16th World Conference on Earthquake Engineering, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 2289

Registration Code: S-V1463034299

GENERATION OF UNIFORM HAZARD FLOOR RESPONSE SPECTRA FOR LINEAR MDOF STRUCTURES

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Abstract

This paper presents a probabilistic seismic demand model for predicting the pseudo-acceleration response of a linear nonstructural component attached to a linear structure. The model relates the response of the component with the pseudo-acceleration response of the generic mode of vibration of the supporting structure. Interaction between component and structure is ignored. Independency of the model on the specific characteristics of seismic hazard at the site is showed. The model is used to develop a method for direct generation of uniform hazard floor response spectra. By using the method floor spectra are determined through a closed-form expression, given the mean annual frequency of interest, the non-structural component damping ratio, the modal properties of the structure, and three uniform hazard spectra representing seismic hazard at the site.

Keywords: nonstructural component; acceleration-sensitive; probabilistic seismic demand model

1. Introduction

Floor response spectra (FRS) may be derived rigorously from floors' acceleration histories, based on structural response-history analysis, or estimated approximately using a predictive equation and a ground response spectrum (GRS). Among the two alternatives, the latter is preferred in common practice, and is widely adopted in seismic codes (e.g., [1, 2]). Its appeal consists in the fact that a response-history analysis of the building is not required, and only a standard response spectrum, rather than a set of ground motion time-series, is needed to model seismic action at the site. In particular, in the case of codes' equations, seismic input is usually represented simply by the peak ground acceleration. The price for this simplicity is the generally poor approximation of the predicted FRS (as shown, e.g., in [3, 4]).

Several researchers have worked on predictive equations, employing a range of approaches, from analytical to numerical, deterministic or probabilistic. Yasui et al. [5] proposed an equation, derived analytically by using the Duhamel integral for the determination of the linear NSC's response supported on a linear structure. Afterwards, the equation was modified by Vukobratović and Fajfar [6] to account for the possible inelastic behavior of the structure. This is explicitly considered also in the empiric equations of Singh et al. [7], Sullivan et all. [8], and Petrone et al. [9]. The common characteristic of these equations is that they can all be considered deterministic models for the nonstructural demand, since they generate FRS by amplifying the structural demand represented by the GRS with a factor which does not account for the record-to-record variability of the amplification.

Probabilistic approaches have evolved in parallel. Many have used random vibration theory to produce probabilistically-characterized FRS (so-called "stochastic" methods; e.g., Singh [10] and Der Kiureghian et al. [11]). More recently efforts have been directed at quantifying the uncertainties in the FRS estimates due to ground motion variability, based on response-history analysis (e.g., [12, 13]). Jiang et al. [14, 15] proposed a probabilistic seismic demand model (PSDM) for the maximum response of the NSC, but only in the case of NSC-structure tuning. In general, the problem inherent with probabilistic approaches is the lack of closed-form expressions for calculating FRS [10, 13], or their limited range of applicability dependent on simplifying assumptions made for the seismic excitation, such as the stationary of the ground motion process [11].

The present work proposes a closed-form PSDM that can be used to predict the (pseudo-) acceleration response of a linear NSC, with any period and damping, attached to a linear structure. The PSDM is derived for light NSCs, whose limited interaction with the structure can be neglected. The model is used to develop a practice-oriented analytical method for direct generation of uniform hazard FRS (UHFRS), namely, of FRS whose ordinates characterized by a given value of the mean annual frequency (MAF) of being exceeded. The method requires seismic input in terms of uniform hazard spectra (UHS) of base motion and can be easily implemented within conventional modal response spectrum analysis.

2. Probabilistic closed-form floor response spectra

Consider a light NSC attached to the fth floor of a MDOF structure excited at the base by a ground acceleration $\ddot{u}_g$. By modeling the NSC as an elastic damped SDOF system, and neglecting dynamic interaction effects between the structure and the NSC, the response of the latter can be obtained by solving the following equation of motion

$$\ddot{u}_{NSC} + 2\xi_{NSC}\omega_{NSC}\dot{u}_{NSC} + \omega_{NSC}^2 u_{NSC} = -\ddot{u}_f^t \tag{1}$$

where $\ddot{u}_f^t$ is the total (absolute) acceleration of the fth floor, u_{NSC} is the relative displacement of the NSC with respect to floor f, $\omega_{NSC}(= 2\pi/T_{NSC})$ and ξ_{NSC} are the circular frequency and the damping ratio of the NSC, respectively. By assuming the behavior of the supporting structure linear, and by applying to its response the modal superposition method, Eq. (1) becomes

$$\ddot{u}_{NSC}^i + 2\xi_{NSC}\omega_{NSC}\dot{u}_{NSC}^i + \omega_{NSC}^2 u_{NSC}^i = -\ddot{q}_i^t \tag{2}$$

$$u_{NSC} = \sum_i \Gamma_i \phi_{i,f} u_{NSC}^i \tag{3}$$

in which Γ_i, $\phi_{i,f}$ and $\ddot{q}_i^t$ denote the participation factor, the shape at floor f (along the direction where the FRS of interest are calculated) and the total acceleration of the ith mode due to $\ddot{u}_g$ (in the considered horizontal direction of the earthquake excitation), respectively. Based on Eq. (2) u_{NSC}^i can be interpreted as the displacement of the NSC attached to a linear SDOF system with the same dynamic properties (i.e., period and damping) of the ith mode of vibration of the structure. In other words, u_{NSC}^i represents the NSC response to the seismic action filtered by the ith mode of vibration of the structure only. Given Eq. (2), the modal contribution to the (pseudo-) acceleration of the NSC can be defined as follows

$$S_{a,NSC}^i = \omega_{NSC}^2 \left| u_{NSC}^i \right|_{max} \tag{4}$$

If the value of $S_{a,NSC}^i$ characterized by a given MAF were known, if it were known the seismic demand hazard in terms of $S_{a,NSC}^i$, the associated ordinate of the UHFRS could be calculated by a simple square root of the sum of the squares (SRSS) combination, or, in alternative, using a complete quadratic combination (CQC) rule [16, 17]

$$S_{a,NSC} = \sqrt{\sum_i \sum_j \rho_{ij} \left(\Gamma_i \phi_{i,f} S_{a,NSC}^i\right) \left(\Gamma_j \phi_{j,f} S_{a,NSC}^j\right)} \tag{5}$$

in a way that matches the evaluation of any other structural demand by means of the modal response spectrum analysis. As it happens, the correlation coefficients ρ_{ij} in Eq. (5) differ from the usual ones used to combine modal contributions to structural responses [16, 17], but they can be calculated using the equations recently proposed by Jiang et al. [14] derived based on random vibration theory[1].

Derivation of a relationship between modal contribution to the floor acceleration spectrum and MAF of exceedance is clearly the missing building block in the procedure to obtain UHFRS based on the conventional modal response spectrum method of analysis used in current practice.

The MAF of $S_{a,NSC}^i$, can be seen as the result of an integral of the probability of exceedance G (complementary cumulative distribution function, CCDF) of $S_{a,NSC}^i$ (the engineering demand parameter EDP) given a level of seismic intensity (the IM), times the absolute value increment of the MAF of exceedance of that IM, over the entire range of its possible values

$$\lambda_{S_{a,NSC}^i}(s_{NSC}) = \int_0^\infty G_{S_{a,NSC}^i}(s_{NSC}|y)\,|d\lambda_{IM}(y)| \tag{6}$$

with $S_{a,NSC}^i = s_{NSC}$ being the demand level in the NSC. Since $\lambda_{IM}(y)$ is the seismic hazard curve (SHC) at the site, which is usually available or it can be easily obtained through a regular probabilistic seismic hazard analysis (PSHA), one needs to develop an EDP-IM relationship, or probabilistic seismic demand model (PSDM), to evaluate the CCDF $G_{S_{a,NSC}^i}$. Under suitable model assumptions for the SHC and the PSDM, the integral in Eq. (6) can be solved in closed form. Several such solutions are available, starting from the initial one by Cornell et al [19], based on linear interpolation in log-space of the hazard, to arrive at the most recent ones based on quadratic interpolation [20]. Using one such closed form, $S_{a,NSC}^i$ can be directly related to the chosen λ value.

[1] Even though the case of a MDOF structure excited by two plan orthogonal components of seismic excitation is outside the scope of this work, it is noted how in such a case Equation (5), similarly to what is done for any other response of interest, shall be applied for each horizontal component of the earthquake, and the obtained spectra shall be combined using an appropriate method (whose selection goes beyond the scope of the present work), such as the percentage rule [18] or the SRSS rule.

Herein, the inverted format of the closed-form expression by Vamvatsikos [20] is adopted in order to calculate $S^i_{a,NSC}$ associated with a given MAF

$$S^i_{a,NSC}(\lambda) = \exp\left[a + \frac{1}{2k_2}\left(-k_1 + \sqrt{\frac{k_1^2}{q} - \frac{4k_2}{q}\ln\frac{\lambda}{k_0\sqrt{q}}}\right)\right]$$
$$q = \frac{1}{1+2k_2\sigma^2} \quad (7)$$

in which k_0, k_1 and k_2 are the coefficients of a quadratic approximation, in the log-space, of the SHC expressed in terms of the spectral (pseudo-) acceleration at the period T_i of the ith mode of vibration the structure, $\lambda_{S_a(T_i)}$, and a and σ are the parameters defining the following PSDM

$$\ln s_{NSC} = a + \ln s + \sigma\varepsilon \quad (8)$$

which relates the seismic intensity level $S_a(T_i)$ = s with the level s_{NSC} of the demand in the NSC. Because ε is a standard normal random variable, the parameter a represents, in the log-space, the median dynamic amplification factor of the NSC's response with respect to that of the structure (i.e., with respect to that of the ith mode of vibration the structure), while σ represents its logarithmic standard deviation (i.e., the record-to-record variability). In general, shape and parameter values of a PSDM depend on the properties of the considered structure, in particular on the extent to which its response enters in the inelastic range and how well the chosen IM captures this nonlinearity, as well as on how much multiple modes contribute to the EDP of interest. Further, parameters may also exhibit a dependence on the site seismicity, represented through the suite of ground motion records selected to support the parameters estimation. Limiting the scope to linear NSCs supported on linear MDOF structures, the parameters a and σ of the PSDM in Eq. (7) depend on the dynamic properties of both the NSC and the mode of vibration of the structure, with negligible dependence on the site seismicity. Their values can be estimated using the following equations [21]

$$a = a^t r^{n_1} \quad r \le 1$$
$$a = a^t + n_2(r^{n_3} - 1) \quad r > 1 \quad (9)$$

$$\sigma = \sigma^t\left[1 - (1 - r)^{n_4}\right] \quad r \le 1$$
$$\sigma = \sigma^t + n_5(r - 1) \quad r > 1 \quad (10)$$

where r = T_{NSC}/T_i; the two coefficients a^t and σ^t represent the "tuning" values (i.e., at r = 1) of a and σ, respectively; the coefficients n_1, n_2, n_3, n_4 and n_5 determine the variation of a and σ for $r \neq 1$. Each of these seven coefficients depends on ξ_{NSC}, and can be calculated through a third order polynomial p of z = $\ln(100\xi_{NSC})$ as follows [21]

$$p = m_0 + m_1 z + m_2 z^2 + m_3 z^3 \quad (11)$$

The values of m_0, m_1, m_2 and m_3 are reported in Table 1 and Table 2 for a^t, n_1, n_2, n_3 and for σ^t, n_4, n_5, respectively. Note that m_0 equals the value of the coefficient at ξ_{NSC} = 1%. Thus, the value of m_0 for the a^t coefficient indicates, for example, that when ξ_{NSC} = 1% and r = 1 the median acceleration response of the NSC is $e^{2.4935} \cong 12$ times that of the ith mode of vibration the structure.

Table 1 – Coefficients of the polynomial given in Eq. (11)
to calculate the parameters of the proposed model for a [21]

a^t			n_1			n_2				n_3			
m_0	m_1	m_2	m_0	m_1	m_2	m_0	m_1	m_2	m_3	m_0	m_1	m_2	m_3
2.4935	-0.3465	-0.0810	2.1504	0.0166	-0.1065	3.2978	0.4937	-0.3367	0.2933	-2.1410	0.7249	0.0819	-0.0369

Table 2 – Coefficients of the polynomial given in Eq. (11)
to calculate the parameters of the proposed model for σ [21]

σ^t		n_4			n_5
m_0	m_1	m_0	m_1	m_2	m_0
0.3338	-0.0737	2.1527	0.1944	-0.1466	0.2000

To conclude, the steps of the method can be summarized as follows.

1. Input seismic action at the base is required in terms of UHS. At least three are needed, for different mean return periods T_R, if the hazard curvature is non negligible. This is the representation of the seismic action usually provided by codes and used by engineers to evaluate structural demands by means of modal response spectrum analysis;
2. Modal analysis of the structure is carried out to yield periods, mode shapes, participation factors and damping ratios for all significant modes;
3. Seismic hazard curve in terms of spectral acceleration S_a for each significant mode can be obtained from the UHFRS, in terms of MAF-IM pairs ($\lambda = 1/T_R$, $S_a(T_i)$). The coefficients $k = (k_0, k_1, k_2)$ of the quadratic approximation are then readily obtained;
4. The "quadratic" hazard curve for each mode, combined with the probabilistic dynamic amplification function, or PSDM, produces the modal contribution to the UHFRS;
5. The UHFRS is finally obtained from a modal combination rule.

It is important to observe that, based on the proposal, UHFRS can be calculated with closed-form expressions (i.e., Eq. (5), (7), (9), (10) and (11)) once a set of 3 (arbitrarily) selected UHS is given, and the modal properties of the structure are known. The additional input parameters which need to be specified are the damping ratio of the NSC, and the MAF of exceeding the FRS. It is also worth noting once again how the method integrates within the usual workflow of structural analysis via the modal response spectrum method, without requiring any response history analysis, being of straightforward implementation within a structural analysis software or even, more simply, in a conventional spreadsheet. Finally, the method is the only one rigorously accounting for the input ground motion uncertainty (including record-to-record variability) within such a simplified analysis framework.

3. Insight into the proposed PSDM

In order to develop the proposed PSDM, a cloud analysis approach was adopted. A suite of ground motion records was filtered through the SDOF system representing the ith mode of vibration of the supporting structure, and responses were then used as input motion to calculate the response of NSCs with varying periods and damping ratios. Mass ratios were kept low, such as to avoid any dynamic interaction issue. The PSDM was estimated via simple linear regression of the obtained $S^i_{a,NSC}$ values on the corresponding values of $S_a(T_i)$. In the remainder of this section, period and damping of the supporting SDOF structure, as well as spectral ordinates $S^i_{a,NSC}$ and $S_a(T_i)$ will be shortened to T_S and ξ_S, $S_{a,NSC}$

In order to explore the variation of the PSDM with period and damping of both the supporting structure and the NSC, the 20x21x2x8=6720 cases were analyzed: $T_S = 0.1s: 0.1s: 2s$, $T_{NSC} = 0: 0.1T_S: 2T_S$, ξ_S = 2%, 5%, and ξ_{NSC} = 1%, 2%, 3%, 5%, 7% 10%, 15% and 20%. According to the definition given in [14, 15], the $T_{NSC} = T_S$ and $\xi_{NSC} = \xi_S$ case will be named hereafter as "tuning case", and the corresponding FRS ordinate denoted with $S^t_{a,NSC}$. This specific NSC-structure system will be used in some of the sections that follow to illustrate results representative of general trends found in all of the considered case studies.

Given the central role played by this PSDM in the proposed method for floor spectra evaluation, the following sub-sections provide details on its derivation and discuss it more in depth. In particular, the ground motions used to support parameter derivation are presented, the functional form is justified, the type of soil and seismic region influence on the PSDM is discussed, and the parameters' variation with periods and damping ratios of both the NSC and the structure is shown. Further details about sufficiency of the chosen conditioning

IM with respect to $S_{a,NSC}$, and about changes in the PSDM when different definitions for S_a are used (i.e., S_a representing the spectral acceleration of the arbitrary ground motion component rather than the geometric mean or the maximum of the two horizontal components), can be found in Lucchini et al. [22].

3.1 Ground motions

Ground motions were selected from the international database by Campbell and Bozorgnia [23]. Exclusion of records showing a recognizable pulse in the velocity trace (identified using the method by Shahi and Baker [24]), and records from earthquakes with moment magnitude Mw smaller than 5 resulted in 715 ground motions records, denoted in the following as "Set 1". For each record, the NSC response was initially obtained using a single arbitrarily selected horizontal component of the ground motion. Regression analyses were then carried out using the entire set of motions (Set 1) or four other subsets: "Set 2", with ground motions of the database recorded in seismic zones worldwide except California (307 records); "Set 3", with ground motions of the database from Californian earthquakes only (408 records); "Set 4", with ground motions from Set 3 and stations characterized by a value of the shear wave velocity V_{S30} lower than 360 m/s (230 records); "Set 5", similar to Set 4 but with ground motions from stations characterized by a V_{S30} value higher than 360 m/s (178 records).

3.2 Functional form

The functional form adopted to predict $S_{a,NSC}$ given S_a is a standard log-log linear model, in which the error term is assumed normally distributed and with an approximately constant standard deviation. This model allows the implementation of the PSDM into the equation of Vamvatsikos [20] for convolving seismic hazard and nonstructural demand. In particular, the two models reported in Fig. 1, fitted using a least squares approach, were investigated: a standard two-parameter model with intercept a and slope b parameters, and a one-parameter model, in which the slope b of the linear regression is fixed equal to 1, and thus only the intercept a has to be estimated.

Regression models

1. $\ln s_{NSC} = a + b \ln s + \sigma\varepsilon$
2. $\ln s_{NSC} = a + \ln s + \sigma\varepsilon$

Comparison
$T = 1s, \xi = 5\%$ case

model	1	2
a	1.65	1.68

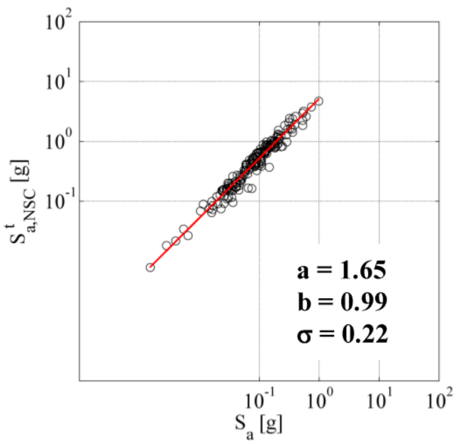

Fig. 1 – Functional form of the regression and analyses results for the case of $S^t_{a,NSC}$ obtained using the set of records 5

As shown by the results reported in Fig. 1 (representative of the results obtained for all of the 6720 considered case studies), the value of the slope was found to be very close to 1. Because of that, the one-parameter model was finally adopted to build the PSDM. In this model, e^a is equal to the estimate of the median value of the $S_{a,NSC}/S_a$ ratio. In other words, as already noted in the previous section of the article, the coefficient a assumes the meaning of the mean (logarithmic) dynamic amplification factor of the NSC's response with respect to that of the structure.

3.3 Dependence on soil conditions and seismic region

Regression results on the subsets of motions previously identified as sets 2 to 5 were used to investigate to what extent the PSDM parameters depend on soil conditions and seismic region. The rationale was that, were the parameters to be found sufficiently independent of these factors, and in particular of the regional seismicity, a unique set of regression coefficients could be supplied for use in any country without the need to perform local fits. The results of these analyses are exemplified through selected plots in Fig. 2. It can be observed that the use of ground motions representative of rock or soil site conditions does not produce a significant change in the result of the regression. This finding agrees with conclusions reached by Bo et al. [15] when investigating the effect of soil characteristics on the $S_a - S_{a,NSC}$ relationship for the NSC-structure tuning case. The same negligible influence is observed if ground motions recorded in a specific seismic region (California, in the considered case) or worldwide are used.

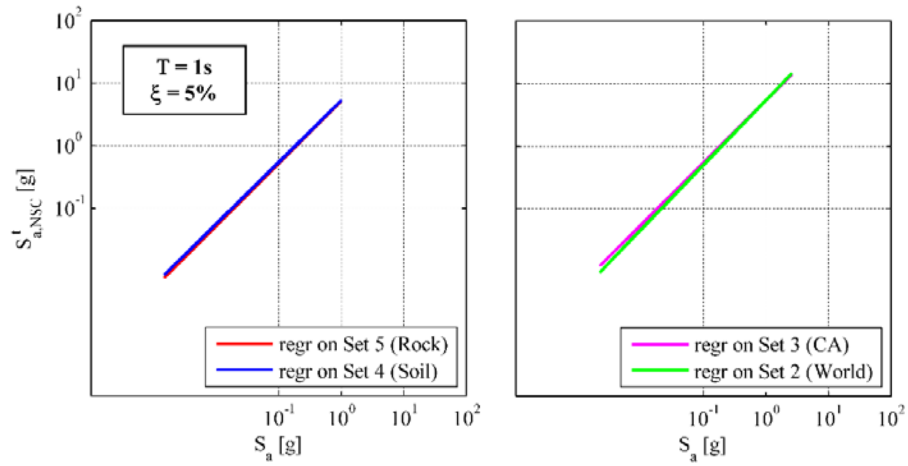

Fig. 2 – Results of regressions on datasets of records representative of different soil conditions and seismic regions (left and right panel, respectively)

To conclude, the proposed PSDM can be considered as site independent for all practical purposes, and can be applied regardless of the specific characteristics of seismic hazard at the site.

3.4 Parameters' variation with periods and damping ratios

The parameters a and σ of the proposed PSDM for $S_{a,NSC}$ as expressed in Eq. (9) and Eq. (10) can be interpreted as follows. They represent, in the log-space, mean and dispersion of a normalized FRS produced by a SDOF supporting structure. Abscissa and ordinate of the spectrum, in fact, are expressed in a normalized form being divided by period and (pseudo-) acceleration response of the structure, respectively. This means that Eq. (9) and Eq. (10) are implicitly based on the assumption that the $S_{a,NSC}/S_a$ ratio, i.e., the dynamic amplification factor of the NSC's response with respect to that of the structure, depends rather than on T_{NSC} and T_S individually, on their ratio r = T_{NSC}/T_S. The goodness of this assumption can be assessed by looking at the plots of Fig. 3, which report the parameters of PSDMs corresponding to different NSC-structure pairs estimated using as exciting ground motions the records from Set 5. The a(r) and σ(r) curves shown in the plots are obtained from the responses of NSCs with different periods of vibration T_{NSC}, attached to structures characterized by the same T_S value. It can be observed that in case the latter changes, the variations of the curves are quite limited. The most significant variations occur, in fact, in cases the amplification of the NSC is small and thus the accurate evaluation of its response is of reduced if not negligible importance. Note that, for example, in the case of T_S = 0.5 s the maximum variation of σ and a occurs near r equal to 0.5 and 2, respectively. The median amplification e^a at these two values of r is about 1.5, while at r = 1.0 it is equal to 11.1. Fig. 3 reports also the a and σ curves calculated with Eq. (9) and Eq. (10). The equations' coefficients are derived from PSDMs built through regression of $S_{a,NSC}/S_a$ values obtained from all the considered T_S cases. In particular, for a^t and σ^t the actual values of the parameters of the PSDM built for the NSCs with $T_{NSC} = T_S$ are adopted. The coefficients n, instead, are calibrated with nonlinear least-squares fitting of the a and σ curves. As it can be noted by observing the right plot of Fig. 3, coefficient n_5 was calibrated by solving the minimization problem in the range 1.5 < r < 2 (and not for r > 1). By using this approach, for r > 1 the value of σ is always overestimated, but the predicted slope of the curve is consistent with the actual value of the observed linear trend.

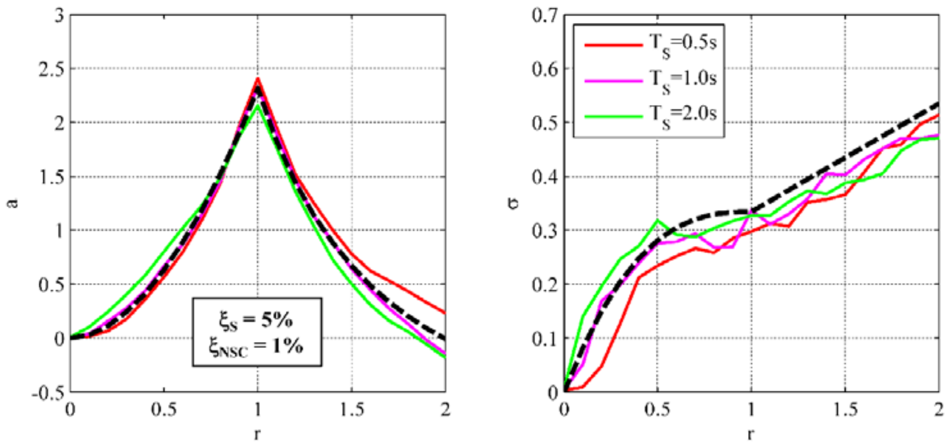

Fig. 3 – PSDM's coefficients expressed as functions of r obtained for different structural periods: values observed and estimated with the proposed Eq. (9)-(10) (denoted with solid and dashed lines, respectively)

Fig. 4 reports the a and σ curves obtained for different damping ratio values of the NSC. In this case, the third order polynomial of $\ln(100\xi_{NSC})$ given in Eq. (11) is used to approximately calculate the coefficients of Eq. (9) and Eq. (10). The values adopted for m_0, m_1, m_2 and m_3 in Eq. (11) are estimated through a least-squares approach as follows: in the case of a^t and σ^t, by fitting the tuning values of a and σ obtained with the different considered ξ_{NSC} ratios; in the case of the coefficients n, instead, by fitting the values of a and σ, observed on both r and ξ_{NSC}, to Eq. (9) and Eq. (10) with a^t and σ^t already calibrated. Comparisons of the results obtained for the two investigated structural damping ratios showed that actually m slightly varies with ξ_S. Because of that, the proposed PSDM was approximately assumed independent on ξ_S, and the values of m reported in Table 1 and Table 2 calibrated using the mean (average) a and σ curves obtained in the two cases of ξ_S = 2% and ξ_S = 5%. Based on these values, it can be also noted that in some cases, e.g. for a^t and σ^t, a polynomial of order lower than three could be actually used to accurately calculate the coefficient. Note that while in the observed a(r) curves the location of the peak depends in general on the actual value of ξ_{NSC}, according to the proposed model the peak always occur at r = 1. The results obtained in the investigated case studies showed, however, that the errors produced by such approximation are negligible.

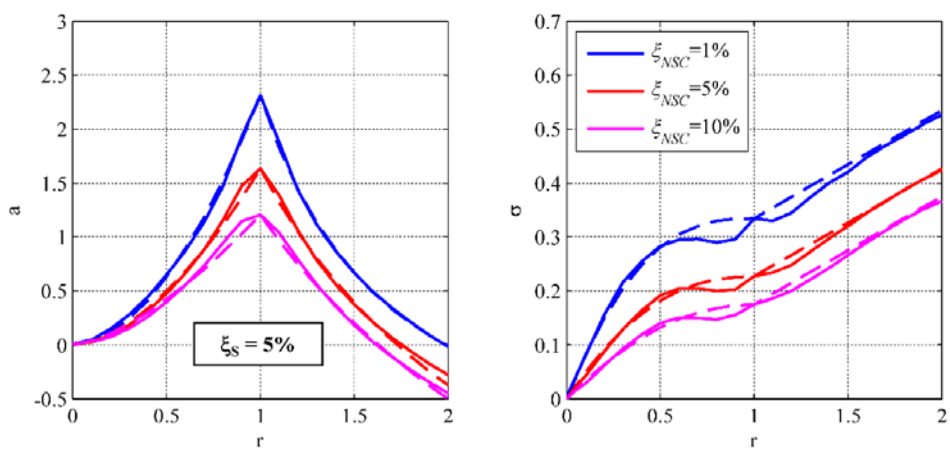

Fig. 4 – PSDM's parameters expressed as functions of the T_{NSC}/T_S ratio r for different NSC's damping ratios: values observed (solid) and estimated (dashed) with Eq. (9)-(10)-(11)

4. Adopted closed-form for the MAF of nonstructural response

As already stated, Eq. (7) that calculates seismic demand for a given value of the MAF is derived by inverting the closed-form of Vamvatsikos [20]. It is easy to demonstrate that Eq. (7) can be used only if the following condition is satisfied

$$\Delta = q\left(k_1^2 - 4k_2 \ln\frac{\lambda}{k_0\sqrt{q}}\right) > 0 \tag{12}$$

In order to obtain Eq. (7), in fact, a quadratic equation with discriminant Δ obtained by transforming the closed-form has to be solved. As underlined in Vamvatsikos [20], the condition (11) is always satisfied "for any limit state of engineering significance". However, when the proposed PSDM for the nonstructural response is used, it can happen in some cases that the condition is not satisfied. This occurs in particular at high values of r, when the values of the parameters a and σ are very low and high, respectively, i.e. when the response of the NSC is significantly deamplified with respect to that of the structure and highly dispersed. Even though such cases are not of engineering interest, they are evaluated when the ordinates of the FRS around the fundamental period of vibration the structure are analyzed and the contribution of the higher modes is calculated. In order to deal with the evaluation of such cases without modifying the expression of Eq. (7) used to calculate $S^i_{a,NSC}$, the following approximation is proposed

$$S^i_{a,NSC}(\lambda, r > 3) \approx S^i_{a,NSC}(\lambda, r = 3) \tag{13}$$

Note that for r > 3, the acceleration level of the NSC is in general very low, being the mean value of the dynamic amplification factor much lower than 1 (e.g., see Fig. 4). Thus, it is reasonable to assume the error produced by such approximation as negligible.

5. Example application

The MDOF structure selected as example case study is a 6-story 3-bay reinforced concrete frame located in Milan, Italy. The modal properties of the structure, i.e., modes' shape and corresponding periods T and participating mass ratios PMR, are reported in Fig. 5. Each mode of vibration is assumed to have a damping ratio equal to 5%.

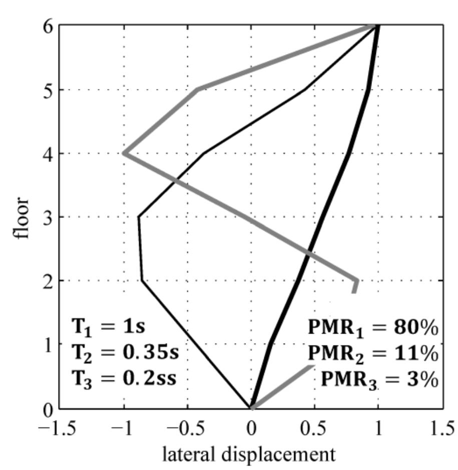

Fig. 5 – MDOF 6-storey structure: modal properties

Seismic hazard at the site is modelled with the SHC reported in Fig. 6, and a suite of hazard-consistent ground motions (e.g., refer to [25] for the definition of hazard consistency). A total of 200 motions (20 records for 10 levels of S_a) have been selected by means of the conditional spectrum method [26], and used in a multiple-stripe analysis [27] to estimate exact PSDMs for the NSCs.

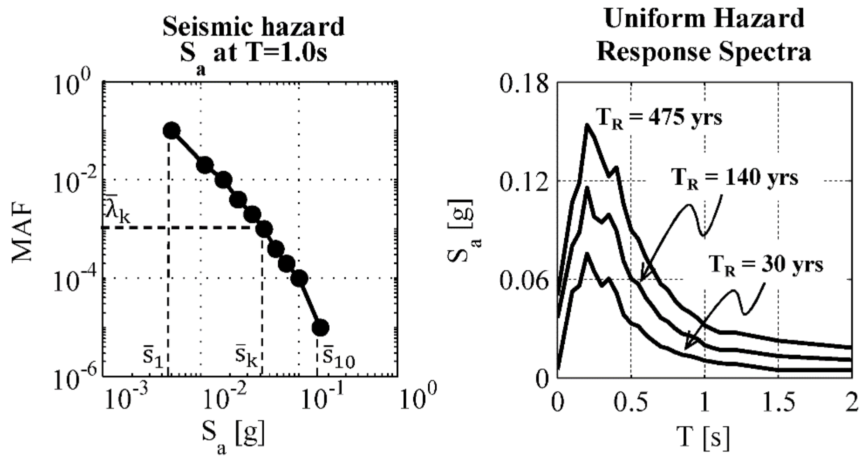

Fig. 6 – MDOF 6-storey structure: SHC with the 10 conditioning intensity levels used to select the records required to run the multiple-stripe analysis (left), and the three computed UHS used in the proposed method (right)

In order for the validation to be correct, by this meaning that differences come only from the method itself and its approximations, results from the proposed method should be computed employing UHS that are consistent with the site seismicity and the ground motion records used in the response-history analyses. Thus, rather than performing PSHA at periods other than the conditioning one T_1 = 1.0 s in order to build the UHS, spectral ordinates at $S_a^* = S_a(T^* \neq T_1)$ have been obtained using the SHC for the conditioning IM, λ_{S_a}, and the conditional distribution of S_a^* given S_a obtained from records, through the expression

$$\lambda_{S_a^*}(s^*) = \int G_{S_a^*|S_a}(s^*|s)\left|d\lambda_{S_a}(s)\right| \cong \sum_{t=1}^{N_S} \widehat{G}_{S_a^*|S_a}(s^*|s_t)\left|\Delta\lambda_{S_a}(s_t)\right| \tag{14}$$

where N_S is the number of seismic intensity levels (i.e., stripes) considered, s_t is the intensity level of the tth stripe, and the CCDF of S_a^* given S_a is approximately obtained from records as

$$\widehat{G}_{S_a^*|S_a}(s^*|s_t) = \frac{1}{N_R}\sum_{l=1}^{N_R} I\left(S_{a,l}^* > s^*|S_a = s_t\right) \tag{15}$$

where N_R is the number of records at the intensity level s_t, $S_{a,l}^*$ is the value of S_a^* obtained from the lth record, and $I(\cdot)$ denotes the indicator function which is equal to 1 for $S_{a,l}^* > s^*$ and 0 otherwise. Fig. 6, on the right, shows the three UHS obtained and used in the proposed method.

Fig. 7 shows the UHFRS for different damping ratios and two different floor levels of the structure, obtained as follows. The exact UHFRS are derived from DHCs, expressed in terms of $S_{a,NSC}$, obtained similarly to the MAF of S_a^*. In the case of the approximate estimates of the UHFRS, the parameters of the PSDMs for the NSCs are determined from Eq. (9)-(10)-(11). Modal contributions to the floor spectra $S_{a,NSC}^i$ are calculated with Eq. (7). For the ith mode of vibration of the structure, the $S_a(T_i)$ hazard curve, used to estimate the values of k_0, k_1 and k_2, is derived from the spectral ordinates at T_i of the set of UHS. Three interpolation points only, corresponding to T_R values equal to 30, 140 and 475 years, respectively, are used to approximate the hazard curves. The simple SRSS rule is finally used to combine the $S_{a,NSC}^i$ values and obtain the UHFRS. The comparisons reported in the figure show the ability of the proposed method to accurately predict UHFRS.

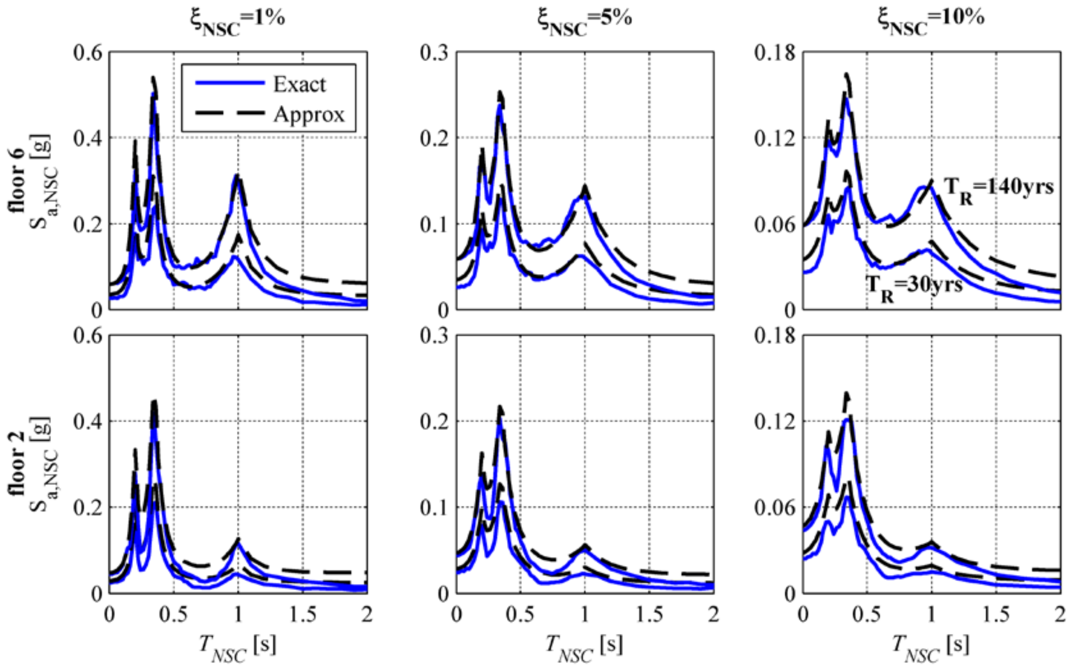

Fig. 7 – MDOF 6-storey structure: exact vs approximated UHFRS

6. Conclusions

A method to generate uniform hazard floor response spectra (UHFRS) for linear MDOF structures was presented. UHFRS are determined through a closed-form expression, given the target mean annual frequency, the nonstructural component damping ratio, the modal properties of the structure and (at least) three uniform-hazard (pseudo-) acceleration response spectra (UHS) at the base of the structure. The method is based on a new proposal of a probabilistic seismic demand model (PSDM) which relates the base spectral acceleration (S_a) with the floor spectral acceleration. Results reported in the paper showed that the PSDM can be considered as site independent. Because of that, the proposed equation to calculate the UHFRS can be applied regardless of the specific characteristics of seismic hazard at the site. Such independency and the use of UHS make the method ready for an easy adoption into international seismic codes.

7. Acknowledgments

Partial funding from the Italian Civil Protection (project DPC-ReLUIS 2014-2016) and Ministry of Education, University and Research (MIUR) are gratefully acknowledged. Opinions, findings, and conclusions or recommendations are the authors' and do not necessarily reflect those of the sponsors.

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