Vibroacoustic optimization using a Statistical Energy Analysis model

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Abstract

In this paper, an optimization technique for medium-high frequency dynamic problems based on Statistical Energy Analysis (SEA) method is presented. Using a SEA model, the subsystem energies are controlled by internal loss factors (ILF) and coupling loss factors (CLF), which in turn depend on the physical parameters of the subsystems. A preliminary sensitivity analysis of subsystem energy to CLF's is performed to select CLF's that are most effective on subsystem energies. Since the injected power depends not only on the external loads but on the physical parameters of the subsystems as well, it must be taken into account under certain conditions. This is accomplished in the optimization procedure, where approximate relationships between CLF's, injected power and physical parameters are derived. The approach is applied on a typical aeronautical structure: the cabin of a helicopter.

Keywords:
Medium-high frequencies, Statistical energy analysis, Coupling loss factors, Input power, Optimization, Helicopter cabin

1. Introduction

Statistical Energy Analysis (SEA) allows to solve medium-high frequency dynamic problems because only the energy averaged on each subsystem is needed when too many DoF's are involved and eigenvalues and eigenvectors loose their significance due to high modal density [1, 2, 3, 4, 5]. Therefore, while classical (FEM, BEM) techniques fail to solve this kind of dynamic problems, SEA gives the energy stored in each subsystem and the energy flow between coupled subsystems. Using a SEA model, the subsystem energies are controlled by coupling loss factors (CLF), internal loss factors (ILF) and injected power, which in turn depend on the physical parameters of the subsystems.

In this paper, an optimization technique for medium-high frequency dynamic problems based on SEA method is presented. A preliminary version of the procedure was presented in conference papers [6, 7]. There have been some attempts to use SEA models in order to control noise: for instance in [8] an analysis of the effect of damping treatment to control noise on offshore platforms is performed, but no optimization procedure is devised. In a previous paper by the authors [9], a sensitivity approach was proposed in order to analyze the propagation of parametric uncertainties in SEA models. The same sensitivity concept is used in this paper to set up an optimization procedure. In literature there are several works [10, 11] dealing with parametric uncertainties of hybrid SEA models using a sensitivity approach: the sensitivity to model parameters can probably be used in an optimization procedure, but this possible development is not foreseen.

The optimization procedure is developed here by considering that CLF's and injected power depend on the physical parameters of the subsystems. Therefore an approximate relation between CLF's, injected power and physical parameters is determined, for instance, by using Design of Experiment (DoE). This relation is the core of the optimization procedure formulated in order to bring the subsystem energies under prescribed levels. Injected power must

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be taken into account under certain conditions, i.e. whenever the physical parameters of subsystems through which power is injected are modified. To decrease the subsystem energies, it is proposed to proceed according to the following steps. (Here it is assumed to use a commercial software for SEA: therefore explicit relations among CLF's and physical parameters are not available). First, the effect of changes to CLF's and ILF's is modeled by using a sensitivity approach [12, 13, 6, 9]. The sensitivity of the energies stored into the subsystems is calculated by considering variations of ILF's and CLF's. The goal is to preliminarily understand how much the energies (SEA solution) depend on changes to CLF's and ILF's. At this stage, it is difficult and unpractical to consider the contribution due to variations of the injected power.

After choosing CLF's and ILF's that are most effective on subsystem energies, a model of the selected loss factors and of the injected powers as function of some chosen physical parameters is developed. The selected physical parameters are those which can be modified to affect the loss factors. Subsequently, a simple mathematical model (Response Surface Model) of how they affect CLF's and ILF's can be obtained using Design of Experiments [14, 15]. Finally, a multi-objective optimization problem is formulated in order to bring the subsystem energies under prescribed levels for any desired frequency band. The variables of the problem are the relative deviations of the selected physical parameters from their nominal value. Upper and lower bounds of these design variables are defined.

In section 4, the procedure described previously is applied to reduce noise inside a helicopter cabin. First, a preliminary sensitivity analysis is performed in order to select the most significant CLF's, to which the considered energies are more sensitive. Then, by using Design of Experiments, an approximate relation between the most sensitive CLF's, the injected powers and the selected physical parameters is obtained. Finally, the results of the optimization provide the values of the physical parameters yielding the desired energy level reduction.

2. SEA equations

In Statistical Energy Analysis, a subsystem is defined as a group of similar modes (i.e. having similar energy, damping, coupling with the other subsystems and being excited by almost the same input power).

Under some hypotheses [3], the power exchanged between two subsystems is assumed to be proportional to a weighted difference of the energies stored in the two subsystems, so that the power P_{ij} transmitted from subsystem i to the subsystem j is:

$$P_{ij} = \omega \left(\eta_{ij} E_i - \eta_{ji} E_j\right) \tag{1}$$

where i and j are indexes of the subsystems, η_{ij} is the coupling loss factor (CLF), E_i is the energy in the i-th subsystem and ω is the central frequency of the considered band.

The power $P_{i,\mathrm{d}}$ dissipated in the subsystem i is:

$$P_{i,\mathrm{d}} = \omega \eta_i E_i \tag{2}$$

where η_i is the internal loss factor (ILF).

Thus, the SEA equations of N_{sub} coupled subsystems can be written as:

$$P_i = \omega \eta_i E_i + \omega \sum_{j=1, j\neq i}^{N_{\mathrm{sub}}} \left(\eta_{ij} E_i - \eta_{ji} E_j\right) \quad i = 1, \ldots, N_{\mathrm{sub}} \tag{3}$$

where P_i is the power injected into the subsystem i. The set of equations (3) represents the energy balance of the subsystems. The energy stored in each subsystem is provided by the solution of the linear system (3). A more convenient notation for the set of equations (3) is adopted:

$$\mathbf{P} = \omega \mathbf{C} \mathbf{E} \tag{4}$$

where $\mathbf{P}$ and $\mathbf{E}$ are vectors gathering the injected powers and the energies, and the coefficients of matrix $\mathbf{C}$ are combinations of ILF's and CLF's as shown in the following equations:

$$\begin{cases} C_{ij} = -\eta_{ji} & i, j = 1 \ldots N_{\text{sub}},\ i \neq j \\ C_{jj} = \eta_j + \sum_{i=1,\, i \neq j}^{N_{\text{sub}}} \eta_{ji} \end{cases} \tag{5}$$

The following reciprocity relationship holds:

$$\eta_{ij} n_i = \eta_{ji} n_j \tag{6}$$

where n_i and n_j are the modal densities of subsystems i and j.

By assuming that only the η_{ji} with $j > i$ are known, it is necessary to take advantage of the reciprocity relations (6), so that Eq. (5) becomes:

$$\begin{cases} \left.\begin{aligned} C_{ij} &= -\eta_{ji} \\ C_{ji} &= -\eta_{ij} = -\eta_{ji} \frac{n_j}{n_i} \end{aligned}\right\} \quad \text{with } j > i \\ C_{jj} = \eta_j - \sum_{i=1,\, i \neq j}^{N_{\text{sub}}} C_{ij} \end{cases} \tag{7}$$

3. Optimization of SEA model to reduce subsystems energies

In SEA equations, CLF's are deterministic functions of the physical parameters. The solution of this deterministic set of equations is given by the energies of the modal groups.

The optimization of the SEA model in order to reduce subsystems energies is performed in three steps. First, a sensitivity analysis allows to identify the CLF's that are the most effective in changing the subsystems energies. Second, physical parameters that can be modified to vary the previously identified CLF's are selected, and a simplified model of CLF's as functions of the selected physical parameters is derived. Third, an optimization procedure is used to find the optimal values of the previously selected physical parameters.

3.1. Sensitivity to SEA parameter variation

The goal is to understand which are the CLF's and ILF's whose variations have significant effects on the subsystem energies (SEA solution). The energy of each subsystem is calculated by solving equation (4), obtaining:

$$\mathbf{E} = \frac{1}{\omega} \mathbf{C}^{-1} \mathbf{P} \tag{8}$$

with the obvious implication that energies depend on the CLF's and the ILF's of the considered system. By defining a range of variability of CLF's and ILF's, a sensitivity approach is used to account for the dependence of the energy on the variations of SEA parameters.

Sensitivity to loss factors is evaluated in correspondence to nominal values $\hat{\eta}$ of the CLF's and of the ILF's.

To compare different sensitivities, it may be assumed that changes $\Delta\eta_{kl}$ in the coupling loss factors are a prescribed fraction, for instance 10%, of the nominal values $\hat{\eta}$:

$$\Delta \mathbf{E}_{kl} = \left.\frac{\partial \mathbf{E}}{\partial \eta_{kl}}\right|_{\eta = \hat{\eta}} \Delta \eta_{kl} \tag{9}$$

and similarly for ILF's.

To find $\partial \mathbf{E} / \partial \eta_{kl}$, it is necessary to differentiate Eq. (8):

$$\frac{\partial \mathbf{E}}{\partial \eta_{kl}} = \frac{1}{\omega} \left(\frac{\partial \mathbf{C}^{-1}}{\partial \eta_{kl}} \mathbf{P} + \mathbf{C}^{-1} \frac{\partial \mathbf{P}}{\partial \eta_{kl}} \right) \tag{10}$$

and similarly if internal loss factors η_k are considered instead of η_{kl}.

The derivative of $\mathbf{C}^{-1}$ can be easily obtained from the identity $\mathbf{C}\,\mathbf{C}^{-1} = \mathbf{I}$

$$\frac{\partial \mathbf{C}^{-1}}{\partial \eta_{kl}} = -\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \eta_{kl}}\mathbf{C}^{-1} \tag{11}$$

where $\partial \mathbf{C}/\partial \eta_{kl}$ can be computed from Eq. (7):

$$\frac{\partial \mathbf{C}}{\partial \eta_{kl}} = \mathbf{e_k}\mathbf{e_k}^T - \mathbf{e_l}\mathbf{e_k}^T - \frac{n_k}{n_l}\mathbf{e_k}\mathbf{e_l}^T + \frac{n_k}{n_l}\mathbf{e_l}\mathbf{e_l}^T \tag{12}$$

where $\mathbf{e_k}$ is the k-th standard unit vector, i.e. it is the k-th column of the unit matrix. Eq. (12) can be written in matrix form, with the four non-zero elements highlighted by a frame. By assuming $k > l$, it is:

$$\frac{\partial \mathbf{C}}{\partial \eta_{kl}} = \begin{bmatrix} 0 & & & \overset{\text{col. } l}{0} & & & & \overset{\text{col. } k}{0} & & & 0 \\ & \ddots & & \vdots & & & & \vdots & & \iddots & \\ & & 0 & 0 & & & & 0 & 0 & & \\ 0 & \cdots & 0 & \boxed{\frac{n_k}{n_l}} & 0 & \cdots & 0 & \boxed{-1} & 0 & \cdots & 0 \\ & & & 0 & 0 & & 0 & 0 & & & \\ & & & \vdots & & \ddots & & \vdots & & & \\ & & & 0 & 0 & & 0 & 0 & & & \\ 0 & \cdots & 0 & \boxed{-\frac{n_k}{n_l}} & 0 & \cdots & 0 & \boxed{1} & 0 & \cdots & 0 \\ & & 0 & 0 & & & & 0 & 0 & & \\ & \iddots & & \vdots & & & & \vdots & & \ddots & \\ 0 & & & 0 & & & & 0 & & & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \text{row } l \\ \\ \\ \\ \text{row } k \\ \\ \\ \\ \end{matrix}$$

and similarly for $\partial \mathbf{C}/\partial \eta_k$:

$$\frac{\partial \mathbf{C}}{\partial \eta_k} = \mathbf{e_k}\mathbf{e_k}^T \tag{13}$$

The derivative of $\mathbf{P}$ satisfies the following condition:

$$\frac{\partial \mathbf{P}}{\partial \eta_{kl}} = 0 \qquad \text{if } P_k = 0 \text{ and } P_l = 0 \tag{14}$$

and can be non zero only with respect to CLF's of subsystems through which power is injected, because P_k depends on the physical parameters of the subsystem k, and η_{kl} depends on physical parameters of subsystems k and l. For instance, it is:

$$\frac{\partial P_k}{\partial \eta_{kl}} \neq 0 \qquad \text{and} \qquad \frac{\partial P_l}{\partial \eta_{kl}} = 0 \tag{15}$$

if $P_k \neq 0$ and only physical parameters of subsystem k are changed, while physical parameters of subsystem l are left unchanged. In such cases, $\partial P_k/\partial \eta_{kl}$ can not be computed analytically, and finding appropriate numerical approximations is unpractical. Therefore, the contribution of $\partial \mathbf{P}/\partial \eta_{kl}$ is usually neglected in Eq. (10). However, the subsystems through which power is injected are always considered amenable to modifications, even if they are not highlighted by sensitivity analysis. In fact, CLF's involving these subsystems are always considered in the subsequent optimization problem.

3.2. CLF's and ILF's as functions of physical parameters

After neglecting CLF's and ILF's that are less effective on subsystem energies, it is necessary to develop a model of the remaining SEA coefficients as function of selected physical parameters. Note that, as stated at the end of the previous section, the CLF's and ILF's that are related to subsystems through which power is injected must not be discarded. Therefore, a subset of CLF's and ILF's that are amenable to modification is obtained involving both CLF's and ILF's highlighted by the sensitivity analysis and CLF's and ILF's of subsystems through which power is injected.

First of all, physical parameters are selected that affect CLF's and ILF's amenable to modifications; subsequently, a simple mathematical model of how they affect CLF's, ILF's and injected powers can be obtained using Design of Experiments (see Appendix A). Here DoE is used as an efficient numerical interpolation method: the 'observations' correspond to CLF's, ILF's and injected powers provided by the SEA software by changing the physical parameters, and the interpolation model is such as to exactly reproduce the observations.

At the end of this stage, CLF's and ILF's that are amenable to modifications are expressed as:

$$\eta_{ji} = \eta_{ji}(\mathbf{x}) \tag{16}$$

where $\mathbf{x}$ contains the relative deviations of the selected physical parameters from their nominal values.

Therefore, from Eq. (5) or Eq. (7) it is possible to express $\mathbf{C}$ as:

$$\mathbf{C} = \mathbf{C}(\mathbf{x}) \tag{17}$$

and $\mathbf{P}$ as:

$$\mathbf{P} = \mathbf{P}(\mathbf{x}) \tag{18}$$

Finally, the subsystem energies can be obtained as functions of $\mathbf{x}$ as:

$$\mathbf{E}(\mathbf{x}) = \frac{1}{\omega} \mathbf{C}(\mathbf{x})^{-1} \mathbf{P}(\mathbf{x}) \tag{19}$$

3.3. Modification of physical parameters to reduce subsystems energies

A constrained optimization problem can be defined in order to reduce the energy level $\hat{E}_{lk}$ of some critical subsystems l at frequency band k under a prescribed level E^*_{lk}, by varying the selected physical parameters. Specifically, a multi-objective goal attainment problem [16] can be defined, in which the objectives can be set as constraints that can be under or over-achieved according to the value of an additional slack variable γ. According to the Matlab® implementation, the 'fgoalattain' function is used and the problem can be formulated as:

$$\begin{aligned} &\min_{\mathbf{x},\gamma} \quad \gamma \\ &\text{subject to} \quad \begin{cases} \hat{E}_{lk}(\mathbf{x}) - w_{lk}\gamma \leq E^*_{lk} & l = 1,\ldots,N_E \quad k = 1,\ldots,N_f \\ x_{Li} \leq x_i \leq x_{Ui} & i = 1,\ldots,N_p \end{cases} \end{aligned} \tag{20}$$

where:

- the slack variable γ indicates the degree of goal attainment, i.e. if $\gamma = 0$ all objectives are satisfied exactly, if $\gamma > 0$ all the objectives are under-achieved, if $\gamma < 0$ all the objectives are over-achieved (note that γ depends on $\mathbf{x}$, but its dependence is not explicitly known);
- $\hat{E}_{lk}(\mathbf{x})$ is the energy of subsystem l at the frequency band k that must be lower than a prescribed value, E^*_{lk}, where
  - $l = 1,\ldots,N_E$, being N_E the number of subsystems for which it is required to reduce or control the energy level;
  - $k = 1,\ldots,N_f$, being N_f the number of frequency bands;
- the weights w_{lk} (≥ 0) control the relative degree of under or over-achievement of each goal, i.e. if $w_{lm} < w_{rs}$ and $\gamma > 0$, then the goal defined for subsystem l and frequency band m is better attained than the goal defined for subsystem r and frequency band s;

- $\mathbf{x}$ is a vector that contains the relative deviations of the selected physical parameters from their nominal values;
- N_p is the total number of physical parameters that can be modified;
- $\mathbf{x}_L$ and $\mathbf{x}_U$ are the upper and lower bounds of the design variables $\mathbf{x}$, so that the solution is always in the range $\mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U$.

4. Results

A simplified model of a helicopter is considered. The SEA model is built by the software VA One of Esi Group. The SEA subsystems are: 17 external aluminum shells 1 mm thick, 8 glass windows 5 mm thick, 3 acoustic cavities and 4 internal aluminum bulkheads 1 mm thick. Table 1 reports the list of subsystems with associated numbers and names.

1	Front Cavity	17	Left Fuselage Bottom
2	Back Cavity	18	Right Fuselage Bottom
3	Cabin Cavity	19	Floor Bulkhead
4	Tail Bulkhead	20	Left Front Panel
5	Left Tail	21	Right Front Panel
6	Right Tail	22	Back Bulkhead
7	Left Cockpit Window	23	Bottom Front Bulkhead
8	Right Cockpit Window	24	Top Front Bulkhead
9	Left Front Cabin Window	25	Left Cockpit Door
10	Right Front Cabin Window	26	Right Cockpit Door
11	Left Back Cabin Window	27	Left Front Cabin Door
12	Right Back Cabin Window	28	Right Front Cabin Door
13	Left Nose	29	Left Back Cabin Door
14	Right Nose	30	Right Back Cabin Door
15	Left Windshield Frame	31	Left Roof
16	Right Windshield Frame	32	Right Roof

Table 1: Subsystems numbering.

Figures 1-3 are the sketches of the system where the numbers of the subsystems are shown. Front Bulkheads (systems 23, 24) separate Front Cavity and Cabin Cavity, Back Bulkhead (system 22) separates Cabin Cavity and Back Cavity.

Let us assume to be interested in studying the helicopter cabin in correspondence with the pilots and the passengers accommodation. Figure 4 shows the coupling between SEA subsystems in graph format. Figure 5 shows the same reduced graph involving only the cabin subsystems. Using VA One, the CLF's of the SEA model are calculated. The vibroacoustic solution (the energy of each subsystem) and the injected powers are also computed by VA One, by assuming a point force of amplitude of 1 N applied both on the Right Roof and the Left Roof in the frequency range 1000–10000 Hz.

Using the proposed sensitivity approach, the variation of the energy of the subsystem 3 and 19 is considered when changes of the CLF's of $\pm 10\%$ around the nominal values are taken into account.

To show the sensitivities in a compact form, their norm over all the frequency bands is computed. The result is shown in figures 6 and 7: the most significant sensitivity values, together with the CLF index and the involved subsystems are also reported in Table 2. A first observation is that, being the model almost symmetric with respect to the longitudinal mid-plane, the sensitivities are almost symmetric too. For instance, the sensitivity of cabin cavity to CLF denoted by index 105, involving the junction among subsystems 3 (Cabin Cavity) and 31 (Left Roof) is almost equal to the sensitivity to the CLF denoted by index 115, involving the junction among subsystems 3 (Cabin Cavity) and 32 (Right Roof). A second observation is that the most effective CLF's involve the following subsystems:

- 3 (Cabin Cavity)

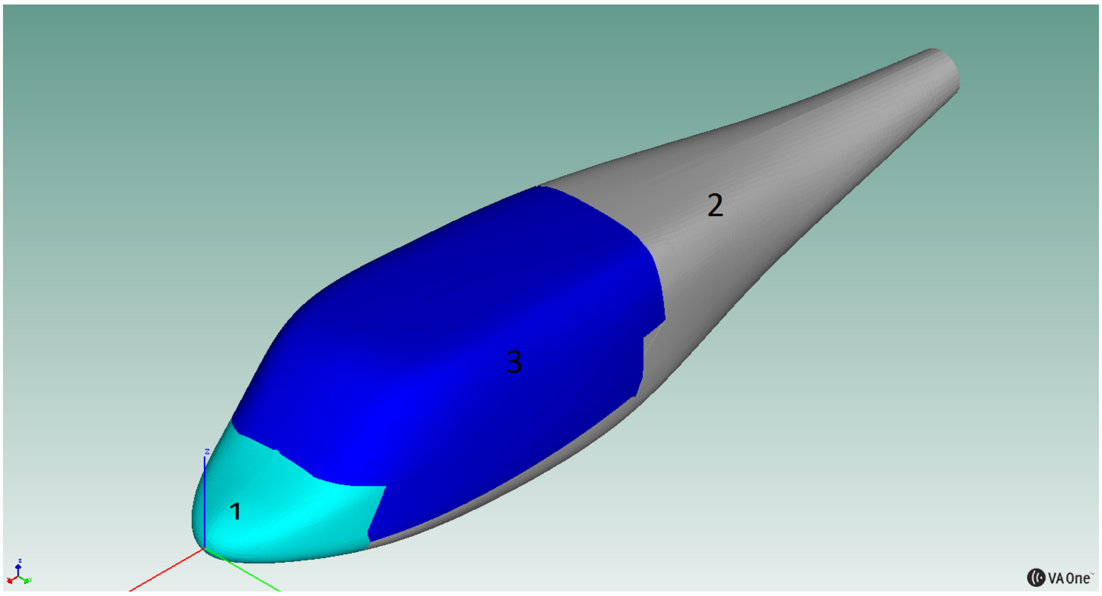

Figure 1: VA One - SEA model of a helicopter: acoustic cavities

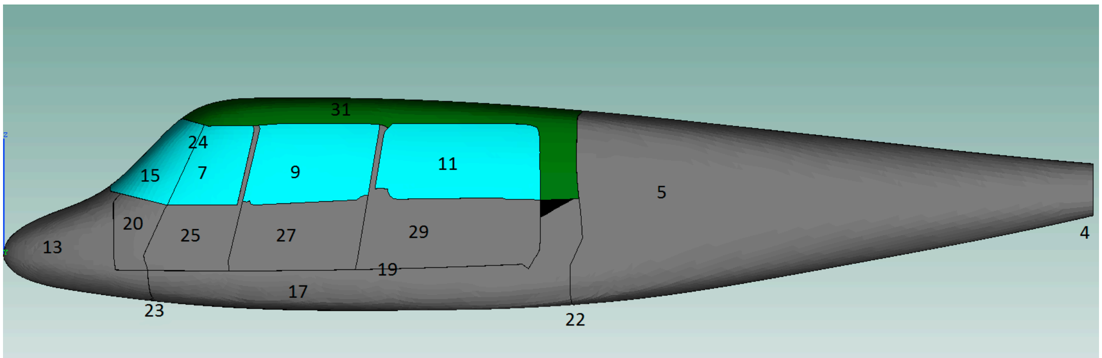

Figure 2: VA One - SEA model of a helicopter: left side external shells and bulkheads

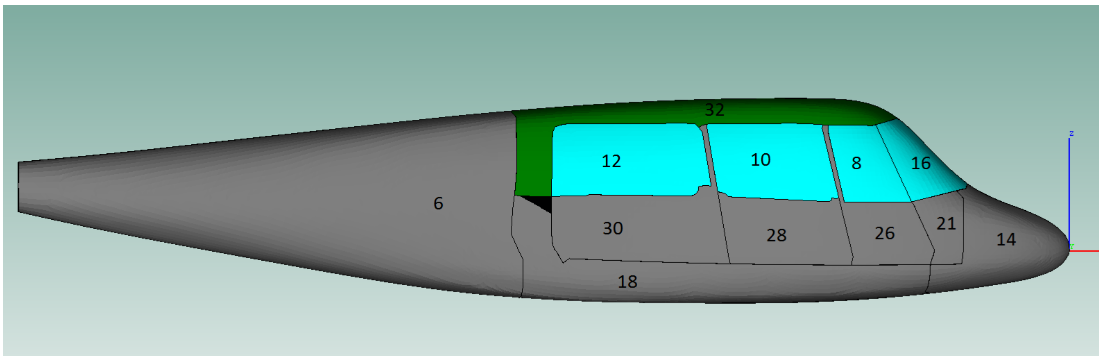

Figure 3: VA One - SEA model of a helicopter: right side external shells

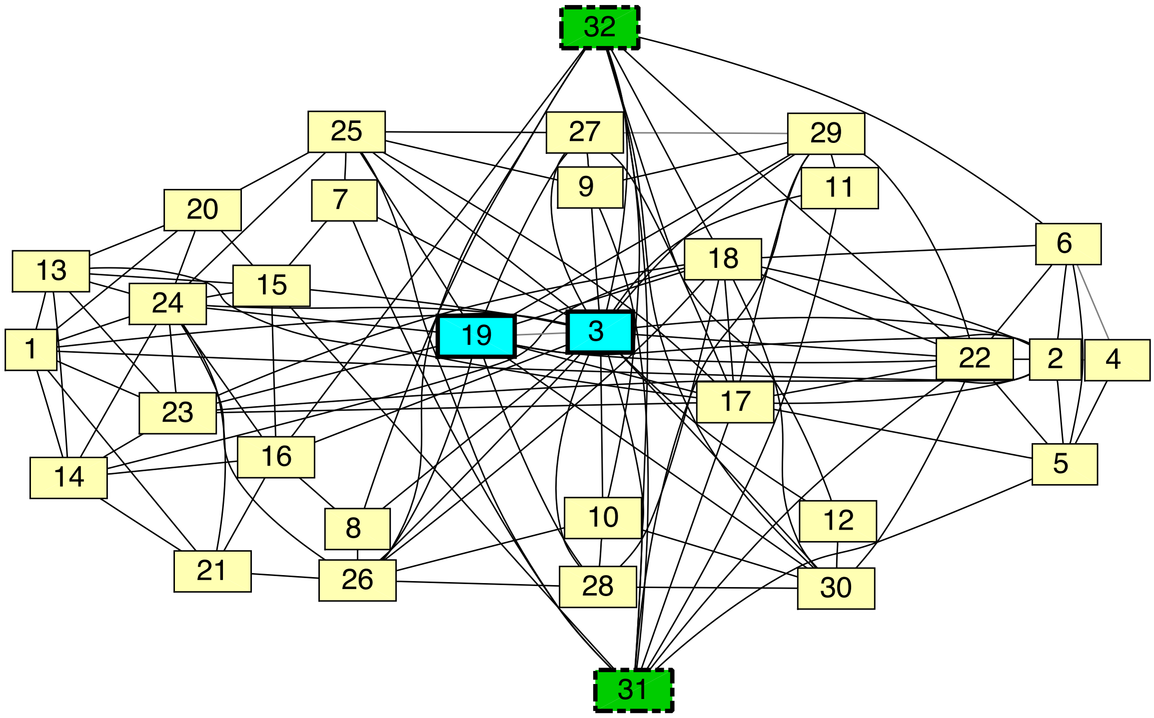

Figure 4: Graph sketch of the whole system: nodes represent the subsystems and edges are the couplings between subsystems (thick dashed contour: source(s), thick continuous contour: receiver(s)).

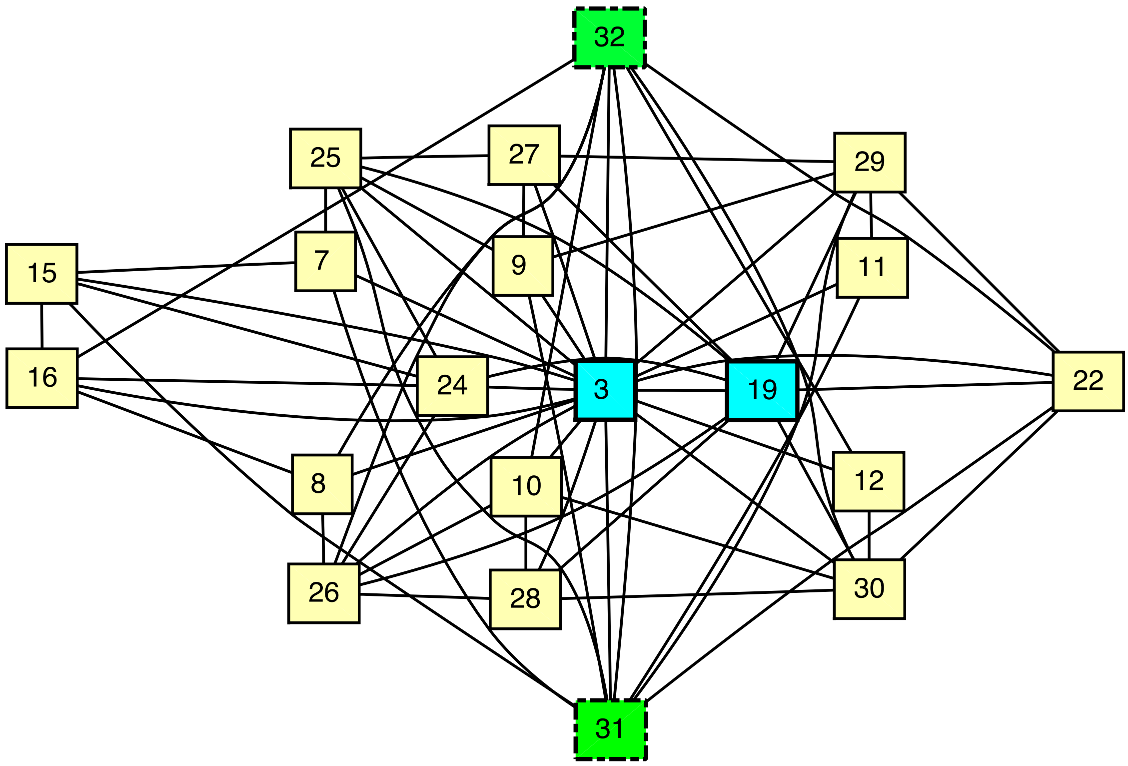

Figure 5: Sketch of the cabin subsystem (thick dashed contour: source(s), thick continuous contour: receiver(s)).

index	η_{ji}		ΔE_3	ΔE_{19}
49	**22**	**19**	5.17E-09	**3.75E-07**
95	**29**	**19**	5.70E-09	**2.22E-07**
102	**30**	**19**	5.60E-09	**2.21E-07**
105	**31**	**3**	**7.74E-06**	6.35E-08
112	**31**	**22**	2.14E-07	**2.13E-07**
115	**32**	**3**	**7.70E-06**	5.91E-08
122	**32**	**22**	2.12E-07	**2.12E-07**

Table 2: Selected results of the sensitivity analysis

- 19 (Floor Bulkhead)
- 22 (Back Bulkhead)
- 29 and 30 (Back Cabin Door, Left and Right)
- 31 and 32 (Roof, Left and Right)

A third observation is that the most effective CLF's do involve the subsystems 31 (Left Roof) and 32 (Right Roof) through which power is injected into the system. If subsystems 31 and 32 were not highlighted by sensitivity analysis, they should have been considered anyway.

Therefore, the physical parameters that are considered amenable to modifications are:

- x_1: relative deviation from nominal value of the thickness of the Roof, Left and Right
- x_2: relative deviation from nominal value of the thickness of the Back Bulkhead

A single parameter, x_1, is selected for the right and left side of the roof to enforce symmetric modifications, because the original system is almost symmetric.

In order to determine the relationships between CLF's, injected powers and physical parameters, a DoE analysis is performed using Central Composite Design (see Eq. A.3) with $p = 2$, giving rise the following model:

$$\begin{aligned} \eta_{ji} &= \alpha_0^{(ji)} + \alpha_1^{(ji)} x_1 + \alpha_2^{(ji)} x_2 + \alpha_{11}^{(ji)} x_1^2 + \alpha_{12}^{(ji)} x_1 x_2 + \alpha_{22}^{(ji)} x_2^2 \\ P_i &= \alpha_0^{(i)} + \alpha_1^{(i)} x_1 + \alpha_2^{(i)} x_2 + \alpha_{11}^{(i)} x_1^2 + \alpha_{12}^{(i)} x_1 x_2 + \alpha_{22}^{(i)} x_2^2 \end{aligned} \tag{21}$$

Being $p = 2$, $2^p + 2p + 1 = 9$ observations are necessary. The values of x_1 and x_2 according to Central Composite Design are shown in Table 3.

Factors	Values								
x_1	-1	+1	-1	+1	0	-1	+1	0	0
x_2	-1	-1	+1	+1	0	0	0	-1	+1

Table 3: Observations in 2-factors Central Composite Design

By considering a variation of the physical parameters of ±10% around their nominal value, the value $x_1 = 1$ is associated with a +10% variation of the roof thickness, $x_1 = -1$ with a −10% variation of the roof thickness, and similarly for x_2. CLF's and injected powers in each frequency band are evaluated using VA One software for each pair of physical parameters. Results are stored and used to identify the coefficients α appearing in equation (21) through Eq. (A.6).

The multi-objective optimization technique described in section 3.3 is performed in order to cut 20% off the energy of the subsystems 3 (Cabin Cavity), and 19 (Floor Bulkhead). The weights w_{lk} are selected all equal to one because all energy reduction objectives are considered equally important. The starting values of the x_l parameters are the

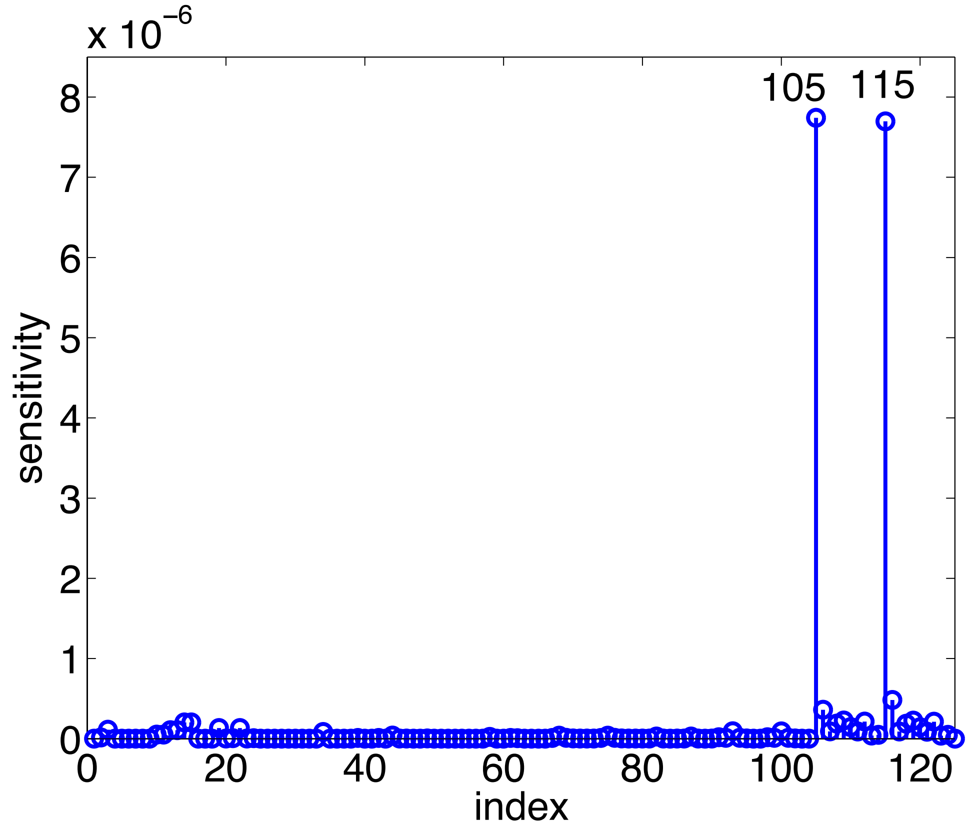

Figure 6: Sensitivity of cabin cavity to CLF changes

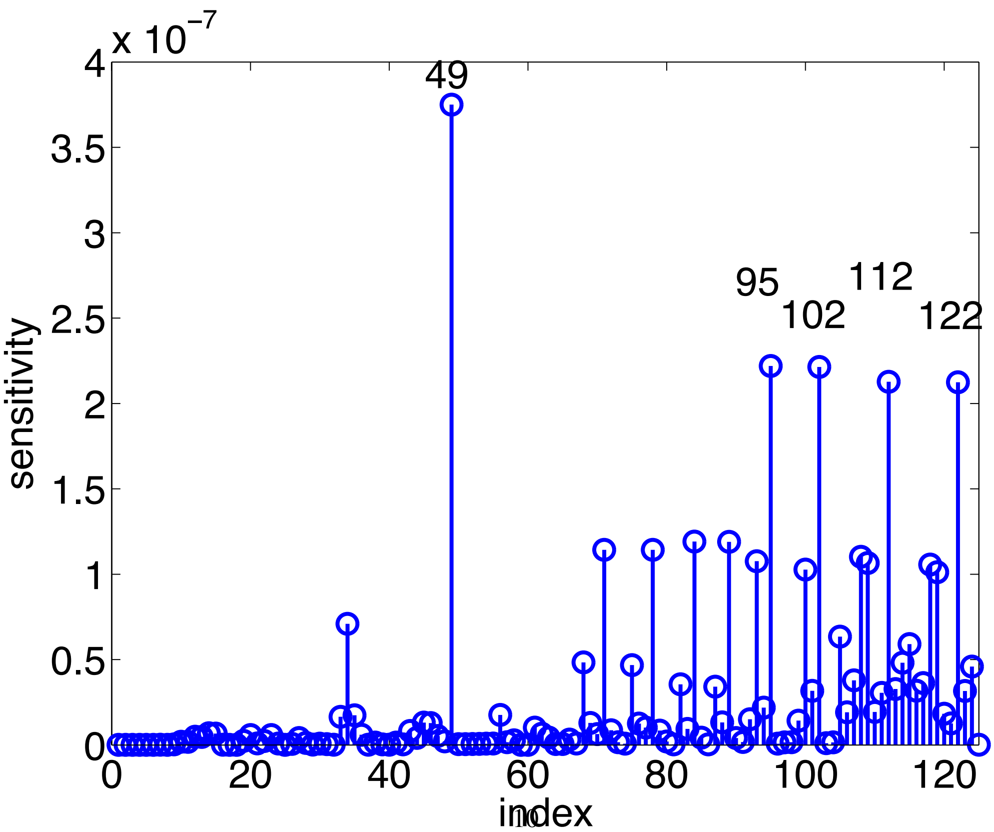

Figure 7: Sensitivity of floor bulkhead to CLF changes

nominal values. The upper and lower bounds for the x_l parameters are ±10% of the nominal values. The result of the optimization procedure is an increase of the Roof thickness (subsystems 31 and 32) by 10% and a decrease of the Floor Bulkhead thickness (subsystem 22) by 10%. It can be noticed (see Table 4) that the goal of decreasing the energy level by 20% can not be completely attained, because both the optimization variables, x_1 and x_2, reach their bounds. However, for the cabin cavity, the decrease up to 1.6 kHz is almost equal to the desired goal, there is a less significant decrease up to 6.3 kHz, but there is an increase at 8 and 10 kHz; for the floor bulkhead, there is an overall decrease around 15%, evenly spread throughout all the third octave bands. Energy values are shown in Figures 8 and 9 for subsystems 3 and 19. It can be noticed that the energy values provided by the approximate model (coupling loss factors and injected powers given by Eq. 21) are practically identical to those computed by introducing the modified physical parameters into the VA-One model.

Table 4: Results of optimization procedure

[Hz]	$\Delta E_3[\%]$	$\Delta E_{19}[\%]$
Overall	0.27	-14.91
1000	-18.59	-17.10
1250	-18.56	-15.90
1600	-18.08	-14.66
2000	-11.67	-14.69
2500	-9.27	-14.85
3150	-9.31	-14.57
4000	-9.62	-14.31
5000	-9.17	-14.10
6300	-5.77	-13.96
8000	5.12	-13.84
10000	107.92	-16.05

5. Discussion

One could try to attain the required 20% energy decrease by broadening the upper and lower bounds. An attempt is made by considering a variation of the physical parameters up to ±30%. In this case, a new DoE must be performed, although the approximate model provided by DoE with Central Composite Design might be unable to reproduce strong non-linearities of the CLF's with physical parameters. The result of the optimization is an increase of the Roof thickness (subsystems 31 and 32) by 7.2% and a decrease of the Back Bulkhead thickness (subsystem 22) by 30%. Energy values are shown in Figures 10 and 11 for subsystems 3 and 19. The goal seems to be fully attained for subsystem 19 and partly for subsystem 3 (see Table 5). By visually comparing (Figures 10 and 11) the energy levels computed by introducing the modified physical parameters into the VA-One model with those given by the approximate model, one would conclude that the approximation provided by DoE is satisfactory. However, this is a lucky circumstance, because one of the physical parameters varies less than 10%, whilst the other reaches its bound: at the bound, which is an observation point, the CCD provides the correct value and no error appears. In fact, as shown in Figs. 12 and 13, the behaviour of CLF's $\eta_{22,19}$ and $\eta_{19,22}$ is strongly nonlinear and could not be described by Central Composite Design at intermediate points.

6. Conclusion

Since optimization techniques based on classical FE models or BE models can not be applied at medium high frequencies, a SEA model is used to formulate an optimization procedure in order to bring the subsystem energies under prescribed levels. In this paper a multi-objective optimization problem is defined on a simplified helicopter model. The variables of the problem are the relative deviations of the selected physical parameters from the nominal values. Upper and lower bounds of these design variables are defined. Finally, the energy of the subsystems are

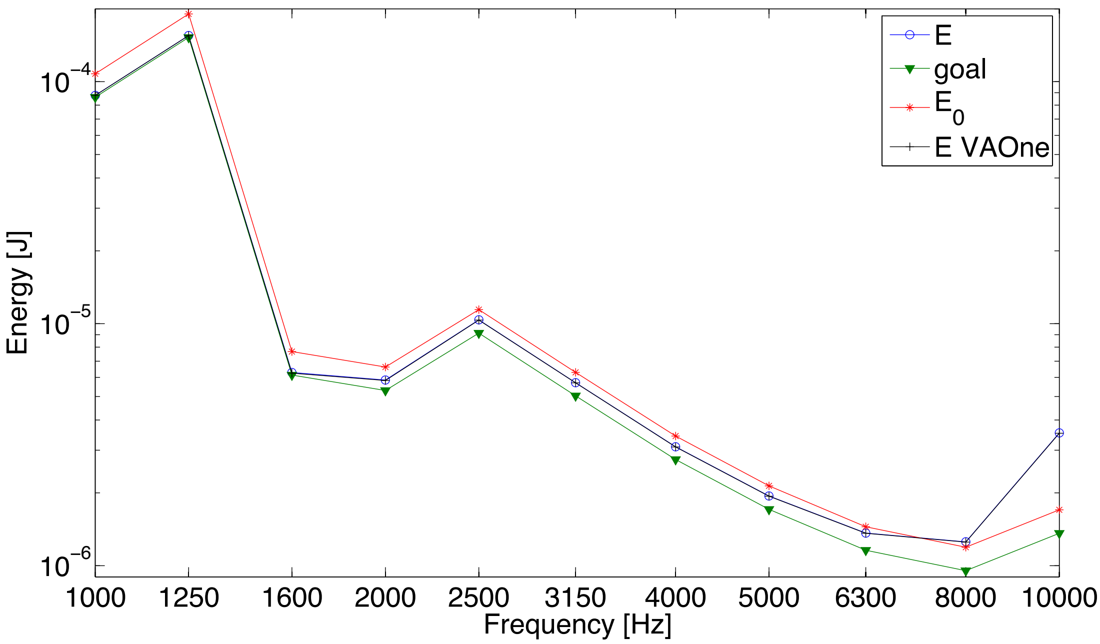

Figure 8: Energy of subsystem 3: nominal (—∗—), goal (—▼—), after optimization approximate model (—○—), after optimization VA-One model (—+—)

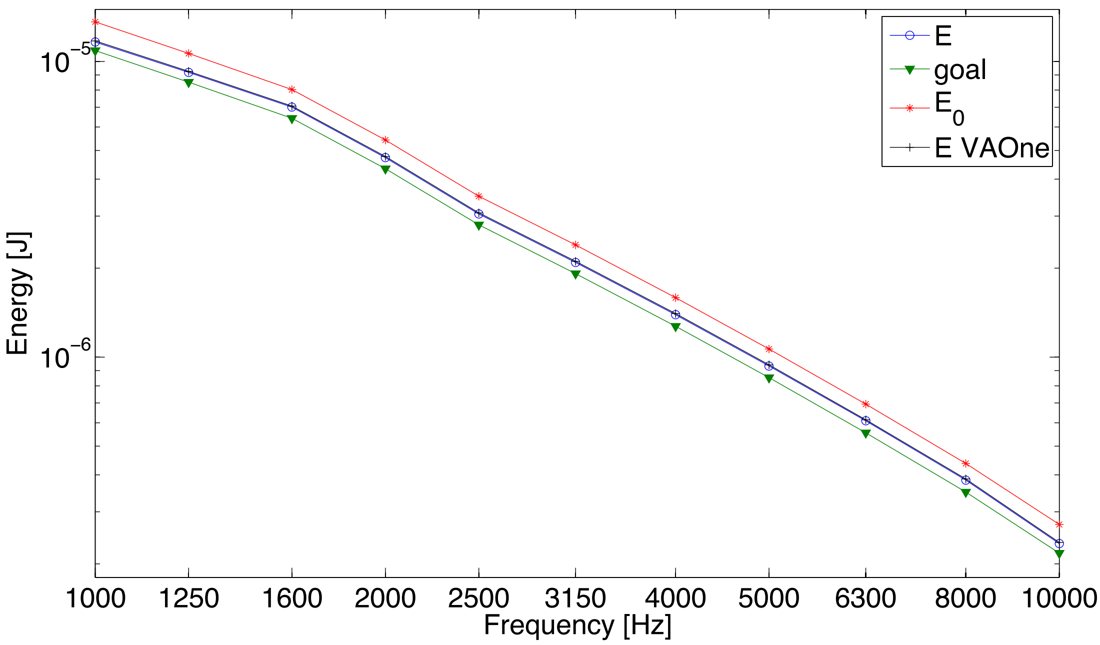

Figure 9: Energy of subsystem 19: nominal (—∗—), goal (—▼—), after optimization approximate model (—○—), after optimization VA-One model (—+—)

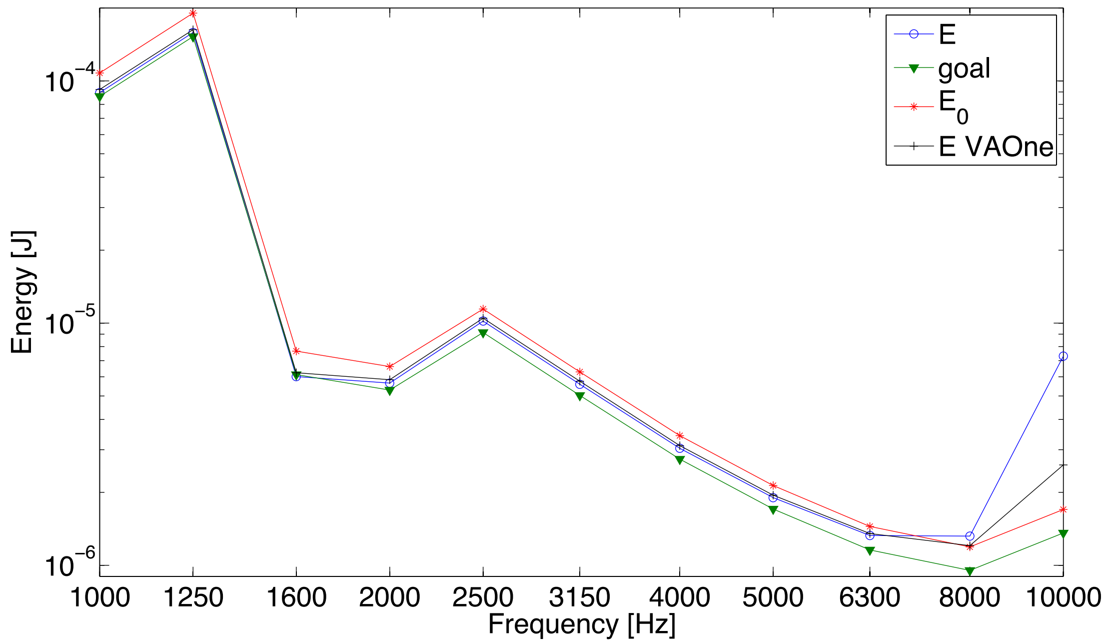

Figure 10: Energy of subsystem 3 with ±30% bounds: nominal (—∗—), goal (—▼—), after optimization approximate model (—○—), after optimization VA-One model (—+—)

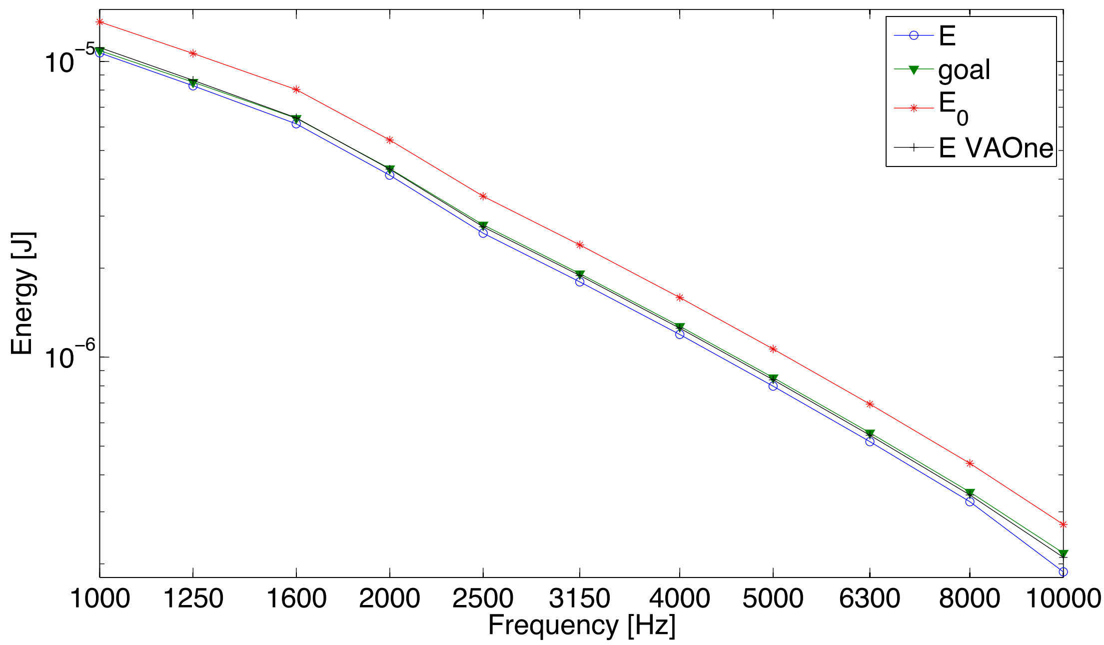

Figure 11: Energy of subsystem 19 with ±30% bounds: nominal (—∗—), goal (—▼—), after optimization approximate model (—○—), after optimization VA-One model (—+—)

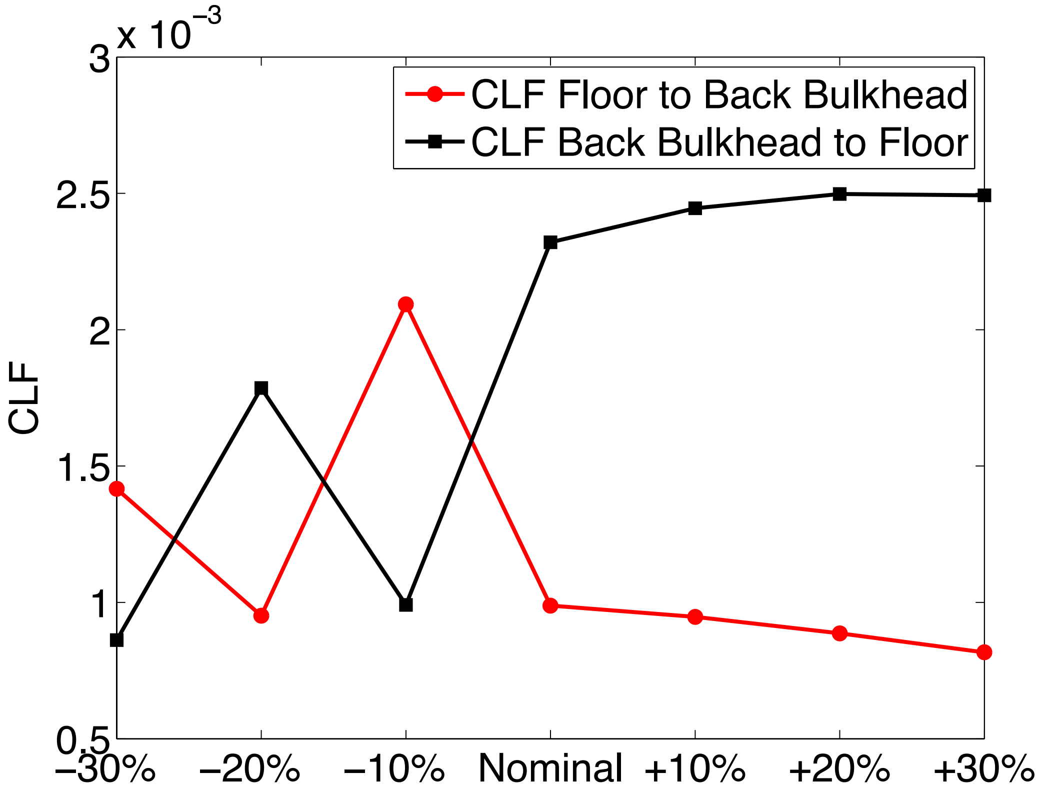

Figure 12: $\eta_{22,19}$ (—■—) and $\eta_{19,22}$ (—•—) computed by VA-One at 1 kHz 3rd octave band, for variations of the Back Bulkhead thickness in the range ±30% with ±10% steps.

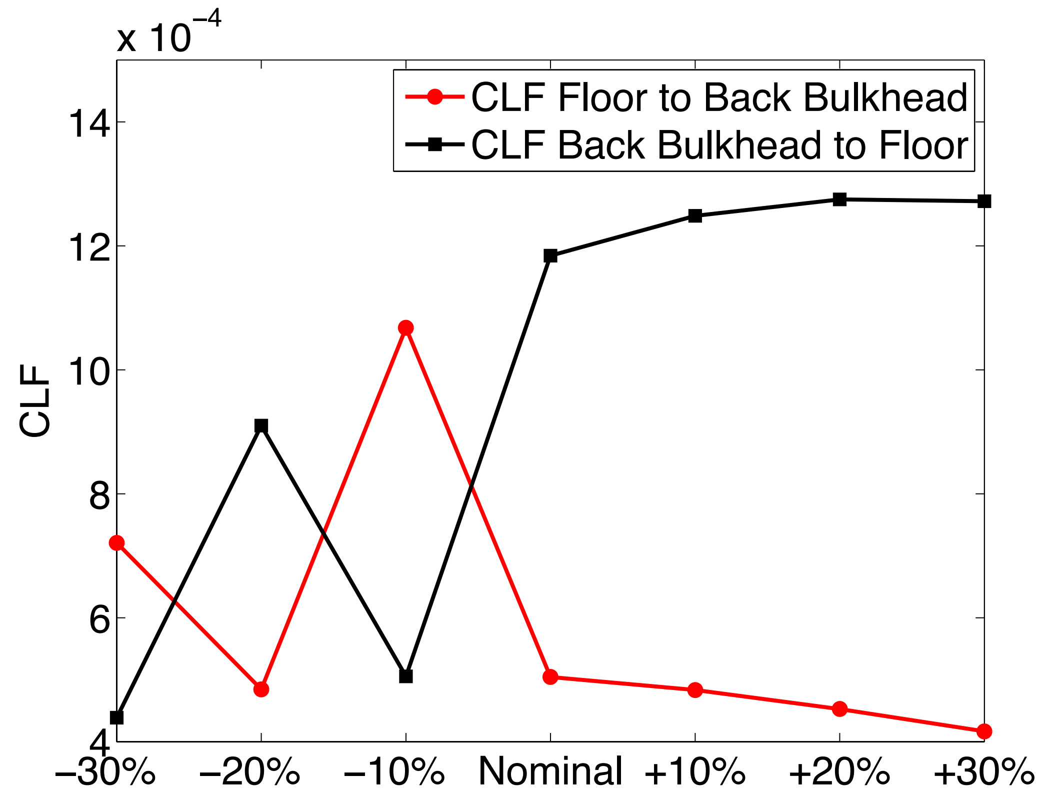

Figure 13: $\eta_{22,19}$ (—■—) and $\eta_{19,22}$ (—•—) computed by VA-One at 4 kHz 3rd octave band, for variations of the Back Bulkhead thickness in the range ±30% with ±10% steps.

Table 5: Results of optimization procedure with ±30% bounds.

[Hz]	ΔE_3[%]	ΔE_{19}[%]
Overall	19.86	-24.84
1000	-17.03	-21.46
1250	-16.88	-22.18
1600	-21.49	-23.43
2000	-14.48	-24.00
2500	-10.84	-25.02
3150	-11.04	-24.96
4000	-11.33	-25.01
5000	-10.90	-25.16
6300	-8.23	-25.37
8000	10.77	-25.80
10000	329.89	-30.87

constrained to be lower than prescribed values. A preliminary sensitivity analysis is performed in order to select the more significant CLF's. By using DoE, an approximate relation between the more sensitive CLF's and the selected physical parameters is obtained. Finally, not all energies satisfy the prescribed constraints because some optimization variables reach the upper or lower bounds.

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Appendix A. A brief reminder on Design of Experiment

In Design of Experiments (DoE) [14], the values of the variables that affect an output response are appropriately modified by a series of tests, to identify the reasons for changes in the response. This does not prevent from performing numerical tests whenever this may be convenient for a better understanding of the numerical problem under investigation.

Since many experiments involve the study of the effects of two or more variables or factors, it is necessary to investigate all possible combinations of the levels of the factors. This is performed by factorial designs which are very efficient for this task.

A regression model representation of a factorial experiment with two factors A and B at two levels could be written as:

$$f = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \varepsilon \tag{A.1}$$

where the α's are parameters whose values are to be determined, the variables x_1 and x_2 are defined on a coded scale from -1 to $+1$ (the low and high levels of A and B) and ε is an error term.

Specifically, if p factors at two levels are considered, a complete series of experiments requires 2^p observations and is called a two-level 2^p full factorial design. Usually, each series of experiments should be replicated several times using the same value of the factors to average out the effects of noise. Of course, this is unnecessary if experiments are numerical.

A feature of two-level factorial design is the assumption of linearity in the effect of each single factor and of multi-linearity in interactions among factors, see for instance the term $\alpha_{12} x_1 x_2$ in Eq. (A.1). To account for possible non linear effects, quadratic terms can be introduced, as in the following regression model for two factors:

$$f = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \varepsilon \tag{A.2}$$

where the α's are parameters whose values are to be determined, the variables x_1 and x_2 are defined on a coded scale from -1 to $+1$ (the low and high levels of the two factors) and ε is an error term.

Of course, a three level (low level -1, intermediate level 0, high level $+1$) factorial design, involving 3^p observations, is a possible option if quadratic terms are important. However, a more efficient alternative is the Central Composite Design (CCD) that starts from the 2^p design augmented with the *center point* i.e. a single observation with all factors at intermediate level, and *axial runs* where each factor is considered at two levels (the low level -1 and the high level $+1$) while the remaining factors are at the intermediate level, for a total of $2p$ observations (Fig. A.14).

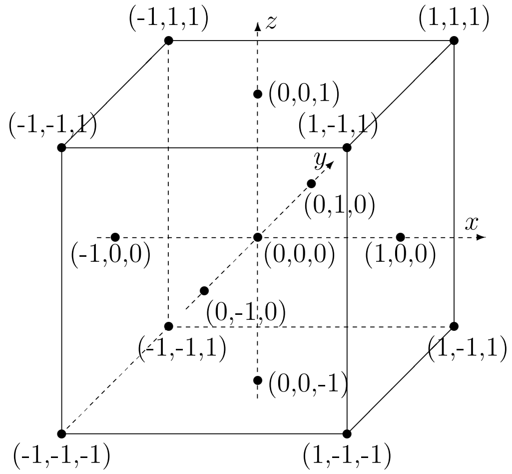

Figure A.14: Central Composite Design for $p = 3$

Overall, a central composite design for p factors requires $n = 2^p + 2p + 1$ observations instead of 3^p observations required by the three level factorial design, with advantages for $p \geq 3$.

For p control factors, the experimental response can be expressed as a regression model representation of a 2^p full factorial experiment (involving 2^p terms), augmented with p quadratic terms:

$$\begin{aligned} f = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_i + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \alpha_{ji} x_j x_i + \ldots + \\ + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \cdots \sum_{n=1}^{m-1} \alpha_{nm\cdots ji}\, x_n x_m \cdots x_j x_i + \sum_{i=1}^{p} \alpha_{ii} x_i^2 + \varepsilon \end{aligned} \tag{A.3}$$

The expression contains $2^p + p$ parameters α, each one providing an estimate of the effect of a single factor (linear or quadratic) or of a combination of them.

Note that Eq. (A.3) is linear in the parameters α, and it can be rewritten as:

$$f = \begin{bmatrix} 1 & x_1 & \cdots & x_p^2 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{pp} \end{Bmatrix} + \varepsilon \tag{A.4}$$

having arranged the parameters in a vector α. A different equation can be written for each observation by varying the factors $(x_1, \ldots, x_p)$ as indicated by CCD.

By arranging the experimental responses in a vector $\mathbf{f}$, a linear relationship between $\mathbf{f}$ and α can be expressed in matrix notation as:

$$\mathbf{f} = \mathbf{X}\alpha + \varepsilon \tag{A.5}$$

where $\mathbf{X}$ is a $(2^p + 2p + 1) \times (2^p + p)$ matrix. The least square estimate of α is:

$$\hat{\alpha} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{f} \qquad \Rightarrow \qquad \hat{\mathbf{f}} = \mathbf{X}\hat{\alpha} \tag{A.6}$$

where $\hat{\mathbf{f}}$ is the fitted regression model.

The difference between the actual observations vector $\mathbf{f}$ and the corresponding fitted model $\hat{\mathbf{f}}$ is the vector of residuals $\mathbf{e} = \mathbf{f} - \hat{\mathbf{f}}$. The residuals account both for the modelling error ε and for the fitting error due to the least square estimation.

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