LEVEL DEPENDENCE OF THE ADJUSTMENT FOR UNBALANCE AND INEQUALITY FOR THE HUMAN DEVELOPMENT INDEX

Abstract. As remarked by Ravallion (2012), the recent switch from arithmetic to geometric mean in the aggregation of the United Nations' Human Development Index has caused a more severe inequality penalization of the index for less developed countries, with outlying consequences. We clarify and explain this fact and propose an aggregation function, the Trichotomy Mean, that depends on two parameters: one regulates the overall penalization of disequilibria (among or within dimensions) in analogy with Atkinson's inequality aversion parameter for power means; the other modulates the Level Dependence of the Adjustment, a novel concept describing the behavior—decreasing, increasing, or constant—of penalization of given disequilibria for increasing index level. Unlike the geometric mean (which, incidentally, has decreasing LDA type), the TM remains valid for zero or negative—and does not distort for small positive—values of the input variables, thus permitting less restrictive raw-variable normalizations and to overcome the need for exogenous lower bounds. We compare the three versions of TM with the geometric mean in an empirical analysis on the HDI 2014 data. We finally illustrate the contributions of the TM to the development literature debate.

1. Introduction

The United Nations Development Programme's *Human Development Index* (*HDI*), published yearly in the Human Development Report (HDR) and perhaps the most important index used in development analysis, aggregates three fundamental dimensions of human development: income, education, and health. Since its introduction in 1990, the index has undergone several methodological revisions, mostly summarized in Klugman et al. (2011), involving raw variable choice and/or indexing procedure for education and income, normalization bounds, and aggregation. This last issue will constitute the focus of this article.

Before 2010, the HDI aggregated the three dimensions by the arithmetic mean of their normalized values, without taking into account any kind of unbalance among them or any inequality (across the population or groups thereof) within single dimensions. In order to overcome these deficiencies the following HDR (UNDP, 2010) started aggregating using the geometric mean instead: thereby, the new HDI would adjust for unbalances among dimensions, whereas the newly-issued *Inequality-adjusted Human Development Index* (*IHDI*) would also adjust for within-variable inequalities. In either case, the adjustment can be regarded as a penalization to be applied to the arithmetic mean and it increases with the disequilibrium.

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Several aspects of aggregation procedures for synthetic indices (especially in development studies) and, in particular, of adjustments for unbalances and inequalities have been investigated in the literature (UNDP, 1993; Anand and Sen, 1995; Hicks, 1997; Foster et al., 2005; Stanton, 2006; Seth, 2009; UNDP, 2010; Grimm et al., 2010). For a general analysis of composite indicators with unbalance adjustment see Casadio Tarabusi and Guarini (2013), building on previous work (Palazzi and Lauri, 1998; Casadio Tarabusi and Palazzi, 2004; Corsi and Guarini, 2011). Inequality among countries and dynamic changes in the HDI and its various components from a relative standpoint have been studied by Sengupta and Ghosh (2010, 2013).

Ravallion (2012, §2) remarks that one effect of the switch from arithmetic to geometric mean in the HDI calculation is a more severe penalization for less developed countries, with possible outlying consequences. We try to clarify and explain this fact by studying the features and shortcomings of the geometric mean and to analyze in general terms the behavior of penalization with respect to the human development level. More explicitly, we introduce the *Level Dependence of the Adjustment* (*LDA*), that correlates the absolute level of the composite indicator and the amount of adjustment it undergoes due to the disequilibrium of (namely, unbalance among or inequality within) its components. We distinguish three kinds of LDA: for any given disequilibrium among dimensions, the adjustment may decrease with the level of the index (*decreasing LDA*), or increase with it (*increasing LDA*), or be independent from it (*constant LDA*).

After formalizing the concept of LDA, we derive the corresponding conditions on generalized means, a class of aggregation functions already considered, e.g., in Seth (2009, pp. 378–379). These conditions, applied to the current HDI's and IHDI's aggregation (geometric means), classifies it as decreasing LDA (whereas the old HDI's arithmetic mean does not adjust for disequilibria at all, so it is a degenerate case of constant LDA). This may or may not be a desirable situation for specific kinds of development analysis, because it may depend on its context, method, scope and theoretical and policy framework.

For this reasons we propose a new aggregation function belonging to the class of generalized means, called *Trichotomy Mean* (*TM*), that depends on a parameter a (level-dependent disequilibrium aversion) according to whose value we have decreasing (for $a < 0$), increasing (for $a > 0$), or constant (for $a = 0$) LDA. By adjusting the value of a the user can choose which of the three alternatives fits best the purpose of their analysis; indeed the absolute value of a may even be used to calibrate the rate of decrease or increase of the adjustment with respect to the index level. The TM also improves on the geometric mean with respect to another issue, which provokes the above-mentioned undesirable effects denounced by Ravallion: the geometric mean is only defined on positive values of the variables, which may unduly constrain the preceding normalization step of the index evaluation procedure and thereby distort the index results when the values of some variables are small, while the TM is defined on all possible values.

The proposed TM also depends on a second user-adjustable parameter b (level-independent disequilibrium aversion) that modulates the amount of an additional, index-independent adjustment for disequilibria. Parameter b plays the same role as Atkinson's inequality aversion parameter in power means (Atkinson, 1970, §3), whose value for the geometric mean (used in the current HDI as well as in the

IHDI) is 1 (Klugman et al., 2011, §6). In the same spirit we choose to select and suggest specific values for the two TM parameters, namely $a = -1$ for decreasing LDA, $a = 1$ for increasing LDA, $a = 0$ for constant LDA, along with $b = 1$ in all cases.

The empirical section of this article is devoted to an analysis and comparison of the TM (for each of the three suggested parameter pairs) versus geometric mean aggregation of the 2014 HDI normalized input data. There result significant rating and ranking differences between the TM and the HDI's geometric mean: according to the fact that the geometric mean has decreasing LDA, the discrepancy is minimum for $a = -1$, maximum for $a = 1$, and intermediate for $a = 0$.

We conclude with a discussion of the theoretical, methodological and policy implications of our remarks and proposals on the ongoing debate in the human development literature.

2. Aggregation analysis of the Human Development Index

A composite index is built in terms of n individual dimensions $x_1, \ldots, x_n$ given for the elements of a prescribed set (or units). Each dimension x_i is usually assumed to be normalized with respect to some reference interval I_i of real numbers; in the sequel we shall assume $I_1 = \cdots = I_n = [0, 1]$, although the whole discussion can easily be adapted to the general case. A positive weight w_i is assumed assigned to each dimension x_i to account for its relative relevance, normalized so that

$$\sum_{i=1}^{n} w_i = 1.$$

If unspecified, such as in HDI and IHDI as well as in the sequel, the tacit assumption is $w_1 = \cdots = w_n = 1/n$. Aggregation of individual indices is obtained via a function F of n variables (which also incorporates the weights as parameters), so that on a unit with dimension vector $x = (x_1, \ldots, x_n)$ the composite indicator takes the value

$$F(x) = F(x_1, \ldots, x_n).$$

With an axiomatic type of approach as in Chakravarty (2003), the focus of this article will be on aggregation, that is, on the properties of function F, without discussing all other aspects of the indexing analysis (choice of individual variables, their definition and weighting, normalization methods and parameters, et cetera); a comprehensive survey of the complete procedure to produce composite indices is provided in OECD (2008).

For the HDI the units are world countries, while $n = 3$, and x_1 is the Health Index, x_2 the Education Index, and x_3 the Income Index, currently defined as, respectively: life expectancy at birth; the simple average of mean years and expected years of schooling; the natural logarithm of the real gross national income per capita in United States dollars adjusted for purchasing power parity; all normalized with the min-max method to the interval $[0, 1]$. The choice made for the raw maxima and minima used in the normalization is such that for all three variables the minima across all countries of the normalized values are somewhat greater than 0, whereas the maxima are rather close to 1 and the average is significantly greater than .5. The weights chosen are $w_1 = w_2 = w_3 = 1/3$. A deep account of further details and a history of changes for HDI can be found in Klugman et al. (2011). Before

2010 the aggregation for HDI was performed by using the arithmetic mean μ_{arith}:

$$\mu_{\mathrm{arith}}(x) = \sum_{i=1}^{n} w_i x_i,$$

whose functional form, though, does not take into account any kind of unbalance among the indices. In order to tackle this shortcoming, the new HDI replaced the arithmetic mean with the geometric mean μ_{geom}:

$$\mu_{\mathrm{geom}}(x) = \prod_{i=1}^{n} x_i^{w_i}$$

(provided all x_i's are positive numbers; this constraint will be further discussed later). As aggregation functions, both the arithmetic and the geometric mean enjoy the following properties:

Property (i): *continuity.* The function F is continuous on its domain. So, for instance, there is no "jump" in the index values.

Property (ii): *positive monotonicity.* $F(x_1, \ldots, x_n) \geq F(x'_1, \ldots, x'_n)$ if $x_i \geq x'_i$ for every i. The index is (weakly) increasing in each component.

Property (iii): *idempotence.* If $x_1 = \cdots = x_n$ (the *equilibrium locus*, as defined below) then $F(x_1, \ldots, x_n) = x_1$.

Property (iv): *symmetry in dimensions.* (This holds for μ_{arith} and μ_{geom} only if all weights are equal, as happens with the HDI.) We have $F(x) = F(Sx)$ whenever S is an $n \times n$ permutation matrix (each column and each row have one 1 and 0 elsewhere). Each variable in the index has the same relevance.

Property (v): *non-regressive compensability.* For any i, i', the rate of marginal compensation (or of substitution) of variable x_i with variable $x_{i'}$ (this rate equals $(\partial F/\partial x_i)/(\partial F/\partial x_{i'})$ under differentiability assumptions) is weakly increasing with $x_{i'}$, provided the index value and the remaining variables are kept constant.

This last property for the geometric mean is an immediate consequence of the weak concavity; the preceding ones are straightforward.

3. Disequilibria and adjustment

The *equilibrium locus* (which is the *balance locus* if across dimensions, or the *equality locus* if across individuals) is the set of points with equal coordinates. The *disequilibrium* of x is the skew-symmetric matrix

$$D(x) = (x_i - x_{i'})_{i,i'=1,\ldots,n}$$

of differences between all pairs of its coordinates. The *adjustment* (called *penalization* in Casadio Tarabusi and Guarini (2013)) of F for disequilibrium is defined as

$$A(x) = \mu_{\mathrm{arith}}(x) - F(x) \qquad \text{for every } x;$$

under Properties (i)–(v) it is continuous, everywhere non-negative, and equals zero on the equilibrium locus. The adjustment may be interpreted as the additive correction that the arithmetic mean $\mu_{\mathrm{arith}}(x)$ undergoes, due to the disequilibrium of x, in order to yield $F(x)$. According to Property (v) on F, the adjustment increases with disequilibrium for fixed value of $\mu_{\mathrm{arith}}(x)$ (more precisely: $A(x + ty)$ increases with $t \geq 0$ if x is in the equilibrium locus and $y = (y_1, \dots, y_n)$ is a zero-sum vector). In stead, for fixed disequilibrium the adjustment may depend on $\mu_{\text{arith}}(x)$ in several ways, as we shall discuss more extensively below. As already mentioned, the above properties hold for $F = \mu_{\text{geom}}$; in particular, as is well-known,

$$\mu_{\text{geom}}(x) \leq \mu_{\text{arith}}(x) \qquad \text{for every } x,$$

and the inequality is strict unless all components of x are equal.

Throughout this article we shall assume the adjustment to be non-negative, which penalizes disequilibria for any index that evaluates a desirable quantity such as human development, and is consistent with the non-regressive compensability Property (v). Nevertheless the entire discussion may easily be converted (simply replacing $F(x)$ with $-F(-x)$) to non-positive adjustment, fit for indices constructed to estimate non-desirable quantities such as human poverty, as long as the word "increasing" is replaced by "decreasing" in Property (v); in this case disequilibria are penalized by assigning a possibly greater index than the plain arithmetic mean.

The arithmetic mean may be regarded as the aggregation function that satisfies Properties (i)–(v) and least adjusts for disequilibria, in a situation of *perfect substitutability* among variables. On the other hand, the one that most adjusts is the minimum function

$$F(x) = \min(x_1, \dots, x_n)$$

(used for instance in the Rawlsian analysis of well-being), in a situation of *perfect complementarity.* All other aggregation functions with those properties, such as the geometric mean, are intermediate between these two extremes. For instance, for $n = 2$, if $x = (.5, .5)$, in the equilibrium locus, then $\min(x) = \mu_{\text{geom}}(x) = \mu_{\text{arith}}(x) = .5$, whereas the point $x = (.3, .7)$, obtained from the previous choice by transferring a quantity .2 from the first to the second coordinate, has the same arithmetic mean .5 (no adjustment), while the minimum drops to .3 (a sharp adjustment of .2) and the geometric mean decreases to approximately .46 (a milder adjustment of about .04). The level sets of μ_{arith} for $n = 2$ are parallel straight lines at 45 degrees, while each of those of min is made up of two half-lines parallel to the coordinate axes and pointing in the positive direction; the level sets of an intermediate aggregation function is somewhere between these two, as in Figure 1. For further details on adjustment for disequilibria see Casadio Tarabusi and Guarini (2013) and Klugman et al. (2011).

We want to investigate another feature of the geometric mean, and of general aggregation functions satisfying the idempotence property, that does not seem to have been studied in the literature: how the adjustment behaves when the disequilibrium is kept constant, but the absolute index level varies.

Two points x and x' share the same disequilibrium, that is $D(x) = D(x')$, if and only if the difference $x'_i - x_i$ is the same for all $i = 1, \dots, n$ (geometrically, they lie on the same parallel line to the equilibrium locus); in this case $(x'_1, \dots, x'_n) = (x_1 + c, \dots, x_n + c)$ for a common c. Given, as observed earlier, that under progressive compensability the adjustment of an aggregation function F increases with disequilibrium, we investigate how the adjustment behaves for fixed disequilibrium (i.e., as the point moves along a parallel to the equilibrium locus). In order to compare the values of F (hence of its disequilibrium adjustment) between x and x', we first observe that under idempotence we have

$$F(x_1 + c, \dots, x_n + c) = F(x_1, \dots, x_n) + c \qquad \text{if } x_1 = \cdots = x_n.$$

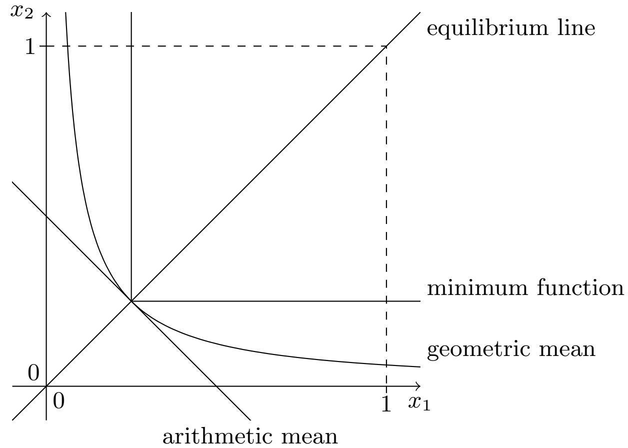

FIGURE 1. Level sets of value .25 for the minimum function, the geometric mean and the arithmetic mean (for n=2)

Comparing the two sides of this equality for x outside the equilibrium locus gives rise naturally to different kinds of dependence of the adjustment on disequilibrium:

Property (vi): *level-dependence of the adjustment.* The *Level Dependence of the Adjustment* (*LDA*) may be:

$$\begin{cases} \textit{decreasing} & \text{if } F(x_1+c,\ldots,x_n+c) > F(x_1,\ldots,x_n)+c, \\ \textit{increasing} & \text{if } F(x_1+c,\ldots,x_n+c) < F(x_1,\ldots,x_n)+c, \\ \textit{constant} & \text{if } F(x_1+c,\ldots,x_n+c) = F(x_1,\ldots,x_n)+c, \end{cases}$$

for every x outside the equilibrium locus and every $c > 0$. For F smooth, we can list respective sufficient conditions: we have

$$\begin{cases} \text{decreasing LDA} & \text{if } \sum_{i=1}^n \partial F/\partial x_i > 1, \\ \text{increasing LDA} & \text{if } \sum_{i=1}^n \partial F/\partial x_i < 1, \\ \text{constant LDA} & \text{if } \sum_{i=1}^n \partial F/\partial x_i = 1. \end{cases} \tag{3.1}$$

(The three cases do not encompass every possible aggregation function F, which may fulfill various of the above requirements in different x-regions.) In terms of the adjustment A, the three kinds respectively translate into:

$$\begin{aligned} A(x_1+c,\ldots,x_n+c) &< A(x_1,\ldots,x_n) \qquad \text{(decreasing LDA)}, \\ A(x_1+c,\ldots,x_n+c) &> A(x_1,\ldots,x_n) \qquad \text{(increasing LDA)}, \\ A(x_1+c,\ldots,x_n+c) &= A(x_1,\ldots,x_n) \qquad \text{(constant LDA)}, \end{aligned}$$

for every $c > 0$; this justifies the terminology. The constant LDA case is sometimes called *stability for translations* in the literature. As shown by Figure 2 for $n = 2$ variables, the joint increment by c of all variables translates into the increment $F(x') - F(x)$, where $x' = (x_1 + c, x_2 + c)$; this increment is always greater than (respectively, smaller than, or equal to) c according if the LDA is decreasing, increasing, or constant. In the first case, the level curves approach those of perfect

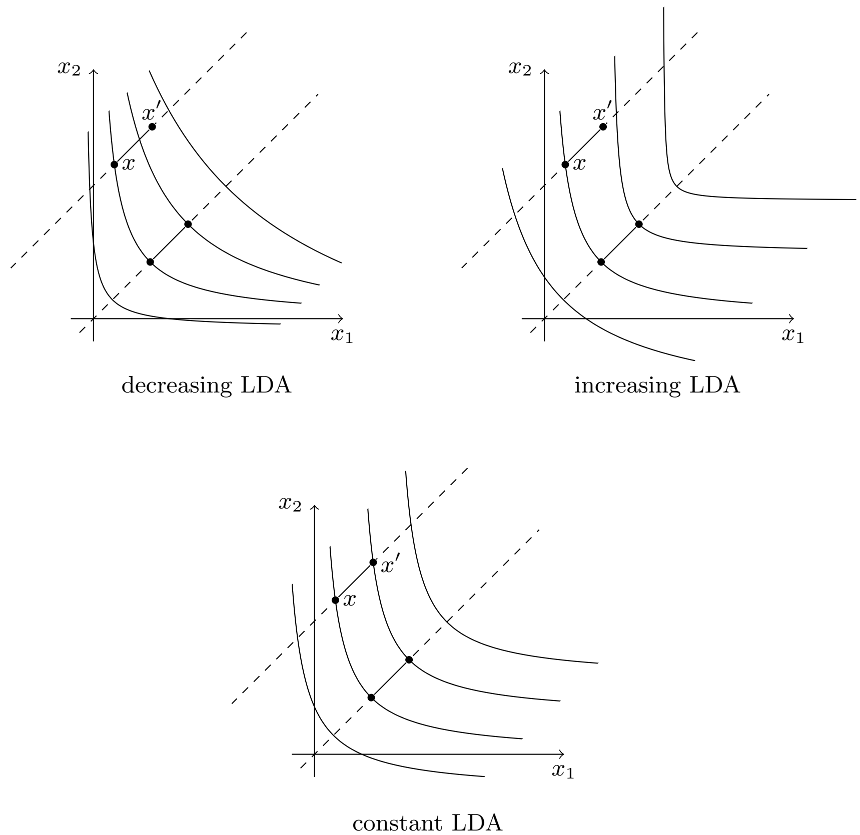

FIGURE 2. Comparison of different level sets according to the LDA for typical aggregation functions

substitutability as the index value increases, while they approach those of perfect complementarity as the index value decreases; the reverse happens in the second case; while all level curves are rigid translates of each other in the third case.

The arithmetic mean $F = \mu_{\text{arith}}$ and the minimum function $F = \min$ are easily seen to have constant LDA. Instead, the geometric mean $F = \mu_{\text{geom}}$ turns out to fall within the case of decreasing LDA, as emerges from the plot of its level sets in Figure 3 and will be proved shortly. An example with $n = 2$ variables may illustrate the situation. If $x = (.1, .3)$ and $x' = (.7, .9)$, two vectors with the same disequilibrium .2 between coordinates, then $c = .7 - .1 = .9 - .3 = .6$, while $F(x') \approx .794$ and $F(x) \approx .173$, so that $F(x') > F(x) + c$, as predicted; equivalently, the additive adjustment at x, namely $\mu_{\text{arith}}(x) - F(x) \approx .2 - .173 = .027$, is greater than at x', namely $\mu_{\text{arith}}(x') - F(x') \approx .8 - .794 = .006$.

4. Generalized means

In order to determine conditions more readily usable for detecting which LDA alternative an aggregation function possibly falls into, we restrict attention to a wide class of aggregation functions: generalized means. For each continuous and strictly

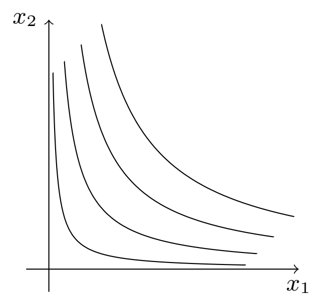

FIGURE 3. Some level sets of the geometric mean

increasing (or decreasing) function f on the real line—or at least on a real interval J, usually the positive half-line—the *weighted generalized* (or *quasi-arithmetic*) *mean* of x is defined as

$$\mu(x) = f^{-1}(\mu_{\text{arith}}(f(x_1), \dots, f(x_n))) = f^{-1}\left(\sum_{i=1}^{n} w_i f(x_i)\right), \tag{4.1}$$

as long as $x_1, \dots, x_n$ are in J; see Hardy et al. (1988, Chap. III). We keep assuming that $w_1 = \cdots = w_n = 1/n$.

Fact 4.1 (Hardy et al. (1988, §83))**.** Two different functions f and g give rise to the same generalized mean μ if and only if they only differ by an additive and a multiplicative constant; that is, if $g = k_1 + k_2 f$ for constants k_1 and k_2 with $k_2 \neq 0$ (in particular, replacing f with $-f$ does not change μ). Therefore a generalized mean μ determines the function f up to multiplication and addition of constants.

A generalized mean μ may be used as an aggregation function, provided the interval J contains the ranges of all the individual variables. Properties (i)–(iv) are automatically satisfied.

Fact 4.2 (Hardy et al. (1988, §92))**.** The generalized mean $\mu(x)$ is a concave function of x if and only if f is concave increasing or convex decreasing. This is exactly when Property (v) holds for the aggregation function $F = \mu$. Then μ, as already noted for the geometric mean, satisfies

$$\mu(x) \le \mu_{\text{arith}}(x) \qquad \text{for every } x,$$

with equality if and only if x is in the equilibrium locus.

One noteworthy case of generalized mean is for $f(t) = t^\gamma$ (equivalently, $f(t) = k_1 + k_2 t^\gamma$, as per Fact 4.1) for any real constant γ, which gives the *power mean* μ_γ *of order* γ, with the following special cases:

- if $\gamma = 1$, we have the *arithmetic mean* μ_{arith}, corresponding to $f(t) = k_1 + k_2 t$ (here J is the entire real line);
- if $\gamma = 0$, the limit of μ_γ for $\gamma \to 0$ is the *geometric mean* μ_{geom}, corresponding to $f(t) = k_1 + k_2 \log t$ (here J is the positive half-line);

- the limit of μ_γ for $\gamma \to -\infty$ is the *minimum function* (a degenerate, limit case of generalized mean).

We can derive the conditions on f for the corresponding generalized mean μ to have decreasing, increasing, or constant LDA.

Theorem 1. *We can suppose the function f to be increasing, up to replacing it, if necessary, with $-f$ by Fact 4.1. If f is also smooth and its derivative f' never vanishes, then the following conditions are equivalent:*

- *the corresponding generalized mean μ has decreasing (respectively, increasing or constant) LDA;*
- *the composite function $f' \circ f^{-1}$ is strictly convex (strictly concave, affine);*
- *f''/f' is strictly increasing (strictly decreasing, constant);*

Proof. By (4.1), setting $g = f' \circ f^{-1}$ and $y_i = f(x_i)$ for all $i = 1, \dots, n$ we obtain

$$\sum_{i=1}^{n} \frac{\partial \mu}{\partial x_i}(x) = \frac{\sum_{i=1}^{n} w_i f'(x_i)}{f'\big(f^{-1}\big(\sum_{i=1}^{n} w_i f(x_i)\big)\big)} = \frac{\sum_{i=1}^{n} w_i g(y_i)}{g\big(\sum_{i=1}^{n} w_i y_i\big)}.$$

By (3.1) we have decreasing LDA when the left-hand side—thus the right-hand side—of this last equality is > 1 for every $x = (x_1, \dots, x_n)$. Since $y_1, \dots, y_n$ are arbitrary, this happens if and only if g is strictly convex.

The third condition is that g', or $g' \circ f$, is strictly increasing. (A sufficient condition is that g'', or $(f')^3(g'' \circ f)$, is > 0, which translates into $f'f''' > (f'')^2$.)

The two other cases of LDA are treated similarly. □

In particular, if $f(t)$ is the increasing function $\log t$ then $f''(t)/f'(t) = -1/t$ is itself strictly increasing, so that the corresponding generalized mean, viz. the geometric mean, satisfies the decreasing LDA, as anticipated.

5. The Trichotomy Mean and the Standard Adjustment

The aggregation function F we propose is the generalized mean μ arising from a function f with some prescribed features. We require f to be defined on the whole real line, smooth, increasing with non-vanishing derivative (that is, $f' > 0$), and concave, in fact with $f'' < 0$ (alternatively, replacing f by $-f$ by Fact 4.1, it must be decreasing, convex, and satisfy $f' < 0 < f''$). Therefore f''/f' is negative; its composition with the decreasing function $\log(-u)$ reverses its monotonicity, so, by the third condition of Theorem 1, the LDA kind of F depends on whether the function

$$L(t) = \log\left(-\frac{f''(t)}{f'(t)}\right)$$

is decreasing, increasing or constant. Indeed the values of L near t reflect the intensity of the adjustment that F applies to x whose arithmetic mean is t.

Since affine functions $at+b$ are the simplest decreasing/increasing/constant functions on the real line, we shall impose that f satisfy the second-order ordinary differential equation

$$L(t) = a(t - \tau) + b \qquad \text{with } \tau = \frac{1}{2}, \tag{5.1}$$

for real parameters a, b. The reference center τ is chosen at the midpoint $1/2$ of the typical reference interval $[0, 1]$ where normalized variables lie; neverthesess the framework naturally allows a different choice for τ. For each pair (a, b), by Fact 4.1

all solutions of the equation give rise to the same generalized mean, which will fulfill decreasing, increasing, or constant LDA according if the parameter a is negative, respectively positive, or zero, while the parameter b will modulate an additional level-independent amount of adjustment for disequilibria, with a similar role as Atkinson's inequality aversion parameter in power means (Atkinson, 1970, §3). The value of Atkinson's parameter selected for the new HDI and IHDI is 1 and corresponds to the geometric mean (Klugman et al., 2011, §6). Likewise, for simplicity—the same principle followed in the choice of equal weights among variables (ul Haq, 1995, Chap. 4)—we recommend specific values for the two parameters, namely $a = -1$ for decreasing LDA, $a = 1$ for increasing LDA, $a = 0$ for constant LDA, and $b = 1$ in all cases.

Up to two constants of integration (one additive, one, non-zero, multiplicative), whose choice, as observed before, does not affect the resulting generalized mean, and setting the constants

$$\begin{aligned} B &= e^b, \\ C &= -\frac{B}{ae^{\tau a}} \qquad \text{if } a \neq 0, \end{aligned}$$

the solution of (5.1) is

$$f(t) = \begin{cases} e^{-Bt} & \text{if } a = 0, \\ \mathrm{Ei}(Ce^{at}) & \text{if } a \neq 0, \end{cases}$$

where Ei is the exponential-integral function, see, e.g., Abramowitz and Stegun (1964, §5.1). The inverse function of f is

$$f^{-1}(s) = \begin{cases} -\dfrac{\log s}{B} & \text{for } s > 0, \text{ if } a = 0; \\ \dfrac{1}{a} \log \dfrac{\mathrm{Ei}^{-1}(s)}{C} & \text{for any } s, \text{ if } a < 0; \text{ for } s < 0, \text{ if } a > 0; \end{cases}$$

where the inverse on the right-hand sides is intended of the restriction of Ei to the negative half-line if $a > 0$, to the positive half-line if $a < 0$.

We define the *Trichotomy Mean* (*TM*) as the generalized mean $\mu_{a,b}$ determined by such f, namely

$$\mu_{a,b}(x) = f^{-1}\left(\sum_{i=1}^{n} w_i f(x_i)\right).$$

The qualitative behavior of the TM for the three choices of the pair (a, b) suggested above, namely $(-1, 1)$, $(1, 1)$, and $(0, 1)$, for $n = 2$ is represented in Figure 2 by the graphs about decreasing, respectively increasing, and constant LDA.

Notice that, of these choices, the function $\mu_{-1,1}$ turns out to resemble the geometric mean the most, while $\mu_{1,1}$ is the most different of the three, in accordance with the fact that the geometric mean has decreasing LDA.

Turning to constant LDA, standard calculus steps show that the arithmetic mean and the minimum function are limit cases as b tends to $-\infty$ or $+\infty$, respectively, of the TM for $a = 0$. Again for $a = 0$ and the recommended value $b = 1$, the function $\mu_{0,1}$ may be taken as a prototype of a disequilibrium-adjusting aggregation function that is stable for translations. In this sense, the adjustment $A_S(x)$ it provides for x may be taken as a measure of the disequilibrium of x, therefore we call it *Standard Adjustment*. Indeed: A_S vanishes exactly on the equilibrium locus; it is strictly positive elsewhere; it is invariant by addition of the same quantity to all components (in other words, it is independent of the index value); and $A_S(x+ty)$ increases with $t \geq 0$ if x is in the equilibrium locus and $y = (y_1, \dots, y_n)$ is a zero-sum vector.

The above framework is flexible enough to possibly encompass more general situations such as mixed LDA types—for instance, decreasing LDA for low values (i.e., for poorer countries) and increasing LDA for high values (richer countries). Indeed, the LDA type is correlated to the behavior of the function on the right-hand side of equation (5.1), which can be assigned as needed.

We discuss two further features for inequality-adjusting functions.

Property (vii): *unrestricted domain.* The function F is defined on R^n.

This is relevant firstly because it allows the use of any normalization on raw data, regardless of the range of normalized values. The HDI instead, being built with the geometric mean, is only defined for positive values of normalized dimensions. In fact, the authors of the HDR have to preliminarily solve the problem of zero and negative values that make the geometric mean lose sense; see, e.g., UNDP (2010, p. 218) (cf. also the comment about the asymmetry between normalized maxima and minima made here in Section 2). Secondly, the index behavior becomes extreme and unstable when some variables take small positive values, and, as pointed out by Klugman et al. (2011, §4) the choice of pre-normalization minima acquires undue relevance. If, instead, standardization rescaled to the $[0, 1]$ interval were used as normalization, the presence of negative outliers could not be excluded a priori.

In Casadio Tarabusi and Guarini (2013, Properties (vii.a),(vii.b)) another feature of aggregation is introduced: complete or incomplete compensability. In the former case, in order to maintain the same index level, any loss in one dimension is always compensable with suitable gains in the other dimensions; in the latter case, a large enough decrease of one dimension cannot be compensated with any increases in the others. The geometric mean has complete compensability, and so does the TM for $a < 0$ (the decreasing LDA case); instead, the TM for $a \geq 0$ (the constant and the increasing LDA cases) enjoys incomplete compensability.

6. The Trichotomy Mean for the Inequality-adjusted Human Development Index

In the literature there have been several contributions on the adjustment of the HDI for inequalities within any specific dimension across individuals (or groups thereof, or regions, et cetera), such as UNDP (1993), Anand and Sen (1995), Hicks (1997), Foster et al. (2005), Stanton (2006), Grimm et al. (2008), Seth (2009), Grimm et al. (2010), and UNDP (2010). To overcome these criticisms, the HDR 2010 introduced the Inequality-adjusted Human Development Index (IHDI), that contemplates adjustments both for unbalance among variables and inequality within single variables.

A possible model is the following. For $i = 1, \dots, n$ and $j = 1, \dots, m$ denote: by x_{ij} the value of variable i for individual (or group) j; by X the whole matrix of such values; by $x_{i\,\cdot} = (x_{i1}, \dots, x_{im})$ its i-th row; by $x_{\cdot\,j} = (x_{1j}, \dots, x_{nj})$ its j-th column. A positive weight w_{ij} is assumed assigned to each index pair (i, j), and all are so that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} = 1.$$

Consequently, the i-th row takes the overall weight $w_{i\,\cdot} = \sum_{j=1}^{m} w_{ij}$ for each i, whereas the j-th column is assigned the overall weight $w_{\cdot\,j} = \sum_{i=1}^{n} w_{ij}$ for each j; of course, $\sum_{i=1}^{n} w_{i\,\cdot} = 1 = \sum_{j=1}^{m} w_{\cdot\,j}$. Specifically, for IHDI we have $w_{1\,\cdot} = w_{2\,\cdot} = w_{3\,\cdot} = 1/3$. (A possible simplifying assumption is that $w_{ij} = w_{i\,\cdot}\, w_{\cdot\,j}$ for every i and j.) When some mean within row i (respectively, column j) is to be computed, the weights w_{ij} are preliminarly renormalized by dividing them by $w_{i\,\cdot}$ (respectively, $w_{\cdot\,j}$). Aggregation is performed by a function F of $n \times m$ variables, so that on a unit with matrix X the composite indicator takes the value

$$F(X) = F\begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}.$$

While the number of variables n is presumed fixed and chosen once and for all, the number m of individuals may need to be left undetermined, in which case F needs to be defined on $n \times m$ matrices for every m.

The IHDI is built by taking the geometric mean of the values x_{ij} of each row i separately, and then the geometric mean of the results across i, namely

$$\mu_{\text{geom}}(\mu_{\text{geom}}(x_{1\,\cdot}), \ldots, \mu_{\text{geom}}(x_{n\,\cdot})). \tag{6.1}$$

In the same fashion, the aggregation function F we propose in this setting is

$$\mu_{a,b}(\mu_{a,b}(x_{1,\,\cdot}), \ldots, \mu_{a,b}(x_{n,\,\cdot})),$$

where $\mu_{a,b}$ is the TM for given (a, b). Most of the discussion of the preceding sections about the HDI, the geometric mean and the TM carries over to the IHDI and the double-subscript situation. In the sequel we list some properties that only make sense in the present setting and that hold true for any generalized mean; in particular, for the geometric mean and for the TM for any (a, b).

Property (viii): *symmetry in people.* $F(X) = F(XS)$ whenever S is an $m \times m$ permutation matrix. Thus each individual (or group) in the index has the same importance.

Property (ix): *replication invariance.* $F(X) = F(X, \ldots, X)$, where k copies of the matrix X are juxtaposed, for every $k \geq 2$. Thanks to this property it is possible to compare the index on populations with different sizes.

Property (x): *subgroup consistency.* If the population is divided into two non-overlapping groups A, B, if the matrices X, X' are correspondingly partitioned into X_A, X_B, respectively X'_A, X'_B, and if $F(X_A) > F(X'_A)$ while $F(X_B) = F(X'_B)$, then $F(X) > F(X')$. This means that the overall index increases if it increases for one group of individuals while not changing for the rest of the population.

Property (xi): *path independence.* The same index value results both if aggregation occurs first across individuals and then across dimensions, or conversely, or all at once (assuming separate row-wise and column-wise aggregations make sense). The results are indifferent to aggregation order (Foster and Shneyerov, 2000, §3).

Path independence holds if F is any generalized mean μ:

$$\begin{aligned} \mu(\mu(x_{1\,\cdot}), \ldots, \mu(x_{n\,\cdot})) &= \mu(\mu(x_{\cdot\,1}), \ldots, \mu(x_{\cdot\,m})) \\ &= \mu(x_{11}, \ldots, x_{1m}, \ldots, x_{n1}, \ldots, x_{nm}). \end{aligned}$$

Table 1

Differences between μ_{geom} and $\mu_{a,1}$	$a=-1$	$a=0$	$a=1$
Percentage with rank difference ≤ -2 or ≥ 2	9.1	21.9	35.3
Maximum absolute rank difference	3	6	8
Standard deviation of rank differences	.92	1.27	1.93

Applying this to the geometric mean $\mu = \mu_{\text{geom}}$, this provides two more possible formulas to compute the IHDI besides (6.1).

7. An application of the Trichotomy Mean to the HDI

In this section we apply the three cases of TM (with decreasing, constant and increasing LDA) on the data used in UNDP (2014), freely downloadable from `http://www.undp.org`, and compare the outcomes versus the HDI. The database lists the education, health and income indices for 194 world countries (although the data for 7 of them are not complete and will not be included here) and some country groups. The three dimensions are aggregated by the geometric mean to obtain the HDI, and are aggregated by the TM to obtain the synthetic index proposed in this article. For decreasing, constant, and increasing LDA we take the recommended values of a to be -1, 0, 1 respectively, along with $b = 1$ in all cases. The results appear in a table in the Appendix, where for each country are listed rating and ranking for the arithmetic mean (namely, the pre-2010 HDI aggregation method), the geometric mean (the actual HDI), each of the three versions of TM, and the Standard Adjustment defined in Section 5.

In Table 1 we collect synthetic results of differences between the geometric mean and each of the TMs (again with the recommended value pairs of parameters), namely the maximum absolute rank difference, the standard deviation of the rank difference and the percentage of countries whose ranks change by two or more (in either direction). It appears clear that the decreasing LDA case is the most similar to the geometric mean, while the increasing LDA case is the least similar. Overall, the above rating and ranking differences are comparable to those between the current HDI's geometric mean versus the previous HDI's arithmetic mean.

Countries that lose the highest numbers of rank positions in some of the TMs with respect to the geometric mean are Oman, Singapore, and United Arab Emirates, which share a fairly high HDI value and a large unbalance among dimensions; in concordance with the definitions, their rank loss worsens when passing from decreasing to increasing LDA. The opposite happens to Latvia, which shows a pretty high HDI and the smallest unbalance. Lesotho, a very low HDI country with a very small unbalance, gains the highest total number of rank positions, the same across the three LDAs.

In accordance with the theoretical framework, the TM value for increasing LDA is smaller (respectively, larger) than for decreasing LDA for high (respectively, low) index values. In fact, $a = 1$ applies a stronger (weaker) adjustment than $a = -1$ to disequilibria for high (low) index values. Correspondingly there is a high negative correlation (about $-.52$) between the arithmetic mean and the difference between TMs for increasing and decreasing LDA.

It may also be interesting to note that there is a rather high negative correlation (namely about $-.45$) between the HDI computed by the arithmetic mean and the standard adjustment. A similar evidence was found for the Index of African Governance by Casadio Tarabusi and Guarini (2013, §7).

8. Contributions of the Trichotomy Mean to the development literature debate

As described earlier, until 2010 the aggregation function used for the HDI was the simple arithmetic mean. Because such function allowed perfect substitutability among the input variables, the simplest choice for a function that would, instead, penalize disequilibria arose from the Atkinson inequality index at the parameter value 1: starting in the HDR 2010 the geometric mean was used as the aggregation function for the HDI. Nevertheless, Ravallion (2012, §2) observed that this change had provoked a substantial downward revision for countries with low HDIs, concentrated for the vast majority in a single world region (Sub-Saharan Africa). Examining the HDR 2010 data, the author singled out the outlying case of Zimbabwe. From the values of its input variables—Education Index .519, Health Index .427, Income Index .012—the resulting HDI had dropped from .319 to .138 in the switch, whereas all other countries had undergone a much smaller decrease.

Our theoretical framework enables a clear explanation for this extreme phenomenon, which is due to two further intrinsic characteristics of the geometric mean and of its generating function as a generalized mean (the logarithm): *decreasing penalization* and *constraint at zero*. The TM introduced in this article provides significant advantages over the geometric mean with respect to each of the two characteristics.

8.1. **Decreasing penalization.**

As argued above, the geometric mean happens to fall in the decreasing LDA category, therefore, for a given disequilibrium among (or within) dimensions, the lower the human development level of a country, the greater its penalization. This characteristic does not appear to have been observed or taken into consideration in the literature before, neither in the UNDP reports, nor in subsequent theoretical and methodological contributions concerning this switch and the related tradeoff issues, such as Klugman et al. (2011) and Ravallion (2012).

In the TM the decreasing penalization represents just one of three kinds of adjustment behavior, besides increasing and constant LDA, and this is modulated via the parameter a. In this way the decreasing LDA, from an unintentional consequence of a simplistic choice of the geometric mean, becomes one of three possible explicit intentional options within the TM framework. The provision of a palette of different LDA types is strongly relevant because each such option entails a different approach to development policies: Is a given disequilibrium—unbalance among or inequality within dimensions—less desirable within a context of low human development (this would be reflected by decreasing LDA) or of high human development (increasing LDA), or are they equally serious (constant LDA)?

On the one hand, decreasing LDA may appear preferable because the lower is the development level of a country, the more serious the consequences of a same amount of disequilibrium may be deemed to be. On the other hand, constant LDA, especially in a disequilibrium-adjusting instance such as the TM for $(a, b) = (0, 1)$ (recall that the arithmetic mean has constant LDA, but fails to adjust for disequilibria), may be considered the best choice because, in this case, a same disequilibrium inflicts the same adjustment to the unadjusted development level, regardless of the development level itself. With this choice, unbalanced poor countries are penalized twice—by the low value of their input variables and by the adjustment for their unbalance—but not a third time by a stiffer adjustment due, again, to their low overall level (as happens, instead, with decreasing LDA), and this may be regarded as fairer. A further advantage of introducing the TM for $(a, b) = (0, 1)$ (and the class of constant LDA aggregation functions in general) is to use it to define a level-independent measure of inequality, that we called the Standard Adjustment. Finally, human development analyses conducted on particular regional contexts may, consistently with the spirit of the capability approach, call for choices of variables that are different from the HDI proper, as in Bubbico and Dijkstra (2011). The balance among the non-standard variables might be more fragile for high rather than low values, therefore increasing LDA would be the natural option in this case.

Whichever the answer to the previous question, it should be kept in mind that the overall effects of penalization tend to be stronger for the group of less developed countries, in view of the high negative correlation between disequilibrium and human development level, as noted in Section 7. For all that has been argued, the TM would contribute to make the use and significance of the HDI more clear and transparent, hence more effective as an instrument for the elaboration and monitoring of development policies. Such goals are coherent with the capability approach perspective that is the ground for the human development framework. In fact, the LDA type selection may be performed in the same spirit as Anand and Sen affirm about weight determination:

> Any choice of weights should be open to questioning and debating in public discussions. (Anand and Sen, 1997)

In general, as argued in Klugman et al. (2011, §5), the capabilities approach is a partial theory of well-being, hence reference to the HDI does not compel the ways and motivations for pursuing better performance as gauged by the index, thus the LDA type choice itself may fall within the tasks left to theorists and policy-makers.

8.2. **Constraint at zero.** The geometric mean is undefined for negative or zero values of dimensions and, furthermore (and inevitably), for small positive values it produces effects on the resulting index that become more and more distorting as the values approach zero; indeed the geometric mean tends to the minimum function as any of the input variables tends to zero, so that a small value for one variable turns out to outweigh possibly much higher values for the others, as happens in the above-mentioned case of Zimbabwe. Already in the HDR 2010, introducing the adoption of the geometric mean, these problems were acknowledged and partially overcome with questionable ad hoc patches:

> The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. (UNDP, 2010, p. 218)

In the same direction, some changes were applied in the subsequent editions of the HDR to the pre-aggregation steps of the HDI construction so that the distributions of some normalized variables in the interval $[0,1]$ would be moved away from the troublesome area of values close enough to zero: one change was to lower from 163 to 100 the minimum of Gross National Income per capita (Purchasing Power Parity US \$) in the min-max normalization; another was, in the HDR 2014, to switch back from geometric to arithmetic mean in the aggregation of input dimensions that constitute the Education Index. All changes and patches may artificially contribute to compress the theoretical range $[0,1]$ of indices to significantly narrower actual ranges for the individual normalized variables and of the ensuing HDI values, a priori excluding a non-negligible neighborhood of zero. The last-mentioned change has also brought to an unexplainedly hybrid overall aggregation scheme, with the arithmetic mean (ensuring perfect substitutability) within the Education Index and the geometric mean (conveying imperfect substitutability) for the final HDI.

On the other hand, the TM is defined for all values of the input variables, hence the choice of the normalization and other pre-aggregation steps is not unduly affected or constrained by the need of avoiding prohibited or distorting values. For instance, applying the TM with the suggested pair of parameters for the decreasing LDA case (thus the same LDA type as the geometric mean) to the Zimbabwe 2010 values quoted at the beginning of this section one obtains .226, substantially reducing (indeed almost halving) the penalization towards the arithmetic mean with respect to the use of the geometric mean. Using on the same input values the TM with the suggested pairs of parameters for the two other LDA types one obtains .249 in the constant LDA case and .265 in the increasing LDA case; the progressive increase of the resulting index value is consistent with the respective LDA types, the lowness of this country's values, and the choices made for all parameters a, b, and τ.

In accordance with Ravallion (2012, §1), we finally point out that the methodological aspects of the HDI construction have significant theory and policy implications that are worth being made fully explicit.

9. Conclusions

Stimulated by the observation of Ravallion (2012) that the 2010 switch from arithmetic to geometric mean in the computation of the United Nation Development Programme's yearly Human Development Index has introduced excessive penalizations for less developed countries, in this article we have defined and studied a new feature of aggregation functions for indices aiming to measure human development: the Level Dependence of the Adjustment (LDA). From previous work (Casadio Tarabusi and Guarini, 2013) we have recalled the concept of adjustment for disequilibrium—namely, for unbalance within variables, or for inequality across variables—by studying its analytical features. The geometric mean adjusts for disequilibrium in an intermediate way between the two extremes: the arithmetic mean, that does not adjust at all and implies perfect substitutability among variables or individuals; and the minimum function, that applies the maximum possible adjustment and implies perfect complementarity. Turning the attention from absolute disequilibrium adjustment to its relative behavior with respect to the HDI level, we have introduced three possible types of LDA: decreasing, constant, and increasing;

and we have recognized that the geometric mean falls in the first one. Within the family of generalized means, which includes the arithmetic and the geometric mean, we have defined a new aggregation function, called Trichotomy Mean (TM). The TM fulfills all the desirable properties of this family of functions and, unlike the geometric mean, it is valid for all possible input values, thus avoiding distortions for low values. The TM depends on two parameters a and b: the former modulates the dependence of the adjustment on the level; the latter (as Atkinson's inequality aversion parameter) adds to the adjustment an additional component that depends on disequilibrium but not on level. We have identified specific pairs of values for the parameters—namely $a = -1$ for decreasing LDA, $a = 0$ for constant, $a = 1$ for increasing, and $b = 1$ for all cases)—and defined the standard adjustment as the adjustment for the constant case above, a satisfactory measure for disequilibrium. Decreasing LDA appears more adequate to the HDI setup, yet the two other options may in principle be applicable to this and other social indicators. The main theoretical and methodological contribution of this article is the provision of a single framework that articulates in three distinct possible (categories of) options. Thus, users are given the opportunity to choose which LDAs fit best their judgment of values and policy perspectives. The choice of the aggregation function (be it the arithmetic mean, the geometric mean, the TM for any value of its parameters, or other) or more generally among the three LDA types carries significant theoretical and methodological implications, therefore (also in view of the respective features described in this article) it should be consistent with the assumptions and the object of the analysis.

We have also treated the case of Inequality-adjusted Human Development Index, identifying a further set of desirable properties, and proving that the TM may also be used as an aggregation function in this context. We have performed an empirical comparison between the TMs and the geometric mean on HDI 2014 data. The analysis has confirmed the main theoretical properties of the three versions of the TM. In particular, the decreasing LDA has turned out to be the closest of the three to the HDI's geometric mean, while the increasing LDA is the farthest, in terms both of rating and of ranking. We have observed a negative correlation between human development level and unbalance across variables. We have concluded by showing how our findings may contribute to the ongoing debate on human development literature.

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Appendix: Comparison of indices on the HDR 2014 data

Legend:

- E: HDR 2014 Education Index
- H: HDR 2014 Health Index
- Y: HDR 2014 Income Index
- new (or n.) HDI: current HDI (computed with the geometric mean)
- old (or o.) HDI: pre-2010 HDI (computed with the arithmetic mean)
- $\mathrm{TM}(a,1)$: TM with $b=1$ and the given value of a (namely -1, 0, or 1)
- A_S: Standard Adjustment
- *: Rank variation from old HDI to new HDI.

	Rating									Ranking						Rank variation from n. HDI to			
	input indices			HDI		$\mathrm{TM}(a,1)$, $a=$			1000	HDI		$\mathrm{TM}(a,1)$					$\mathrm{TM}(a,1)$		
Country	E	H	Y	new	old	-1	0	1	$\cdot A_S$	n.	o.	-1	0	1	A_S	o.*	-1	0	1
Australia	.979	.961	.911	.950	.950	.950	.949	.949	1.1	1	1	1	1	1	170	0	0	0	0
Norway	.909	.946	.976	.943	.944	.943	.943	.942	1.0	2	2	2	2	2	171	0	0	0	0
New Zealand	.956	.940	.874	.923	.923	.922	.922	.921	1.7	3	3	3	3	3	160	0	0	0	0
Switzerland	.843	.963	.950	.917	.919	.916	.915	.913	4.0	4	4	4	5	5	138	0	0	-1	-1
Netherlands	.894	.939	.914	.915	.916	.915	.915	.915	.5	5	5	5	4	4	183	0	0	1	1
U.S.A.	.888	.907	.946	.913	.914	.913	.913	.912	.8	6	6	6	6	6	175	0	0	0	0
Germany	.883	.935	.916	.911	.911	.911	.911	.910	.6	7	7	7	7	7	180	0	0	0	0
Ireland	.903	.934	.878	.905	.905	.905	.904	.904	.7	8	9	8	8	8	179	1	0	0	0
Canada	.852	.946	.912	.902	.903	.902	.901	.900	2.1	9	10	9	9	9	154	1	0	0	0
Singapore	.768	.959	.995	.902	.907	.898	.893	.887	14.1	10	8	12	13	15	52	-2	-2	-3	-5
Iceland	.866	.955	.885	.901	.902	.901	.900	.899	1.9	11	11	10	11	11	159	0	1	0	0
Denmark	.873	.914	.916	.901	.901	.901	.900	.900	.5	12	12	11	10	10	182	0	1	2	2
Sweden	.829	.951	.917	.898	.899	.897	.895	.894	3.7	13	13	13	12	12	141	0	0	1	1
U.K.	.860	.931	.885	.892	.892	.891	.891	.890	1.2	14	17	14	14	13	169	3	0	0	1
R. of Korea	.866	.947	.863	.891	.892	.891	.890	.889	2.0	15	17	15	15	14	156	2	0	0	1
Hong Kong	.767	.975	.946	.891	.896	.888	.884	.878	12.0	16	14	17	18	19	66	-2	-1	-2	-3
Japan	.808	.978	.892	.890	.893	.888	.886	.883	6.5	17	16	16	16	17	110	-1	1	1	0
Liechtenstein	.763	.921	1	.889	.895	.885	.881	.875	13.5	18	15	19	20	22	56	-3	-1	-2	-4
Israel	.853	.951	.861	.887	.888	.887	.886	.885	2.6	19	19	18	17	16	151	0	1	2	3
France	.814	.951	.892	.884	.886	.883	.881	.879	4.3	20	20	20	19	18	134	0	0	1	2
Belgium	.813	.932	.903	.881	.883	.880	.879	.877	3.6	21	23	21	21	20	142	2	0	0	1
Luxembourg	.763	.931	.963	.881	.886	.878	.875	.870	10.9	22	20	24	24	25	78	-2	-2	-2	-3
Austria	.793	.941	.916	.881	.883	.879	.877	.875	5.9	23	22	22	23	23	118	-1	1	0	0
Finland	.816	.931	.895	.879	.881	.878	.877	.876	3.2	24	24	23	22	21	146	0	1	2	3
Slovenia	.863	.917	.845	.874	.875	.874	.874	.873	1.3	25	25	25	25	24	168	0	0	0	1
Italy	.789	.960	.874	.872	.874	.870	.868	.865	6.6	26	26	26	26	26	108	0	0	0	0
Spain	.795	.955	.864	.869	.871	.867	.866	.863	5.8	27	27	27	27	27	120	0	0	0	0
Czech R.	.866	.888	.831	.861	.862	.861	.861	.861	.8	28	28	28	28	28	177	0	0	0	0
Greece	.798	.935	.832	.853	.855	.852	.851	.849	4.5	29	31	29	29	29	132	2	0	0	0
Brunei	.693	.901	.991	.852	.862	.846	.840	.831	21.9	30	28	30	31	34	27	-2	0	-1	-4

(continued)	Rating									Ranking						Rank variation from n. HDI to			
	input indices			HDI		TM$(a,1)$, $a=$			1000	HDI		TM$(a,1)$					TM$(a,1)$		
Country	E	H	Y	new	old	-1	0	1	$\cdot A_S$	n.	o.	-1	0	1	A_S	o.*	-1	0	1
Qatar	.687	.898	1	.851	.862	.845	.838	.829	23.8	31	28	31	33	36	22	-3	0	-2	-5
Cyprus	.776	.921	.844	.845	.847	.844	.842	.840	4.7	32	32	32	30	30	128	0	0	2	2
Estonia	.858	.838	.824	.840	.840	.840	.840	.840	.3	33	34	33	32	31	186	1	0	1	2
Saudi Arabia	.723	.854	.945	.836	.841	.832	.829	.825	11.4	34	33	36	37	37	71	-1	-2	-3	-3
Lithuania	.877	.802	.826	.834	.835	.834	.834	.833	1.3	35	37	34	34	33	167	2	1	1	2
Poland	.824	.868	.811	.834	.834	.834	.834	.833	.8	36	38	35	35	32	173	2	1	1	4
Andorra	.672	.941	.907	.831	.840	.825	.819	.812	20.5	37	34	39	39	42	30	-3	-2	-2	-5
Slovakia	.803	.852	.836	.830	.830	.830	.830	.830	.6	38	40	37	36	35	181	2	1	2	3
Malta	.733	.919	.846	.829	.833	.827	.825	.822	8.1	39	39	38	38	38	97	0	1	1	1
Un. Arab Emir.	.673	.874	.961	.827	.836	.821	.816	.808	20.4	40	36	40	43	46	31	-4	0	-3	-6
Portugal	.726	.922	.829	.822	.826	.819	.817	.814	8.7	41	42	42	42	41	94	1	-1	-1	0
Chile	.746	.922	.806	.821	.825	.820	.818	.815	7.0	42	43	41	40	40	107	1	1	2	2
Hungary	.804	.840	.809	.818	.818	.817	.817	.817	.3	43	46	43	41	39	184	3	0	2	4
Bahrain	.713	.871	.872	.815	.819	.813	.811	.808	7.9	44	44	45	45	47	99	0	-1	-1	-3
Cuba	.743	.912	.799	.815	.818	.813	.811	.809	6.5	45	45	44	44	45	109	0	1	1	0
Kuwait	.646	.835	1	.814	.827	.806	.799	.789	28.2	46	41	49	49	49	8	-5	-3	-3	-3
Croatia	.769	.878	.793	.812	.813	.811	.810	.809	2.9	47	47	46	47	44	149	0	1	0	3
Latvia	.814	.802	.816	.811	.811	.811	.811	.811	.1	48	48	47	46	43	187	0	1	2	5
Argentina	.782	.866	.778	.808	.809	.807	.806	.806	2.2	49	49	48	48	48	153	0	1	1	1
Uruguay	.714	.880	.785	.790	.793	.788	.787	.785	6.2	50	51	50	51	52	112	1	0	-1	-2
Bahamas	.713	.850	.811	.789	.791	.788	.787	.785	4.6	51	52	52	52	51	130	1	-1	-1	0
Montenegro	.772	.843	.754	.789	.790	.788	.788	.787	2.0	52	53	51	50	50	158	1	1	2	2
Belarus	.819	.768	.770	.785	.786	.785	.785	.785	.7	53	55	53	53	53	178	2	0	0	0
Romania	.748	.828	.780	.785	.785	.784	.784	.783	1.5	54	56	54	54	54	164	2	0	0	0
Libya	.697	.851	.812	.784	.787	.782	.781	.779	6.0	55	54	55	55	55	116	-1	0	0	0
Oman	.604	.870	.913	.783	.796	.775	.769	.761	26.9	56	50	59	62	64	13	-6	-3	-6	-8
Bulgaria	.751	.824	.761	.778	.779	.778	.777	.777	1.4	57	57	56	56	56	165	0	1	1	1
Russian Fed.	.779	.738	.819	.778	.779	.778	.777	.777	1.5	58	57	57	57	57	162	-1	1	1	1
Barbados	.741	.852	.742	.777	.778	.776	.775	.774	3.6	59	59	58	58	58	143	0	1	1	1
Palau	.787	.806	.733	.775	.775	.774	.774	.774	1.3	60	62	60	59	59	166	2	0	1	1
Antig. & Barb.	.680	.861	.791	.774	.777	.771	.770	.767	7.6	61	60	61	60	61	100	-1	0	1	0
Malaysia	.669	.846	.814	.772	.776	.770	.768	.765	8.4	62	61	63	63	62	95	-1	-1	-1	0
Mauritius	.717	.825	.774	.771	.772	.770	.769	.769	2.6	63	64	62	61	60	150	1	1	2	3
Trinid. & Tob.	.702	.767	.836	.766	.768	.765	.764	.763	4.1	64	67	64	64	63	137	3	0	0	1
Panama	.658	.885	.770	.765	.771	.762	.759	.756	11.6	65	65	65	66	67	70	0	0	-1	-2
Lebanon	.630	.923	.769	.765	.774	.759	.755	.750	19.1	66	63	67	68	70	36	-3	-1	-2	-4
Venezuela	.681	.841	.776	.763	.766	.761	.760	.758	5.9	67	68	66	65	65	117	1	1	2	2
Costa Rica	.655	.922	.735	.763	.771	.758	.755	.750	16.1	68	66	68	69	69	45	-2	0	-1	-1
Turkey	.653	.850	.788	.759	.764	.756	.754	.752	9.4	69	69	70	70	68	91	0	-1	-1	1
Kazakhstan	.763	.716	.796	.758	.758	.757	.757	.756	1.5	70	72	69	67	66	163	2	1	3	4
Seychelles	.636	.818	.832	.756	.762	.753	.751	.748	11.3	71	71	71	71	71	73	0	0	0	0
Mexico	.639	.885	.765	.756	.763	.752	.749	.746	13.6	72	70	72	72	74	55	-2	0	0	-2

(continued)	Rating									Ranking						Rank variation from n. HDI to			
	input indices			HDI		TM$(a,1)$, $a=$			1000	HDI		TM$(a,1)$					TM$(a,1)$		
Country	E	H	Y	new	old	−1	0	1	$\cdot A_S$	n.	o.	−1	0	1	A_S	o.*	−1	0	1
Sri Lanka	.738	.835	.684	.750	.752	.748	.747	.746	5.2	73	74	73	73	73	123	1	0	0	0
St. Kitts & N.	.638	.824	.801	.750	.754	.747	.745	.742	9.7	74	73	76	76	78	88	−1	−2	−2	−4
Iran	.682	.832	.740	.749	.751	.747	.746	.745	5.1	75	75	75	75	75	124	0	0	0	0
Azerbaijan	.701	.781	.764	.748	.749	.747	.747	.747	1.6	76	76	74	74	72	161	0	2	2	4
Jordan	.699	.829	.715	.745	.748	.744	.743	.742	4.4	77	78	77	78	77	133	1	0	−1	0
Grenada	.726	.812	.701	.745	.746	.744	.743	.743	3.0	78	81	78	77	76	148	3	0	1	2
Serbia	.694	.832	.714	.744	.747	.743	.742	.740	4.8	79	80	79	79	79	126	1	0	0	0
Brazil	.662	.830	.749	.744	.747	.742	.741	.739	6.4	80	79	80	80	80	111	−1	0	0	0
Georgia	.770	.835	.639	.743	.748	.741	.739	.736	9.2	81	77	81	81	81	92	−4	0	0	0
Peru	.664	.843	.714	.737	.740	.734	.733	.731	7.4	82	82	82	82	83	102	0	0	0	−1
Ukraine	.796	.747	.666	.734	.736	.733	.732	.731	3.9	83	84	83	83	82	139	1	0	0	1
Macedonia	.643	.849	.720	.733	.737	.730	.728	.725	9.5	84	83	85	85	85	90	−1	−1	−1	−1
Belize	.691	.829	.686	.732	.735	.731	.730	.728	5.7	85	86	84	84	84	121	1	1	1	1
Bosnia & Herz.	.654	.867	.687	.730	.736	.727	.725	.722	11.2	86	85	87	87	87	75	−1	−1	−1	−1
Armenia	.702	.839	.661	.730	.734	.728	.726	.725	7.5	87	87	86	86	86	101	0	1	1	1
Fiji	.766	.766	.646	.724	.726	.722	.722	.720	4.5	88	91	88	88	88	131	3	0	0	0
Tunisia	.622	.860	.702	.721	.728	.718	.715	.712	12.7	89	88	90	91	92	60	−1	−1	−2	−3
Thailand	.607	.837	.739	.721	.728	.718	.716	.713	12.2	90	89	89	90	91	64	−1	1	0	−1
St. Vinc. & Gr.	.656	.808	.701	.719	.722	.717	.716	.715	5.4	91	94	91	89	89	122	3	0	2	2
China	.608	.851	.716	.718	.725	.714	.712	.709	13.2	92	93	93	93	94	58	1	−1	−1	−2
Dominica	.609	.887	.684	.718	.727	.713	.709	.705	17.6	93	90	95	96	96	41	−3	−2	−3	−3
Algeria	.642	.785	.730	.717	.719	.715	.714	.713	4.8	94	97	92	92	90	127	3	2	2	4
Albania	.610	.883	.683	.717	.725	.712	.708	.704	17.0	95	92	96	97	97	42	−3	−1	−2	−2
Jamaica	.667	.823	.665	.715	.718	.713	.711	.710	7.1	96	98	94	94	93	106	2	2	2	3
St. Lucia	.632	.843	.684	.714	.720	.711	.709	.707	10.4	97	96	97	95	95	82	−1	0	2	2
Ecuador	.595	.869	.696	.711	.720	.706	.703	.700	16.7	98	95	99	99	99	44	−3	−1	−1	−1
Colombia	.603	.831	.717	.711	.717	.707	.705	.703	11.7	99	99	98	98	98	68	0	1	1	1
Suriname	.590	.785	.758	.705	.711	.702	.700	.698	10.6	100	101	101	101	102	80	1	−1	−1	−2
Tonga	.722	.810	.600	.705	.711	.702	.701	.698	10.2	101	102	100	100	100	83	1	1	1	1
Dominican R.	.592	.822	.708	.701	.707	.698	.695	.693	11.9	102	103	103	104	104	67	1	−1	−2	−2
Turkmenistan	.680	.699	.717	.699	.699	.698	.698	.698	.3	103	105	102	102	101	185	2	1	1	2
Mongolia	.693	.731	.670	.698	.698	.697	.697	.697	.9	104	106	104	103	103	172	2	0	1	1
Maldives	.546	.891	.697	.697	.711	.690	.685	.680	26.1	105	100	106	106	107	15	−5	−1	−1	−2
Samoa	.702	.818	.582	.694	.701	.690	.688	.686	12.5	106	104	105	105	105	63	−2	1	1	1
Palestine	.663	.819	.596	.687	.693	.683	.681	.679	11.3	107	107	107	107	109	74	0	0	0	−2
Indonesia	.603	.782	.679	.684	.688	.682	.681	.679	7.2	108	108	108	109	108	104	0	0	−1	0
Botswana	.618	.683	.755	.683	.685	.682	.681	.680	4.2	109	110	109	108	106	135	1	0	1	3
Egypt	.574	.787	.702	.682	.688	.679	.677	.675	10.5	110	109	110	110	110	81	−1	0	0	0
Paraguay	.587	.804	.654	.676	.682	.673	.671	.669	10.7	111	111	111	111	111	79	0	0	0	0
Gabon	.588	.669	.776	.673	.678	.671	.670	.668	7.9	112	112	112	112	112	98	0	0	0	0
Bolivia	.673	.727	.607	.667	.669	.666	.666	.665	3.3	113	114	113	113	113	145	1	0	0	0
Moldova	.654	.752	.592	.663	.666	.661	.660	.659	5.8	114	116	114	114	114	119	2	0	0	0

(continued)	Rating									Ranking						Rank variation from n. HDI to			
	input indices			HDI		TM$(a,1)$, $a=$			1000	HDI		TM$(a,1)$					TM$(a,1)$		
Country	E	H	Y	new	old	-1	0	1	$\cdot A_S$	n.	o.	-1	0	1	A_S	o.*	-1	0	1
Uzbekistan	.653	.742	.598	.662	.664	.660	.660	.659	4.7	115	117	115	115	115	129	2	0	0	0
El Salvador	.553	.809	.647	.661	.670	.657	.655	.653	14.6	116	113	117	117	118	49	−3	−1	−1	−2
Philippines	.611	.749	.628	.660	.663	.658	.658	.657	4.9	117	118	116	116	116	125	1	1	1	1
Syria	.553	.839	.613	.658	.668	.652	.649	.646	19.0	118	115	119	119	119	37	−3	−1	−1	−1
South Africa	.694	.568	.720	.657	.661	.655	.654	.653	6.2	119	119	118	118	117	113	0	1	1	2
Iraq	.467	.760	.747	.642	.658	.634	.632	.628	26.4	120	120	121	121	121	14	0	−1	−1	−1
Viet Nam	.514	.861	.588	.638	.654	.631	.627	.623	27.4	121	121	122	122	123	11	0	−1	−1	−2
Guyana	.581	.712	.627	.638	.640	.637	.636	.635	3.9	122	124	120	120	120	140	2	2	2	2
Cape Verde	.483	.848	.627	.636	.653	.627	.624	.620	29.0	123	122	123	124	126	6	−1	0	−1	−3
Micronesia	.610	.754	.544	.630	.636	.627	.626	.625	10.0	124	125	124	123	122	85	1	0	1	2
Guatemala	.484	.802	.639	.628	.642	.622	.619	.616	22.5	125	123	127	127	127	24	−2	−2	−2	−2
Kyrgyzstan	.657	.731	.515	.628	.634	.624	.623	.622	11.2	126	126	125	125	124	76	0	1	1	2
Namibia	.521	.684	.683	.624	.629	.622	.621	.620	8.3	127	131	126	126	125	96	4	1	1	2
Timor-Leste	.472	.731	.691	.620	.631	.615	.613	.611	18.5	128	129	128	128	128	39	1	0	0	0
Honduras	.506	.828	.562	.618	.632	.611	.608	.604	24.3	129	127	130	131	131	19	−2	−1	−2	−2
Morocco	.469	.784	.640	.617	.631	.611	.609	.606	22.5	130	130	129	129	129	25	0	1	1	1
Vanuatu	.594	.794	.495	.616	.628	.610	.608	.605	19.9	131	132	131	130	130	35	1	0	1	1
Nicaragua	.485	.844	.567	.615	.632	.606	.603	.599	29.0	132	127	132	132	134	5	−5	0	0	−2
Tajikistan	.641	.727	.482	.608	.617	.604	.602	.601	14.3	133	133	133	133	132	51	0	0	0	1
Kiribati	.602	.752	.495	.607	.616	.603	.602	.600	14.7	134	134	134	134	133	48	0	0	0	1
India	.472	.714	.595	.585	.594	.582	.580	.579	13.2	135	137	135	135	135	57	2	0	0	0
Cambodia	.496	.799	.504	.585	.600	.578	.575	.573	24.2	136	136	136	136	137	20	0	0	0	−1
Bhutan	.421	.743	.637	.584	.600	.577	.575	.573	25.1	137	135	137	137	136	18	−2	0	0	1
Ghana	.553	.633	.538	.573	.575	.573	.572	.572	2.3	138	141	138	138	138	152	3	0	0	0
Lao	.437	.743	.570	.570	.583	.564	.563	.561	20.7	139	138	139	140	140	29	−1	0	−1	−1
Congo	.512	.597	.588	.564	.566	.564	.564	.564	2.0	140	143	140	139	139	155	3	0	1	1
Zambia	.592	.586	.509	.561	.562	.560	.560	.560	2.0	141	144	141	141	141	157	3	0	0	0
S. Tome & Pr.	.471	.713	.519	.559	.568	.555	.554	.553	13.9	142	142	142	142	142	54	0	0	0	0
Bangladesh	.448	.779	.499	.558	.575	.551	.549	.547	25.9	143	140	143	143	143	16	−3	0	0	0
Eq. Guinea	.416	.509	.815	.557	.580	.547	.545	.542	35.3	144	139	144	144	144	2	−5	0	0	0
Nepal	.451	.745	.467	.539	.554	.533	.532	.531	22.3	145	146	146	146	146	26	1	−1	−1	−1
Pakistan	.371	.716	.580	.536	.556	.529	.528	.527	27.7	146	145	147	148	148	10	−1	−1	−2	−2
Kenya	.516	.642	.464	.536	.541	.534	.533	.533	7.3	147	148	145	145	145	103	1	2	2	2
Swaziland	.551	.446	.606	.530	.534	.528	.528	.528	6.1	148	149	148	147	147	114	1	0	1	1
Angola	.473	.491	.626	.526	.530	.524	.524	.524	6.0	149	150	149	149	149	115	1	0	0	0
Myanmar	.372	.695	.557	.524	.541	.518	.517	.517	23.9	150	147	150	150	150	21	−3	0	0	0
Rwanda	.477	.678	.399	.505	.518	.501	.500	.500	17.6	151	152	153	153	153	40	1	−2	−2	−2
Cameroon	.486	.539	.490	.504	.505	.504	.504	.504	.8	152	156	151	151	151	176	4	1	1	1
Nigeria	.423	.500	.601	.503	.508	.501	.501	.501	7.1	153	155	152	152	152	105	2	1	1	1
Yemen	.339	.663	.555	.500	.519	.493	.494	.494	25.4	154	151	154	154	154	17	−3	0	0	0
Madagascar	.459	.688	.391	.498	.513	.493	.492	.492	20.3	155	153	155	155	155	32	−2	0	0	0
Papua N. Guin.	.377	.653	.483	.492	.504	.487	.487	.487	16.9	156	157	157	157	157	43	1	−1	−1	−1

(continued)	Rating									Ranking						Rank variation from n. HDI to			
	input indices			HDI		TM$(a,1)$, $a=$			1000	HDI		TM$(a,1)$					TM$(a,1)$		
Country	E	H	Y	new	old	-1	0	1	$\cdot A_S$	n.	o.	-1	0	1	A_S	o.*	-1	0	1
Zimbabwe	.498	.613	.388	.491	.500	.488	.488	.488	11.4	157	160	156	156	156	72	3	1	1	1
Solomon Isl.	.406	.733	.397	.491	.512	.483	.483	.482	29.4	158	154	161	161	162	4	−4	−3	−3	−4
Tanzania	.426	.639	.428	.488	.498	.485	.485	.485	12.6	159	161	159	159	159	62	2	0	0	0
Comoros	.449	.629	.410	.487	.496	.484	.484	.484	11.6	160	162	160	160	160	69	2	0	0	0
Lesotho	.505	.453	.503	.486	.487	.486	.486	.486	.8	161	166	158	158	158	174	5	3	3	3
Mauritania	.351	.639	.513	.486	.501	.482	.482	.482	18.9	162	158	162	162	161	38	−4	0	0	1
Senegal	.369	.668	.465	.486	.501	.481	.481	.481	19.9	163	159	164	164	164	34	−4	−1	−1	−1
Uganda	.480	.603	.391	.484	.491	.481	.481	.481	10.0	164	164	163	163	163	84	0	1	1	1
Benin	.412	.605	.430	.475	.482	.473	.473	.473	9.7	165	168	165	165	165	89	3	0	0	0
Togo	.516	.562	.366	.473	.481	.471	.471	.472	9.8	166	169	166	166	166	87	3	0	0	0
South Sudan	.306	.647	.534	.473	.496	.466	.468	.469	28.1	167	163	168	167	167	9	−4	−1	0	0
Haiti	.374	.663	.422	.471	.486	.466	.466	.466	19.9	168	167	167	168	168	33	−1	1	0	0
Afghanistan	.365	.630	.445	.468	.480	.464	.464	.464	15.8	169	170	169	169	169	46	1	0	0	0
Djibouti	.304	.643	.519	.466	.489	.460	.461	.463	27.2	170	165	170	170	170	12	−5	0	0	0
Côte d'Ivoire	.391	.473	.502	.453	.455	.452	.452	.452	3.1	171	172	171	171	171	147	1	0	0	0
Gambia	.346	.597	.415	.441	.453	.438	.438	.439	14.4	172	173	172	172	172	50	1	0	0	0
Ethiopia	.316	.671	.388	.435	.458	.429	.430	.430	28.8	173	171	173	173	173	7	−2	0	0	0
Malawi	.440	.543	.297	.414	.427	.411	.413	.414	13.9	174	175	174	174	174	53	1	0	0	0
Liberia	.366	.624	.305	.411	.432	.407	.408	.409	23.7	175	174	175	175	175	23	−1	0	0	0
Mali	.306	.539	.409	.407	.418	.405	.406	.407	12.1	176	177	176	176	176	65	1	0	0	0
Guinea-Bissau	.327	.528	.361	.396	.405	.395	.395	.396	9.9	177	180	177	177	177	86	3	0	0	0
Guinea	.295	.556	.368	.392	.406	.390	.391	.392	15.5	178	179	179	179	178	47	1	−1	−1	0
Mozambique	.371	.465	.349	.392	.395	.391	.392	.392	3.3	179	182	178	178	179	144	3	1	1	0
Burundi	.371	.525	.304	.390	.400	.388	.389	.390	11.1	180	181	180	180	181	77	1	0	0	−1
Burkina Faso	.252	.559	.419	.389	.410	.386	.389	.391	21.4	181	178	181	181	180	28	−3	0	0	1
Eritrea	.227	.659	.369	.381	.418	.375	.378	.381	40.1	182	176	182	182	182	1	−6	0	0	0
Sierra Leone	.305	.393	.438	.374	.379	.374	.374	.375	4.2	183	184	183	183	183	136	1	0	0	0
Chad	.256	.480	.421	.373	.386	.371	.373	.375	12.6	184	183	184	184	184	61	−1	0	0	0
Central Afr. R.	.317	.464	.268	.340	.350	.339	.341	.342	9.0	185	187	185	185	186	93	2	0	0	−1
Congo (D. R.)	.373	.461	.225	.338	.353	.337	.340	.342	13.1	186	186	186	186	185	59	0	0	0	1
Niger	.197	.591	.327	.336	.372	.334	.338	.342	33.7	187	185	187	187	187	3	−2	0	0	0