Experimental damage evaluation of open and fatigue cracks of multi-cracked beams by using wavelet transform of static response via image analysis

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SUMMARY

In this study, a method for crack detection and quantification in beams based on wavelet analysis is presented.
The static deflection is measured at particular points along the length of (i) real damaged structures, using few displacement transducers and a laser sensor, and (ii) simulated structures, using closed-form analysis, for give location of a concentrated load along the beam. Furthermore, the measurement of the beam displacements in a large number of spatially distributed points is made by processing digital photographs of the beam. The smoothed deflection responses of the cracked beams are then analyzed using the wavelet transform. For this purpose, a Gaus2 wavelet with two vanishing moments is utilized. The wavelet transform spikes are used as indicators to locate and quantify the damage; furthermore, the multi-scale theory of wavelet is employed, in order to eliminate or at least reduce the spurious peaks and enhance the true ones. Simply supported beams with single and double cracks are used to demonstrate the devised methodology. Open and fatigue cracks of different sizes and locations have been used in the examples. In a closed-form analysis, the damage is modeled as a bilinear rotational spring with reduced stiffness in the neighborhood of the crack location. Damage calibration of simply supported steel beams with open and fatigue cracks has been carried out experimentally using this technique. A generalized curve has been proposed to quantify the damage in a simply supported beam. Based on the experimental study, the spatial wavelet transform is proven to be effective to identify the damage zone even when the crack depth is around 3% of the height of the beam.

KEY WORDS: damage evaluation; experimental investigations; static deflection; image analysis; wavelet transform

1. INTRODUCTION

The detection of crack-like defects in mechanical systems and civil engineering structures is a problem that received considerable attention from engineering researchers in the last decades [1-4]. The research effort devoted to this topic is due to its practical importance. For example, one of the most important requirements for the proper maintenance of machinery or civil infrastructure systems is the detection of crack-like damage at the early stages of growth. This leads to the necessity of developing nondestructive techniques for crack detection that are both practical and accurate [5-10].

To ensure high system performance, structural safety and integrity, and low maintenance cost, structural health monitoring (SHM) has emerged as a reliable, efficient, and economical approach to monitor the system performance, detect damage, assess/diagnose the structural health condition, and make corresponding maintenance decisions [11]. Damage is often observed in many engineering structures during their service life. The damage may be caused by various factors, such as excessive response, cumulative crack growth, wear-and-tear of working parts, and impact by a foreign object. The detection of damage in civil engineering and mechanical systems is a problem that received considerable attention from researchers in the recent years. The existing diagnostic techniques (ultrasonic, radiography, acoustic emission, etc.) require not only *a priori* knowledge of the damaged region, but also accessibility to the vicinity of damage.

In the last decade, wavelet theory has been one of the emerging and fast-evolving mathematical and signal processing tools [12]. An important feature of the wavelet transform (WT) is the ability to characterize the local irregularity of a function and to react to subtle changes of the signal structure. A crack in a structure introduces singularities to the first derivative of the static deflection curve (i.e., rotation of the beam cross-section). These small defects cannot be identified directly from the structure response, but observed on wavelet transforms because local abnormalities in the signal result in large wavelet coefficients (WCs) in the neighborhood of the damage [13, 3]. The crack location is indicated by a peak in the variations of the WCs along the length of the beam. The static deflection measurements at sparse intervals are relatively easier and cost-effective compared to dynamic response measurements. Thus, for damage detection in beams, methods where the WT is applied on spatial signals, static displacement responses were most commonly employed [14]. In Garstecki et al. [15] static and dynamic numerical examples were used to identify damages in structure by discrete wavelet transformation; beyond the identification of the (open crack) damage, the aim of the paper was to determine the minimum number of measurements points sufficient for the goal, also in presence of noise. Spanos et al. [14] built a WT modulus map where boundary effects were eliminated and damage-related local maxima were clearly identifiable. They also conducted numerical tests on the (simulated) pseudo-experimental response of beams featuring up to three open cracks. Wavelet analysis has been performed on the simulated static deflected shape for locating the damage and wavelet-kurtosis-based calibration of the extent of damage and an experimental validation of this method has been carried out by Pakrashi *et al*. [16, 17]. The proposed method has been validated experimentally by performing wavelet analysis on the damaged shape of a simply supported freely vibrating aluminium beam by Pakrashi *et al*. [16]. Damage calibration of a simply supported phenolic beam with an open crack has been carried out experimentally using this technique by Pakrashi *et al*. [17]. A method to detect the location and also to quantify the open crack using the deflection response of the damaged beams alone has been proposed by Umesha *et al*. [11]. The deflection was measured at a particular point for various locations of a concentrated load on the beam. This static deflection profile was used as the input signal for wavelet analysis. Rucka and Wilde [18] presented a method to localize damage in a polystyrene cantilever beam using static deflection for the purpose of damage (one open crack) detection via Gaussian and Coiflet wavelet families; the measurement of the beam displacements of spatially distributed points was obtained by processing digital photographs. In Rucka and Wilde [19] a continuous wavelet transform was applied to detect experimentally vibration based damage both for beams and plates; for the one-dimension problem (one open crack) Gaussian wavelets were considered, whereas for two dimensional structure reverse biorthogonal wavelet was applied. Shi *et al*. [20] used a computer vision camera to capture the static deformation profiles of one-crack and two-crack cantilever beams under loading; the profiles were then processed to reveal the existence and location of the irregularities (slots) on the deformation profiles by applying wavelet transform. In Rucka [21] higher vibration modes and the influence of their order were studied for damage detection; vibration measurement techniques were considered on a steel cantilever beam with rectangular notches of different depth, identified by Gaussian wavelets Gaus4, Gaus6, and Gaus8. Wu and Wang [22] employed a laser profile sensor to measure the deflection profile of a cracked cantilever aluminum beam subjected to a static displacement at its free end; the smoothed static profile of the cracked beam was analyzed with Gabor wavelet to identify the open crack with different depths. Wavelet transform combined with inverse analysis was used in Piatkowska and Garbowski [23] for the detection of concentrated defects in structural health monitoring of a cantilever steel beam; the authors proposed a “pseudo experiment” to minimize the discrepancy between the wavelet representation of real measurable quantities and the numerically computed ones.

The aim of the present work is to detect, locate and evaluate damage at beams under static load based on discrete wavelet analysis. To this end, the static displacement of damaged simply supported beams under a load applied at the beam center is measured at few locations along the beam axis; then the measurement of the beam displacements in a large number of spatially distributed points is made by processing digital photographs of the beam. The smoothed displacement profile of the beam is used as the input of continuum wavelet transform (CWT) analysis to detect subtle discontinuities in the response, namely rotations of beam cross-sections. For this purpose, Gaus2 wavelet is utilized. The peak of CWT coefficient is used as indicator to identify damage. Effect of crack depth, position, number (one or two cracks) and type (open or fatigue cracks) are discussed based on the peak value of the wavelet coefficients. Furthermore, the single crack is modeled as a bilinear rotational spring characterized by a reduced stiffness in beam element and comparison is made between experimental and analytical results.

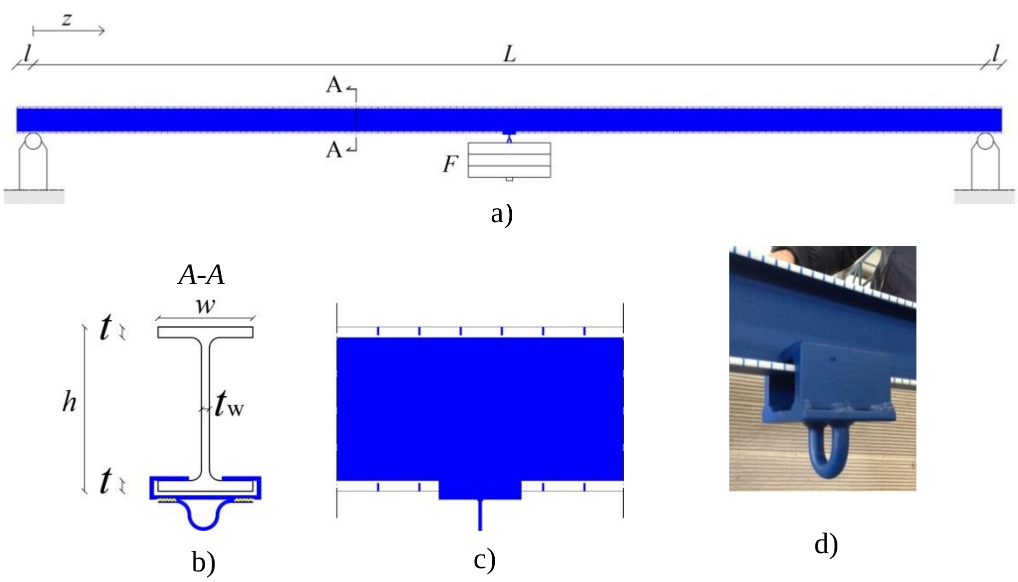


Figure 1. Design of the experimental setup. a) Lateral view; b) cross section and sliding device for load suspension (front view); c) sliding device for load suspension (lateral view); d) real picture.

2. EXPERIMENTAL STUDIES OF DAMAGE DETECTION USING WAVELET ANALYSIS

2.1 Experimental set-up

Four types of simply supported beam were adopted in the experiment for verification purposes, Fig. 1a. All of them were cut from the same steel sheet to avoid any variation in material properties, and machined to the same dimensions of L = 290 mm and I-shaped cross-section IPE80 (height h = 80 mm, flange thickness t = 5.2 mm, flange width w = 46 mm, web thickness t_w = 3.8 mm), Fig. 1b. The only difference is that one beam specimen (I) has a machined slot, another one (II) has a fatigue crack, the third one (III) has two machined slots, and the fourth one (IV) has a machined slot and a fatigue crack. The open cracks are created by sawing notches into the lower section of the beam of cross section. The fatigue cracks are generated by cycling the sample to maximum and minimum loads. The locations of the non-propagating vertical cracks are situated at abscissas z_1 = 1.93 m and z_2 = 0.97 m from the left hand support of the beam. The beam is subjected to a static weight at the center. The considered damage conditions were: crack depth d_1= 0.5 t, 0.7 t, 0.85 t, 0.95 t, 1.0 t, 0.3 h at z_1 for the I specimen; crack depth d_1= 0.3 h at z_1 for the II specimen; crack depth d_2= 0.3 h at z_2 and crack depth d_1= 0.4 t, 0.7 t, 1.0 t at z_1 for the III specimen; crack depth d_2= 0.3 h at z_2 and crack depth d_1= 0.25 h at z_1 for the IV specimen. As indicated by the schematic in Figure 1, the beams were simply supported at both ends using roller devices fixed on a work table. In particular, Figures 1b,c,d show the details of the annular support and the rectangular bracket that connects it to the lower flange of the beam, which were suitably designed.

Figure 2 shows the general arrangement of the experimental setup. A static weight of magnitude F has been carefully put on the beam to deflect the beam for elastic deformation. A digital displacement transducer (Mitutoyo Digimatic Indicator, 543-682, ID-S1012M), analogical displacement transducers (Mitutoyo Model 2050F Dial Indicator) and a laser profile sensor (optoNCDT ILD 1302-50) are employed to measure the deflection profile of the beams.

a) b) c)

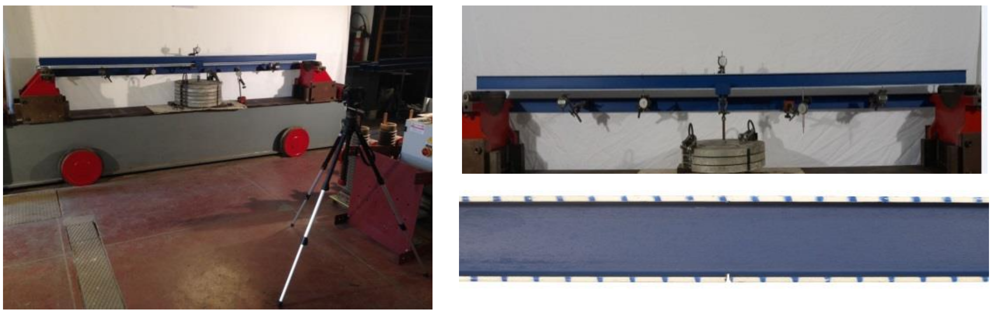


Figure 2. Experimental setup. a) Overall view; b) lateral view; c) crack detail across the centre of the lower flange.

The value of the Young's modulus E = 203.481 GPa and the processed profile data were identified in terms of the magnitude of the measured displacements of the healthy beam, Fig. 3a:

$$u = \frac{Fz(-3L^2+z^2)}{6B}, \quad 0 \le z \le L \tag{1}$$

where $B = EI$ is the bending stiffness and I the moment of inertia of the cross-section of the healthy beam. Figure 3a shows one such calibration for a static load 3 kN. In a previous work [24] the authors formulated an analytical model for simulating the static behavior of multi-cracked beams and a WT based method for damage identification. The influence of a fatigue crack at abscissa z_c was accounted for by means of a bilinear rotational spring, the stiffness k_c of which was affected by a suitable damage parameter ε_c:

$$v = \begin{cases} \frac{Fz(-6L\alpha_c - 3L^2\varepsilon_c + z^2\varepsilon_c)}{6B\varepsilon_c}, & 0 \le z \le z_c \\ \frac{F[z^3\varepsilon_c - 3L^2(2\alpha_c{}^2 + z\varepsilon_c)]}{6B\varepsilon_c}, & z_c \le z \le L \end{cases} \tag{2}$$

with $\alpha_c = z_c / L$, $\varepsilon_c = k_c / B$. The values of such a parameter were identified on the basis of the displacements measured in the damaged beams. For sample's sake, Figure 3b reports the results of the above mentioned identification in the case of crack depth = 1.0 t at z_2 for a static load 2 kN.

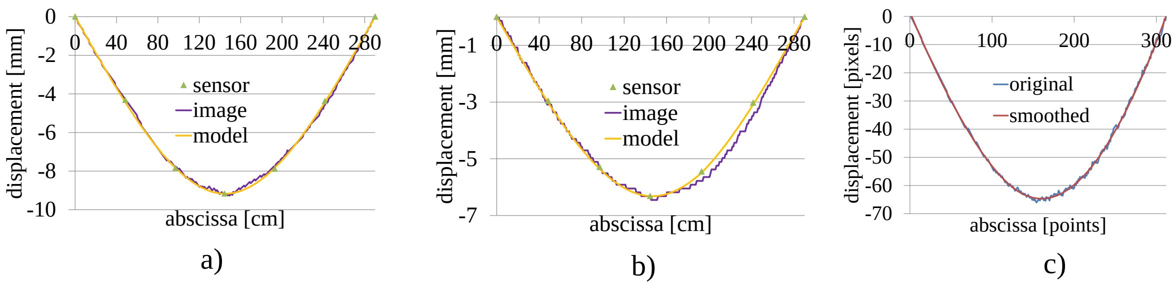


Figure 3. Identification of material, damage and image data. a) Young's modulus; b) damage parameter; c) signal smoothing.

2.2 Image analysis

The deflected photographs has been recorded in the spatial domain by an Sony NEX-3N digital camera with resolution of 16 MP (http://www.sony.co.uk/support/en/product/nex-3n). The images were recorded in RAW, which allows the post-production correction via editing (Sony Image Data Converter v. 4.2.04), and in ready-to-use JPEG compression format. The latter have shown the optimal compromise between accuracy and speed in image recording throughout 4912×2760 pixels size. Eventually to increase this resolution, each image was interpolated by the open-source software GIMP (GNU Image Manipulation Program) v. 2.8.2, thus obtaining images of size 15000×9968. Figure 4 shows the captured image of a beam under load. The lower and upper edges of the beam were detected from the image by a pattern recognition scheme simulating a graduated ruler. It can be seen that a white background was used in experiment to provide good contrast against the blue surface of the steel beam. Since the cracks were made to simulate the general internal (often invisible) damages in a structure, direct observation does not serve the general purpose of this study. As such, in the experiment, the profiles were actually captured only along the top surfaces as shown in Fig. 4. All the images were processed by γ-correction, with γ=0.8, thus obtaining brighter data, [25].

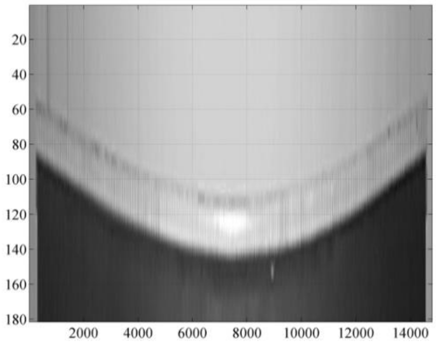


Figure 4. Magnification of a zone of the photograph.

To determine the deformation of the beam under load, both when it was undamaged and when cracks were present, each image was first segmented. To segment an image means to determine a partition of the data with respect to a specific characteristic of the elements to be identified, for example, the gray level, the color, the shape and so on; the number of classes in which the data should be partitioned determines the level of detail in which the segmented data will be represented. In the considered images the beam appeared as a rectangular object with uniform gray level and therefore a segmentation with respect to gray level was adopted.

Many different methods exists to segment images, depending on the specificity of the acquired data; here the Otsu method was chosen [26] also for its efficiency. It provides the optimal threshold to separate the objects from the background. It is based on the assumption that the histogram of the data is bi-modal and therefore two classes of pixels are considered, the object and the background. The optimal threshold is obtained by minimizing the intra-class variance:

$$\sigma_\omega^2(t_\mathrm{h}) = \omega_1(t_\mathrm{h})\sigma_1^2(t_\mathrm{h}) + \omega_2(t_\mathrm{h})\sigma_2^2(t_\mathrm{h}) \tag{3}$$

where σ_k^2 , $\mathrm{k} = 1{,}2$ are the variances of the classes and the weights ω_k , $\mathrm{k} = 1{,}2$ represent the probabilities of the two classes separated by the threshold t_h.

In order to detect the shape of the beam, a 4-levels segmentation was advisable; it has been obtained by applying hierarchically the binarization described above. The beam corresponded to the second value among the four obtained, when sorted in ascending order; after area filtering it was easily identified.

Assuming a fixed horizontal step, the corresponding values v_i of the measured vertical displacements of the upper beam were evaluated; therefore a vector **V** with the "n" measures v_i was available. The first measurement point was the first left point of the beam and the step was the same among all the analysed images; this was crucial to perform a comparison among all the data collected on beams under different load conditions.

2.3 Wavelet analysis

A wavelet Φ is a L^2- function with compact support and zero average; the function Φ is called the mother wavelet and by dilation and shifting parameters a family of wavelets may be obtained.

The correlation between the signal $f(z)$, representing the values of the vertical measurements taken on the upper beam, and the scaled wavelet

$$\Phi,\ C(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(z)\Phi\left(\frac{z-b}{a}\right)dz \tag{4}$$

is the continuous wavelet transform (CWT); *a* and *b* are the scale and position parameters respectively. If the wavelet is compressed, that is $a \gg 1$, only a limited portion of the signal $f(z)$ is approximated, whereas a global approximation is obtained with $a \ll 1$. The result of the CWT is the coefficient *C* representing the similarity between the analyzed signal and the chosen wavelet function. The specific kind of wavelet should be selected in order to facilitate the detection of the signal's features of interest. Abrupt changes in signal can be detected by wavelets centered around the discontinuity; more precisely, the set of coefficients of CWT increases with scale, whereas the precise localization of the discontinuity can be obtained at the small scales. Therefor a peak in the coefficient *C* can indicate the location of a crack.
The vanishing moments

$$\int_{-\infty}^{+\infty} t^k\Phi(z)dz,\ \ k=0,1,2,...,n-1 \tag{5}$$

are important in the detection of discontinuities. A wavelet with *n* vanishing moments is orthogonal to polynomials up to degree *n-1*. Following [27] and [18], it can be shown that a wavelet with *n* vanishing moments can be interpreted as the *n*-th derivative of a function ϑ:

$$\Phi(z)=\frac{d^n\vartheta(z)}{dz^2}; \tag{6}$$

therefore the corresponding CWT may be written as:

$$C(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(z)\Phi\left(\frac{z-b}{a}\right)dz=\frac{a^n}{\sqrt{a}}\int_{-\infty}^{+\infty} f(z)\frac{d^n}{dt^n}\vartheta\left(\frac{-(z-b)}{a}\right)dz=\frac{a^n}{\sqrt{a}}\frac{d^n}{dz^n}\int_{-\infty}^{+\infty} f(z)\vartheta\left(\frac{-(z-b)}{a}\right)dz \tag{7}$$

Therefore the CWT is the *n*-th derivative of the signal $f(z)$ suitably smoothed by the wavelet

$$\vartheta(-x/a)/\sqrt{a}\,. \tag{8}$$

In [28] an interesting review on wavelets' analysis and a comparison on their capability for damage detection were presented; it was stressed that, when changing scale, Gaussian wavelets don't interrupt the propagation of the maxima [29]. Moreover, from Eq. (4), once the parameter *a* is adequately small in the domain of interest, the CWT is proportional to the *n*-th derivative of the signal $f(z)$. Therefore, for the purpose of this paper, the wavelet Gaus2, with two vanishing moments, was chosen to detect the presence of a discontinuity in the analyzed beam; the chosen wavelet is available in the Matlab Toolbox [30] which will be the software used later to implement the identification methodology.

2.4 Deflection profile of the bending beam and the denoising process

To erase the noises from the original signal of the deflection profile for a better detection purpose, a smoothing algorithm provided by Matlab R2007a, ‘rloess’, is imported in the data processing before the wavelet transform [22]. The method assigns zero weight to data outside six mean absolute deviations; a span value of 45% showed to be the most efficient in all cases. For sample’ sake, the case of crack depth = 1.0 t at z_1 from the left hand support of the beam, for a static load 2 kN, is shown in Fig. 3c: it can be seen that the original signal is smoothed effectively for a possible detection with wavelet transform.

2.5 Damage detection using wavelet analysis

To determine the position of the crack a wavelet analysis was performed. The second order derivative of a Gaussian, W, is a wavelet useful to identify discontinuity in signals. In the present situation for the undamaged beam with load there is a discontinuity in the third order derivative in the middle of the beam; when the beam is damaged is present another discontinuity is present in its location in the 4th order derivative.

Each signal **V** representing the deformation is convolved with a wavelet W with suitable variance. When a discontinuity in the 3rd order derivative of the vector **V** is present the convolution showed a peak. Some other peaks may be present, due to the presence of noise.

To recognize the peaks due to a true discontinuity the multi-scale theory of wavelet is used. In [31, 27], as already recalled, the authors observed that the Gaussian wavelets propagate the maxima, when their scale is changed, so the peaks corresponding to a true discontinuity do not change position, whereas the peaks due to noise may change high or position. This consideration suggested us to consider the wavelet Gaus2 with three different scales, consider the convolution and then study the product. The result eliminated or at least reduced the spurious peaks and enhanced the true ones. The choice of the scale was mainly related with the length of the domain of interest; since the support of the wavelet with scale a is about [-2a, 2a] and the number of measurements points was 312, a reasonable choice for the scale of three wavelets was 3, 6 and 9, Fig. 5a; so, the confusion between two possible adjacent discontinuities was avoided and it was possible an accurate position detection.

Based on the calculation shown by Eq. (4), the crack position can be discerned by conducting the smoothed discrete deflection signals from the captured image with wavelet transform. To avoid any experimental errors usually happened at the measurement border, the mid 312 (= n) points out of 15000 from the deflected photograph are selected. Chang and Chen [32] pointed out that the WT may exhibit local maxima at the beam ends, even if no damaged section is located therein. These boundary effects, relevant for any function defined on a finite interval, represent a non-negligible limitation of the approach, since no reliable diagnosis can be formulated on the damage state at the beam ends. In [18] to avoid large discrepancy at the boundaries, the signal was extended outside its original support by a cubic spline extrapolation. Spanos et al. [14] observed that boundary effects can be eliminated if the WT is applied on the difference between the displacement responses of the damaged and undamaged beam, subject to the same load. In [33], the problem of the damage masked by CWT border distortions was discussed and a new signal extension polynomial method to enhance damage detection by spatial CWT is proposed. The method is based on high-order polynomial functions that fit the original data and its first derivative so as to extend smoothly the signal and its derivatives. For their simplicity the signal boundary constraint approaches are preferred. Traditional extending methods as zero padding, periodic padding, symmetric padding and linear padding (e.g., see MATLAB Wavelet Toolbox [30]), are usually employed in WA.

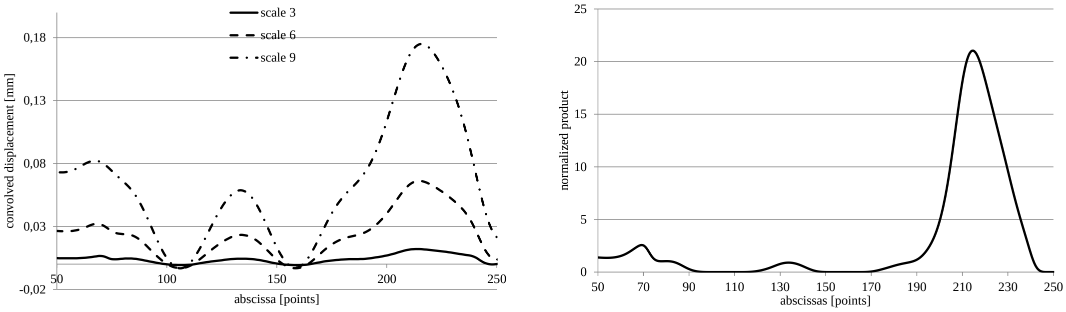
In more detail, the CWTs were normalized with respect to the chosen scales of WT as follows, Fig. 5b. Indicating the different wavelet scales with p_1=3, p_2=6, and p_3=9, the normalized CWT was defined as

$$\mathrm{CWT_{norm}} = \mathrm{CWT} \cdot (p_1 \cdot p_2 \cdot p_3) \cdot \sqrt{2\pi} \cdot \sqrt[3]{\mathrm{CWT_{max}}} \tag{9}$$

where

$$\mathrm{CWT} = |\mathrm{CWT}(p_1)| \cdot |\mathrm{CWT}(p_2)| \cdot |\mathrm{CWT}(p_3)| \tag{10a}$$

$$\mathrm{CWT_{max}} = \mathrm{Max}_z(\mathrm{CWT}) \tag{10b}$$



a) b)

Figure 5. Wavelet transforms. a) Convolutions at different scales; b) normalized product.

2.6. Single open crack (First specimen)

2.6.1 Damage location

The damage detection of the open crack with the depth d_1= 0.5 t located at z_1 is first studied (Fig. 6).

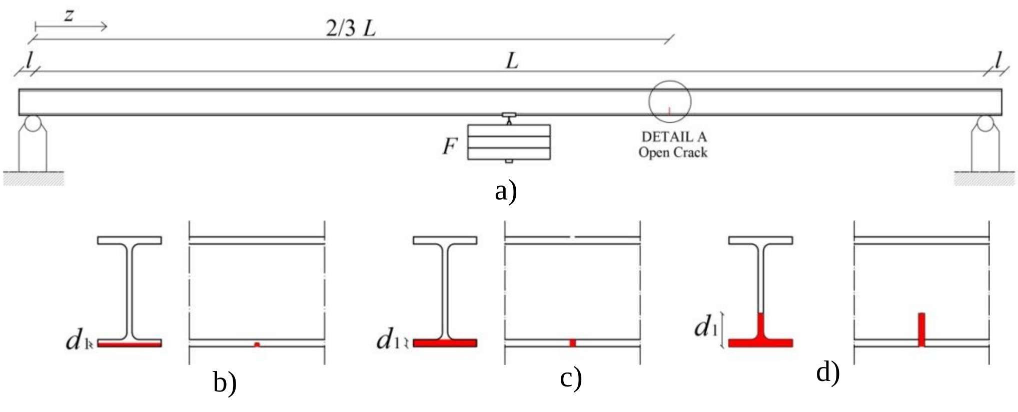


Figure 6. a) Single open cracked beam; b) open crack detail A for d_1=0.5t (0.7t, 0.85t, 0.95t); c) open crack detail A for d_1=t; d) open crack detail A for d_1=0.3 h.

Figure 7a shows the calculated wavelet coefficients (CWT) of the smoothed deflection profile (difference between the displacement responses of the damaged and undamaged beam, subject to the same load) around the crack area of the steel beam with the product of three different wavelets Gaus2 with scales 3, 6, 9. Although small perturbations can be seen at some regions which are owing to noises, an apparent perturbation at z_1 is intransigently un-changeable when the product of wavelets of different scales is calculated. Although the crack depth is around 50% of the flange thickness, the crack position still can be detected successfully by the spatial wavelet transform method experimentally From these experimental studies, it is found that the crack, whose depth is equal or more than 50% of the flange thickness, can be evidently detected by the proposed experiment set and spatial wavelet transform. To further show the effectiveness of the damage detection method using the spatial wavelet transform, the detection of transverse cracks with larger depths d_1=0.7 t, 0.85 t, 0.95 t, 1.0 t, 0.3 h, which is located at the position z_1 from the fixed end of the same beam structure, is conducted. Figures 7b,c,d,e,f illustrates the wavelet coefficients after the transform of the deflection profile around the crack area, as above with the product of the wavelets with scales 3, 6, 9.

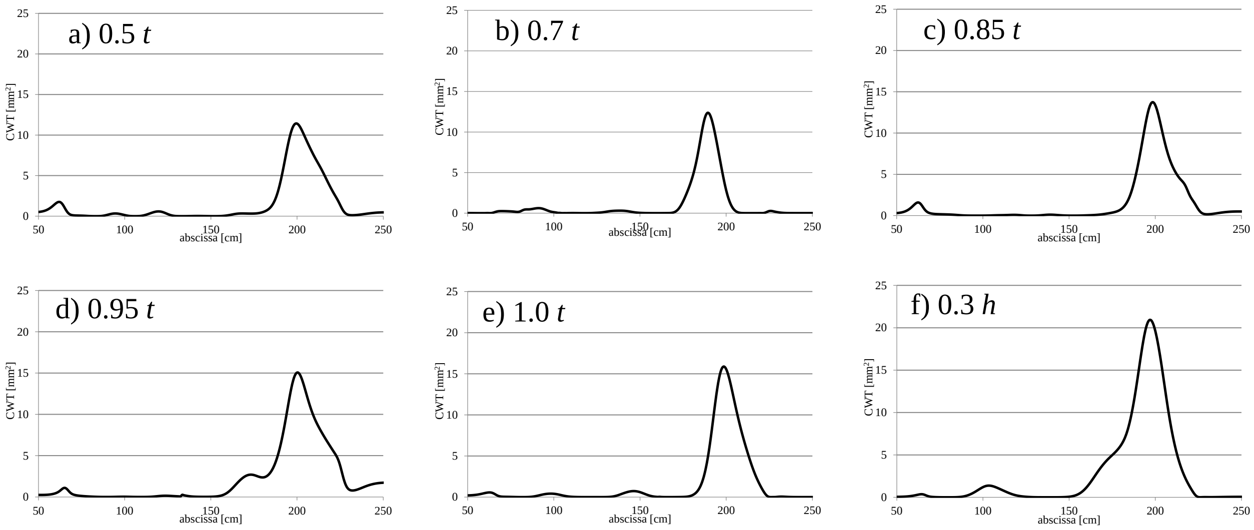


Figure 7. Normalized CWTs for a single open crack.

2.6.2 Damage evaluation

A suggestive curve has been constructed, Fig. 8, to quantify the damage in a simply supported beam by taking envelop of all maximum WCs of the deflection response measured at damage points. The damage severity s is defined as:

$$s = 1 - \frac{I_d}{I} \tag{11}$$

where I_d is the moment of inertia of the damaged cross-section. Applying the analytical model proposed in [24] leads to a slightly good agreement with the results obtained via the experimental tests, Fig. 9.

Polynomial interpolation of the experimental curve of Fig. 8:

$$\mathrm{P}(s) = 20.9275 + \{16.8591 + [\![22.4049 + [-0.00485656 + (0.0649298 + 0.698705 \cdot \langle -0.635793 + s)) \cdot (-0.405768 + s)] \cdot (0.594049 + s)]\!] \cdot (-0.274852 + s)\} \cdot (-0.83809 + s) \tag{12}$$

allows to evaluate the actual crack depth d starting from the known peak value $\overline{\mathrm{P}}$; in fact, the corresponding severity $\bar{s}$ can be calculated by solving the algebraic equation

$$[\ \mathrm{P}(s) = \overline{\mathrm{P}}\ ] \ \rightarrow \ \bar{s} \tag{13}$$

The moment of inertia of the damaged cross-section is given by Eq. (11):

$$I_\mathrm{d} = (1 - \bar{s})I \tag{14}$$

The actual crack depth $\bar{d}$ can be calculated by solving one of the following two equations that relate I_d and depth of the crack, depending on the fact that the flaw affects only the flange:

$$I_{d1}(d) = (w - t_w)\, t^3/3 + t_w\,(h-t)^3/3 + w\,t^3/12 + wt(h - t/2)^2 - w\,d^3/12 - wd(h - d/2)^2 - A_1(d)[y_1(d)]^2 \qquad (15)$$

where:

$$A_1(d) = wt + t_w(h - 2t) + w(t - d) \qquad (16a)$$
$$S_1(d) = (w - t_w)\,t^2/2 + t_w\,(h-t)^2/2 + w(t-d)[(t-d)/2 + (h-t)] \qquad (16b)$$
$$y_1(d) = S_1(d)/A_1(d) \qquad (16c)$$

or penetrates in the web of the cross-section:

$$I_{d2}(d) = (w - t_w)\,t^3/3 + t_w\,(h-d)^3/3 - A_2(d)[y_2(d)]^2 \qquad (17)$$

where:

$$A_2(d) = (w - t_w)t + (h - d)t_w \qquad (18a)$$
$$S_2(d) = (w - t_w)\,t^2/2 + (h-d)^2/2 \qquad (18b)$$
$$y_2(d) = S_2(d)/A_2(d) \qquad (18c)$$

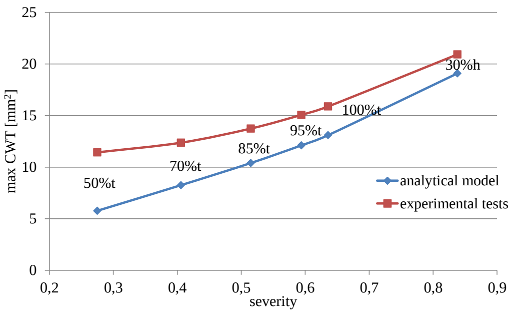


Figure 8. Envelop curves of wavelet peaks.

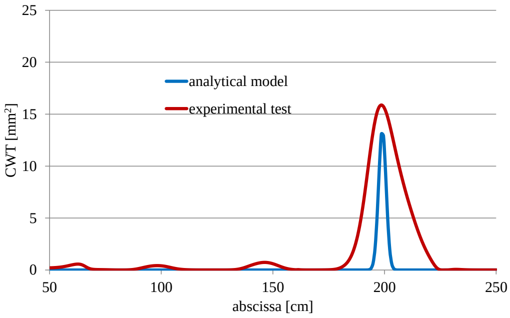


Figure 9. Single open crack, crack position 2/3L $s_1 = t$: comparison between the CWTs obtained by the analytical model and the experimental test.

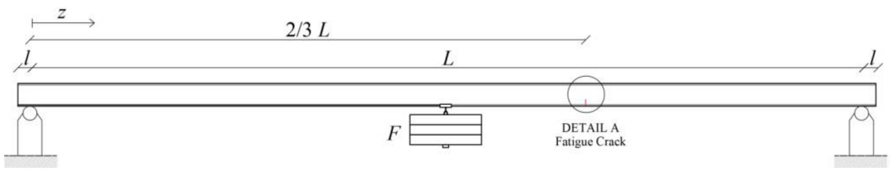
In more detail, the analytical results were obtained by applying the WT (product of the wavelets with scales 3, 6, 9) to the signal given by the difference between the deflection profiles of the damaged (Eq. (2)) and healthy (Eq. (1)) beams respectively:

$$S_o = v - u \qquad (19)$$

2.7 Single fatigue crack (second specimen)

The damage detection of the fatigue crack with the depth $d_1 = 0.3\,h$ located at z_1 is now studied, Fig. 10. The problem can be easily traced back to the scenario already addressed in the preceding Subsect. 2.4.1. In fact, in the load condition which involves the opening of the crack fatigue, it behaves as an open crack (slot), and then is attributable to the situation of

§2.4.1 shown in Fig. 7f. On the other hand, in the loading condition that causes the closing of the fatigue crack, the beam behaves as healthy and thus the WT does not indicate any damage.



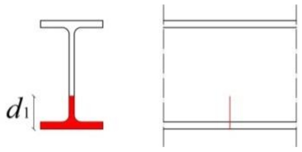


Figure 10. a) Single fatigue cracked beam; b) fatigue crack detail A for d_1=0.3 *h*.

2.8 Double cracked beam, two open cracks

The damage detection of two open cracks located at z_1 and z_2 is depicted in Fig. 11.
The depth of the second crack is d_2=0.3*h*, while for the first crack different depths are considered: d_1=0.4*t*, 0.7*t*, *t*. In analogy to what was done in the Subsect. 2.4.1 and shown in Fig. 7, the normalized CWTs are plotted in Fig. 12.

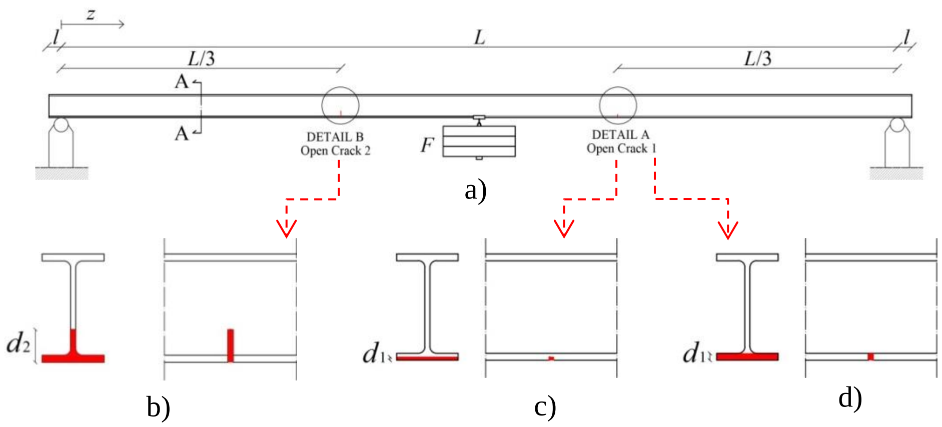


Figure 11. a) Double open cracked beam; b) second open crack detail B for d_2=0.3h; c) first open crack detail A for d_1=0.4 t (0.75t) d) first open crack detail A for d_1=t.

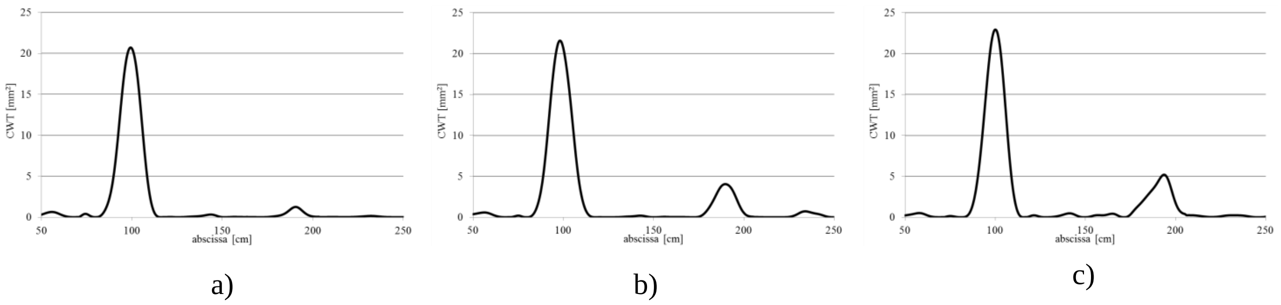


Figure 12. Double cracked beam, two open cracks. a) d_2=0.3h, d_1=0.4t; b) d_2=0.3h, d_1=0.7t; c) d_2=0.3h, d_1=t.

2.9 Double cracked beam, one open crack and one fatigue crack

The damage detection of one open crack and one fatigue crack located at z_1 and z_2 and with depths d_2=0.3 h and d_1=0.25 h is sketched in Fig. 13.

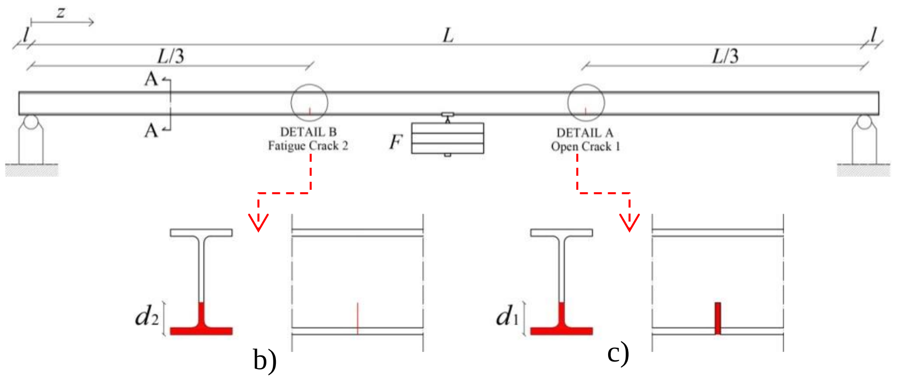
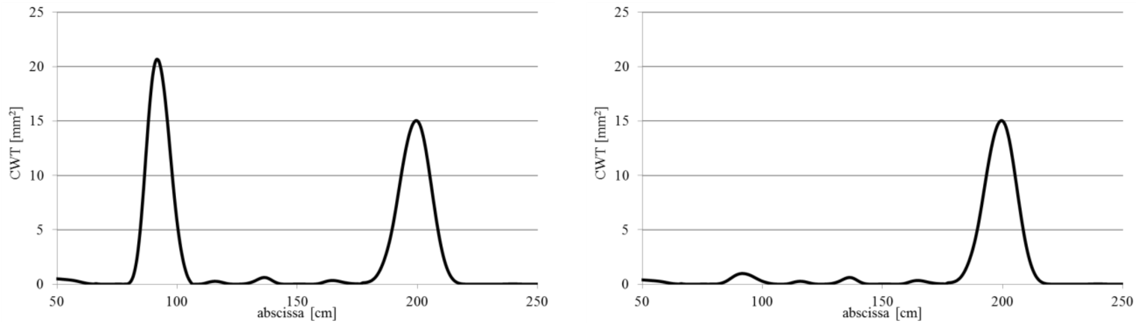


Figure 13. a) Double cracked beam; b) second fatigue crack detail B for d_2=0.3 h; c) first open crack detail A for d_1=0.25 h.

In order to highlight the existence and the depth of both cracks and to distinguish the fatigue from the open crack, we adopt the procedure proposed in [24], where it was defined, in addition to the signal S_o in Eq. (19), also the signal

$$S_f = v^+ - v^- \tag{20}$$

where v^+ indicates the deformed shape of the beam damaged obtained with a load applied in one direction and v indicates the deformed shape obtained with the same load applied in the opposite direction. In the test that is presented here, the load has been directed downward, while the beam is trivially been rotated by an angle equal to π around its longitudinal axis. Also apply in this case the WT, as described in the previous sections, to the signals S_o and S_f. In Fig. 14a the signal S_o indicating the presence of both cracks, in that they open both under the load directed downward. In Fig. 14b, the signal S_f declared the existence of only the fatigue crack, since it behaves as healthy in v^- with the load directed upwards and therefore is not filtered by the signal S_f, unlike the open crack that manifests its influence in both v^+ both v^- and then is eliminated by calculating the difference $v^+ - v^-$.



a) b)

Figure 14. Double cracked beam, one open crack (z_1) and one fatigue crack (z_2). a) signal S_o; b) signal S_f.

3. CONCLUSION

An experimental method for crack identification in beams based on the spatial wavelet transform applied to static deflection of the beam is presented. Unlike other damage identification techniques, wavelet- based methods can be applied not only to structural members but also to full structures. In addition, the damage can be detected using the static response of the structures. This is a very useful feature of the method since it is much easier and more inexpensive to measure the static response compared to the dynamic one. Another fact that makes the wavelet-based methodology easy to implement in practice is that it can be used for structural monitoring at the expected areas of damage only. All these properties make the method a potentially reliable and cost-effective assessment technique that can be applied to the maintenance of the built structures.

The experiments were conducted using an optical method which allowed simultaneous measurements of static deflection lines in a large number of spatially distributed points. The proposed method provides very fast, noncontact, simple measurements that require only a personal computer and a commercially available camera. The accuracy of this method depends on the picture resolution. In experimental studies, a commercial camera is employed to capture the static deflection profile of multi-cracked simply supported beams subjected to a static load at its middle point. Such a static deflection profile, in which the discontinuity of rotation of the beam cross-section at the crack position is too small to be perceived, is conducted by the Gaus2 wavelet transform to discern or amplify the perturbation at the crack position. Denoising process of the original deflection signal given by the camera and padding treatments of the images to reduce edge effects for enhanced damage identification are critical for an effective detection using wavelet analysis. In order to eliminate the spurious peaks and enhance the true ones, the application of the product of three Gaus2 wavelets transforms generated using three suitably selected scales to the difference between the damaged and the healthy deflection profiles has proved particularly effective. In this respect, the detection method is proven to be practically feasible and effective through our experimental studies even for a minor crack with a depth of around 3% of the beam height. The use of the Gaussian wavelet with two vanishing moments provides a clear and precise way of detecting the crack position by maxima lines. The wavelet detection technique makes it possible to detect cracks which require input data easy to be obtained, i.e. only the static response of the structure.

The location of damage is precisely determined by the peak of CWT wavelet coefficient. It is observed that the sign of the peak of CWT coefficients can be used to help identifying the crack location whether on the upper or lower side of the beam. The crack depth and load location affect the wavelet coefficient, where deeper the crack size the wavelet coefficients slightly increase. Hence, the present method can be applied as an indicator to predict the location and quantify damage. Detection and calibration of open and fatigue cracks in beams have been studied in detail experimentally and analytically. A local damage model has been considered to model open and fatigue cracks and the static profile has been simulated for both single and double cracks. A calibration of damage on steel simply supported beams with progressively increasing cracks has been experimentally performed. The generalized curve/envelop of maximum CWTs has been proposed to quantify the damage. The quantity of the damage is evaluated by mapping the maximum CWT onto the generalized curve. The proposed method can easily be applied to beams with other boundary conditions and it can be extended to identify multiple damages in single- and multi-span beams. Hence, the present method can easily be adopted as a structural health monitoring tool for civil and mechanical engineering structures.

AKNOWLEDGEMENTS

The research was funded by the Italian Ministry of University and Research, under the Scientific Research Program of Relevant National Interest: Year 2010-2011, Protocol 2010MBJK5B-005, Title “Dynamics, Stability and Control of Flexible Structures”.

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