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Formulation of the Transit Link Transmission Model

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Abstract

In this paper we present a new kind of dynamic assignment model for public transport, including transit and pedestrian networks, that is capable of representing single runs, whose schedule is possibly affected by congestion. To this end we avoid introducing explicitly a diachronic graph and rely just on a spatial/functional graph, like in frequency based models.
More specifically, we here extend to transit networks the framework of the Link Transmission Models, so far applied only to road networks (Yperman, 2007; Gentile, 2010; Gentile, 2015). The focus of LTM is the propagation, affected by congestion, of flows on the network, for given route choices. In this framework, both schedule based and frequency based passenger behaviors can be implicitly simulated by properly setting time-varying splitting rates.
The core of the proposed Transit Link Transmission Model (TLTM) is the node model, which aims at reproducing the flow conflicts (congestion) occurring at bus stops and rail platforms, such as: passengers that fail-to-board or fail-to-sit due to vehicle overcrowding, vehicles queueing to serve a stop, doors opened longer to allow passengers alighting and boarding.
The formulation and implementation of the TLTM will be presented in the following, while the application of the model on test and real networks will be presented in a forthcoming paper.

Keywords: dynamic network loading; public transport network; vehicle capacity constraints, passenger queue at stop; schedule based assignment

1. Introduction

Modelling the within-day dynamic of public transport systems is crucial for optimal real-time operations and predictive passenger information, as well as for off-line planning of the service.

In these contexts, the classical frequency based models (Nguyen and Pallottino, 1988; Spiess and Florian, 1989; Gentile et al., 2005) do not match some crucial requirements, such as:

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complying with strict capacity constraints, representing the congestion of passengers and vehicles at stops, providing the loads of single runs.

Schedule-based models using space-time networks, or diachronic graphs (Nuzzolo and Russo, 1998), offer a proper option for operation, but are not suited to reproduce congestion phenomena that affect travel times, because they are founded on the assumption that timetables are reliable. On the other hand, strategies and frequency based behavior are observed in practice when users have no possibility or convenience in timing their arrival at stops with those of vehicles (Bell et al., 2012; Trozzi et al., 2014).

Some efforts have been made in the past to cope with single issues (a review of those can be found in Gentile and Noekel, 2016), but a comprehensive framework where to represent in a fully macroscopic model all relevant phenomena is still missing. The only available methods are based on agent microsimulation (Cats, 2011) and mesoscopic simulation (Leurent et al., 2012), which present some intrinsic limits of stochasticity, complexity and calibration.

2. Model framework

2.1. Time discretization

The *assignment period* $[\tau_0 , \tau_n]$ (e.g. one day) is discretized into n subsequent *intervals* of equal temporal duration $\delta = (\tau_n - \tau_0) / n$ (e.g. one second) separated by an ordered set of *instants* whose generic integer index is $t\in[0, n] = T$ and whose generic clock *time* is $\tau_t = \tau_0 + \delta \cdot t$. By convention we refer to an interval $t\in T$ as to its final instant, therefore interval $t = 0$ represents the initial state. To represent events occurring after the assignment period, e.g. a discharging phase, an additional instant n+1, with $\tau_{n+1} = \infty$, and the corresponding interval are introduced.

2.2. Travel demand

Land is partitioned into a set Z of *zones* and all socio-economic activities located in a zone are assumed to be concentrated in one single point, called *centroid*, where trips start and end.

In a macroscopic model passengers are represented, not as individual entities, but as particles of a mono-dimensional partly compressible fluid.

Then, travel *demand* is given as a fixed (but time varying) flow d_{odt} of passengers departing during interval $t\in T$ from origin $o\in Z$ and directed toward destination $d\in Z$.

2.3. Base network: nodes and arcs

The topology of the network constituting the transport *supply* is represented by means of a *directed multigraph* (N, A), where N is the set of nodes and A is the set of arcs. In a directed multigraph there can be multiple arcs between the same ordered pair of nodes, so that each arc is identified with a triplet $(i\in N, j\in N, x)$, where x is a label that allows to distinguish them (which is omitted, if not necessary).

The initial node of the generic arc $a\in A$ is referred to as *tail* and denoted $N_a^-\in N$, while the final node is referred to as *head* and denoted $N_a^+\in N$. The set of arcs exiting the generic node $i\in N$ is referred to as its *forward star* and denoted $A_i^+ = \{a\in A: N_a^- = i\}$. Symmetrically, the set of arcs entering node $i\in N$ is referred to as its *backward star* and denoted $A_i^- = \{a\in A: N_a^+ = i\}$.

The zone centroids are a subset of these nodes: $Z \subseteq N$. To exclude paths that traverse a centroid one should split it in two distinct nodes, one as origin and one as destination; but this is not a major issue for transit networks.

Infrastructures, such as roads and rails, are described through the *base network* (A^{base}, N^{base}), that is a sub-graph of (N, A). Usually, each node of the base network has geographic coordinates and base arcs are described by polylines with intermediate points, mainly to allow map representations.

The relevant attributes of the generic base arc $a\in A^{base}$ are:

- l_a^{base} length
- v_a^{walk} walking speed; non-walkable arcs have a null speed (e.g. railway support arcs)
- x_a^{walk} sidewalk width; non-walkable arcs have a null width

2.4. Public transport: stops and lines

Public transport consists of a set S of stops between which services operate. A *stop* $s \in S$ is indeed a unique location (with geographic coordinates) where passengers can board and/or alight from transit services, e.g. a particular platform or sidewalk. By definition, transfers within a single stop take zero walking time and waiting passengers are guaranteed to observe all services departing from there including the related dynamic information, if present.

Transit services are organized in a set L of lines. A *line* $\ell \in L$ serves in one direction an ordered set of stops, its stop sequence or *itinerary*, denoted $S_\ell \subseteq S$, with no repetitions (circular lines and side-trips are excluded for the sake of simplicity). The part of a line between one stop s and the successive one is called *line segment*. Each line segment s is associated with an acyclic path on the base network, whose *support* arcs are denoted $A_{\ell s} \subseteq A^{base}$; this is essential to plot the line on a map.

Clearly, more lines can share the same stop. Stops are also nodes: $S \subseteq N$.

The relevant attributes of the generic stop $s \in S$ are:

- k_s^{pax} platform passenger capacity
- k_s^{veh} platform vehicle capacity

The relevant attributes of the generic line $\ell \in L$ are, with reference to a single carrier:

- k_ℓ^{seat} seating capacity
- k_ℓ^{stand} standing capacity
- k_ℓ^{alight} alighting door capacity
- k_ℓ^{board} boarding door capacity

2.5. Scheduled service: runs and timetables

Each line $\ell \in L$ is served by an ordered set of runs, called its *run sequence* and denoted R_ℓ . The line of run $r \in R$ is denoted $L_r \in L$.

A *run* $r \in R_\ell$ is constituted by one vehicle serving all stops of its line in order. As before, the part of a run between one stop s and the successive one is called a *run segment*.

We assume that each run $r \in R_\ell$ has a schedule, with an *arrival time* τ_{rs} and a *departure time* θ_{rs} for each stop $s \in S_\ell$, at least in the form of a working timetable defined by the operator. However, this can be affected by poor regularity and not known to passengers, who may then perceive the service only in terms of frequency. In this case, we will use as a model input only the departure time of each run from the first stop and the scheduled running times between subsequent stops, but not the dwelling times at stops, which may actually depend on flows of alighting and boarding passengers as well as of vehicles occupying the platform. As a consequence, the actual arrival and departure times at stops are a result of the simulation.

The subset of *frequency based* lines is denoted $L_{FB} \subseteq L$, the other lines are *scheduled based*.

3. The Link Transmission Model

3.1. Relation to Dynamic Network Loading models

The Transit Link Transmission Model is an extension of the LTM to the case of public transport networks. This is a Dynamic Network Loading model, with fixed but possibly time-varying route choices in the form of arc conditional probabilities, for each destination.

To make the model applicable to very large networks also in real-time, we assume that the demand flows are not assigned separately by destination, but are instead propagated jointly by aggregating all arc conditional probabilities into so-called *splitting rates* or turning fractions. Basically, at each node and for each time interval we consider as an input the share of flow that exits from an arc and enters to an adjacent arc.

Travel demand is injected on the networks at origins consistently with the generation of the o-d matrix, and extracted from destinations consistently with the local splitting rates The flow pattern resulting from the LTM will be consistent with the o-d matrix, if the splitting rates are constant in time, not necessarily otherwise.

3.2. Main features of the LTM

The major components of the LTM are the *link model* and the *node model*.

The link model propagates flow states forward (vehicles and passengers) and backward (spaces) on each arc consistently with the theory of kinematic waves. This produces future sending flows and receiving flows from current entry flows and exit flows, respectively.

The node model instead solves flow conflicts and provides entry and exit flows for given sending and receiving flows at the current interval.

The LTM model can be solved in chronological order under the assumption that the propagation of forward and backward traffic states along each arc takes more than a time interval.

The model is capable of simulating the daily evolution of flows and travel times, including queues and spillback, for given origin flows, splitting rates and possibly time-varying supply.

On road networks, the time-varying elements of the supply are the traffic lights. This can be fully exogenous or can be driven by flows in adaptive control systems. On transit networks, the time-varying elements of the supply are the service schedules of the lines.

The extension of the LTM to public transport is then achieved by:

- describing the topology of the transit network, so as to distinguish the different trip phases;
- modelling the availability of services in time and the priorities of different passenger flows at stops, by properly setting time-varying sending capacities that reproduce a sort of semaphore.

3.3. Formulation of the LTM

The main variables of the Link Transmission Model are the following cumulative flows (upper case), for each arc $a \in A$ and instant $t \in T$, and the corresponding interval variations (lower case):

- F_{at} , f_{at} entry flow, made of users that actually get in the arc,
- E_{at} , e_{at} exit flow, made of users that actually get out of the arc,
- H_{at} , h_{at} sending flow, made of users that are ready to exit the arc,
- G_{at} , g_{at} receiving flow, made of spaces that are ready to be occupied by users entering the arc.

Each arc $a \in A$ is characterized by the following constant attributes:

- l_a length
- v_a free-flow speed
- k_a link capacity
- j_a jam density
- w_a jam wave speed

Moreover, each arc $a \in A$ is characterized by the following time-varying attributes in the generic interval $t \in T$:

- η_{at} sending capacity rate, which may be an input or depend endogenously from other model variables (e.g. actuated traffic lights, transit gating – see later)
- p_{at} splitting rate, obtained by aggregating route choices of an assignment model for all destination or directly from GPS data

For each arc $a \in A$, the sending flow h_{at} and the receiving flow g_{at} during the generic interval $t \in T$ are given by:

$$h_{at} = H_{at} - E_{at-1} \quad , \quad g_{at} = G_{at} - F_{at-1} \ . \tag{1}$$

Sending and receiving are only potential flows that are bounded from above by capacities:

$$\hat{h}_{at} = Min\left(h_{at}-, k_a \cdot \eta_{at}\right) \quad , \quad \hat{g}_{at} = Min\left(g_{at}, k_a\right) \ . \tag{2}$$

For each node $i \in N$, the exit flow eat of its backward arcs and the entry flow f_{bt} of its forward arcs are obtained by solving the Node Model at interval $t \in T$. Below we provide a specific formulation based on the FIFO rule, where the *congestion level* $(1\text{-}\rho_a) \in [0,1]$ of each incoming arc $a \in A_i^-$ is given by the lowest ratio between the (supply) receiving flow and the (demand) sending flows of all used ($p_{ab} > 0$) outgoing arcs $b \in A_i^+$; references to the interval are here omitted, as the node model is always solved for a given t, while more informative turning fractions p_{ab} are be considered instead of splitting rates p_b:

$$\rho_a = Min\left(1, \frac{\hat{g}_b}{\sum_{c \in A_i^-} \hat{h}_c \cdot p_{cb}} : b \in A_i^+, p_{ab} > 0\right) \quad ,a \in A_i^- \qquad \begin{aligned} q_{ab} &= \hat{h}_a \cdot p_{ab} \cdot \rho_a \quad ,a \in A_i^-, b \in A_i^+ \\ e_a &= \sum_{b \in A_i^+} q_{ab} \quad ,a = A_i^- \\ f_b &= \sum_{a \in A_i^-} q_{ab} \quad ,b = A_i^+ \end{aligned} \quad . \tag{3}$$

The above model (5) implicitly assumes no conflicting maneuvers and that the receiving flow g_b of each outgoing arc $b \in A_i^+$ is partitioned among the incoming arcs $a \in A_i^-$ proportionally to the sending flows $\hat{h}_a \cdot p_{ab}$.

Then, for each arc $a \in A$, the cumulative entry flow F_{at} and the cumulative exit flow E_{at} at time $t \in T$ can be updated as follows:

$$F_{at} = F_{at-1} + f_{at} \quad , \quad E_{at} = E_{at-1} + e_{at} \ . \tag{4}$$

We assume here a triangular fundamental diagram, which allows for a very simple solution of the resulting fluid model through the kinematic wave theory based on cumulative flows. The generalization to concave fundamental diagrams, which requires to solve conflicts among waves, has been developed for road networks, but is less critical for transit networks, although pedestrian flows might be affected by hypocritical congestion.

In the Link Model we propagate forward the entry flow and backward the spaces left by the exit flow to obtain the cumulative sending flow H_{az} and the cumulative receiving flows G_{az} at later times $z \in T$, respectively:

$$\begin{aligned} H_{az} &= F_{at} - Mod\left(l_a / v_a, \delta\right) \cdot f_{at} \qquad &, \quad z = t + Div\left(l_a / v_a, \delta\right) \\ G_{az} &= l_a \cdot j_a + E_{at} - Mod\left(l_a / w_a, \delta\right) \cdot e_{at} \quad &, \quad z = t + Div\left(l_a / w_a, \delta\right) \end{aligned} \ . \tag{5}$$

However, some arcs (like running arcs of lines) may be conveniently characterized by a given (possibly time-varying) travel time, although this is not consistent with the Kinematic Wave Theory:

- t_{at} travel time of arc $a \in A$ at instant $t \in T$

In this case, in equation (5) v_a is replaced by l_a / t_{at} .

Random travel times can be easily introduced into this (otherwise deterministic) supply model to reproduce the effect of casual events that further affect (beside congestion) service regularity.

The overall model is solved by computing equation (1)-(5) for each interval $t = 1, 2, \dots, n$ in chronological order.

Interestingly, only the cumulative sending and receiving flows actually require to keep several temporal instances of the variable during the computation. The number of required components (additional to the current status) for each arc is equal to the number of intervals spanned by the corresponding kinematic waves. Implementation through a circular vector does the job. All other variables can be aggregated and recorded at the desired temporal discretization for result analysis.

To inject travel demand at origins and to extract flows at destinations, two dummy arcs for each zone centroid are introduced. These arcs are taken into account only in the node model, and not in the link model, as part of the backward and forward star, respectively. Their link model does not involve propagation; rather the cumulative sending flow of the generic origin arc is updated with the flows generated in each interval and the cumulative sending flow of the generic destination arc a is set to infinity.

4. Extension of the LTM to public transport

4.1. The transit network

A trip by public transport consists in general of several phases:

- accessing a stop from the origin, usually by walking;
- waiting at that stop for a vehicle;
- boarding a dwelling vehicle;
- dwelling on-board the vehicle at that stop ;
- running on-board the vehicle to the next stop;

- (possibly) repeat the phases from dwelling to running a certain number of times;
- alighting a dwelling vehicle;
- (possibly) transferring between two stops, usually by walking;
- (possibly) repeat the phases from waiting to transferring a certain number of times;
- and finally, egress from a transit stop to reach the destination, usually by walking.

Each one of the above trip phase is represented by an arc on the transit network, composed by:

- the pedestrian network, including centroids, connectors and (walkable) arcs of the base network;
- the line network, with a sub-network for each line, articulated in boarding, running, dwelling and alighting arcs, plus the stops shared by several lines;
- intermodal arcs at each stop to connect the pedestrian network with the line network.

The topology of the transit network includes then several types of nodes:

- the zone centroids Z ;
- the base nodes Nbase ;
- the *stop nodes* S ; each stop $s\in S$ is associated with a base node $B_s\in N^{base}$;
- the *line nodes* N_ℓ , with one layer for each line $\ell\in L$.

Two nodes for each stop of line $\ell\in L$ are introduced, so as to consistently represent dwelling, running, boarding and alighting as separate trip phases:

- the *arrival node* $N_{\ell s}{}^{arr}\in N_\ell$, $\forall s\in S_\ell$;
- the *departure node* $N_{\ell s}{}^{dep}\in N_\ell$, $\forall s\in S_\ell$.

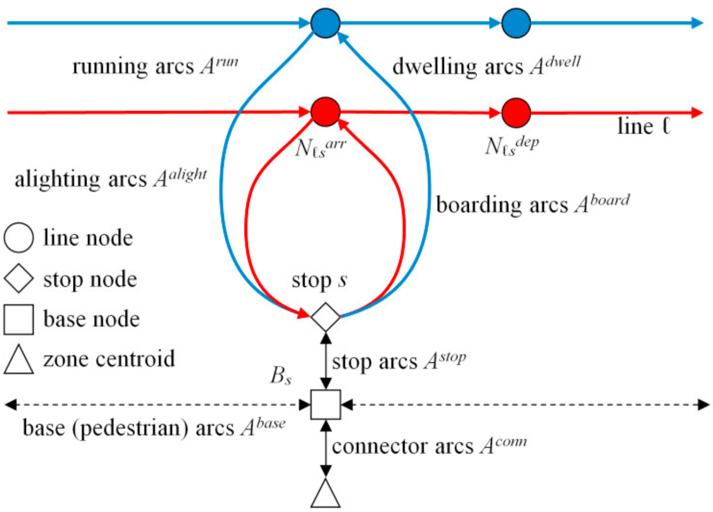

Fig. 1. Topology of the transit network at a given stop $s\in S$.

To build-up the transit network, the above nodes are connected by arcs of different types, as described in Fig. 1.

For each arc $a\in A^{run}\cup A^{dwell}\cup A^{board}\cup A^{alight}$, $L_a\in L$ denotes the line associated to it.

For each arc $a\in A^{stop}\cup A^{dwell}\cup A^{board}\cup A^{alight}$, $S_a\in S$ denotes the stop associated to it. The stop $S_a\in S$ associated with each arc $a\in A^{run}$ is that of the head.

Note that in many transit assignment models the boarding arc is headed at the departure node, if any. However, heading the boarding arc at the arrival node is here functional to using the dwelling arc as a gating facility.

4.2. Network specification for the Transit LTM

To separate user components and to hold distinguished variables for their flow and performance, in the TLTM each dwelling and running arc is actually split in 3 elements:

- 1. the seated arc, used by passengers who can find a seat on board the vehicle;
- 2. the standing arc, used by passengers who cannot find a seat and thus have to stand;
- 3. the vehicle arc, used by the carriers that are operating the line.

In addition, the network model requires a proper interface with travel demand, which is provided by dummy arcs: *origin* arcs A^{orig} and *destination* arcs A^{dest}, where trips are generated and attracted.

Finally, only the following turns are permitted at line nodes:

- $A^{r\text{-}seat} \rightarrow A^{d\text{-}seat}, A^{alight}$; $A^{r\text{-}stand} \rightarrow A^{d\text{-}standt}, A^{d\text{-}seat}, A^{alight}$; $A^{r\text{-}veh} \rightarrow A^{d\text{-}veh}$; $A^{r\text{-}board} \rightarrow A^{d\text{-}seat}, A^{d\text{-}stand}$; $A^{d\text{-}seat} \rightarrow A^{r\text{-}seat}$; $A^{d\text{-}stand} \rightarrow A^{r\text{-}stand}$; $A^{d\text{-}veh} \rightarrow A^{r\text{-}veh}$

The default values of the relevant attributes for the generic arc $a \in A$ are listed below:

- $l_a = \delta$, $v_a = 1$, $k_a = \infty$, $j_a = \infty$, $w_a = 1$.

These are changed accordingly with the arc type:

- $l_a = l_a^{base}$, $v_a = v_a^{walk}$, $k_a = 2 \cdot x_a^{walk}$, $j_a = 5 \cdot x_a^{walk}$, $w_a = 0.5 \cdot v_a^{walk}$ $\quad a \in A^{base}$
- $j_a = k_{S_a}{}^{pax} / \delta$ $\quad a \in A^{stop}$
- $k_a = k_{L_a}{}^{board}$ $\quad a \in A^{board}$
- $k_a = k_{L_a}{}^{alight}$ $\quad a \in A^{alight}$
- $v_a = l_a / t_{at}$ $\quad a \in A^{r\text{-}veh} \cup A^{r\text{-}seat} \cup A^{r\text{-}stand}$
- $t_{at} = \tau_{r S_{\ell s}^+} - \theta_{rs}$, $\ell = L_a$, $s = S_a$, $r \in R_\ell : \theta_{rs} \leq \tau_t < \theta_{r+1s}$ $\quad a \in A^{r\text{-}veh} \cup A^{r\text{-}seat} \cup A^{r\text{-}stand}$
- $j_a = k_{S_a}{}^{veh} / \delta$ $\quad a \in A^{d\text{-}veh}$
- $j_a = k_{L_a}{}^{seat} / \delta$ $\quad a \in A^{d\text{-}seat}$
- $j_a = k_{L_a}{}^{stand} / \delta$ $\quad a \in A^{d\text{-}stand}$

Once the network and the arc attributes are specified, the TLTM is a direct application of the Link Transmission Model. But we miss yet to properly specify one crucial variable: the sending capacity ratio η_{at} . Moreover, the splitting rates of turns whose head is a dwelling, seated or standing, arc are set dynamically according with a strategic behavior.

4.3. The sending capacities

The sending capacity ratio η_{at} reduces the capacity k_a of the generic arc $a \in A$ by setting the width of the bottleneck at its final section. It allows to regulate the exit flow, usually by opening or closing the arc at each interval t, based on specific conditions occurring at adjacent arcs, that are different for different arc types:

- the running vehicle arc of a scheduled based line is open if there is a run arrival at the head stop during the interval;
- the running vehicle arc of a frequency based line is always open;
- the running seated and standing arcs are open if at the beginning of the interval there is a dwelling vehicle at the head stop;
- the dwelling vehicle arc of a scheduled based line is open if there is a run departure at that stop during the interval;
- the dwelling vehicle arc of a frequency based line is open if: there is a dwelling vehicle at the stop, no more passenger can alight or board or the dwelling time τ_t-t_a^d has reached the maximum dwelling time t_ℓ^{mdt} (t_a^d is updated each time the dwelling vehicle arc is opened);
- the dwelling seated and standing arcs are open if the corresponding dwelling vehicle arc is open;
- the boarding arc is open if: there is a dwelling vehicle at the stop, no vehicle is departing, no passenger is alighting.

This system of semaphores provides a meaningful functioning of the stop, with the exception of one issue: when a vehicle reaches another vehicle of the same line at a stop (pairing) the model allows its passengers to pass to the front vehicle giving them priority wrt the waiting passengers, which cannot be in reality.

4.4. Strategic splitting rates for seating

The probabilities of alighting from a running, seated or standing, arc are externally provided by the route choice model (they are here a given input) and may differ because the expected travelling comfort of the two conditions is different; indeed, when carriers are crowded standing passengers do not know with certainty if and when they will be able to get a seat.

The probabilities of turns whose head is a dwelling, seated or standing, arc dynamically adapt to the number of seats actually available based on the following priority rule: first the passengers who come from the running seated arc, second those who come from the running standing arc, third those who come from the boarding arc:

$$p_{abt} = Min\left(1, \frac{\hat{g}_{bt} - \hat{h}_{ct} \cdot p_{cbt}}{\hat{h}_{at} \cdot (1 - p_{adt})}\right) \quad , \quad p_{fbt} = Min\left(1, \frac{\hat{g}_{bt} - \hat{h}_{ct} \cdot p_{cbt} - \hat{h}_{et} \cdot p_{ebt}}{\hat{h}_{ft}}\right)$$
$$p_{cbt} = 1 - p_{cdt} \quad , \quad p_{aet} = 1 - p_{adt} - p_{abt} \quad , \quad p_{fet} = 1 - p_{fbt} \quad . \tag{6}$$
$$a \in A^{r-stand}, b \in A^{d-seat}, c \in A^{r-seat}, d \in A^{alight}, e \in A^{d-stand}, f \in A^{board}$$

This implies the assumption that passengers behave strategically and take the opportunity of seating whenever possible.

4.5. Two different use cases

In the context of a Dynamic User Equilibrium, the main output of the TLTM are the travel times under a given flow pattern, rather than the passenger route choices, which are actually taken as an input (in terms of splitting rates). The main performance indicators that characterize the service, which are line frequency and regularity at each stop, can be retrieved from the simulated arrival and departure times (Gentile and Noekel, 2016). Indeed, the waiting times depend from the latter, while the running times are here assumed fixed but time-varying.

In the context of a Dynamic Network Loading, the main output of the LTM are instead the arc flows, besides the travel times.

The travel time of each arc $a \in A$ is obtained as the horizontal distance between the cumulative temporal profiles of exit flow $E_a(\tau)$ and entry flow $F_a(\tau)$ assuming a piecewise linear form.

References

Bell M.G.H., Trozzi V., Hosseinloo S.H., Gentile G., Fonzone A. (2012) Time-dependent hyperstar algorithm for robust vehicle navigation. Transportation Research A 46, 790-800.

Cats O. (2011) Dynamic modeling of transit operations and passenger decisions. PhD Thesis, KTH Royal Institute of Technology, Stockholm, Sweden.

Gentile G. (2010) The General Link Transmission Model for Dynamic Network Loading and a comparison with the DUE algorithm. New developments in transport planning: advances in Dynamic Traffic Assignment (selected papers from the DTA 2008 Conference, Leuven), ed.s L.G.H. Immers, C.M.J. Tampere, F. Viti, Transport Economics, Management and Policy Series, Edward Elgar Publishing, MA, USA, 153-178.

Gentile G. (2015) Using the general link transmission model in a dynamic traffic assignment to simulate congestion on urban networks. Transportation Research Procedia 5, 66-81.

Gentile G., Noekel K. (2016) Modelling public transport passenger flows in the era of Intelligent Transport Systems: COST Action TU1004 (TransITS). Springer Tracts on Transportation and Traffic 10, Springer, Switzerland.

Gentile G., Nguyen S., Pallottino S. (2005) Route choice on transit networks with online information at stops. Transportation Science 39, 289-297.

Leurent F., Chandakas E., Poulhes A. (2012) A passenger traffic assignment model with capacity constraints for transit networks, Procedia - Social and Behavioral Sciences 54, 772-784.

Nguyen S., Pallottino S. (1988) Equilibrium traffic assignment for large scale transit networks. European Journal of Operational Research 37, 176-186

Nuzzolo A., Russo F. (1998) A dynamic network loading model for transit services. Proceedings of TRAISTAN III, San Juan, Puerto Rico.

Spiess H., Florian M. (1989) Optimal strategies: A new assignment model for transit networks, Transportation Research B 23, 83-102.

Trozzi V., Gentile G., Bell M.G.H., Kaparias I. (2013) Dynamic user equilibrium in public transport networks with passenger congestion and hyperpaths. Transportation Research B 57, 266-285.

Yperman I. (2007) The Link Transmission Model for Dynamic Network Loading. PhD Thesis, Katholieke Universiteit Leuven.