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An integer black-box optimization model for repairable spare parts management

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ABSTRACT

Spare parts management affects significantly costs and service level for supply chains. This paper deals with an inventory management problem for multi-item repairable systems via a systemic perspective based on a new efficient integer black-box optimization model. With respect to the traditionally used marginal allocation that considers items individually, the proposed black-box optimization model is a holistic approach in the fact that it exploits relationships among items. The authors propose a derivative-free algorithm specifically tied to the application which exploits a new selection strategy for choosing entire subsets of items with the aim to get the best expected improvement in the objective function. The approach has been tested on a real case study for optimizing stocks in an airline's inventory network. The case study provides evidence about the good behavior of the exploratory geometry of the proposed approach in finding quickly a feasible and optimal solution for inventory control.

INTRODUCTION

Modern organizations are fully dependent on readily available spare parts to maximize operational capability in case of failures. Managing repairable inventories - which are spares normally characterized by high market value - represents an important managerial domain for improving operational readiness and reducing life-cycle costs for equipment (Yerpude et al., 2019). As a component downtime can be very costly, inventories are required to keep the stock-out time as low as possible. Nevertheless, an excessive number of spare parts may generate not negligible holding costs. Spare parts shall thus be thoroughly optimized to balance high system availability requirements and low cost of allocation (Sleptchenko et al., 2018).

Repairable items are usually managed following a one-for-one replenishment policy, where the part is ordered after each substitution in lots of one. This situation is usually represented as a (S-1, S) policy where S is the optimum number of items in the inventory, and S-1 is the re-order level, i.e. the number of items below which activates the need for a re-ordering. This latter remains meaningful for parts characterized by high inventory cost and low demand, where the economic order quantity tends to a size of one (Diaz and Fu, 1997). In case of failure, the defective part is removed from the equipment and substituted with a functioning one. In the meantime, the original defective part is sent to a maintenance facility to be repaired.

Traditionally, inventory is balanced following an item-approach process, i.e. inventory levels for each item are set independently (Wong et al., 2006), failing to give a holistic optimization (Tripathi and Misra, 2012). The assignment of over-simplified constraints and requirements at the item level becomes increasingly less adequate for the needs of the so-called HA-HCLDS (High Availability, High Cost and Low Demand Systems) (Costantino et al., 2018), which represent the focus of the current manuscript. Starting from the original contribution of Sherbrooke (1968), it is possible to set items' stock levels jointly, adopting a systemic optimization, as for the so-called system-approach. The system-approach allows a holistic perspective on the system, being fed by systemic variables (e.g., total inventory budget, overall system's availability), and supporting the identification of system-wide parameters (e.g., the budget required for an overall service level, the effect of a stock reduction on the overall system service level).

The dominant system-approach model for repairable items is the Multi-Echelon Technique for Recoverable Item Control (METRIC), which relies on Palm's theorem (Sherbrooke, 1968). Since the METRIC aims to respect a systemic perspective, it has to take into account a large number of variables, e.g. for each item at each local warehouse, the demand rate, the on-site repairing time, the turn-around-time, the reparability level. The approach should be also subjected to constraints related to holding costs, and availability requirements. The corresponding algorithmic computational complexity – which is non-linearly increasing with the number of items - forces the analysts to develop and adopt approximated optimization solutions.
Traditionally, METRIC approaches adopt heuristics based on the so-called marginal allocation algorithm, as initially proposed by (Sherbrooke, 1968). The marginal allocation generates acceptable stock level solutions in a limited time interval, counting on the incremental benefits related to the combinatorial placement of an additional item in stock.

After reviewing the literature on optimization algorithms for the METRIC, this paper explores possible enhancements for the marginal allocation heuristics in order to define an alternative optimization algorithm for solving the system-approach model for repairable items. The main contribution of this paper consists thus of advancing the existing literature on operational research for multi-item inventory systems through an enhanced time-effective optimization algorithm. More specifically, the METRIC-based model aims at defining the stock level for a single site that allows minimizing the holding costs whilst satisfying availability constraints.

The discussed problem can be considered a non-linear optimization problem, where functions are not available in closed form but only as the output of a black-box system, which implies expensive evaluation and not available derivatives. Therefore, this manuscript describes an original black-box derivative-free algorithm (Audet & Hare, 2017) for solving such a problem which fully exploits the peculiar aspects of the application. The present document points out that our derivative-free approach allows tackling the non-linearity as is, without any decomposition in subproblem and without any approximation or necessity to check the feasibility of the solution.

The algorithm is inspired by pattern search algorithms (Audet & Dennis, 2003) and it includes specific features both to exploit integrality of the variables and to locally explore promising feasible subregions by using suitable tailor-made rules. In particular, the algorithm considers an enhanced approach for the selection parameters based on the ratio between holding costs of each item at a local warehouse and the absolute value of the availability variation associated with a change in the stock of the same item at the local warehouse. The tailored selection rules allow improving performance in terms of needs of function evaluations which represent the main computational costs in black-box optimization.

The remainder of the paper is organized as follows. Section 2 provides a literature review about METRIC with a focus on optimization approaches. Section 3 clarifies the analytic formulation of METRIC, as applied to spare management of a fleet of aircraft. Section 4 specifies the innovative black-box algorithm proposed for its optimization, and Section 5 details its application as a walk-through application in a simple case study, comparing the results obtained via the proposed approaches with the ones obtained via the traditional marginal allocation and a pattern search algorithm . Lastly, the conclusions summarize the outcomes of the paper and pave the way to future joint multi-disciplinary research combining advanced optimization algorithms and logistics.

LITERATURE REVIEW

Multi-component systems such as aircraft fleets, nuclear power plants, oil refineries, etc., demand for a thorough analysis of system requirements before deciding how many spares should be kept in each warehouse. The analysis should consider multi-variate non-linear relations, which require an analytical formulation hard to made explicitly (Marseguerra et al., 2005).

The METRIC starts from the assumption of a Poisson-distributed demand and of independent and identically distributed repair time, characterized by any distribution with a specified mean. Both these assumptions are representative of HA-HCLDS, and in analytical terms, they allow adopting Palm's theorem. Once respected the above-mentioned assumptions, the theorem states that the steady-state probability distribution for the number of units in repair is still a Poisson distribution, where the mean can be calculated as a simple product of the mean demand (following a Poisson) and the mean repair time (Sherbrooke, 2004). From a logistics perspective, the theorem represents an important milestone that can significantly ease the mathematical formulation for the allocation problem, motivating the dominant role of the METRIC as one of the most used system-approaches, especially considering its reduced degree of mathematical sophistication (Sherbrooke, 2004).

In terms of optimization, the METRIC traditionally adopts a marginal allocation algorithm, whose applications in literature confirm it to be a flexible method suitable for a variety of problems. As mentioned in the introduction, the marginal allocation can be originally found in the work of Sherbrooke (1968), who started promoting its advantage compared to a random trial-and-error stock assignment procedure (Sherbrooke, 1986; 2004). The concept of marginal analysis dates back to Gross (1956), and it has been re-organized by Sherbrooke to find the optimal stock solution which maximizes the backorder reduction-versus-cost increment when marginally adding a spare individually to each item. It remained a widely used approach in several early METRIC applications; see (e.g.) the multi-echelon METRIC model assuming a compound Poisson processes for modeling the demands (Graves, 1985); or the extended MOD-METRIC for multi-indenture systems by Muckstadt (1973). Furthermore, Kline & Bachman (2007) use the marginal allocation for their inventory optimization problem, where the volume of spares is calculated from the functioning time percentage requested for the system to work. Similarly, in the context of performance-based logistics, Nowicki et al. (2008) adopt a traditional marginal analysis for inventory optimization. De Smidt Destombes et al. (2009) use a marginal allocation as well for a joint optimization problem on the frequency of maintenance activities, the ability to repair and the spare level. A similar marginal analysis has been employed by Costantino et al. (2013) to solve a military inventory problem in a multi-item system with non-equal maintainability certification levels, imposing a weighted availability constraint on the number of equipment at each local warehouse. The adoptions of the marginal analysis can be also confirmed in the work of Xu et al. (2015), who in their METRIC approach relax the hypothesis of an infinite supplier's capacity, rather assuming a prioritized maintenance service. The approach has been also used to optimize the stock level in a METRIC-based model relying on the Discrete Weibull distribution (Patriarca et al., 2019). Lastly, Basten & Van Houtum (2014) in their review about spare parts inventory indicate the METRIC as a dominant technique for rotable spare parts management, and present a greedy algorithm, which is largely based on the traditional marginal analysis. Their paper also refers to software in place for the adoption of METRIC, or its variant VARI-METRIC.

More recently, even considering the benefits arising from the adoption of the METRIC, alternative optimization approaches have been discussed and explored in literature, mainly focused on genetic and pattern search algorithms. Kapoor et al. (2016) develop a simulation approach referring to METRIC theory for a two-echelon problem in a public transport fleet counting 9000 buses. Their version of a genetic algorithm provides optimal review periods and the level of spare parts for each site. Patriarca et al. (2016b) propose a Real Coded Genetic Algorithm, the MI-LXPM, to increase the randomness of a Mixed-Integer (MI) solution employing Laplace Crossover (LX) and Power Mutation (PM). Similarly, Patriarca et al. (2016a) adopt a similar genetic algorithm for a more computationally demanding problem, i.e. including also lateral transshipment of items. The advantage of these MI genetic algorithms is the possibility of providing integer values as outcomes of the optimization, which is meaningful outcome for any inventory problem optimization. In general terms, an integer solution obtained by approximating the real numbers to the nearest integer values may be not feasible or produce quite different results in terms of costs. Alternatively, Duran & Perez (2014) develop a hybrid Particle Swarm Optimization algorithm combined with local research to solve a multi-item problem. The explorative nature of this work is acknowledged by the authors themselves, who suggest to further consider different initialization strategies, fitness definitions, and replacement strategies. Another explorative contribution is the one developed by Costantino et al. (2014), who test a Pattern Search algorithm to determine the items to stock, and their quantity in a multi-item multi-indenture problem. The solution is obtained by implementing a Generalized Pattern Search (GPS) (Audet & Dennis, 2003), where however no specific parameters refinement is proposed. Even though not explicitly related to the METRIC, Nickel et al. (2006) address a similar problem by a pattern search algorithm for spare parts allocation. In their work, the target cost function is linear, while the constraints are highly non-linear and considered as black-box functions, i.e. very difficult (and expensive) to evaluate, whose derivatives are not available. The (resource-expensive) solution adopted in this case consists of performing a second pattern search algorithm for each solution point that is found by the initial pattern search at each iteration, to find a good integer solution. Other relevant approaches for inventory optimization can be linked to the work conducted by Topan et al (2017), and Wong et al. (2007), both in multi-echelon systems. Both the researches propose variations to the standard marginal allocation approach, but with a partly different focus: the former aimed at defining the order quantities, the reorder points at the central warehouse, and the base-stock levels at the local warehouse; the latter offers a multi-echelon representation which does not necessarily hold the same results for the single-echelon proposed in this research.

Based on this review, it emerges from the literature the continuous interest in spare parts optimization over the years. Nevertheless, recent research in the area confirms an increasing interest in developing alternative optimization techniques for such complicated problems (Nowicki et al., 2012), aimed at exploring the benefits of pattern search, genetic algorithms, and particle swarm optimization.
These algorithms, however, are not widespread in the literature, partly due to their recent introduction and partly due to the large difference in terms of ease of implementation and required computational efforts.

These pieces of evidence motivate the development for a heuristic for the METRIC to be both effective and efficient if compared (at least) with the marginal analysis. The black-box algorithm presented in this paper aims to provide optimal solutions in a reduced computational time. This original and efficient solution remains significant to allow quick systemic parametric analyses, in order to test multiple managerial options and allocations in a non-invasive approach.

THE INVENTORY MANAGEMENT MODEL

This section presents a black-box optimization model for a single-echelon multi-item inventory system. The authors consider a problem based on the METRIC for the minimization of the holding costs of items at a single site, whilst guaranteeing a required availability level.

The single-echelon multi-item problem and assumptions

The authors refer to the single-echelon multi-item system described in Figure 1, where:

- CD (Central department)
- MD (Maintenance Department)
- LW_j (Local Warehouse j = 1, ..., J, J= total number of LW)
- LRU_i (items, Line Replaceable Unit i =1, …, I, I = total number of LRU)

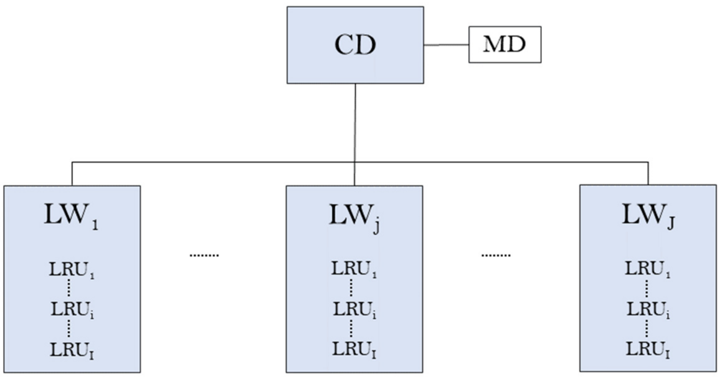

Figure 1. Echelons structure for multi-item system.

In Figure 1, it is important to clarify the authors assume the CD has no stock levels, and it is used only as a maintenance center with infinite capacity, i.e. it always holds an appropriate resources level to execute any needed operation. Furthermore, each LW_j does not have a maintenance center. Hence each LRU_i that requires maintenance at LW_j must be sent to the CD. For these reasons, in analytical terms, the problem can be interpreted as a single-echelon optimization problem.

The proposed formulation does not consider lateral transshipment among LW_j; it further assumes that the back-order queues generated by each LW_j at the CD are unrelated. Under these assumptions the authors can decompose the spare parts management problem of the LRUs on subproblems on each LW_j for all j=1, …, J. The remainder of this section focuses on the definition of the optimization problem for each LW_j for the detection of an optimal stock level sj* such as to minimize an objective cost function $C_j(s)$ subject to nonlinear restriction which enforces the availability of vehicles at each LW_j.

Analytic formulation

This section describes the analytical model at the local warehouse LW_j.
As a first step, the authors define the decision variables which are the stock level $s_{i,j}$ for all LRU_i, i =1, …, I, at the local warehouse LW_j. The goal is the minimization of the holding cost for all LRUs i at the j-th local warehouse LW_j. Denoting $c_{i,j}$ the unit cost for stocking the i-th item in the j-th local warehouse the authors get the overall holding cost for the local warehouse LW_j.

$$C_j = \sum_{i=1}^{I} s_{i,j} \cdot c_{i,j} \qquad (1)$$

At any time, the stock level $s_{i,j}$ of LRU_i at LW_j can be split into the sum of three components:

$$s_{i,j} = DI_{i,j} + OH_{i,j} - BO_{i,j} \qquad (2)$$

where:

- $DI_{i,j}$ is the number of Due In items of LRU_iwaiting for repair by the CD; Under the hypothesis of no queue at CD, they are the items being repaired in the exact moment of the calculation
- $OH_{i,j}$ is the number of On Hand items of LRU_i currently available at LW_j
- $BO_{i,j}$ is the number of Back Orders of LRU_i due to request arrived when the inventory was already out of stock at the LW_j

The feasible values of $s_{i,j}$ are thus constrained by two nonlinear constraints. The first constraint imposes a lower bound $A^{target} > 0$. In addition, the availability at site A_j^* has to verify the target availability constraint:

$$A_j^* \geq A^{target} \qquad (3)$$

The availability at site refers to the availability of the fleet planned for the site. In this case, the authors consider a fleet of machines, so that each site can be modelled as a passive redundancy system constituted by N_j machines, when M_j machines are active, the remaining $N_j - M_j$ are put in cold stand-by ready to substitute the active machine in case of failure.
The total availability A_j^* , is described by the following formula (Costantino et al., 2013):

$$A_j^* = {A_j}^{M_j} \sum_{k=0}^{N_j - M_j} \frac{\left[-M_j \ln{(A_j)}\right]^k}{k!} \qquad (4)$$

The availability of the single machine A_j depends on the availability of the LRUs that affect it, so that a machine is available when all the items that contribute to form it are available. The mathematical relation is here simplistic described as a series:

$$A_j = \prod_{i=1}^{I} A_{i,j} \qquad (5)$$

where $A_{i,j}$ is the availability of the single item LRU_i at site j and it is expressed by:

$$A_{i,j} = 1 - \frac{E[BO_{i,j}]}{N_j} \qquad (6)$$

where $E[BO_{i,j}]$ is the expected value of the Back-Order corresponding to the stock level s_{ij} and the ratio between $E[BO_{i,j}]$ and N_j is the unavailability level.

Since $s_{i,j}$ can only take non-negative integer values, at least one of $BO_{i,j}$ and $OH_{i,j}$ is zero, the Expected Back Order is the positive part of the delta between the Due In $DI_{i,j}$ and the stock level $s_{i,j}$ (Costantino et al., 2018):

$$EBO_{i,j}(s_{i,j}) = E\left[BO_{i,j}\right] = E\left[\left(DI_{i,j} - s_{i,j}\right)^{+}\right] \quad (7)$$

Hence the expected value of backorders BO_{ij} is obtained, following the assumption of Poisson distribution, as:

$$E\left[BO_{i,j}\right] = \sum_{x=s_{i,j}+1}^{\infty} \left(x - s_{i,j}\right) f_{i,j}(x) \quad (8)$$

where $f_{i,j}$ represents the fraction of demand due to item i at site j (Sherbrooke, 2004):

$$f_{i,j} = \frac{m_{i,j}\left(1 - r_{i,j}\right)}{m_{i,0}} \quad (9)$$

In the last relation $m_{i,j}$ is the share of LRU_i sent from LW_j to the CD with probability $1 - r_{i,j}$ with respect to the total demand $m_{i,0}$ of LRUs of all local warehouses. Since it is assumed that the local warehouses have no maintenance center the probability of a repair is $r_{i,j} = 0$.
Since the availability of the single item $A_{i,j}$ is non-negative, from the formula (6), a second non-linear constraint is needed:

$$E[BO_{i,j}] \leq N_j. \quad (10)$$

The whole formulation of the problem has to deal with is the following integer nonlinear constrained minimization for all local warehouse j=1, ..., J:

Minimize $C_j = \sum_{i=1}^{I} s_{i,j} \cdot c_{i,j}$ (1)

Subject to

$$A_j^* \geq A^{target} \quad (3)$$

$$E[BO_{i,j}(s_{i,j})] \leq N_j \quad (10)$$

$$lb \leq s_{i,j} \leq ub \quad (11)$$

$$s_{i,j} \in Z \quad (12)$$

The lower bound and the upper bound (11) limit the values that can be assumed by the inventory level. The lower bound lb is usually set to 0 to avoid those solutions that present one or more negative - unrealistic - values. The upper bound, on the other hand, can model different requirements such as a limitation in the space of storage or a precise restriction about the quantities in stock for one or more items. Since the authors consider indivisible goods, the authors must insert the last constraint (12) ensuring that the stock levels are integer.

The authors note that constraints (3) and (10) link the values of the stock levels $s_{i,j}$, and are tackled explicitly in our algorithm scheme so that feasibility is retained at every trial solution.

In table 1 the authors summarize the main parameters involved in the definition of the mathematical model.

Tab. 1 Variables and parameters of the mathematical model.

I	Number of items (LRU)
J	Number of Local Warehouse (LW)
i	Item index / LRU index $i = 1{:}I$
j	Warehouse index of the LW $j = 1{:}J$
$m_{i,0}$	Annual average demand of LRU i at CD
$m_{i,j}$	Annual average demand of LRU i at LW_j
$s_{i,j}$	Stock level of LRU i at LW_j
s_j	$[s_{1,j}, s_{2,j}, \ldots., s_{I,j}]$ vector of stock levels of LRU i
$r_{i,j}$	Repair rate of LRU i at LW_j
N_j	Number of vehicles at LW_j
M_j	Number of active vehicles at LW_j
c_{ij}	Unit inventory holding cost of LRU i at LW_j
A_j^*	Fleet availability at LW_j
A_j	Aircraft availability at site j
$A_{i,j}$	Availability of LRU_i at site j
A^{target}	Target value of availability for each LW
$BO_{i,j}$	Back Order level for LRU I at LW j
lb, ub	Lower and Upper bounds on stock level of LRU i at LW_j
Z	The set of integers

T_{max}	Number of items which may be considered all together during the optimization. It may limit the computational effort that is machine-dependent.

Optimization using the Marginal Analysis

At this step, it is possible to solve the above-described problem via marginal analysis, which represents the most common approach for METRIC-like problems. The following 7 steps summarize the main phases of a marginal analysis approach applied to inventory optimization:

1. Set $k = 0$ and choose the starting point (zero - vector in our case). k is an index used for the iteration.
2. Calculate the expected backorder
$$E[BO_{i,j}]$$
3. Calculate the rating vector, where $\rho_{i,j}$ is a variable created to prioritize the solution (as for step 4)
$$R_j = \left\{\rho_{i,j} = \frac{E[BO_{i,j}]}{c_{i,j}} : i = 1,..I\right\}$$
4. Select the index $\hat{\imath}$ which returns the $\max_{i \in R_j} \rho_{i,j}$
5. Calculate s_j^{k+1} as $s_{ij}^{k+1} = \begin{cases} s_{ij}^k + 1 & : for\ i = \hat{\imath} \\ s_{ij}^k & : otherwise \end{cases}$
6. Evaluate the availability A_j^* in S_j^{k+1}
7. Stopping criterion
   If $A_j^* < A_j^{target}$ Set $k = k + 1$ and repeat from 2.
   If $A_j^* \geq A_j^{target}$ then evaluate the objective function and ends.

The marginal analysis provides an incremental solution that does not consider complex re-combinations of different items: once an item is assigned, it cannot be removed from the optimal solution. Nevertheless, the solution provided by the marginal analysis can be used as a good starting point for more accurate and complex optimization algorithms, i.e. to define a feasible point. In this regard, one can note that the service level constraint (3) and the upper bound on the back order expected value (10) are highly non-linear constraints involving all the variable $s_{1,j}$ which represent the hard constraints in the spare management problem.

THE DETERMINISTIC BLACK BOX INTEGER FEASIBLE OPTIMIZATION

In this section, the authors present the Derivative Free Optimization (DFO) method, which is an innovative deterministic feasible algorithm to deal with the integer black-box constrained problem represented by the inventory problem at hand. It is well known that the complexity of finding an optimal solution of black-box optimization problems increases exponentially with the number of variables (Vavasis, 1995). Indeed, an exhaustive search is not conceivable because of the time-consuming evaluation of the objective function and of the constraints due to the simulations involved. Examples of recent efficient algorithms for black-box optimization are the NonMonotone Black-Box Optimization

Algorithm (NM-BBOA) developed by Liuzzi et al. (2018), or the Genetic Algortihm integrated with a Bounding Restart technique and derivate free Local Searchers (GABRLS) (Romito, 2017).

The algorithm proposed in this paper is inspired by the pattern search framework and it includes specific features both to exploit integrality of the variables and to locally explore promising areas of the feasible region by using tailor-made rules. A generic DFO pattern search algorithm starts from a first feasible solution and at each iteration produces points that lie on a rational lattice. Elementary directions are combined and scaled with a step length parameter on a finer and finer grid to meet, where possible, convergence requirements. Elementary displacements are movement along one direction with unit step-length. The set of used directions must constitute a positive spanning set (Regis, 2016) such as in any positive basis method (Coope & Price, 2002). In our algorithm, the set of chosen directions coincides with the Cartesian axes $\pm e_i$, where e_i represents the i-th column of the identity matrix I.

Key elements that influence the performance of a DFO algorithm are the rules for selecting the i-th elementary displacement and the number of trial points along the selected direction.
The proposed contribution stays in the definition of tailor-made rules for selecting the mesh of trial points that exploit the relationship among constraints and objective function. The tailored selection rules allow improving performance in terms of needs of function evaluations which represent the main computational costs in black-box optimization.

DeB2IFO (Deterministic Black Box Integer Feasible Optimization)

As previously discussed, the main innovative aspect of the proposed algorithm is to explore the feasible integer mesh as best as possible using tailor-made rules. Indeed, the authors aim to avoid expensive function computations at points where the expected value of the objective function is locally worse than the current best solution. Thus, the authors introduce a parametric selection strategy for choosing subsets of items with the best expected improvement in the objective function. The selection strategy of an item i is based on the trade-off between its holding cost and the percentage availability variation due to a unit variation of the stock s_i. The idea of using similar ratios has been also explored in (Wong et al., 2007) within a greedy procedure to find a feasible solution.

Formally the authors introduce the selection indicator (13) for each item i-th and the local warehouse LWj

$$k_{i,j} = \frac{c_{i,j}}{\Delta A_{i,j}} \tag{13}$$

which is the ratio between the storing cost of the item i at LW_j and the value of the change in availability $\Delta A_{i,j}$ due to a change of the stock $\Delta s_{i,j}$ of the item i at the same LW_j. The authors discuss some issues in order to explain the role played by $k_{i,j}$ in the construction of the new trial point.

Two scenarios may occur following a unit variation of stock $\Delta s_{i,j}$:

- when $\Delta s_{i,j} = +1$, namely the stock increases, the availability $A_{i,j}$ increases too so that the difference $\Delta A_{i,j}$ is positive and the cost of storage increases by $c_{i,j}$ (the unit storage price of the item i-th in LW_j)
- when $\Delta s_{i,j} = -1$, namely the stock decreases, the availability of the item $A_{i,j}$ is reduced, so the difference $\Delta A_{i,j}$ is negative and the cost of storage is reduced by $c_{i,j}$

Hence for each LW_j the authors construct two vectors $k_j^+ = [k_{i,j}]_{i:k_{i,j}>0}$, $k_j^- = [k_{i,j}]_{i:k_{i,j}<0}$

corresponding respectively to positive or negative variations on all the components $\Delta s_{i,j}$, with components sorted in ascending order, namely such that

$$(k_j^+)_h < (k_j^+)_{h+1} \text{ and } (k_j^-)_h < (k_j^-)_{h+1} \tag{14}$$

The authors can heuristically select combinations of items that seem to be more promising to provide a decrease in the cost in order to obtain an improvement of the objective function. The authors select the items corresponding to the first T_{max} components of k_j^+ and k_j^-, which are linked respectively to a positive and negative unit variation of availability, and therefore to an elementary displacement on the integer mesh. Our algorithm moves along a grid defined by selecting a bunch of items of cardinality $2\ T_{max}$. The authors denoted with $\hat{k}_j$ the vector made up of the $2\ T_{max}$ selected items

$$\hat{k}_j = [k_j^+, k_j^-] \tag{15}$$

Note that, at first glance, using items i-th corresponding to the values in k_j^+may appear to be useless. In fact, if the stock increases, the cost function increases too, thus deteriorating the objective function. However, it is essential to consider the positive variations due to the presence of the service level constraint. Merely decreasing the stock could lead to a reduction in availability, with a possible violation of the corresponding constraint. What is of interest now are $A_{i,j}^+$ and $A_{i,j}^-$ which represent respectively the increase and reduction in availability due to a change in stock of items in k_j^+, k_j^-. These availability factors are combined in order to maintain feasibility. The core idea of this approach consists of choosing items corresponding either to low reduction in availability at a high cost or high increase in availability at a low cost, thus reducing the total cost whilst satisfying the service level constraint.

The parameter T_{max} plays a crucial role in the computational effort needed by the algorithm. The authors explore the neighbourhood of feasible points combining in all possible feasible ways the changes in the stock of at most $2T_{max}$ items corresponding to different components of vectors k_j^+, k_j^-. It makes no sense to consider combinations related to the same item which will lead either to a null displacement or a double displacement which is not allowed in the algorithm. The number of combinations is exponential in T_{max}. In principle the authors use an iterative incremental strategy to select the value T_{max} starting from a minimum number T_{min} of two items, in order to limit the computational effort at the first iterations when the authors are far from the optimal solution. All search points are therefore determined for the current iteration as the most promising subset of the mesh points in a neighborhood of s_j^0 with radius T_{min}.
Now the authors proceed to the last step, the Direct Search. Consider a starting point s_j for the current iteration with value of the objective function C_j. This value of the objective function is the target for the considered iteration. The algorithm plans to move to the search points earlier identified. For each of them, indicated by s_j^{trial} , the value of the objective function $C_j(s_j^{trial})$ is calculated.

Only if

$$C_j(s_j^{trial}) \leq C_j \tag{16}$$

the correspondence of the point s_j^{trial} to the bounds and to the constraints is verified, while if

$$C_j(s_j^{trial}) > C_j \tag{17}$$

the point is discarded.

In the case s_j^{trial} satisfies both the constraints and the bounds, the value of the objective function $C_j(s_j^{trial})$ is stored in a matrix, called Cost Matrix, while s_j^{trial} is put in a second matrix, called Stock Matrix.

Once this procedure has been repeated for all the selected displacements, the stock level is chosen from the Stock Matrix related to the respective lowest value in the Cost Matrix. This level of inventory is imposed as the starting point of the next iteration and the corresponding value of the objective function becomes the new target. The number of elements taken for the combinations must be increased in those cases where it would not be possible to determine a point better than the starting point, i.e. one of the following situations does not hold:

- value of the objective function $C_j(s_j^{trial})$ lower or equal than the target value C_j^0 of the current iteration
- s_j^{trial} satisfies the bounds
- s_j^{trial} satisfies the constraints

The algorithm stops when the number of elements taken to construct the combinations is equal to the number of items T_{max} previously identified as reasonable choice for computational resource limitation. The authors point out that by setting T_{max} equal the number of items, the algorithm is able to perform an exhaustive search in the feasible domain.
The evaluation of the cost function is the first step because the cost function is much less onerous, in terms of computational time, than the constraint functions. This allows to avoid the calculation of the latter for those points which, in any case, will not be chosen at the end of the iteration, since they present a value of the cost function greater than the target one. The following section shows formally the algorithm steps for each LW$_j$.

Optimization using the DeB2IFO

In agreement with the described black-box approaches, the following steps detail the algorithm as applied to the inventory optimization problem at hand. This section presents the approach intended to enhance the traditional optimization based on the marginal analysis.

1. Set $l = 0$ as an index for the iteration of the algorithm, $T^l = T_{min}$ for the number of items to be optimized jointly, and find the starting point s_j^l , through marginal analysis and evaluate its availability

$$s_j^l = \{ s_{i,j}^l \ : \ i = 1, \dots, n\}$$
$$A_j^l = \{A_{i,j}(s_{i,j}^l) \ : \ i = 1, \dots, n\}$$

2. Perturb s_j^l and find

$$s_j^{l+} = s_j^l + 1$$
$$s_j^{l-} = s_j^l - 1$$

3. Calculate the corresponding availability A_j^{l+} and A_j^{l-}

$$A_j^{l+} = \{A_{i,j}(s_{i,j}^{l+}) \ : \ i = 1, \dots, n\}$$
$$A_j^{l-} = \{A_{i,j}(s_{i,j}^{l-}) \ : \ i = 1, \dots, n\}$$

4. Calculate the delta availability w.r.t. the base scenario ΔA_j^{l+} and ΔA_j^{l-}

$$\Delta A_j^{l+} = A_j^{l+} - A_j^l$$
$$\Delta A_j^{l-} = A_j^{l-} - A_j^l$$

5. Calculate the ratio to rank the solutions k_j^{l+} and k_j^{l-} for all the items

$$k_j^{l+} = \{k_{i,j}^{l+} \mid k_{i,j}^{l+} = c_{i,j}/\Delta A_{i,j}^{l+} \quad per\ i = 1, \dots, n\} \ .$$
$$k_j^{l-} = \{k_{i,j}^{l-} \mid k_{i,j}^{l-} = c_{i,j}/\Delta A_{i,j}^{l-} \quad per\ i = 1, \dots, n\}$$

6. Sort k_j^{l+} and k_j^{l-} in ascending order to facilitate the ranking

7. Create vector $\hat{k}_j^l$ by selecting a subset of items, i.e. the first T^l values of k_j^{l+} and k_j^{l-}

$$\hat{k}_j^l = \left[\left(k_j^{l+}\right)_1, \dots, \left(k_j^{l+}\right)_h, \dots, \left(k_j^{l+}\right)_{T^l}, \left(k_j^{l-}\right)_1, \dots, \left(k_j^{l-}\right)_h, \dots, \left(k_j^{l-}\right)_{T^l}\right]$$

8. Create all the possible combinations of values of $\left(A_j^{l+}\right)_h$ and $\left(A_j^{l-}\right)_h$ corresponding to the components of $\hat{k}_j^l$ to be used for explore the feasible space

9. Eliminate combinations involving two or more displacements along the same direction.

10. Evaluate the expected availability for each remaining combination $p = 1, \dots, P$

$$A_j^{l,p} = \left\{ \begin{array}{ll} A_{i,j}^{l,p} \mid A_{i,j}^{l,p} = A_{i,j}\left(s_{i,j}^{l+}\right) \ : \ i \in I, i \text{ linked to } h \ \rightarrow \left(k_j^{l+}\right)_h \in \hat{k}_j^l & \\ A_{i,j}^{l,p} \mid A_{i,j}^{l,p} = A_{i,j}\left(s_{i,j}^{l-}\right) \ : \ i \in I, i \text{ linked to } h \ \rightarrow \left(k_j^{l-}\right)_h \in \hat{k}_j^l \ , & i = 1, \dots, n \\ A_{i,j}^{l,p} \mid A_{i,j}^{l,p} = A_{i,j}\left(s_{i,j}^{l}\right) \ : \ i \in I, i \text{ not linked to any } h \text{ in } \hat{k}_j^l & \end{array} \right\}$$

11. Eliminate configurations whose expected availability $A_j^{l,p}$ is less than the target value A^{target}
12. Construct complex trial displacements $s_j^{l,p}$, $p = 1, \dots, P$
13. Evaluate and compare trial mesh points $s_j^{l,p}$ w.r.t. the current best solution $\hat{s}_j^l$

    If $\left(C_j(s_j^{l,p}) \leq C_j(\hat{s}_j^l)\right)$ and $\left(s_j^{l,p} \text{ is feasible}\right)$

$$\hat{s}_j^l = s_j^{l,p}$$

    end
14. Stopping criterion

    If $T^l < T_{max}$

        Set $l = l + 1$, $T^l = T^l + 1$

        Repeats from 2

    else

        Return an optimal (local) solution $\hat{s}_j^* = \hat{s}_j^l$

    end

CASE STUDY

In this section, the authors illustrate an application of the DeB2IFO for a single-site case study for a fleet of aircraft.

Scenario description

The system under investigation is a multi-item system constituted by one CD and 3 LWs. This problem becomes a single-echelon scenario where each LW is responsible for a different fleet of aircraft. The case study is intended to test the proposed optimization process for a subset of items (23 items) which constitute the main LRU for an aircraft flight system. The items considered in this explorative research are linked to an aircraft hydraulic plant.
Each site has the possibility to activate cold stand-by aircraft. This situation is representative of several civil aviation real operating contexts, in which the stand-by aircraft can be used to guarantee fleet operability and deal with unexpected failures generating the so-called AoG (aircraft on ground), i.e. a problem serious enough to prevent an aircraft from flying.
For each LW, the efficient number of aircraft must satisfy an availability target equal to 0,96. In operating contexts, the availability level is usually set by the decision-maker based on the company scheduled service level and it depends on the market competitiveness, as well as customers' expectations.
The sites are characterized respectively N_1 =97, M_1= 96; N_2 = 23, M_2 =22; N_3 = 202; M_3 = 201, while the holding cost are distributed between 104 €/piece and 5705 €/piece, and an average value of about 1300 €/item.

Note that the proposed scenario remains representative also of an MRO company (Maintenance, Repair, Overhaul) network. In this case, the company remains responsible for a subset of maintenance interventions for an airline, or a pool of airlines, and has to manage the spare parts to be located in 3 LWs intended for maintenance operations.
The aim of the computation experiment presented in this case study is twofold without the ambition of being exhaustive. First, the authors wanted to compare the DeB2IFO with the traditional marginal analysis; secondly, the authors chose one of the most efficient and effective metaheuristics, the Pattern Search, to assess the quality of the tailored grid search around feasible solutions.

Results

The authors start from the results of marginal analysis and then compare the classical direct search alternatives and the proposed approach on 23 demand scenarios. The authors focus on Pattern Search algorithms as a common alternative to marginal analysis to show the gain obtained through the proposed approach. The software environment the authors have used is MATLAB (https://it.mathworks.com/help/gads/patternsearch.html).
Since the Pattern Search algorithms integrated into the MATLAB optimization toolbox allow several parametrizations, the authors have made a preliminary analysis to select the most robust tuning. The fundamental three parameters the authors have tuned concern the search directions:

- Poll method (Generalized Pattern Search, Generating Set Search, Mesh Adaptive Direct Search (Audet & Dennis, 2006) combined to positive basis 2N).
- Polling order (random, success, consecutive)
- Complete search (on/off)

To ensure an optimal setting of parameters, twelve different configurations have been considered setting an overall computational time (limited to 1h) and the total cost on the 23 demand scenarios. The best configuration corresponds to MADS algorithm combined to a positive basis 2N as reported in table 2. This configuration is used in the following comparisons.

Table 2 Selected Configuration.

Poll Method	Polling Order	Complete Search
MADS 2N	Random	Off

Now it is possible to compare the results of the proposed algorithm with the ones of the marginal analysis and the Pattern Search indicated in Table 2. The authors adopt the default stopping criterion for the Pattern Search (MATLAB options) while setting $T_{min} = T_{max} = 4$ (i.e. the maximum number of items to be considered jointly during the optimization) in the DeB2IFO algorithm.
Fig. 2 shows the total amount of the costs for the 23 demand scenarios per each LW, while Table 3 summarize the total costs on all LW and the saving w.r.t. the best solution found. It is interesting to observe how the DeB2IFO ensures a cost reduction of approximately 3.6% w.r.t. MADS 2N, or about 6.6% w.r.t. a traditional marginal analysis.
A comparative table with detailed results on the 23 demand scenarios is provided in the appendix.

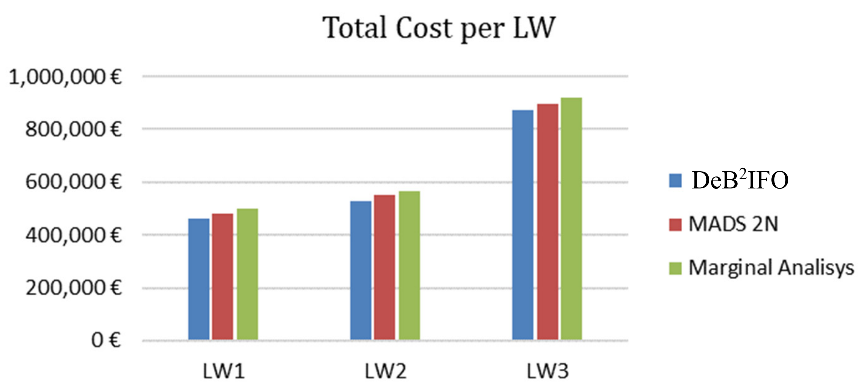

Figure 2. Total costs for each LW (23 items)

Table 3 Total costs and cost effort w.r.t. DeB2IFO

	DeB2IFO	**MADS 2N**	**Marginal Analysis**
Total Cost	€ 1863360,70	€ 1929780,69	€ 1985996,33
Δ cost w.r.t. min cost	-	€ 66540,21	€ 122755,85
% cost w.r.t. min cost	-	% 3,57	% 6,59
Total time (sec.)	3427	1832	281
% time w.r.t. min time	11,20	5,52	-

All these results are highly satisfactory and confirm the goodness of the optimization procedure and the effectiveness of the proposed approach.

CONCLUSIONS

A novel strategy to reduce the costs associated with spare parts management has been presented. The proposed approach started by modeling the spare parts management problem as an integer constrained minimization of a linear objective function. Then the focus of the manuscript was devoted to the introduction of the DeB2IFO algorithm, a deterministic interior method that takes into account the relationships between different items and a robust selection strategy for subsets of items with the most promising impact in reducing the total cost function.

The advantages of DeB2IFO are the ability to handle integer solutions always satisfying feasibility and the tailor-made criterion of choosing only subsets of promising directions that avoid costly evaluations of the objective function and constraints. The results of comparison with the traditional marginal analysis and another common black-box algorithm were satisfactory both in terms of quality of the solutions and computational time. Nevertheless, the focus of this paper was mainly devoted to the definition of the analytical formulation of the algorithm itself and as such, there are several possibilities for further research. Firstly, the algorithm might be tested in more complex logistic networks, with other logistic solutions (e.g.) multi-echelon, lateral transshipment, cannibalization. It could be also relevant to expand the proposed solution enhancing the objective function via other ordering costs related to transportation or administrative aspects. The proposed algorithm could be also used for optimization related to additional METRIC-like solutions referred to the management of performance-based contract, such as the PBC-METRIC (Patriarca et. al., 2016b). Moreover, with respect to multi-echelon scenarios, it will be relevant to compare, and possibly integrate the results of this research with other algorithms developed as a variation of the greedy algorithm, or Dantzig-Wolfe decomposition and Lagrangian heuristics (Wong et al., 2007; Topan et al., 2017). There might be also possible to combine route optimization algorithms (Linqiao et al., 2016), or to test the algorithm in multi-indenture systems: systems whose LRUs are made up of multiple SRUs (Shop Replaceable Units), i.e. items at a lower level of the bill of material. In this case, it is expected to further increase the effects of reduced computational efforts for the proposed solution. Lastly, as for the general formulation, the DeB2IFO remains conceptually suitable for a wide range of spare parts optimization problems, where items are subject to 1 by 1 replenishment policy (S-1, S) to guarantee high service levels.

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Appendix

The following table provides the details of item's holding costs at each LW.

Demand	LW	DeB^{2}IFO	MADS 2N	Marginal Analysis
1	LW1	€ 23566,89	€ 24634,95	€ 28714,21
	LW2	€ 22251,05	€ 22381,09	€ 22881,35
	LW3	€ 40547,17	€ 41303,13	€ 42436,93
	Total	€ 86365,11	€ 88319,17	€ 94032,48
2	LW1	€ 18478,07	€ 19250,1	€ 20324,77
	LW2	€ 19648,67	€ 21950,27	€ 22216,94
	LW3	€ 40354,59	€ 41201,48	€ 44397,73

	Total	€ 78481,33	€ 82401,86	€ 86939,44
3	LW1	€ 23566,89	€ 27804,65	€ 28714,21
	LW2	€ 22251,05	€ 22323,77	€ 22881,35
	LW3	€ 40547,17	€ 41303,13	€ 42436,93
	Total	€ 86365,11	€ 91431,55	€ 94032,48
4	LW1	€ 18478,07	€ 18668,77	€ 20324,77
	LW2	€ 19648,67	€ 21586,64	€ 22216,94
	LW3	€ 40354,59	€ 42238,63	€ 44397,73
	Total	€ 78481,33	€ 82494,03	€ 86939,44
5	LW1	€ 19533,26	€ 19786,78	€ 19786,78
	LW2	€ 25537,68	€ 28269,57	€ 29388,45
	LW3	€ 35999,3	€ 36175,46	€ 36175,46
	Total	€ 81070,25	€ 84231,81	€ 85350,69
6	LW1	€ 19888,64	€ 19993,07	€ 19993,07
	LW2	€ 23223,02	€ 25071,17	€ 26092,98
	LW3	€ 36661,16	€ 36652,83	€ 37771,71
	Total	€ 79772,82	€ 81717,07	€ 83857,76
7	LW1	€ 17026,2	€ 17026,2	€ 17026,2
	LW2	€ 25289,88	€ 26761,21	€ 28052,14
	LW3	€ 41502,06	€ 41502,06	€ 41502,06
	Total	€ 83818,14	€ 85289,47	€ 86580,41
8	LW1	€ 23566,89	€ 24036,82	€ 28714,21
	LW2	€ 22251,05	€ 22323,77	€ 22881,35
	LW3	€ 40547,17	€ 42436,93	€ 42436,93
	Total	€ 86365,11	€ 88797,52	€ 94032,48
9	LW1	€ 18478,07	€ 20108,45	€ 20324,77
	LW2	€ 19648,67	€ 21229,59	€ 22216,94
	LW3	€ 40354,59	€ 42181,94	€ 44397,73
	Total	€ 78481,33	€ 83519,98	€ 86939,44
10	LW1	€ 19896,23	€ 20082,71	€ 20082,71
	LW2	€ 23463,68	€ 24177,38	€ 25252,05
	LW3	€ 29185,68	€ 29523,72	€ 29804,3
	Total	€ 72545,59	€ 73783,81	€ 75139,07
11	LW1	€ 23566,89	€ 25838,95	€ 28714,21
	LW2	€ 22251,05	€ 22705,19	€ 22881,35
	LW3	€ 40547,17	€ 41929,7	€ 42436,93
	Total	€ 86365,11	€ 90473,84	€ 94032,48
12	LW1	€ 18478,07	€ 20058,1	€ 20324,77
	LW2	€ 19648,67	€ 22056,57	€ 22216,94
	LW3	€ 40354,59	€ 41304,58	€ 44397,73
	Total	€ 78481,33	€ 83419,25	€ 86939,44
13	LW1	€ 20941,92	€ 21467,49	€ 21907,11

	LW2	€ 23791,91	€ 23829,2	€ 23829,2
	LW3	€ 38853,24	€ 38853,24	€ 38853,24
	Total	€ 83587,07	€ 84149,93	€ 84589,55
14	LW1	€ 23462,76	€ 25741,03	€ 25741,03
	LW2	€ 25987,32	€ 25987,32	€ 25987,32
	LW3	€ 35194,15	€ 36935,04	€ 37132,71
	Total	€ 84644,23	€ 88663,39	€ 88861,06
15	LW1	€ 18458,65	€ 18709,53	€ 18885,69
	LW2	€ 25279,49	€ 27236,69	€ 27236,69
	LW3	€ 34999,58	€ 36151,27	€ 36943,91
	Total	€ 78737,72	€ 82097,49	€ 83066,29
16	LW1	€ 19497,57	€ 19497,57	€ 19497,57
	LW2	€ 23458,77	€ 26143,33	€ 26334,03
	LW3	€ 42952,91	€ 43070,86	€ 43070,86
	Total	€ 85909,25	€ 88711,76	€ 88902,46
17	LW1	€ 24420,26	€ 24596,42	€ 24890,19
	LW2	€ 21168,16	€ 21168,16	€ 21168,16
	LW3	€ 34985,39	€ 35440,78	€ 37627,36
	Total	€ 80573,81	€ 81205,36	€ 83685,71
18	LW1	€ 17382,66	€ 17382,66	€ 17382,66
	LW2	€ 22690,73	€ 23282,41	€ 25316,91
	LW3	€ 33593,27	€ 34093,53	€ 34093,53
	Total	€ 73666,65	€ 74758,6	€ 76793,1
19	LW1	€ 17977,21	€ 18137,58	€ 18137,58
	LW2	€ 23985,26	€ 26654,01	€ 27528,09
	LW3	€ 40660,19	€ 40660,19	€ 40660,19
	Total	€ 82622,65	€ 85451,78	€ 86325,87
20	LW1	€ 17539,25	€ 17992,39	€ 18096,82
	LW2	€ 20521,95	€ 22440,21	€ 25397,01
	LW3	€ 36946,9	€ 38673,65	€ 41996,76
	Total	€ 75008,1	€ 79106,25	€ 85490,59
21	LW1	€ 18972,43	€ 19244,69	€ 19442,36
	LW2	€ 25488,18	€ 26233,23	€ 26233,23
	LW3	€ 37843,51	€ 38481,3	€ 39038,88
	Total	€ 82304,11	€ 83959,22	€ 84714,47
22	LW1	€ 24296,02	€ 24993,76	€ 25568,12
	LW2	€ 23999,27	€ 24444,95	€ 24556,84
	LW3	€ 31173,85	€ 34168,79	€ 35387
	Total	€ 79469,14	€ 83607,5	€ 85511,96
23	LW1	€ 16432,94	€ 16432,94	€ 16630,6
	LW2	€ 24501,81	€ 24501,81	€ 25059,38
	LW3	€ 39310,65	€ 41255,31	€ 41549,67

	Total	€ 80245,39	€ 82190,05	€ 83239,66
Total		**€ 1863360,7**	**€ 1929780,69**	**€ 1985996,33**
Δ w.r.t. min value		-	€ 66540,21	€ 122755,85
% w.r.t. min value		-	% 3,57	% 6,59