

10th ERME TOPIC CONFERENCE (ETC10)

Mathematics Education in the Digital Age (MEDA)

16-18 September 2020 in Linz, Austria

PROCEEDINGS

Edited by:

Ana Donevska-Todorova, Eleonora Faggiano, Jana Trgalova, Zsolt Lavicza, Robert Weinhandl, Alison Clark-Wilson, and Hans-Georg Weigand

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**PROCEEDINGS of the
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Introduction

The fifth ERME Topic Conference for Mathematics Education in the Digital Age (MEDA), held in September 2018 in Copenhagen was inspired by the contributions to the Thematic Working Groups 15 and 16 at CERME 10 in Dublin, which highlighted the diversity of current research and its overlaps with other TWG themes. MEDA was an interdisciplinary, multifaceted collaboration that brought together participants who would normally attend a range of CERME Thematic Working Groups to provide the opportunity for further in-depth discussion and debate. The successful conference experience resulted in an intensive communication and collaboration, which continued through the collegial work that culminated in the publication of a post-conference book in the ERME Series published by Routledge. Moreover, inspired by the contributions to the Thematic Working Groups 15 and 16 in the last CERME 11 in Utrecht, the second conference, MEDA2, provides the opportunity for further in-depth discussion and debate. In particular, MEDA2 is of interest to the following TWGs:

TWG 18	Mathematics Teacher Education and Professional Development
TWG 22	Curricular Resources and Task Design in Mathematics Education
TWG 21	Assessment in Mathematics Education
TWG17	Theoretical Perspectives and Approaches in Mathematics Education Research

The conference welcomed theoretical, methodological, empirical or developmental papers (8 pages) and poster proposals (2 pages) in relation to the following themes:

- Theme 1: Mathematics teacher education and professional development in the digital age
- Theme 2: Mathematics curriculum development and task design in the digital age
- Theme 3: Assessment in mathematics education in the digital age
- Theme 4: Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

Theme 1 - Mathematics teacher education and professional development in the digital age

- The specific knowledge, skills and attributes required for efficient/effective mathematics teaching with digital resources, to include digital mathematics resources, which we define as resources that afford or embed mathematical representations that teachers and learners can interact with by acting on objects in mathematical ways.
- The design and evaluation of mathematics teacher education and professional development programmes that embed the knowledge, skills and attributes to teach mathematics with digital resources.

Theme 2 - Mathematics curriculum development and task design in the digital age

- The design of resources and tasks (e.g. task features, design principles and typologies for e-textbooks);
- The evaluation and analysis of resources and tasks (e.g. determining quality criteria for curricular material, resources and methods of analysis);

- The interactions of teachers and students with digital curriculum materials (e.g. appropriation, amendment, re-design), both individually or collectively. This includes the consideration of teacher learning/professional development in their work with digital resources.

Theme 3 - Assessment in mathematics education in the digital age

- New possibilities of assessment (formative, summative, etc.) in mathematics education brought by digital technology
- Use of digital technology to support students to gain a better awareness of their own learning
- Assessment of learners' mathematical activity in digital environment

Theme 4 - Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

- Theories for research on technology use in mathematics education (e.g. design theories, prescriptive theories, theories linking research and practice, theories addressing the transfer of learning arrangements to other learning conditions etc.)
- The linking of theoretical and methodological approaches and the identification of conditions for productive dialogue between theorists, within mathematics education and beyond (e.g. developing collaborative research with educationalists, including teachers and educational technologists).

The conference particularly welcomed contributions linking some of these four themes at any level of mathematics education: pre-school, primary, lower- and upper-secondary or tertiary.

The Conference Proceedings of the 10th ERME Topic Conference MEDA 2020 are rich in the variety of content-formats and are therefore structured in two parts. They include the contributions of the plenary speakers and all the 67 reviewed and accepted submissions from participants, organised as four chapters according to the aforementioned themes.

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Jana Trgalova, Zsolt Lavicza, Robert Weinhandl,
Alison Clark-Wilson, and Hans-Georg Weigand

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