AN AXIOMATIC APPROACH TO DECOUPLING INDICATORS FOR GREEN GROWTH

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Abstract. The concept of decoupling was introduced to measure and analyze the controversial trade-off between economic development and environmental sustainability; in particular, several empirical studies concern the construction and use of decoupling indicators. We elaborate on a descriptive comparison by Conte Grand (2016) of the three main ones, respectively D_O by OECD (2002), D_ϵ by Tapio (2005), and D_t by Lu, Wang, and Yue (2011), and introduce an axiomatic approach into the subject that articulates in the identification of some properties that appear indispensable or at least desirable for any decoupling indicator and in the assessment of their validity for the indices under scrutiny and/or in the construction of new indices that satisfy them. A graphical examination of the aggregation function level sets in the Cartesian plane is a relevant part of the method. Under such analysis, the index D_O turns out to show milder defects than D_ϵ and D_t. We then propose a suitable modification D_N in order to remove the defects and fulfill all the given compatible axioms; in particular, D_N is cumulative over sub-periods.

It may also be opportune for a decoupling index to differentiate the treatment of the case when, during economic growth, environmental stress (e.g., polluting emissions) decreases from the case when it increases, although less than economy (the so-called absolute and relative decoupling, respectively), as well as to capture the rebound effect phenomenon, whereby the efforts to reduce environmental intensity may eventually result in a smaller overall environmental improvement than predicted or intended. To this end we build another index D_P by applying to D_N a correction (that can be calibrated via a global parameter) for the distance from what we define symmetric decoupling, the case when the variations of economy and of environmental pressure are inversely proportional.

We conclude by testing the novel indices D_N and D_P against D_O on data from OECD (2017) of 103 world countries for the most recent completely available decade 2003–2013.

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1. Introduction

Since its introduction by Zhang (2000) in the study of environmental costs of China's economic take-off, the term *decoupling* refers to the breaking of the link between economic development and ecological unsustainability. Sometimes also referred to as *delinkage* or *delinking* (de Bruyn, 2000), the concept is now widespread in the political and institutional context as a desirable goal in view of the ever increasing challenges posed by climatic change. Indeed an intense debate is in progress on the implementation of effective strategies to reduce the potential trade-off between economic progress and environmental protection (Selin, 2016).

The environmental stress caused by economic activities can be assessed by focusing either on the consumption of primary raw materials such as water, minerals, and fossil fuels, or on the environmental impact of water, land or air pollutants. Correspondingly (UNEP, 2011) one speaks about *resource decoupling*, respectively *impact decoupling*.

In a situation of economic growth, another distinction (OECD, 2002) is between *absolute* (or *strong*) *decoupling* and *relative* (or *weak*) *decoupling*, according if the environmental stress decreases or it increases, although less than economy. In the economic growth literature the former alternative is also referred to as *green growth* to emphasize a sustainable path for economic development.

A phenomenon to be taken into account when dealing with decoupling is the *rebound* (or *take-back*) *effect* (Sorrell, 2009), according to which the efforts to reduce energy intensity may eventually result in a smaller overall energy saving than predicted or intended. For instance, the diminution of energy cost per production unit of a certain good can reduce its price and thereby stimulate the demand either of that good, now less expensive, or of other goods with the saved money. The additional energy consumption thus generated may partially or totally compensate—or even overcompensate (the so-called Jevons' paradox)—the initial decrease.

Several factors are involved in the above-mentioned issues, such as consumer behavior (Huang and Rust, 2011), innovation (Rennings, 2000), and market regulation (Porter and van der Linde, 1995), and such complexity requires a careful study of the quantification of decoupling. To this end there are contributions about the estimation of the environmental Kuznets curve (Vehmas, Luukkanen, and Kaivo-oja, 2007), a decomposition analysis of the environmental pressure (Diakoulaki and Mandaraka, 2007), and decoupling indicators, which are the object of the present investigation.

The three most popular decoupling indicators, respectively D_O by OECD (2002), D_ϵ by Tapio (2005), and D_t by Lu, Wang, and Yue (2011), are compared in a recent descriptive analysis by Conte Grand (2016). In the present article we change the methodological perspective by adopting an axiomatic approach. As happened in the debate on the operationalization of the human development concept through the study and revision of the United Nations' HDI Index (UNDP, 2010), see for instance Klugman, Rodríguez, and Choi (2011), Zambrano (2014), and Casadio Tarabusi and Guarini (2016), the normative characterization of an index may enable to better assess its measurement capability, coherence with the intended characteristics of the phenomenon under scrutiny, correct interpretation, and practical usability. Consequently such an approach tends to be all the more useful as the phenomenon is theoretically complex and politically sensitive, as is the case with decoupling; in particular, it permits to detect relevant features and possible defects of existing indicators and propose new ones that better satisfy the identified axioms. A relevant part of the method followed here consists in the graphical examination of level sets of the aggregation function in the Cartesian plane.

The indices D_ϵ and D_t, mutual algebraic complements to 1, turn out to suffer from several significant defects, among which the instability of index values in case of economic stagnation, the incomplete monotonicity with respect to input variables (greater economic growth coupled with greater environmental improvement may yield worse index values), and the impossibility for index values to yield meaningful rankings because they are unable to distinguish what may be called *brown degrowth* (economic decline with increasing environmental pressure—the least desirable combination) from green growth (the most desirable one); numerical values need to be complemented, and were indeed introduced, with categorical labels in order to separate various types of decoupling situations. These and other structural problems make the two indices unfit for further consideration in our axiomatic approach.

The index D_O does not present the same defects, yet it shows some less severe weaknesses such as metric inhomogeneity (the significance of a difference between index values depends on the values location on the real line) or non cumulativeness (the index values of consecutive periods do not add up for the overall period). We illustrate the analyzed defects of each index with examples from OECD (2017).

By suitably modifying D_O we propose a novel decoupling index D_N that overcomes these and other disadvantages and fulfills the corresponding axiomatic properties; in particular, while yielding the same meaningful rankings as D_O, it is both metrically homogeneous and cumulative.

In the aforementioned literature on HDI and, in particular, on the trade-offs among its input variables it was proposed to adjust the original synthetic index, a simple arithmetic mean, with a penalization that increases with the disequilibrium among input variables. In the same vein, we propose another decoupling indicator D_P obtained by applying to D_N the larger correction, the farther the situation from what we define as *symmetric decoupling*, the case where economy varies in inverse proportion to environmental pressure. The amount of the correction can be overall adjusted by means of a real parameter c (the value $c = 1$ is recommended); in the limit case $c = 0$ the index D_P reduces to the simpler, uncorrected D_N. The index D_P, unlike D_O, manages to take into account the duality of absolute versus relative decoupling, penalizing the former less than the latter, as well as the rebound effect described above, that similarly results in less symmetric decoupling than intended or expected, thence in stiffer index penalization. The indicators D_N and D_P may constitute an improvement in terms of policy relevance and analytical soundness, "key principles in selecting indicators to monitor progress with green growth" (OECD, 2011, Box 1 in §1).

Finally we test the novel indices D_N and D_P against D_O and the resulting rankings on data again from OECD (2017) for 103 world countries in the period 2003–2013, the most recent decade for which the data are presently complete.

The contribution of the present article to the study of decoupling indicators is manifold: an axiomatic approach is introduced, with an important graphical examination side; a list of desirable properties is compiled; their validity is tested for the three most used indices; two novel indices are proposed that better fulfill such properties. The structure is the following: in Section 2 the three indicators are defined and analyzed critically; in Section 3 some axiomatic properties are introduced and new indices are proposed; in Section 4 various indicators are compared by applying them to real data; the conclusions are in Section 5.

2. The main decoupling indicators

2.1. **Definitions.** All the sequel may indifferently be applied to either impact or resource decoupling. For a given country at time j, let Y_j be the Gross Domestic Product (or a similar index related to economic progress), H_j the level of environmental pressure, and $T_j = H_j/Y_j$ the resulting environmental intensity. The three quantities are all intrinsically positive. The respective variation rates with respect to time $j-1$ are

$$\begin{aligned} y &= \frac{Y_j}{Y_{j-1}} - 1, \\ h &= \frac{H_j}{H_{j-1}} - 1, \end{aligned} \qquad t = \frac{T_j}{T_{j-1}} - 1 = \frac{h+1}{y+1} - 1. \tag{2.1}$$

The pair (y, h) (in terms of which the variable t can be uniquely obtained) can be represented on a Cartesian plane as in Figure 2.1, which shows a subdivision corresponding to the list of thirteen cases described in Conte Grand (2016, Table 5, where the two variables are denoted by g, e respectively):

- six open regions into which the plane is divided by the two coordinate axes and by the line $y = h$ of *perfect coupling*, where the two variation rates coincide;
- six half-lines into which those three lines are parted by the axes origin;
- the origin itself (where $y = h = 0$).

The figure depicts the yearly variations y of real GDP and h of production-based CO_2 emissions of 103 world countries in the year 2009 (with respect to the preceding year) according to OECD (2017). The cloud of points extends for a substantial number of units over each of the six open plane regions (and some points may be located arbitrarily close to each of the six half-lines as well as to the origin), showing that each theoretically possible combination of signs for y, h and their difference $y - h$ may actually occur and cannot be neglected. The variables y and h are only bound by the constraints

$$y, h > -1 \tag{2.2}$$

following by (2.1).

We now present three of the main decoupling indicators used in the literature. The first was introduced in OECD (2002) and widely used thereafter, e.g., in Lu, Lin, and Lewis (2007), de Freitas and Kaneko (2011), Yu, Chen, Zhu, and Hu (2013), and Conrad and Cassar (2014):

$$D_O = -t = 1 - \frac{h+1}{y+1}, \tag{2.3}$$

therefore subject to the constraint

$$D_O < 1.$$

The second indicator that we shall consider, appeared in Tapio (2005) and applied in several other studies, such as Zhang and Wang (2013), Tang, Shang, Shi,

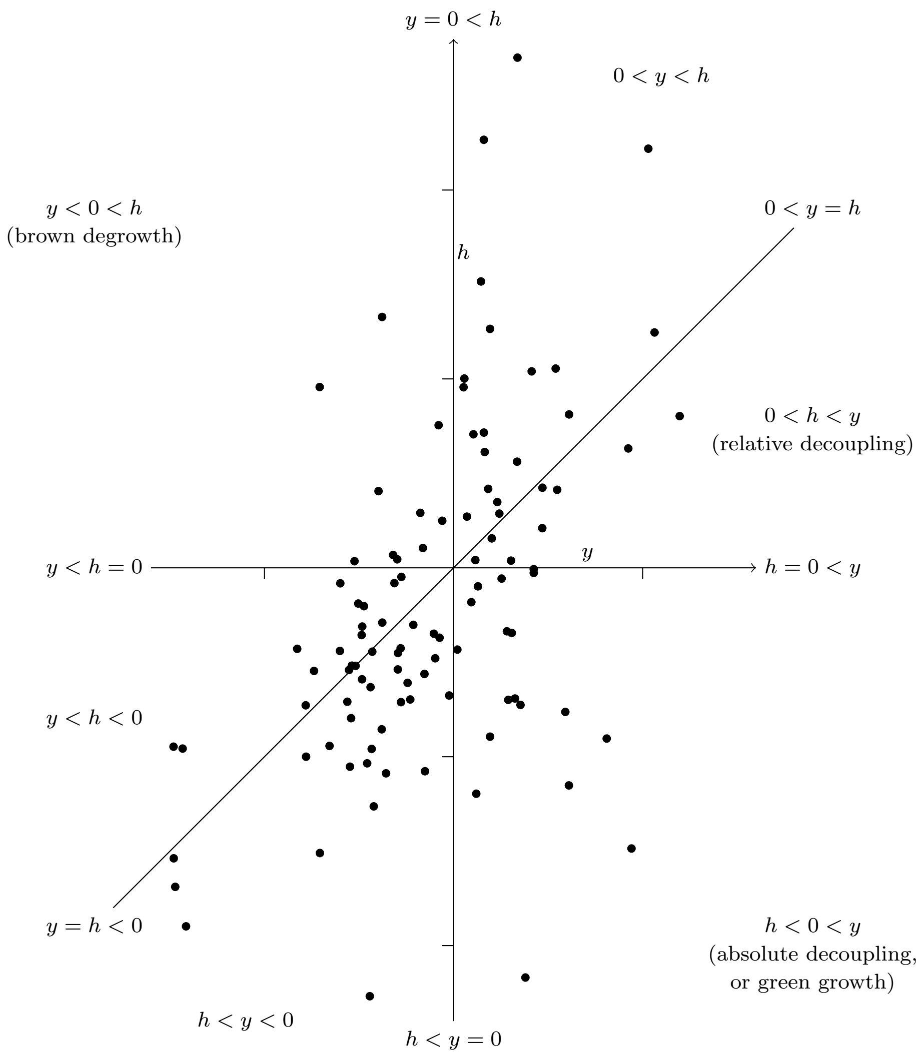


FIGURE 2.1. The (y, h)-plane subdivision and the cloud of world countries data for the year 2009 from OECD (2017). (Ticks are .1 units apart on both axes.)

Liu, and Bi (2014), Lu, Yang, Huang, Chuai, and Wu (2015), Zhang, Song, Su, and Sun (2015), Wan, Wang, and Ng (2016), and Zhao, Zhang, and Shao (2016), is

$$D_\epsilon = \frac{h}{y}, \tag{2.4}$$

without theoretical constraints. In economic terms it is the elasticity of environmental pressure with respect to economic growth.

The third decoupling indicator, proposed in Lu, Wang, and Yue (2011) and employed in Wang, Hashimoto, Yue, Moriguchi, and Lu (2013), may be defined as the ratio of the environmental intensity variation rate t over the GDP inverse variation rate $Y_{j-1}/Y_j - 1$. As shown in Conte Grand (2016, equation (14)), it can

TABLE 2.1. Some data from OECD (2017) to illustrate some defects of the indices D_ϵ and D_t. (All values rounded to the second decimal.)

Defect	Country	Year	y	h	D_ϵ	D_t
(a)	Malta	2003	+.13%	+12.50%	+94.63	−93.63
	Azerbaijan	2011	+.07%	+12.37%	+187.59	−186.59
	Denmark	2003	+.39%	+10.37%	+26.60	−25.60
	Trinidad and Tobago	2010	−.09%	+10.76%	−115.85	+116.85
	Mexico	2001	−.03%	+.24%	−7.28	+8.28
	Netherlands	2002	+.10%	+.19%	+1.80	−.80
(b)	Kyrgyzstan	2002	−.02%	+26.06%	−1500.05	+1501.05
	Germany	2002	.00%	−1.63%	N.A.	N.A.
	Ukraine	2013	.00%	−3.22%	N.A.	N.A.
(c)	Uruguay	1994	+7.28%	−7.31%	−1.00	+2.00
	Paraguay	2009	−3.97%	+4.06%	−1.02	+2.02
(d)	Panama	2007	+15.32%	−3.50%	−.23	+1.23
	Slovak Republic	2007	+10.80%	−2.14%	−.20	+1.20
	Finland	2007	+5.18%	−3.00%	−.58	+1.58
	Ireland	2007	+3.80%	−1.68%	−.44	+1.44

be written

$$D_t = 1 - \frac{h}{y} = 1 - D_\epsilon, \tag{2.5}$$

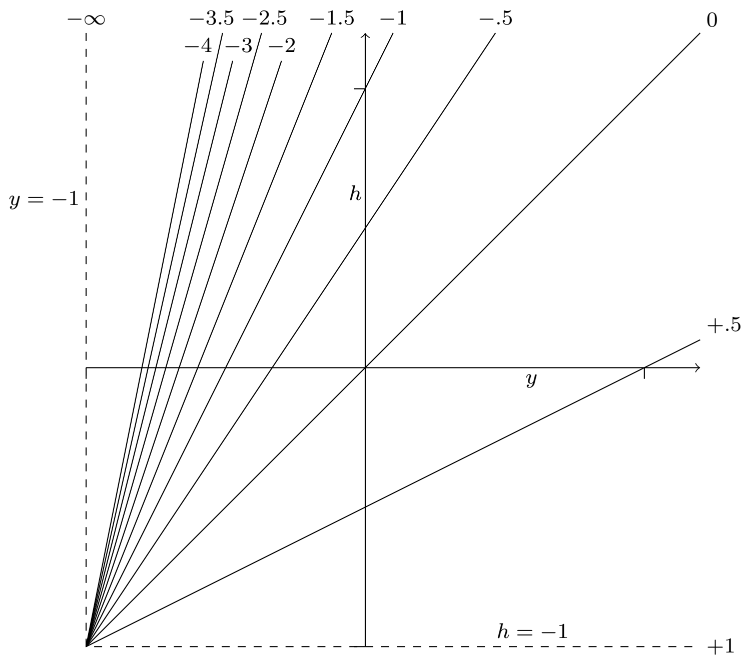
thereby it is equivalent to D_ϵ up to a linear transformation, although a higher value of D_ϵ corresponds to a lower value of D_t and vice versa. Hence every consideration on numerical indices automatically carries over from D_ϵ to D_t with the obvious changes.

In Figure 2.2 some equispaced level sets for each of the indices D_O (panel A) and D_ϵ (panel B) are depicted in the (y, h)-plane, keeping in mind constraints (2.2) and relation (2.5). In the sequel we discuss some corresponding features of the three indices.

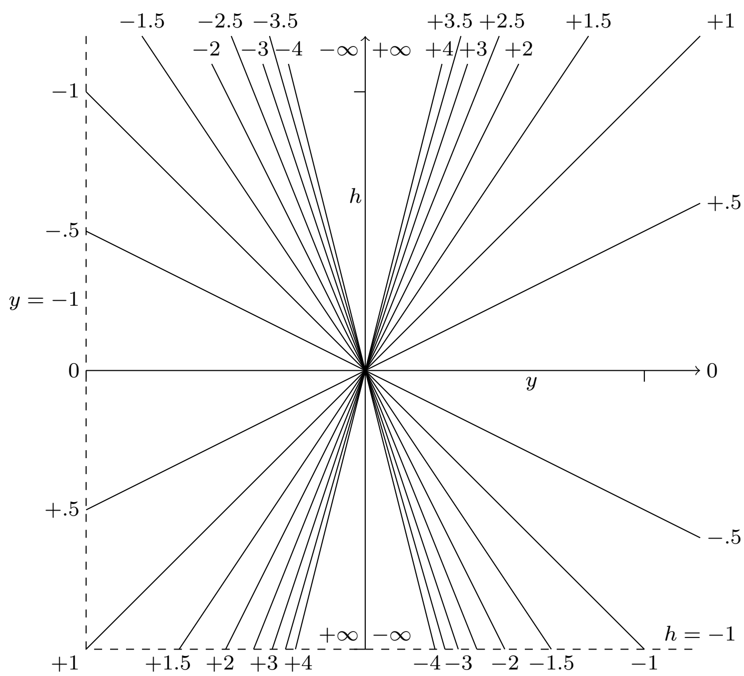
2.2. **Critical analysis of the indicators by Tapio and by Lu, Wang, and Yue.** The indices D_ϵ and D_t, due to their definitions (2.4) and (2.5) and as can be seen in Figure 2.2B, are affected by relevant defects that impair their usability. In order to exemplify these points, again we use some data of production-based CO_2 emissions for h and real GDP for y taken from OECD (2017) and listed in Table 2.1.

(a) *Instability for small growth/degrowth.* Both indices are very unstable when y (positive or negative) is close to zero. In Figure 2.2B the level sets for index values that tend to $+\infty$ or $-\infty$ accumulate (from opposite sides) around the h axis.

If h is not itself close to zero, the value of either index takes a (possibly very) large value, positive or negative, and is extremely unstable for small variations of y. For example, Malta in the year 2003 and Azerbaijan in 2011 have similar input values y and h, but the resulting indices are large and very different. This lack of robustness may unduly amplify the effect on the statistical results of any measurement errors, unavoidable in the collection of input values.



A. The index D_O



B. The index D_ϵ

FIGURE 2.2. Some equispaced levels of the indices D_O (upper panel) and D_ϵ (lower panel) in the (y, h)-plane. (Ticks are 1 unit apart on both axes.)

Even worse, in the case of Denmark in 2003 and Trinidad and Tobago in 2010, which have similar variations for emissions and small y values of opposite signs, the corresponding large values of either index also have opposite signs.

The least robust situation occurs when h is also close to zero, because then either index can take any possible value (not only large ones—cf. Figure 2.2B where all level lines intersect at the axes origin) and a small variation of either variable may translate into an unpredictable jump in the index value, as happens comparing the similar performances of Mexico in 2001 and the Netherlands in 2002.

Therefore such indices are little usable during periods of economic crisis and near-zero growth such as the current one.

(b) *Non-existence for zero growth.* Besides the possibility of abnormally large index values with non-outlying variable values, as with Kyrgyzstan in 2002, it may even happen that D_ϵ and D_t lose sense due to a division by zero, as for Germany in 2002 or Ukraine in 2013, which had no variation in the GDP.

(c) *Same values to green growth and brown degrowth.* Another defect, again due to their definitions, is that the two indices may take the same respective values in, and are thus unable to distinguish between, two opposite situations: one is the ideal, green growth situation of $h < 0 < y$ (lower right quadrant in Figure 2.1), where decreasing environmental impact accompanies economic progress, while the other is the worst scenario of $y < 0 < h$ (upper left quadrant in Figure 2.1), when pollution increases during an economic recession, a situation which may be called brown degrowth. Indeed, as visible in Figure 2.2B, the level set of each negative index value stretches across both quadrants.

For instance, Uruguay in the year 1994 (in green growth) and Paraguay in the year 2009 (in brown degrowth) receive almost identical values of D_ϵ as well as of D_t. Thus the employment of either of these two numerical indices may convey ambiguous information. This aspect weakens their policy relevance which would require them to "be easy to interpret and transparent, i.e. users should be able to assess the significance of the values associated with the indicators" (OECD, 2011, Box 1 in §1).

The confusion between the two diametrically opposed situations carries over to the categorical classification that accompanies the index D_t: indeed both conditions are labeled "absolute decoupling", with no hint to the respective different desirability. Instead, the index D_ϵ tries to overcome the numerical ambiguity by framing the two situations into different categories: "strong decoupling" if $h < 0 < y$ and "strong negative decoupling" if $y < 0 < h$. Yet, this addition does not solve the problem: from the theoretical point of view, an index value by its own nature is supposed to contain sufficiently exhaustive information in itself, without the need for additional data such as verbal labels, sign of input values, or anything else; from the practical point of view, any addition is likely to make the building of an index-based ranking cumbersome if at all possible.

(d) *No ranking for green growth.* Even worse, also if one restricts attention to green growth situations for $h < 0 < y$, it is unclear whether a higher value of either index is preferable to a lower one or vice versa. This can be read out from Figure 2.2B noting that the slope of level sets in the green growth quadrant—as well as in the brown degrowth quadrant—ought to be opposite (lower left to upper right), as it is in the other two quadrants.

As an instance, Panama, the Slovak Republic, Finland, and Ireland all are green-growth cases in the year 2007, yet Panama's situation is certainly more desirable than the other three, it having the largest growth along with the greatest environmental improvement, whereas, for opposite reasons, Ireland's situation is the least desirable in this group; nevertheless, Panama's and Ireland's values of D_ϵ and of D_t are intermediate between Finland's and the Slovak Republic's. Similar examples may easily be made for $y < 0 < h$. This fact makes the derived rankings meaningless, because the best (or the worst) performing unit in a given group in terms of decoupling may get an intermediate index value between the extremes, instead of the highest or the lowest as expected. Thus the indices D_ϵ and D_t fail to comply with another requirement, namely to "provide a basis for comparisons across countries" (OECD, 2011, Box 1 in §1). For the same reasons, in many cases policy makers would be confused as to whether their objective should be to raise or lower their countries' or regions' index values, so neither index can be used in practice to evaluate or monitor the effectiveness of an environmental strategy because it cannot assess the achievement of policy targets.

The numerical indices D_ϵ and D_t thereby lose most of their sense, and seem to maintain some utility only when h and y are both positive and y is not close to zero, that is, in a case of significant economic growth accompanied by increasing pollution, for which the decoupling can be at most relative. Yet, this scope is too limited because it includes neither any degrowth nor green growth—cf. Figure 2.1 where more than half of world countries for the year 2009 lie in the $y < 0$ area and several have y very close to zero.

2.3. **Critical analysis of the indicator by OECD.** The index D_O does not suffer from any of the disadvantages discussed for D_ϵ and D_t. Indeed:

- it is stable for small growth/degrowth;
- it exists for zero growth;
- it distinguishes between green growth and brown degrowth (taking positive values in the former case, negative in the latter);
- it suitably ranks countries for all values of y and h.

Nevertheless, the index D_O has also some weak points, although less severe than D_ϵ and D_t. We shall refer to Figure 2.2A.

(e) *Bounded range of values.* By (2.3) all values of D_O must be smaller than 1, which provokes a value compression across better decoupling situations; on the opposite side, there is no lower bound on the values of D_O, so that worse decoupling situations are set farther apart from each other by index values. Indeed, level lines in Figure 2.2A rarefy towards the lower right as the index value tends to its upper bound 1, whereas they increasingly accumulate to the upper left as the value tends to $-\infty$. We shall further illustrate this problem in Section 4.

(f) *Metric inhomogeneity.* As stated in Casadio Tarabusi and Guarini (2013, Step 6 of §2), for the construction of any composite indicator it is desirable that its input values enjoy a satisfactory metric homogeneity: for each input variable, like value differences between units should carry like significances independently of the values themselves; in Casadio Tarabusi and Palazzi (2004) some procedures are described that achieve this goal. The variables (2.1) used in the decoupling indices discussed in this article are variation rates, therefore, as such, they are

TABLE 2.2. Some data from OECD (2017) to illustrate some weak points of the index D_O. (All values rounded to the second decimal.)

Defect	Country	Period/Year	y	h	D_O
(g)	Azerbaijan	1990–1995	-58.14%	-39.48%	$-.45$
	Azerbaijan	1995–2000	$+40.67\%$	-15.64%	$+.40$
	Azerbaijan	1990–2000	-41.11%	-48.94%	$+.13$
(h)	Cuba	2006	$+12.07\%$	$+1.56\%$	$+.09$
	Sweden	2006	$+4.69\%$	-5.05%	$+.09$

intrinsically metrically inhomogeneous. For instance, the difference between $y = -50\%$ (a halving of the GDP) and $y = 0$ (unchanged GDP) does not carry the same significance as the difference between $y = 0$ and $y = +50\%$ (when GDP increases by one half); indeed, considering the performance of a country in two successive periods, in order to compensate $y = -50\%$ in the former, one needs $y = +100\%$ in the latter.

An index should itself enjoy a satisfactory metric homogeneity, meaning, for example, that like differences between values of one input variable (assuming it metrically homogeneous and the other variables kept constant) yield like differences between the resulting index values. This is not the case with D_O: for instance, by the preceding discussion, the pair $(y, h) = (0, 0)$ (unchanged values for GDP and emissions) is equidistant from the pairs $(-50\%, 0)$ and $(+100\%, 0)$, but the resulting value of D_O, namely 0, is not equidistant between the resulting values -1 and $+.5$. Note that the existence of a bound to the range of index values, Defect (e), is incompatible with metric homogeneity, because input values are themselves unbounded.

(g) *Non cumulativeness.* Another desirable feature—related to, but distinct from, metric homogeneity—of a decoupling index is cumulativeness, for which the index value for a country over a certain period (for instance a decade) equals the sum of index values for the same country over component sub-periods (for instance the two halves of the decade).

This property is not fulfilled by D_O. Indeed, in the case of Azerbaijan over the period 1990–2000 (see Table 2.2) the algebraic sum of the two index values for the component five-year periods 1990–1995 and 1995–2000 is negative, suggesting that the positive performance in the latter half of the decade has not fully compensated the negative performance of the former half; nevertheless, the index value for the entire decade turns out to be positive, indicating that the compensation has been more than complete. The consequent interpretation of data may therefore be misleading.

(h) *Same values to absolute and relative decoupling.* As observed in Conte Grand (2016), for growing economies (i.e., for $y > 0$) the index D_O takes the same range, between 0 and 1, of values both if emissions increase less than GDP and if they decrease. In Figure 2.2A, this corresponds to the fact that the level set of each positive index value stretches across the lower right (absolute decoupling) quadrant and the lower 45° half (relative decoupling) of the upper right quadrant.

For instance, in the year 2006 Cuba and Sweden nearly shared the same index value (cf. Table 2.2), although Cuba was in relative and Sweden in absolute decoupling.

3. Novel proposals for a decoupling indicator

3.1. **Axiomatic approach.** According to the approach in Conte Grand (2016), the quality of decoupling indicators—individually or comparatively—may be assessed by examining their values in each of the thirteen cases recalled earlier in Subsection 2.1 (and depicted in Figure 2.1) that are distinguished according to the signs of y, h and their difference $y - h$, cf. Conte Grand (2016, Tables 1–5). In spite of (or, perhaps, due also to) the high number of such possible "decoupling situations", this approach does not seem to detect problems such as those highlighted in Section 2 above for the indices considered.

In this article we shall adopt an axiomatic approach analogous to Chakravarty (2003), Casadio Tarabusi and Guarini (2013), and Casadio Tarabusi and Guarini (2016) for the Human Development Index. We shall therefore focus on the properties of the aggregation function that yields the index value as output in terms of the two input variables. In view of the discussion on metric homogeneity made earlier about Defect (f), we implement a standard procedure to achieve metric homogeneity for the input variables, by taking the natural logarithm of the variations; more precisely, rather than y and h as in (2.1), as metrically homogeneous input variables we shall henceforth use

$$\begin{aligned} \tilde{y} &= \log \frac{Y_{j+1}}{Y_j} = \log(y+1), \\ \tilde{h} &= \log \frac{H_{j+1}}{H_j} = \log(h+1). \end{aligned} \tag{3.1}$$

For each decoupling index D we shall denote by F, respectively $\tilde{F}$, the aggregation function that gives the index value in terms of the variable pair (y, h), respectively of $(\tilde{y}, \tilde{h})$, so that

$$D = F(y,h) = \tilde{F}(\tilde{y}, \tilde{h}).$$

With this notation, by (2.3), (2.4), (2.5), and (3.1), the aggregation functions corresponding to the indices D_O, D_ϵ, and D_t discussed heretofore are respectively the following:

$$\begin{aligned} D_O &= F_O(y,h) = 1 - \frac{h+1}{y+1} &&= \tilde{F}_O(\tilde{y}, \tilde{h}) = 1 - e^{\tilde{h}-\tilde{y}}, \\ D_\epsilon &= F_\epsilon(y,h) = \frac{h}{y} &&= \tilde{F}_\epsilon(\tilde{y}, \tilde{h}) = \frac{e^{\tilde{h}} - 1}{e^{\tilde{y}} - 1}, \\ D_t &= F_t(y,h) = 1 - \frac{h}{y} &&= \tilde{F}_t(\tilde{y}, \tilde{h}) = 1 - \frac{e^{\tilde{h}} - 1}{e^{\tilde{y}} - 1}. \end{aligned}$$

Note in passing that the symmetry relation between $\tilde{y}$ and $-\tilde{h}$ of the aggregation function $\tilde{F}$ of an index D given by

$$\tilde{F}(\tilde{y}, -\tilde{h}) = \tilde{F}(\tilde{h}, -\tilde{y}) \tag{3.2}$$

holds for the index D_O but not for the indices D_ϵ and D_t. Thus, for instance, in the green growth case (namely $\tilde{h} < 0 < \tilde{y}$), exchanging the improvements $\tilde{y}$ (for GDP) with $-\tilde{h}$ (for emissions, whose decrease is indeed considered an improvement) does not change the index value D_O.

The variables $\tilde{y}$ and $\tilde{h}$ take the same signs as y, respectively as h. Yet, the possible values of y and h are all larger than -1 by (2.1), whereas the range of $\tilde{y}$ and $\tilde{h}$ is unrestricted, namely the whole set $\mathbf{R}$ of real numbers; so $\tilde{F}$ may be defined on the set $\mathbf{R}^2$ of real pairs.

Also in consideration of the defects listed in Subsection 2.2, the following properties for $\tilde{F}$ appear indispensable:

Property (i): *unrestricted domain.* The function $\tilde{F}$ is defined on the whole of $\mathbf{R}^2$. The index takes a value for every possible pair $(\tilde{y}, \tilde{h})$.

Property (ii): *continuity.* The function $\tilde{F}$ is continuous on its domain. So, for instance, there is no "jump" in the index values.

Property (iii): *strict monotonicity.*

$$\tilde{F}(\tilde{y}_1, \tilde{h}_1) < \tilde{F}(\tilde{y}_2, \tilde{h}_2) \qquad \text{if } \tilde{y}_1 \leq \tilde{y}_2 \text{ and } \tilde{h}_1 \geq \tilde{h}_2, \text{ with } (\tilde{y}_1, \tilde{h}_1) \neq (\tilde{y}_2, \tilde{h}_2).$$

The index is strictly increasing in the variable $\tilde{y}$ as well as strictly decreasing in the variable $\tilde{h}$. (The reverse monotonicity, namely that $\tilde{F}$ decrease in $\tilde{y}$ and increase in $\tilde{h}$, could alternatively be required, but the index behavior would be less intuitive, assigning higher values to worse situations, and the consequent rankings would also be reversed.)

Defect (b) contradicts Property (i); Defect (a) contradicts Property (ii); Defects (c) and (d) contradict Property (iii); consequently none of these properties is satisfied by the indices D_ϵ and D_t. On the other hand, it is easy to verify that all of these properties are fulfilled by the index D_O.

In the light of the discussion in Subsection 2.3, the following additional properties are desirable:

Property (iv): *unbounded range of values.* The function $\tilde{F}$ takes values in the whole of $\mathbf{R}$. In particular, there is no upper or lower bound to the possible index values.

Property (v): *metric homogeneity.*

$$\begin{aligned} \tilde{F}(\tilde{y}_1 + u, \tilde{h}_1) - \tilde{F}(\tilde{y}_1, \tilde{h}_1) &\simeq \tilde{F}(\tilde{y}_2 + u, \tilde{h}_2) - \tilde{F}(\tilde{y}_2, \tilde{h}_2) \\ \tilde{F}(\tilde{y}_1, \tilde{h}_1 + u) - \tilde{F}(\tilde{y}_1, \tilde{h}_1) &\simeq \tilde{F}(\tilde{y}_2, \tilde{h}_2 + u) - \tilde{F}(\tilde{y}_2, \tilde{h}_2) \end{aligned}$$
$$\text{for every pair } (\tilde{y}_1, \tilde{h}_1) \text{ and } (\tilde{y}_2, \tilde{h}_2) \text{ and every increment } u.$$

Assuming metrically homogeneous input variables such as $\tilde{y}$ and $\tilde{h}$ and keeping one of them constant, like differences between values of one input variable yield like differences between the resulting index values. Since the ranges of $\tilde{y}$ and $\tilde{h}$ are unbounded, then so is the range of the index.

Property (vi): *cumulativeness.* If a time period S, for which the variation pair is $(\tilde{y}, \tilde{h})$, is subdivided into sub-periods $S_1, \ldots, S_m$, with respective variation pairs $(\tilde{y}_1, \tilde{h}_1), \ldots, (\tilde{y}_m, \tilde{h}_m)$, then

$$\tilde{F}(\tilde{y}, \tilde{h}) = \sum_{j=1}^{m} \tilde{F}(\tilde{y}_j, \tilde{h}_j).$$

> The index value over a certain period equals the sum of index values over component sub-periods.

Defects (e) contradicts Property (iv); Defect (f) contradicts Property (v); Defect (g) contradicts Property (vi); consequently none of these properties is satisfied by the index D_O. Neither D_ϵ nor D_t fulfill any of them.

3.2. **New decoupling indicators.** We now introduce a decoupling index that satisfies all Properties 1–6. Define D_N by

$$D_N = -\log(1 - D_O), \tag{3.3}$$

so that

$$D_N = F_N(y, h) = \log \frac{h+1}{y+1} = \tilde{F}_N(\tilde{y}, \tilde{h}) = \tilde{y} - \tilde{h}.$$

The aggregation function $\tilde{F}_N$ is easily seen to fulfill Properties 1–6 of Subsection 3.1 and has the advantage of a very simple definition and an easily computable expression, thus the resulting index D_N has the required "analytical soundness": "The indicators should be analytically sound and benefit from a consensus about their validity. They should further lend themselves to being linked to economic and environmental modelling and forecasting" (OECD, 2011, Box 1 in §1). The aggregation function $\tilde{F} = \tilde{F}_N$ also enjoys the same symmetry relation (3.2) as $\tilde{F}_O$; in addition, $\tilde{F}_N$ also fulfills the skew-symmetry relation in the variables $\tilde{y}$ and $\tilde{h}$

$$\tilde{F}(\tilde{y}, \tilde{h}) = -\tilde{F}(\tilde{h}, \tilde{y}). \tag{3.4}$$

This relation, not enjoyed by $\tilde{F}_O$, means that, if the variations of GDP and emissions are exchanged, then the absolute value of the resulting index simply changes sign. A plot of some equispaced levels for $\tilde{F}_N$ is given in Figure 3.1, where the metric homogeneity of D_N reflects in the fact that they are the same graphical distance apart.

Notice that D_N yields the same ranking as D_O because, by (3.3), the former is a strictly increasing function of the latter.

Turning to Defect (h), we can prove the following:

Theorem 1. *No index whose aggregation function $\tilde{F}$ satisfies Properties (1)–(3) can distinguish absolute from relative decoupling.*

Proof. Assume that the function $\tilde{F}$ takes values larger than a real threshold β for absolute decoupling, namely when $\tilde{h} < 0 < \tilde{y}$, and values smaller than β for relative decoupling, that is if $0 < \tilde{h} < \tilde{y}$. Then, by continuity, $\tilde{F}$ must take the same value β along the whole positive half-axis $\tilde{y}$, thus contradicting strict monotonicity. □

Therefore Defect (h) cannot be fully overcome without giving up any of Properties 1–3. In the sequel we modify the aggregation function of D_N to obtain another novel index D_P in order to alleviate the defect.

The best relation, in terms of decoupling, between $\tilde{y}$ and $\tilde{h}$ occurs when emissions vary in inverse proportion to increasing GDP, hence, by (3.1), we define the locus of *symmetric green growth* as the half-line $\tilde{y} = -\tilde{h} > 0$. With a similar approach as in Casadio Tarabusi and Guarini (2013, §4) and Casadio Tarabusi and Guarini (2016, §3), for $(\tilde{y}, \tilde{h})$ in the half-plane $\tilde{y} > \tilde{h}$ we subtract to the aggregation function $\tilde{F}_N(\tilde{y}, \tilde{h})$ a penalization that vanishes for $\tilde{y} = -\tilde{h}$ and that increases with $|\tilde{y} + \tilde{h}|$

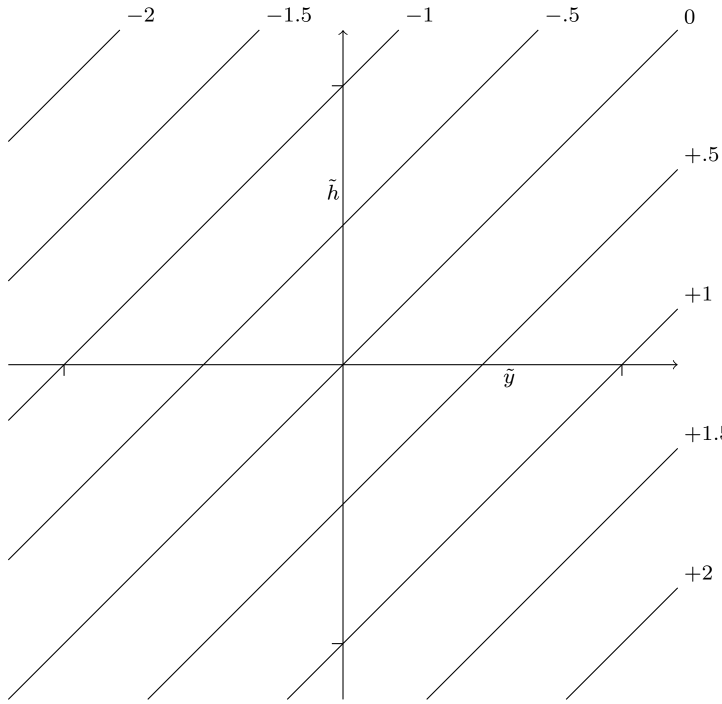


FIGURE 3.1. Some equispaced levels of the index D_N in the $(\tilde{y}, \tilde{h})$-plane. (Ticks are 1 unit apart on both axes.)

for fixed value of $\tilde{F}_N(\tilde{y}, \tilde{h})$; this is ensured if the resulting function is concave in that half-plane.

Symmetrically, opposite circumstances occur in the half-plane $\tilde{y} < \tilde{h}$. Indeed, the worst decoupling relation between $\tilde{y}$ and $\tilde{h}$ is when emissions vary in inverse proportion to decreasing GDP, that is, on the half-line $\tilde{y} = -\tilde{h} < 0$, the locus of *symmetric brown degrowth*. For $(\tilde{y}, \tilde{h})$ in this half-plane we add to the aggregation function $\tilde{F}_N(\tilde{y}, \tilde{h})$ a premium that vanishes for $\tilde{y} = -\tilde{h}$ and that increases with $|\tilde{y} + \tilde{h}|$ for fixed value of $\tilde{F}_N(\tilde{y}, \tilde{h})$; in this case, this holds if the modified function is convex in that half-plane.

Therefore the value of the index D_P resulting from this procedure is obtained by starting with the value of D_N and subtracting a penalization from it (if the variation of GDP exceeds that of emissions) or adding a premium to it (otherwise) that is all the larger as emissions do not vary in inverse proportion to GDP. In particular, for a given value of D_N, relative decoupling is penalized more than absolute decoupling.

In view of the symmetries in the discussion just made, we shall strive to construct the index D_P in such a way as to preserve symmetry relations (3.2) and (3.4) for its aggregation function $\tilde{F}_P$. Observing that $\tilde{F}_N(\tilde{y}, \tilde{h})$ equals twice the arithmetic mean of $\tilde{y}$ and $-\tilde{h}$, as in Casadio Tarabusi and Guarini (2016, §4) we define $\tilde{F}_P$ via a suitable generalized mean of the variables $\tilde{y}$ and $-\tilde{h}$, namely as

$$\tilde{F}_P(\tilde{y},\tilde{h}) = 2f^{-1}\left(\frac{f(\tilde{y})+f(-\tilde{h})}{2}\right),$$

where f is a monotonic (e.g., increasing) function on the real line with suitable properties. Specifically, in order to have the required properties for $\tilde{F}_P$ we need that f be concave increasing for $x > 0$ (to properly apply a penalization to positive values) and convex increasing for $x < 0$ (to properly apply a premium to negative values), see Hardy, Littlewood, and Pólya (1988, §92), as well as odd, namely $f(-x) = -f(x)$, for every x (in order to obtain (3.4)); a simple such function is the hyperbolic tangent $f(x) = \tanh x$ or, more generally, $f(x) = (1/c)\tanh cx$ for $c > 0$, for which $f^{-1}(x) = (1/2c)\log((1+cx)/(1-cx))$, yielding

$$D_P = \tilde{F}_P(\tilde{y},\tilde{h}) = \frac{1}{c}\log\frac{2+\tanh c\tilde{y}-\tanh c\tilde{h}}{2-\tanh c\tilde{y}+\tanh c\tilde{h}}.$$

This expression tends to $\tilde{F}_N(\tilde{y},\tilde{h})$ as c tends to 0; on the other hand, the larger the parameter c, the larger the penalization/premium the resulting $\tilde{F}_P$ undergoes, the stronger the judgment of situations that are far from symmetric green growth/brown degrowth—that is, when emissions do not vary in inverse proportion to GDP—, the more curved the level set of every value except 0 (which is the symmetric green growth/brown degrowth locus for any c), the more significant the resulting changes in rankings. We recommend a specific value for the parameter c that seems well-balanced between extremes, namely $c = 1$. A plot of some equispaced levels for $\tilde{F}_P$ is given in Figure 3.2.

Notice that the difference between D_N and D_P tends to be small for pairs that are close enough to the symmetric green growth/brown degrowth locus $\tilde{y} = -\tilde{h}$ (because the penalization/premium nearly vanishes) and/or to the line $\tilde{y} = \tilde{h}$ (because this is the boundary between the penalization and the premium area).

4. An application of the novel decoupling indicators to data from OECD (2017)

In this section we apply to actual data the novel decoupling indicators D_N and D_P with the recommended value of $c = 1$, alongside with the indicator D_O by OECD for comparison. A sensitive analysis of decoupling needs a long-term perspective, as in Vehmas, Luukkanen, and Kaivo-oja (2007), Lee and Brahmasrene (2013), Tang, Shang, Shi, Liu, and Bi (2014), and Zhao, Zhang, and Shao (2016), because the relationships between pollutant emission and GDP are affected by deep relevant structural changes in terms of technological processes and production systems, and the impact of possible environmental policies can be better assessed in the long run. For this reason we choose the most recent decade for which the data in OECD (2017) are as complete as possible, namely 2003–2013, and collect the results in a table in the Appendix. The data are relative to 103 world countries from all continents; the columns contain the values of input variables $\tilde{y}$ and $\tilde{h}$, then the ratings and rankings for each index, and finally the ranking difference.

Although the rankings induced by D_O and D_N coincide, as observed in general in Subsection 3.2, their ratings are distributed differently, because the latter index is metrically homogeneous whereas the former unduly brings higher values closer to one another while increasing the separation among lower values; the ensuing

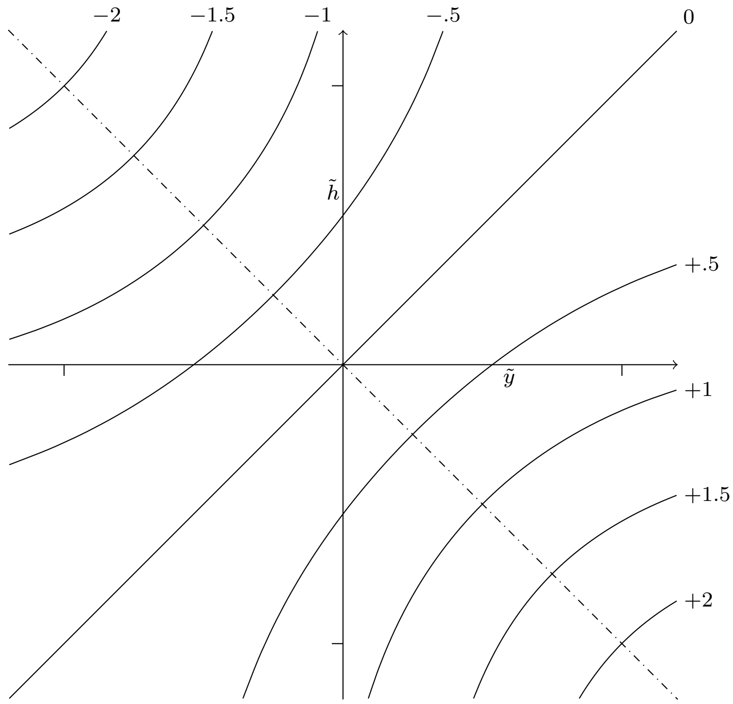


FIGURE 3.2. Some equispaced levels of the index D_P, for $c = 1$, in the $(\tilde{y}, \tilde{h})$-plane. The symmetric green growth/brown degrowth locus is the dashed and dotted line. (Ticks are 1 unit apart on both axes.)

distortion may bias judgments. For instance, the D_N difference between Azerbaijan and Uzbekistan (the two top countries) is nearly three times larger than between Cameroon and Oman (the two bottom countries); nevertheless, the D_O difference within the former pair is even smaller than within the latter, thus conveying the biased perception that, for the decade in question, Cameroon's decoupling situation is no closer to Oman's than Azerbaijan's is to Uzbekistan's.

The ranking that arises from D_P shows significant differences with those of D_O and D_N; indeed the ranking of more than one third of the observed countries changes by more than 2 positions up or down.

Figure 4.1 shows the cloud of observed variable pairs along with some level curves of D_N and of D_P in the $(\tilde{y}, \tilde{h})$-plane and allows the illustration of some relevant features of D_P that were theoretically discussed in Subsection 3.2. For instance, Uzbekistan has a smaller D_N, yet a larger D_P than Azerbaijan, because they are in absolute, respectively in relative decoupling and have similar values of D_N; Moldova and Myanmar show the same respective behavior. Qatar has the highest number of rank changes (namely it loses as many as 37 positions) because it is very distant from symmetric green growth and because numerous countries are, instead, closer to it and, at the same time, they have slightly smaller D_N values. On the other hand, Lybia is the only country in the data set that experiences a brown degrowth; Georgia, with a smaller D_N and a much larger distance than Lybia

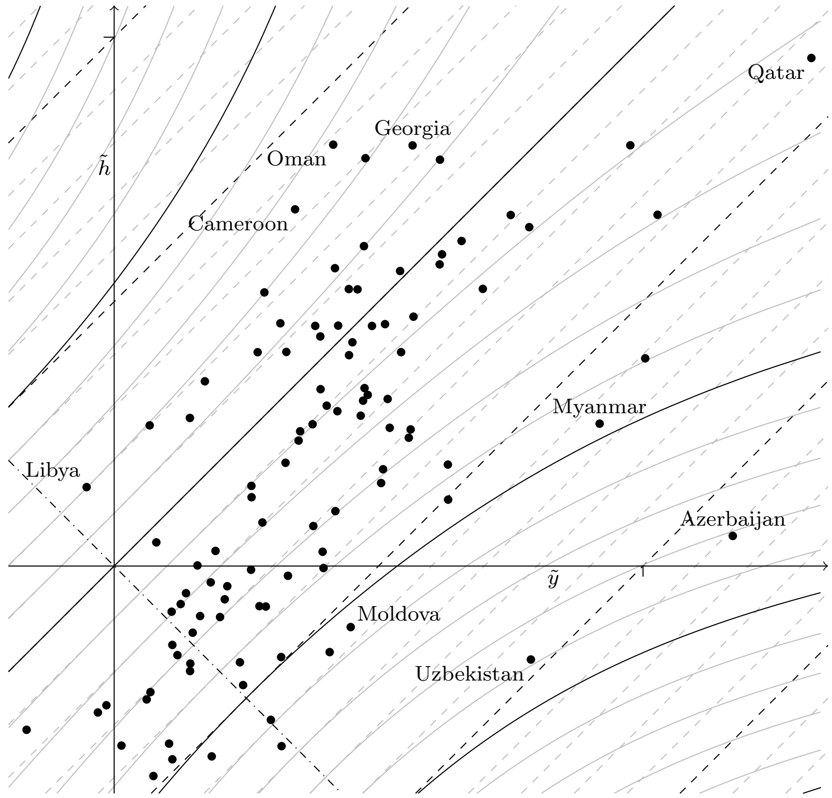


FIGURE 4.1. Some equispaced levels of the indices D_N (dashed lines) and D_P for $c = 1$ (solid curves) in the $(\tilde{y}, \tilde{h})$-plane and the data cloud of world countries for the decade 2003–2013. The symmetric green growth/brown degrowth locus is the dashed and dotted line. (Ticks are 1 unit apart on both axes.)

from symmetric brown degrowth, is rewarded with a higher D_P. Oman obtains the maximum premium (the only premium larger than .1), due to its distance from symmetric brown degrowth as well as its low D_N value; nevertheless its ranking last does not change because of its relative position with respect to the other countries. Finally, as observed in general in Subsection 3.2 also, the values of D_N and D_P are very close for country pairs for which $\tilde{y} \simeq \pm\tilde{h}$.

We can also notice that a large majority (about 80%) of countries are assigned positive values of D_N (or, equivalently, of D_O or of D_P), which confirms a positive long-term trend of eco-efficiency observed by several empirical studies, such as Szigeti, Tóth, and Szabó (2017).

Finally we find a weak positive correlation (about +19%) between D_N and $\tilde{y}$. This is consistent with the weak version of the environmental Kuznets curve (Vehmas, Luukkanen, and Kaivo-oja, 2007), according to which environmental efficiency increases with income level.

5. Conclusions

The studies about decoupling aim to measure and analyze the controversial trade-off between economic development and environmental sustainability. In this article, elaborating on the descriptive comparison by Conte Grand (2016) of the three main decoupling indicators D_O by OECD (2002), D_ϵ by Tapio (2005), and D_t by Lu, Wang, and Yue (2011), we set forth an axiomatic approach through which we analyze them critically and propose two novel decoupling indicators. A relevant part of the method consists in the graphical examination of the aggregation function level sets in the Cartesian plane.

Initially we examine the three pre-existing indicators, describing their defects and illustrating them with examples drawn from actual data. The indices D_ϵ and D_t turn out to suffer from seemingly unrecoverable problems, whereas D_O shows milder defects. Thence we identify some axiomatic properties that appear indispensable or at least desirable for any decoupling indicator. After that, we modify D_O into a new decoupling indicator D_N in order to remove its defects and fulfill the given compatible axioms. In order to take into account the distinction between absolute and relative decoupling as well as to capture the phenomenon of the rebound effect we introduce another decoupling indicator D_P built by applying to D_N a correction (that can be calibrated via a global parameter c) for the distance from what we define as symmetric decoupling, the case when the variations of economy and of environmental pressure are inversely proportional.

We conclude by testing the novel indices D_N and D_P (with the recommended value $c = 1$) against D_O on data by OECD (2017) of 103 world countries for the decade 2003–2013. The indices D_O and D_N, although producing the same ranking, are shown to differ in the distribution of values, because the latter enjoys metric homogeneity whereas the former does not. Furthermore there are significant ranking differences between D_O (or D_N) and D_P arising from the different corrections applied to countries according to their positions with respect to symmetric decoupling.

The novel contribution of this article consists in a critical analysis of the main pre-existing decoupling indicators, the introduction of an axiomatic analysis (with a graphical examination of level sets) into the literature on the subject and to construct new indicators that are mathematically simple but, at the same time, sound and useful to gain a better insight into the phenomenon.

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Appendix: Decoupling indicators of world countries for the decade 2003–2013 from OECD (2017) data

The variables $\tilde{y}$ and $\tilde{h}$ for all listed countries are given by (3.1) in terms of the real GDP and production-based CO_2 emissions, respectively, for the decade 2003–2013. By construction, the index D_N yields the same ranking as D_O. (All values rounded to the fourth decimal.)

	Variables		Ratings			Rankings		
Country	$\tilde{y}$	$\tilde{h}$	D_O	D_N	D_P	D_O	D_P	diff.
Azerbaijan	+1.1698	+.0569	+.6714	+1.1128	+.8087	1	2	−1
Uzbekistan	+.7880	−.1768	+.6190	+.9648	+.8860	2	1	+1
Romania	+.3165	−.3406	+.4817	+.6571	+.6570	3	3	0
Myanmar	+.9180	+.2692	+.4773	+.6487	+.4705	4	9	−5
Turkmenistan	+1.0042	+.3927	+.4575	+.6115	+.3947	5	16	−11
Syrian Arab Rep.	+.2963	−.2909	+.4441	+.5872	+.5872	6	4	+2
Slovak Rep.	+.4075	−.1628	+.4347	+.5704	+.5621	7	5	+2
Moldova	+.4473	−.1155	+.4304	+.5627	+.5479	8	6	+2
Sweden	+.1845	−.3600	+.4199	+.5445	+.5404	9	7	+2
Belarus	+.6315	+.1255	+.3971	+.5060	+.4413	10	13	−3
Bulgaria	+.3158	−.1721	+.3861	+.4880	+.4855	11	8	+3
Finland	+.1100	−.3655	+.3784	+.4755	+.4680	12	11	+1
Denmark	+.0742	−.3971	+.3759	+.4714	+.4595	13	12	+1
Ukraine	+.2438	−.2251	+.3743	+.4689	+.4689	14	10	+4
Hungary	+.1038	−.3359	+.3558	+.4397	+.4339	15	14	+1
Singapore	+.6310	+.1918	+.3554	+.4392	+.3735	16	18	−2

(continued)	Variables		Ratings			Rankings		
Country	$\tilde{y}$	$\tilde{h}$	D_O	D_N	D_P	D_O	D_P	diff.
Czech Rep.	+.2378	−.1821	+.3429	+.4199	+.4196	17	15	+2
Poland	+.3959	−.0038	+.3295	+.3998	+.3850	18	17	+1
Russia	+.3942	+.0270	+.3073	+.3672	+.3515	19	20	−1
Latvia	+.2868	−.0766	+.3047	+.3634	+.3595	20	19	+1
Iraq	+1.0275	+.6642	+.3047	+.3633	+.1924	21	48	−27
Qatar	+1.3184	+.9607	+.3007	+.3576	+.1219	22	59	−37
Jamaica	+.0138	−.3396	+.2977	+.3534	+.3443	23	22	+1
Iceland	+.2747	−.0759	+.2957	+.3505	+.3471	24	21	+3
Dominican Rep.	+.5048	+.1569	+.2938	+.3479	+.3128	25	26	−1
Lithuania	+.3286	−.0185	+.2933	+.3471	+.3390	26	24	+2
Ireland	+.1435	−.1986	+.2898	+.3421	+.3419	27	23	+4
Belgium	+.1440	−.1845	+.2800	+.3285	+.3284	28	25	+3
Cuba	+.5085	+.1830	+.2778	+.3255	+.2898	29	31	−2
Israel	+.4183	+.1038	+.2699	+.3145	+.2942	30	30	0
Venezuela	+.5571	+.2426	+.2698	+.3144	+.2694	31	36	−5
Spain	+.0615	−.2523	+.2693	+.3138	+.3110	32	27	+5
Croatia	+.0686	−.2384	+.2643	+.3070	+.3048	33	28	+5
Indonesia	+.5606	+.2583	+.2609	+.3024	+.2572	34	39	−5
Nicaragua	+.3766	+.0756	+.2600	+.3011	+.2863	35	33	+2
Malta	+.2002	−.0965	+.2567	+.2967	+.2959	36	29	+7
United Kingdom	+.1196	−.1685	+.2503	+.2881	+.2879	37	32	+5
Austria	+.1485	−.1261	+.2401	+.2746	+.2745	38	34	+4
New Zealand	+.2091	−.0629	+.2382	+.2720	+.2706	39	35	+4
Luxembourg	+.2588	−.0071	+.2335	+.2659	+.2618	40	37	+3
Philippines	+.5211	+.2614	+.2287	+.2597	+.2239	41	44	−3
France	+.1100	−.1492	+.2283	+.2591	+.2590	42	38	+4
United States	+.1625	−.0946	+.2267	+.2570	+.2567	43	40	+3
Switzerland	+.2139	−.0382	+.2229	+.2522	+.2503	44	41	+3
Portugal	−.0147	−.2633	+.2200	+.2485	+.2438	45	42	+3
Italy	−.0308	−.2770	+.2182	+.2461	+.2404	46	43	+3
El Salvador	+.1826	−.0309	+.1923	+.2136	+.2124	47	45	+2
Lebanon	+.5172	+.3159	+.1824	+.2013	+.1702	48	51	−3
Australia	+.2803	+.0823	+.1796	+.1980	+.1916	49	49	0
Germany	+.1258	−.0719	+.1794	+.1977	+.1976	50	46	+4
Netherlands	+.1087	−.0864	+.1772	+.1951	+.1951	51	47	+4
Slovenia	+.1358	−.0514	+.1708	+.1872	+.1869	52	50	+2
Paraguay	+.4662	+.2843	+.1663	+.1819	+.1586	53	53	0
China (P. Rep. of)	+.9760	+.7956	+.1650	+.1803	+.0898	54	69	−15
Tajikistan	+.6971	+.5241	+.1588	+.1729	+.1218	55	60	−5
Canada	+.1916	+.0286	+.1504	+.1630	+.1610	56	52	+4
Colombia	+.4707	+.3127	+.1462	+.1581	+.1361	57	56	+1
Norway	+.1575	+.0011	+.1447	+.1563	+.1553	58	54	+4
Argentina	+.4795	+.3236	+.1444	+.1559	+.1333	59	57	+2
Greece	−.1656	−.3097	+.1342	+.1441	+.1362	60	55	+5
Panama	+.7849	+.6408	+.1342	+.1440	+.0901	61	68	−7
Jordan	+.5426	+.4044	+.1292	+.1383	+.1114	62	64	−2

(continued)	Variables		Ratings			Rankings		
Country	$\tilde{y}$	$\tilde{h}$	D_O	D_N	D_P	D_O	D_P	diff.
Turkey	+.4734	+.3365	+.1279	+.1369	+.1167	63	62	+1
Estonia	+.2598	+.1301	+.1217	+.1298	+.1250	64	58	+6
Costa Rica	+.4221	+.2928	+.1213	+.1293	+.1141	65	63	+2
South Africa	+.3240	+.1952	+.1209	+.1288	+.1205	66	61	+5
Guatemala	+.3488	+.2372	+.1057	+.1117	+.1026	67	66	+1
Mexico	+.2595	+.1516	+.1023	+.1079	+.1035	68	65	+3
Korea	+.3752	+.2681	+.1016	+.1071	+.0968	69	67	+2
Honduras	+.4019	+.3032	+.0940	+.0988	+.0874	70	71	−1
Tunisia	+.3517	+.2548	+.0924	+.0969	+.0885	71	70	+1
Armenia	+.5660	+.4715	+.0902	+.0945	+.0730	72	72	0
India	+.7499	+.6638	+.0825	+.0861	+.0542	73	73	0
Thailand	+.3903	+.3344	+.0543	+.0559	+.0491	74	74	0
Bahrain	+.5122	+.4574	+.0533	+.0548	+.0437	75	75	0
Egypt	+.4441	+.3987	+.0444	+.0454	+.0382	76	76	0
Saudi Arabia	+.6155	+.5704	+.0441	+.0451	+.0323	77	78	−1
Kazakhstan	+.6569	+.6147	+.0413	+.0422	+.0289	78	79	−1
Japan	+.0797	+.0447	+.0344	+.0350	+.0348	79	77	+2
Malaysia	+.4875	+.4540	+.0330	+.0335	+.0271	80	80	0
Peru	+.6200	+.5894	+.0301	+.0305	+.0216	81	82	−1
Morocco	+.4505	+.4231	+.0270	+.0274	+.0228	82	81	+1
Uruguay	+.5407	+.5579	−.0173	−.0171	−.0128	83	83	0
Kuwait	+.4236	+.4546	−.0315	−.0310	−.0257	84	84	0
Brazil	+.3898	+.4341	−.0453	−.0443	−.0376	85	85	0
Chile	+.4602	+.5232	−.0650	−.0630	−.0499	86	86	0
Senegal	+.3802	+.4541	−.0767	−.0739	−.0624	87	87	0
Trinidad & Tobago	+.3256	+.4048	−.0824	−.0792	−.0695	88	89	−1
Kyrgyzstan	+.4437	+.5237	−.0833	−.0800	−.0639	89	88	+1
Ecuador	+.4723	+.6049	−.1418	−.1326	−.1005	90	91	−1
Iran	+.2713	+.4043	−.1422	−.1330	−.1189	91	93	−2
Haiti	+.1432	+.2800	−.1466	−.1368	−.1309	92	95	−3
Algeria	+.3143	+.4590	−.1557	−.1447	−.1251	93	94	−1
United Arab Emir.	+.4175	+.5632	−.1569	−.1457	−.1157	94	92	+2
Viet Nam	+.6161	+.7684	−.1644	−.1522	−.0976	95	90	+5
Yemen	+.1715	+.3493	−.1946	−.1778	−.1663	96	97	−1
Brunei Darussalam	+.0672	+.2660	−.2199	−.1988	−.1934	97	98	−1
Libya	−.0521	+.1492	−.2230	−.2013	−.2009	98	100	−2
Georgia	+.5643	+.7954	−.2599	−.2310	−.1505	99	96	+3
Côte d'Ivoire	+.2840	+.5174	−.2629	−.2334	−.1998	100	99	+1
Bolivia	+.4752	+.7712	−.3445	−.2960	−.2060	101	101	0
Cameroon	+.3421	+.6745	−.3943	−.3324	−.2600	102	102	0
Oman	+.4142	+.7968	−.4661	−.3826	−.2719	103	103	0