [10], in particular for microwave and millimeter-wave ranges.

Radially periodic LWAs are two-dimensional (2-D) LWAs consisting of a grounded dielectric slab centrally fed by a coaxial feed and covered with an annular strip grating (see Fig. 2(a)) [24]. The radially periodic character of the structure allows for exciting a *backward cylindrical leaky wave* that is responsible for the near-field Bessel-like character exhibited by the vertical component of the electric field in the diamond-shaped region above the radiating aperture (the interested reader can find more details in [22, 23]). As opposed to *classical* BBs that are generated from a uniform aperture distribution, those generated by radially

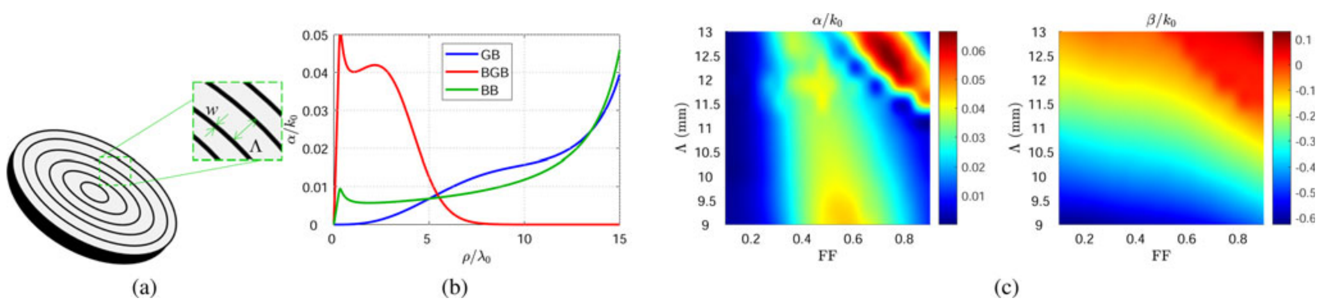

Fig. 2. (a) A perspective view of a radially periodic leaky-wave antenna (LWA) where the annular strip grating is modulated in order to support a BB/BGB/GB. A metallic disk is placed on top of the central coaxial feed for impedance matching purposes. (b) Radial profiles of the leakage rate α/k_0 versus ρ/λ_0 required to obtain a BB (green line), a BGB (red line), and a GB (blue line) (parameters in the text). (c) Color map plots of the leakage rate α/k_0 (left) and leaky phase constant β/k_0 (right) as a function of the period Λ and the filling factor FF.

periodic LWAs intrinsically have a tangential aperture distribution of the type $H_1^{(2)}(k_\rho\rho) \propto \exp(-j\beta\rho)\exp(-\alpha\rho)/\sqrt{\rho}$ [23], where α is the attenuation (or *leakage*) constant of the radial wavenumber $k_\rho = \beta - j\alpha$, β being the leaky phase constant, related to the axicon angle through $\sin\theta_0 = \beta/k_0$. However, this typical exponential decay of leaky waves can easily be converted to recover the aperture distribution in (3), which is required to obtain a BGB. Indeed, by locally *modulating* the strip width w and/or the period Λ of the grating (see Fig. 2(a)) it is possible to produce the local variation of the leakage rate required for obtaining a Gaussian modulation from an exponentially decaying one (the interested reader can find details in [10]). (It is worth noting here that the excitation of a backward cylindrical wave, i.e., with $\beta\alpha < 0$, allows for recovering the required inward character of the Hankel function characterizing the aperture distribution.)

The synthesis technique for obtaining a BGB has been described in [10] taking cues from the standard leaky-wave approach originally applied to 1-D periodic LWAs [25] to obtain a target far-field radiation pattern, and more recently to 2-D radially periodic LWAs [26, 27] to obtain focused beams in the radiative near-field region.

However, to the authors' best knowledge, the leaky-wave approach has not yet been applied to 2-D radially periodic LWAs for synthesizing GBs at microwaves.To this aim, the exponential taper that intrinsically characterizes a leaky wave should be converted to the aperture distribution given by (5), as needed for having an *ideal* GB. However, this would require $\beta = 0$, which in turn implies a periodic LWA with the open stopband (OSB) suppressed [28, 29]. While the possibility to work at $\beta = 0$, with the OSB suppressed, is an important aspect that is worth to be investigated in the future, in this work, we will synthesize GBs operating as close as possible to the OSB, but without employing any technique for its mitigation/suppression [29]. As a consequence, the *synthesized* GB will somehow differ from an *ideal* GB. Nevertheless, we will readily show here that it is still possible to synthesize a Gaussian-like beam, even when $\beta \neq 0$. Indeed, the expression of an ideal GB (4) evaluated for $z = z_0 < 0$ instead of $z = 0$ (as in (5)) yields an aperture field that can be described with rays that point toward the axis of symmetry with an axicon angle $\theta_0 \neq 0°$. More details about the procedure employed to select z_0 will be given in section "Numerical results".

The modulation of the leakage constant required along the radial direction to obtain the BB (green line), the BGB (red line), and the GB (blue line) solution is obtained by means of [10, equation (12)] and reported in Fig. 2(b) assuming $\rho_{\rm ap} = 15\lambda_0 = 15$ cm, at $f_0 = 30$ GHz ($\boldsymbol{\lambda_0}$ being the vacuum wavelength), an axicon angle $\theta_0 = 10°$ (for the BB and the BGB solution), and a normalized beam waist parameter $\tilde{w}_0 = w_0/\rho_{\rm ap} = 0.2$ (for the BGB and the GB solution).

The flexibility of the leaky-wave approach to obtain BBs, BGBs, and GBs can be inferred from Fig. 2(c) where we have evaluated the variation of the normalized leaky attenuation and phase constant for different values of Λ and the filling factor defined as $\mathrm{FF} = w/\Lambda$, by means of an accurate method-of-moments (MoM) analysis [30].

A closer look to Fig. 2(c) reveals that the iso-line $\beta/k_0 = 0$ corresponds to high values of α/k_0 with abrupt variations, thus confirming the presence of an OSB and also preventing the possibility to use these values for accurately generating a GB. Conversely, values of β/k_0 as low as -0.17 correspond to regions where α/k_0 varies smoothly in the range required for the synthesis of BBs, BGBs, and GBs (see Fig. 2(b)); this motivates the choice of $\theta_0 = 10°$ (note that $\sin 10° \simeq 0.17$) for the other examples. In brief, the choice of $\theta_0 = 10°$ proves to be a good compromise among (i) the feasibility of the design process, (ii) the deviation from the paraxial condition, (iii) the focused character that can be obtained with nonparaxial BBs and BGBs.

Finally, the radial profiles of the leakage constant are reconstructed with the values of α/k_0 that provide for the requisite of keeping β/k_0 as constant as possible. The latter condition identifies a path over the color maps in Fig. 2(c) that requires the simultaneous variation of Λ and FF. We should highlight that this is a considerable difference with respect to the approach proposed in [10], where the period was assumed to be fixed (at which corresponds a path that follows a straight horizontal line over the color maps in Fig. 2(c)), and there were no degrees of freedom for reducing the variation of the leaky phase constant.

In the next section "Numerical results", we will numerically evaluate the near-field distribution of $|E_z|^2$ for all the beam solutions described so far, also highlighting their advantages and disadvantages in terms of diffractive and focusing properties when these solutions are generated beyond the paraxial limit.

Numerical results

In this section, we aim at showing and comparing the different beam propagating features of BBs, BGBs, and GBs, as well as the agreement between the expected theoretical distributions and accurate numerical simulations.

Near-field distributions for the vertical component of the electric field E_z will be illustrated over the domain $|\rho| < \rho_{ap}$ and $0 < z \leq z_{\rm ndr}$. Such distributions can be obtained from the aperture fields, through the application of the equivalence theorem, i.e., the Huygens–Fresnel principle [20]. Indeed, the aperture field can be replaced by an equivalent magnetic surface current density M_ϕ equal to $M_\phi = -2E_{\rho,ap}$. Therefore, the vectorial near-field distribution is given by [20]:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= 2\oint_{S_A} \left(E_{\rho,ap}(\mathbf{r}')\mathbf{u}_\rho \times \nabla G(\mathbf{r},\mathbf{r}')\right) d\mathbf{r}', \\ \mathbf{H}(\mathbf{r}) &= \frac{j}{k_0\eta}\nabla \times \mathbf{E}(\mathbf{r}), \end{aligned} \tag{6}$$

where $\mathbf{u}_\rho$ is the radial unit vector, $\mathbf{r}$ ($\mathbf{r}'$) represents the observation (source) points, $G(\mathbf{r},\mathbf{r}') = e^{-jk|\mathbf{r}-\mathbf{r}'|}/(4\pi|\mathbf{r}-\mathbf{r}'|)$ is the free-space Green's function and the integration domain S_A is the aperture plane. Such radiation integrals have been evaluated numerically, carefully handling the singular behavior of the free-space Green's function for observation points at or close to the plane $z = 0$.

In order to make a fair comparison among the three different solutions, numerical results are obtained under the same operating conditions used in the previous section "Leaky-wave approach".

In Figs 3(a)–3(c), the near-field distributions of (a) a truncated BB, (b) an ideal BGB, and (c) an ideal GB are reported. Comparison among Figs 3(a)–3(c) highlights the following aspects:

- For the BB solution (see Fig. 3(a)), the beam waist remains almost constant up to the nondiffractive range, that ray optics (shadow boundaries are reported in white dashed lines)

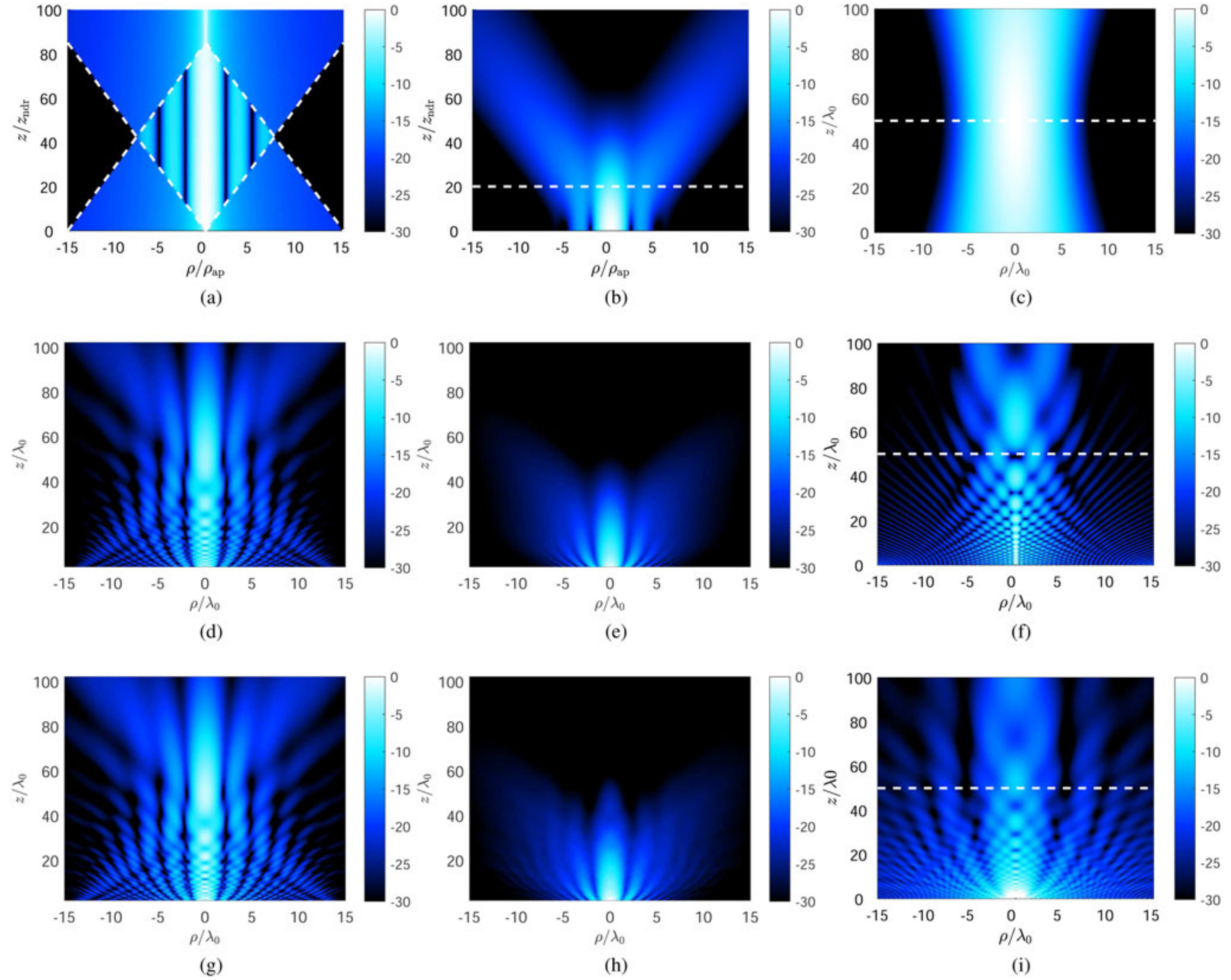

Fig. 3. Color maps of $|E_z|^2$ (normalized to its maximum and in dB scale) generated through an aperture size of radius $\rho_{ap} = 15\lambda_0$ with $\lambda_0 = 1$ cm ($f_0 = 30$ GHz) for (a)–(c) *ideal* BBs, BGBs, and GBs, respectively; (d)–(f) *ideal synthesis* of BBs, BGBs, and GBs, i.e., radiated by the aperture distributions in (2), (3), and (5), respectively; (g)–(i) *leaky-wave synthesis* of BBs, BGBs, and GBs, i.e. reconstructing the aperture distributions in (2), (3), and (5), respectively, with the values of β/k_0 and α/k_0 provided by the synthesis procedure for the planar radially periodic leaky-wave antennas of Fig. 2(a).

would predict to occur at around $80\lambda_0$. However, the field intensity rapidly decays beyond that distance and the beam also starts to considerably widen along the transverse direction.

- For the BGB solution (see Fig. 3(b)), the beam waist is maintained almost constant over a shorter distance with respect to the BB solution, due to the effect of the Gaussian amplitude modulation of the aperture distribution. From the numerical results, one can infer that the effective nondiffractive range is reduced to approximately $20\lambda_0$ (see the white dashed line) in agreement with the formula reported in [10, equation (8)]. However, the BGB solution features a more localized character close to the axis of symmetry, and a considerable reduction of the intensity of the annular rings that instead characterizes the BB solution.
- For the GB solution (see Fig. 3(c)), the beam waist changes size along z according to (4). Specifically, it is manifest from Fig. 3(c) that the beam waist doubles at the Rayleigh range that for a normalized beam waist $\tilde{w}_0 = 0.2$ should occur at around $56\lambda_0$. Since the GB is shifted along the z-axis of $50\lambda_0$ the Rayleigh range is achieved at around $z = 100\lambda_0$. We should stress here that the GB is shifted of $50\lambda_0$ because we decided to synthesize on the aperture (i.e., at $z = 0$), the expression of an ideal GB at $z = -50\lambda_0$, in agreement with the motivation outlined in the previous section "Leaky-wave approach". The specific choice of $z_0 = -50\lambda_0$ will be commented next.

In Figs 3(d)–3(f), near-field distribution of $|E_z|^2$ (normalized to its maximum and in dB scale) has been numerically evaluated through (6) (projected onto the vertical unit vector $\mathbf{u}_z$), using (2) for a BB (see Fig. 3(d)), (3) for a BGB (see Fig. 3(e)), and (5) for a GB (see Fig. 3(f)). The same results, but when the aperture field is synthesized through the leaky-wave approach described in section "Leaky-wave approach" are reported in Figs 3(g)–3(i)) for the BB, the BGB, and the GB solution, respectively.

The appreciable differences between the ideal distributions (see Figs 3(a)–3(c)) and the near-field distributions obtained by radiating the ideal aperture fields (Figs 3(d)–3(f)) are due to the diffraction effects that are tacitly neglected by the ray-optics approximation over which the ideal distributions are based. This disagreement is even more reasonable for a GB which is

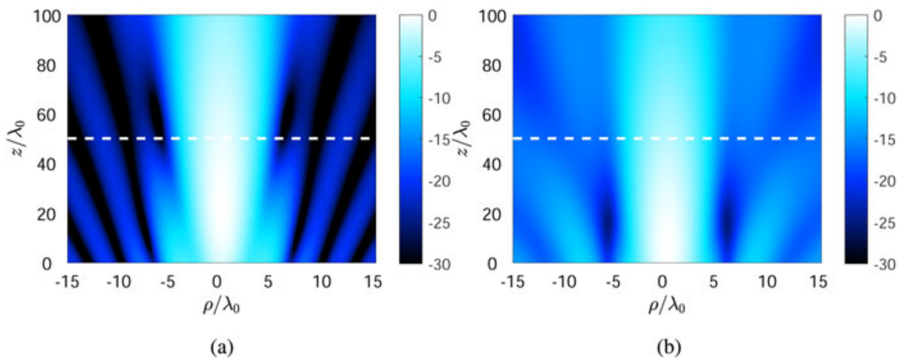

Fig. 4. (a)–(b) As in Figs 3(f) and 3(i), respectively, but now assuming the aperture plane being at $z = -150\lambda_0$.

not a solution of the wave equation, but of its paraxial approximation which leads to a parabolic equation [21]. As a result, the field radiated by a Gaussian distribution is not expected to recover the ideal GB solution, as is manifest from the significant differences between Figs 3(c) and 3(f). Nevertheless, it is still possible to see that, far from the aperture plane (beyond the white dashed line), the radiated field starts to recover the main beam shape GB. In agreement with the choices we made, the beam waist that one would expect to occur at $z = 0$ from the standard definition of a GB is here obtained for $z = 50\lambda_0$ (see the dashed white line).

In this regard, we should recall further comment the choice of $z_0 = -50\lambda_0$ for the aperture-field synthesis. Indeed, if one takes the expression of a GB at $z = z_0$ and calculate the phase gradient, one would obtain an expression for β as a function of ρ that, for $z_0 = -50\lambda_0$ is approximately in the range $-0.17 < \beta_\rho < 0$. Since $\beta = -0.17$ represents a good compromise between the requirement of being not too close to the OSB, and not too far from the paraxial condition, the value of $z_0 = -50\lambda_0$ has been selected. It is also worth noting here that the inward character of the aperture distribution is manifest from Fig. 3(f). This aspect confirms that the discrepancies between Figs 3(f) and 3(c) are mainly due to the deviation from the paraxial condition.

On the other hand, a very good agreement is appreciable between the fields radiated assuming the ideal aperture distributions and those synthesized with the leaky-wave procedure (compare Figs 3(d)–3(f) with Figs 3(g)–3(i)). This aspect further corroborates the accuracy and flexibility of the leaky-wave approach, and also confirms that the appropriate synthesis of the GB solution is mostly limited by the difficulties in achieving the paraxial limit with the proposed structure. This consideration suggests future interesting pathways for the synthesis of GBs at microwaves with periodic LWAs implementing OSB suppression techniques. Finally, we show in Figs 4(a)–4(b), the synthesis of a GB assuming the aperture plane being at $z = -150\lambda_0$. Comparison of these results with those of Fig. 3(c) shows that, far from the aperture plane, it is easier to obtain a good agreement between an ideal GB (Fig. 3(c)), its *ideal synthesis* (Fig. 4(a)), and its *leaky-wave synthesis* (Fig. 4(b)). We should stress that, although the agreement is obtained far from the aperture-plane (approximately after $100\lambda_0$) the reported domain is fully within the radiative Fresnel region, the Fraunhofer distance starting at around $z = 1800\lambda_0$ for the proposed LWA.

Conclusion

The near-field distribution of BBs, BGBs, and GBs has been derived under the frame of a fully vectorial electromagnetic approach useful for the accurate design of limited-diffraction focusing devices in the microwave/millimeter-wave range. Numerical examples have been presented at microwaves to discuss and compare their focusing and diffractive properties. A possible realization of BBs, BGBs, and GBs through radially periodic leaky-wave antennas has finally been proposed and numerically validated.

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