MEMORY EFFECTS IN WAVE PROPAGATION

A. S. Rezaei
Sapienza university of Rome, Rome, Lazio, Italy
E-mail: Amirsajjad.rezaei@uniroma1.it

F. Mezzani
Sapienza university of Rome, Rome, Lazio, Italy
E-mail: Federica.mezzani@uniroma1.it

A. Carcaterra
Sapienza university of Rome, Rome, Lazio, Italy
E-mail: Antonio.carcaterra@uniroma1.it

This paper investigates the response of structures including in their constitutive relationship memory effects. The analysis is carried on both by using modal analysis and a wave approach. Memory effects appears by a convolution integral added to conventional differential terms. The dispersion relation for an infinite waveguide is obtained together with the frequency response function for its finite counterpart, with given boundary conditions. The analysis shows large modifications of the wave propagation characteristics in the low frequency range, both for phase and group velocity, and an associated displacement of the structure natural frequencies.

Keywords: Memory Effect, Wave Propagation, Frequency Response Function, Dispersion Relation, Damping.

1. Introduction

Memory effect is a topic that enjoys a widespread attention due to the variety of applications they are involved in, as wave propagation [1, 2], viscoelastic applications [3] and optics [4].

In this paper, a theoretical background is presented in terms of wave propagation and modal analysis. The simpler case of a one-dimensional system is considered, under a time-dependent load. The memory effect appears as a convolution term involving a characteristic kernel, characterized by a persistency time and an amplitude that modulate its effect in the structural response. The obtained analytical results show this retarded action is the driver of wave attenuation strongly dependent upon the frequency. These effects appear to be important especially in the low frequency region. The analysis is carried on by using both a wave approach and modal analysis. A systematic investigation of the dependence of the structural response in terms of the amplitude and the persistency time of the memory effect is presented.

The analysis represents a first investigation in a more general context related to the presence of memory effects due to the fluid-structure interaction [5]. In several aerodynamic and hydrodynamic problems an elastic structure is in contact with a fluid [5-8]. Typically, two representative cases are of interest: the aerofoil that generates a lift effect due to the release of vortexes and the effect of turbulent boundary layer on an elastic wall [9, 10]. In both cases the vortex is a travelling structure that is conveyed by the flow with two important effects. One is related to the wake presence, along which the vortexes are stepping away from the trailing edge. The vortex is a structure that generates a velocity field in the fluid and an associated pressure field that invests the surrounding region. This pressure field is generated by the foil motion at the time *t*, but this effect is perceived by the foil even at later time as a downwash. This produces in the elastic response of the foil a retarded memory effect. A second element is related to the presence of the vortex pressure field as travelling structure along the elastic wall, that produces travelling pressure loads. In this case this random vortex structure couples time and space because of its convection in the flow, leading to typical effects of the moving loads along an elastic system. The coincidence effect of the vortex convection and the elastic response of the wall generate very interesting phenomena appearing in the turbulent boundary layer noise, when coupled to an elastic wall. More precisely, the coincidence phenomena become critical when the speed of the flow equals the speed of the vibrating modes of the elastic system. This condition, named coincidence, is a recurring topic of many works that tackle the problem of the turbulent boundary layer (TBL). In this regard, TBL has been analysed in many works and several models have been proposed [11-14].

The inclusion of the memory effects due to the vortex convection, modifies the wave propagation in the structure, and a deep modification of the vibration field produced by the travelling pressure field over the structure is expected. We expect the present analysis could be applied in future works in this context.

2. Theory and Formulation

Navier-Cauchy differential equation of motion can be completed with integral terms when including the effects of memory effects and long-range interactions, as in the prototype equation:

$$\rho\ddot{u}(x,t) - \frac{E}{2(1+\nu)}\left[\nabla^2 u(x,t) + \frac{1}{1-2\nu}\nabla\big(\nabla\cdot u(x,t)\big)\right] + \mu\int_{-\infty}^{t} P(t-\tau)\, u(x,t)\, d\tau + \int_{-\infty}^{+\infty} f(|r|) r d\xi = 0$$

where u is the structural axial displacement, ρ, E and μ are the mass density, Young's modulus and the memory gain, respectively. The first integral term introduces memory effects, the second one the long-range interactions in space. In this context, the authors intend to focus only on the memory effects. The response when including long-range interactions has been widely discussed in [15-18].

2.1 Plane waves

An infinite waveguide with a memory effect is considered here. In such systems, the structural response is not only dependent on the instant displacement, velocity and acceleration, but also upon the changes of the structural response along its time-history. We assume, as a reference, the governing equation for longitudinal vibration and for a linear homogeneous rod can be expressed as

$$\rho\ddot{u} - Eu'' + \mu\int_{-\infty}^{t} P(t-\tau)\, u(x,t)\, d\tau = 0 \tag{1}$$

where prime denotes space derivation. The additional term $\int_{-\infty}^{t} P(t-\tau)\, u(x,t)\, d\tau$ brings into the equation the memory effect through the kernel $P(t)$. For $P(t) = e^{-\alpha t} H(t)$, $H(t)$ being the Heaviside step, and $1/\alpha$ the memory persistency time, applying spatiotemporal Fourier transform, the dispersion relation is obtained as:

$$\rho\omega^2 = Ek^2 + \frac{\mu}{\alpha + j\omega} \tag{2}$$

where the memory delay α is a positive real number and j is the imaginary unit. This is a complex equation with complex roots, which proclaims the fact that the low frequency propagation of waves in such systems is distorted with respect to the standard D'Alembert equation. At higher frequency the memory distortion term $\frac{\mu}{\alpha+j\omega}$ tends to vanish. To obtain a generic nondimensional form of the dispersion relation, the following nondimensional parameters are conveniently considered

$$\begin{aligned} \Omega &= \omega/\alpha \\ K &= k\sqrt{E/\rho\alpha^3} \\ \Gamma &= \mu/\rho\alpha^3 \end{aligned} \tag{3}$$

In view of these parameter, Eq. (2) changes as follow

$$\Omega^2 = K^2 + \frac{\Gamma}{1+\Omega^2} - j\frac{\Gamma\Omega}{1+\Omega^2} \tag{4}$$

Here, Γ is a nondimensional parameter, which distinctly characterizes the system's response with respect to the classical one. The roots of this equation are found as

$$K(\Omega) = \pm\frac{\sqrt{-(1+j\Omega)(\Gamma-\Omega^2-j\Omega^3)}}{1+j\Omega} \tag{5}$$

And the associated waveform is

$$u = A\, e^{\mp K_i x}\, e^{i(\pm K_r x-\omega t)} \tag{6}$$

where $K_r(\Omega)$ and $K_i(\Omega)$ are the real and imaginary components of the wavenumber $K(\Omega)$. This implies the response corresponding to an infinite waveguide with memory shows an exponentially decaying trend governed by $e^{\mp K_i x}$ of the travelling disturbance $e^{i(\pm K_r x-\omega t)}$.

2.2 Frequency response analysis

The problem under investigation is the forced vibration of a finite waveguide of length l. The origin load is at the center of the rod and both ends ($x = \pm l/2$) are clamped. The system equation is

$$\rho\ddot{u} - Eu'' + \mu P * u = q(t) \tag{7}$$

where $q(t)$ is a time-varying load. Provided the initial conditions are set to zero, Eq. (7), in the Laplace domain, takes the form

$$\rho s^2\bar{u}(x,s) - E\frac{d^2\bar{u}(x,s)}{dx^2} + \frac{\mu\bar{u}(x,s)}{s+\alpha} = q(s) \tag{8}$$

Considering the assumed boundary conditions, one obtains

$$\bar{u}(x,s) = A\,q(s) - \frac{A\,q(s)}{(e^{Bl}+1)}e^{B(\frac{l}{2}-x)} + \frac{A\,q(s)}{(e^{Bl}+1)}e^{B(\frac{l}{2}+x)} \tag{9}$$

with

$$\begin{aligned} A &= \frac{s+\alpha}{\mu+s^2\rho(s+\alpha)} \\ B &= \frac{1}{\sqrt{E}\,\sqrt{A}} \end{aligned} \tag{10}$$

Hence, the transfer function is

$$H(j\omega) = \frac{\bar{u}(x,s)}{q(s)} = A - \left.\frac{A\left[e^{B\left(\frac{l}{2}-x\right)}+e^{B\left(\frac{l}{2}+x\right)}\right]}{(e^{Bl}+1)}\right|_{s=j\omega\ ,\ x=x_0} \tag{11}$$

2.3 Modal analysis

Let us consider a similar system described by Eq. (7) with arbitrary boundary conditions. Expressing the longitudinal displacement in terms of normal modes ϕ_i, we have

$$u(x,t) = \sum_i^\infty \phi_i(x)q_i(t) \tag{12}$$

Substitution of Eq. (12) into Eq. (7) results in

$$\rho\sum_i \phi_i\ddot{q}_i - E\sum_i q_i{\phi''}_i + \mu\int_{-\infty}^t P(t-\tau)\,\left[\sum_i \phi_i q_i\right]d\tau = q \tag{13}$$

Multiplying the Eq. (13) by ϕ_j and integrating over the length l, in the view of orthogonality relationships, the preceding reduces to

$$\ddot{q}_i + \omega_i^2 q_i + \tilde{\mu}(q_i * P) = \tilde{q}(t)\int_0^l \phi_i\,dx \tag{14}$$

where $\tilde{\mu} = \mu/\rho$ and $\tilde{q} = q/\rho$. Applying Fourier transform yields

$$Q_i(\omega) = \frac{\tilde{Q}(\omega)\int_0^l \phi_i dx}{\omega_i^2 - \omega^2 + \frac{\tilde{\mu}}{\alpha + j\omega}} \quad (15)$$

Thus, the signal may be reconstructed easily in the frequency domain as

$$u(x,\omega) = \tilde{Q}(\omega)\sum_i^\infty \frac{\phi_i(x)\int_0^l \phi_i dx}{\omega_i^2 - \omega^2 + \frac{\tilde{\mu}}{\alpha + j\omega}} \quad (16)$$

3. Results and discussion

This section is devoted to investigating the propagation properties of an infinite waveguide in the presence of memory effects governed by the nondimensional parameter Γ. On the other hand, a frequency response analysis on the finite counterpart of the investigated system is performed.

3.1 Wave propagation

Following the approach presented in Section 2.1, the propagation characteristics of infinite waveguide with non-instant interactions can be investigated. The imaginary component of the nondimensional wave-number i.e. wave attenuation, with respect to the nondimensional frequency is plotted in Fig. 1, for different values of Γ. Based on the figure, such a waveguide may be employed as a potential medium for high-frequency energy transfer since the medium is significantly less attenuating at higher frequencies.

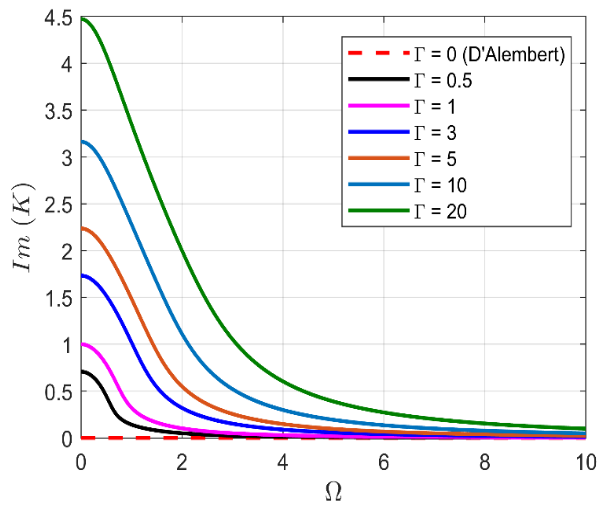

Figure 1: The wave attenuation curves for the waveguide with non-instant interactions for various the nondimensional parameters Γ.

Fig. 2 shows the phase velocity $C_\phi = \Omega/K_r(\Omega)$ against the nondimensional frequency for various values of Γ. Being the most obvious remarks, the system acts similarly to a conventional waveguide at higher frequencies. One may note that the amplitude of C_ϕ at the starting point of curve decreases for higher values of Γ. Furthermore, the phase velocity initially grows in a parabolic fashion with frequency and after reaching a maximum point, it decreases asymptotically to one.

Fig. 3 shows that the group velocity, at first, decreases rapidly to substantially rise straight afterwards. Finally, the value of C_g manifests a monotonic drop immediately after reaching a certain frequency and approaches the response of the conventional waveguide. Note that, the fluctuations corresponding to the group velocity shows rather large variations (the distortion belong to a range 0.5-2 with respect to the conventional speed) that can be purposely used to affect the propagation regimes, for example including control systems based on memory effects [19]. The propagation regimes associated to both time and space memory effect can in fact permit a surprising control of the wave pattern, even inducing wave-stopping or superluminal propagation which are already reported for systems affected by space and time nonlocality [15].

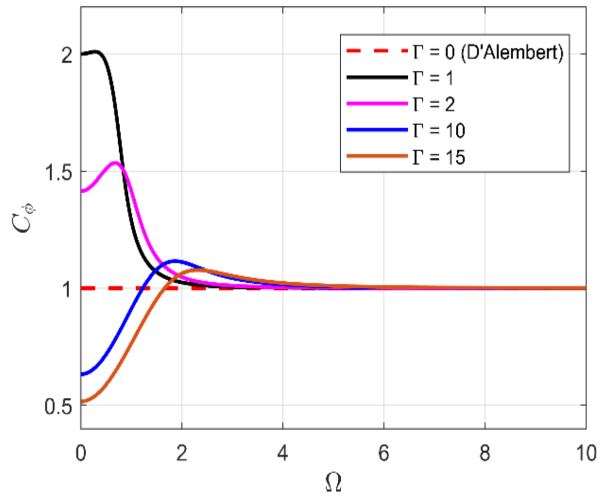

Figure 2: The phase velocity curves for the waveguide with non-instant interactions for various the nondimensional parameters Γ.

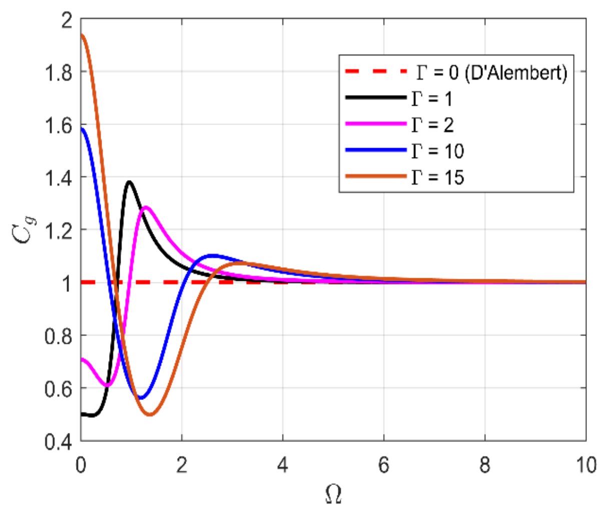

Figure 3: The group velocity curves for the waveguide with non-instant interactions for various the nondimensional parameters Γ.

3.2 Frequency response analysis

Based on the mathematical framework developed in Section 2.2, the following graphical results for a finite waveguide of unit length with Young's modulus $E = 1$ and mass density $\rho = 1$, are provided. Note that the frequency response function is evaluated at the origin. Fig. 4 represents the amplitude of the complex frequency response for different values of α and μ. From the figure, it is clear the value of $|H(j\omega)|$ decreases as the memory effect grows stronger. This, indeed, reflects the influence of induced damping due to the integral term $\int_{-\infty}^{t} P(t-\tau)\, u(x,t)\, d\tau$ when included into the conventional wave equation. The phase response of a waveguide with memory to an input signal with frequency content of sub-50 rad/s is depicted in Fig. 5. The abruptness of the phase shift may be attributed to high damping at the corresponding frequency. Drastic changes in phase response of system is due to an increase in the memory gain μ. Thus, temporal memory can cause a considerable delay of the input signal.

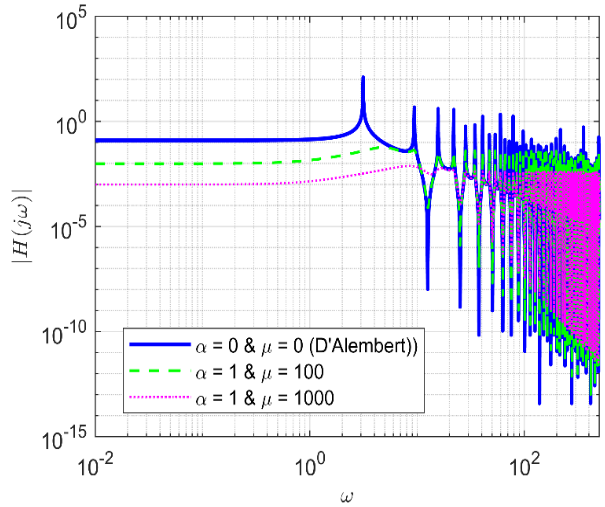

Figure 4: The variation in the amplitude of the transfer function for the central point of a waveguide with non-instant interactions with respect to frequency for various values of α and μ.

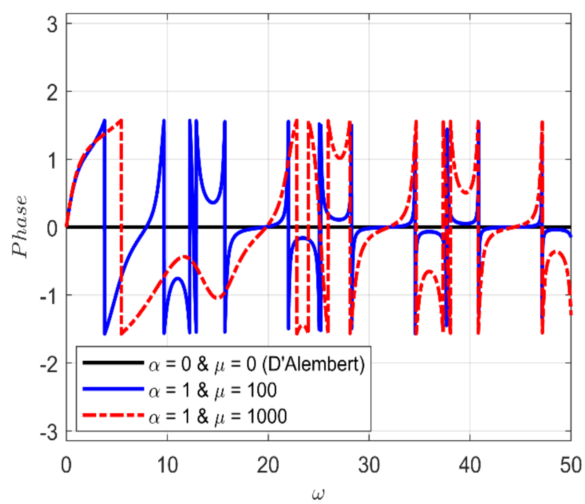

Figure 5: The phase response for the central point of a waveguide with non-instant interactions with respect to frequency for various values of α and μ.

4. Concluding remarks

The frequency response and wave propagation analysis for a waveguide with temporal memory is investigated in this study. The standard one-dimensional wave equation is modified by considering a supplementary term which describes the dependence of the system's response on the whole time-history. The spatiotemporal Fourier transform is applied to study the characteristics of waves in such a system with unbounded length and the dispersion relations is expressed in terms of nondimensional parameters.

Results indicate the attenuation is significantly dependent upon the frequency. In the low frequency region, i.e. for $\Omega = \omega/\alpha$ about less than 5, i.e. for $\omega < 5\alpha$, the wave phase and group speeds are highly affected by the memory effects and can be reduced or increased of a factor up to 2. Furthermore, the imposed forcing term which introduces the memory effects, causes significant variations in phase response as the influence of memory grows stronger.

REFERENCES

1 Wang, I. Dynamic Behavior of a One-dimensional Wave Equation with Memory and Viscous Damping, *ISRN Applied Mechanics*, **2014**, (2014).

2 Lü, Q., Zhang, X. and Zuazua, E. Null Controllability for wave equation with memory, *Journal de Mathématiques Pures et Appliquées*, **108**, 500-531, (2017).

3 Alabau-Boussouira, F., Nicaise, S. and Pignotti, C. Exponential Stability of the Wave equation with Memory and Time Delay, Chapter in *New Prospects in Direct, Inverse and Control Problems for Evolution Equations*, Springer Cham, 1-22, (2014).

4 Freund, I. and Rosenbluh, M. Memory Effect in Propagation of Optical Waves through Disordered Media, *Physical Review Letters*, **61**, 2328, (1988).

5 Carcaterra, A., Dessi, D. and Mastroddi, F. Hydrofoil Vibration Induced by a Random Flow: a Stochastic Perturbation Approach, *Journal of Sound and Vibration*, **283** (1-2), 401-432, (2005).

6 Dessi, D., Carcaterra, A. and Diodati, G. Experimental Investigation versus Numerical Simulation of the Dynamic Response of a Moored Floating Structure to Waves, *Proceedings of the Institution of Mechanical Engineering, Part M: Journal of Engineering for the Maritime Environment*, **218** (3), 153-165, (2004).

7 Carcaterra, A. and Ciappi, E. Hydrodynamic Shock of Elastic Structure Impacting on the Water: Theory and Experiment, *Journal of Sound and Vibration*, **271** (1-2), 411-439, (2004).

8 Lafrati, A., Carcaterra, A., Ciappi, E. and Campana, E. F. Hydroelastic Analysis of a Simple Oscillator Impacting the Free Surface, *Journal of Ship Research*, **44** (4), 278-289, (2000).

9 Ciappi, E., De Rosa, S., Franco, F., Vitiello, P. and Miozzi, M. On the Dynamic Behavior of Composite Panels under Turbulent Boundary Layer Excitations, *Journal of Sound and Vibration*, **364**, 77-109, (2016).

10 Franco, F., De Rosa, S. and Ciappi, E. Numerical Approximation on the Predictive Responses of Plates under Stochastic and Convective Load, *Journal of Fluid and Structures*, **42**, 296-312, (2013).

11 Caiazzo, A., D'Amico, R. and Desmet, W. A Generalized Corcos Model for Modelling Turbulent Boundary Layer Wall Fluctuations, *Journal of Sound and Vibration*, **372**, 192-210, (2016).

12 Chase, D. M. Modelling the Wavevector-frequency Spectrum of Turbulent Boundary Layer Wall Pressure, *Journal of Sound and Vibration*, **70** (1), 29-67, (1980).

13 Williams, J. E. F. Boundary-layer Pressures and the Corcos Model: A Development to Incorporate Low-wavenumber Constraints, *Journal of Fluid Mechanics*, **125**, 9-25, (1982).

14 Smol'yakov, A. V. and Tkachenko, V. M. Models of the Psedosound Turbulent Wall-pressure Field and Experimental Data, *Soviet Physics Acoustics*, **37**, 627-631, (1991).

15 Carcaterra, A., Coppo, F., Mezzani, F. and Pensalfini, S. Long-range Retarded Elastic Metamaterials: Wave-stopping, Negative, and Hypersonic or Superluminal Group Velocity, *Physical Review Applied*, **11**, 01404, (2019).

16 Mezzani, F., Coppo, F. and Carcaterra, A. Long-range Coupling of Waveguides, *Proceedings of the ISMA-USD Conference*, Leuven, Belgium, 7-9 September, (2018).

17 Mezzani, F., Coppo, F., Pensalfini, S., Roveri, N. and Carcaterra, A. Twin-waves Propagation Phenomena in Magnetically Coupled Structures, *Procedia Engineering*, **199**, 711-716, (2017).

18 Coppo, F., Rezaei, A. S., Mezzani, F., Pensalfini, S. and Carcaterra, A. Waves Path in an Elastic Membrane with Selective Nonlocality, *Proceedings of the ISMA-USD Conference*, Leuven, Belgium, 7-9 September, (2018).

19 Paifelman, E., Pepe, G., La Gala, F. and Carcaterra, A. Control of Fluctuations of a Tethered Unmanned Underwater-vehicle, *Proceedings of the ISMA-USD Conference*, Leuven, Belgium, 7-9 September, (2018).