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TRAIN-WEIGHT-IN-MOTION IDENTIFICATION MEASURING TIME-HISTORIES OF RAIL STRAINS

Annamaria Pau[1], **Fabrizio Vestroni**[2]

[1]Department of Structural and Geotechnical Engineering,
Sapienza University of Rome
via Gramsci 53, 00197 Roma, Italy
e-mail: annamaria.pau@uniroma1.it

[2]Department of Structural and Geotechnical Engineering,
Sapienza University of Rome
via Eudossiana 18, 00184 Roma, Italy
e-mail: vestroni@uniroma1.it

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Abstract. *This paper deals with the identification of the weight of a train in motion, based on the measurement of the time-history of the response in terms of strains at the foot of the rail. The direct problem is initially addressed: the response of a rail modelled as a one-dimensional Euler-Bernoulli beam with constant properties, resting on a linear elastic foundation with viscous damping and subjected to a Dirac delta load travelling at constant speed is considered. For the model described, a closed-form expression of the solution can be obtained, which permits to investigate the sensitivity of the response to the main mechanical parameters. Analytical strains are compared to their experimental counterpart, showing their practical ability to describe the real phenomenon. As a second step, the inverse problem consisting in the identification of the loads for a given time-history of measured strains is addressed. The solution of the inverse problem is set up as a minimization problem whose objective function is based on the difference between experimental and model time-histories of strains. This inverse problem is nonlinear, and its solution can be pursued by the Newton method, which requires recursive application of a linearized expression for the evaluation of the optimal parameters. The Bayesian formulation enables to investigate identifiability of parameters and minimum number of measurements, and leads to conclude that the identification process must begin with an improving of the interpretative model. This model updating can be achieved by evaluating the model parameters, using the time-history of a train whose weight is known. After that, the actual identification of the loads can be performed. The procedure proposed is applied to experimental strains recorded at the foot of a rail on a stretch of line run by a locomotor moving at a low constant speed. The identified loads were in good agreement with the expected value, with errors smaller than 4%.*

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1 INTRODUCTION

Increased standards of safety in railway transport require awareness of the loads actually travelling on railway lines, in particular when freight trains running. This enables to timely schedule maintenance and monitor rail wear, as well as to check unbalanced loads which can affect vehicle safety, and requires the development of methods for the identification of travelling loads. This paper deals with the identification of the weight of a train in motion, based on the measurement of the time-history of the response in terms of strains at the foot of the rail.

To solve this inverse problem, it is first useful to address the direct problem. Here, the rail is modelled as a one-dimensional Euler-Bernoulli beam with constant geometrical and mechanical properties, resting on a linear elastic foundation with viscous damping and subjected to a Dirac delta load travelling at constant speed. The same model was used in the past to describe the response of rocket test tracks [1] and train tracks [2]. This model has proved its ability to describe the real experimental response and, since it enables obtaining a closed-form solution, it is used here as a reference model, enabling to investigate the response sentitivity to the main mechanical parameters. More complex models involving 2D descriptions of the elastic foundation were also proposed [3], but they seem not suitable for this inverse problem.

On accepting the simplified modelling of the travelling load as a Dirac delta, its identification involves the evaluation of its amplitude only. We approach this inverse problem using an estimator which minimizes the difference between the experimental response and the response provided by the described mechanical model of the rail. An overview of the different approaches presented in the literature for the solution of load identification problems can be found in the work by Ouyang [4]. Among these, it is worth citing the approach proposed by Trujillo and Busby [5], based on dynamic programming, not only the forcing term which provides the best match is sought, but also that which has a certain degree of smoothness according to Tikhonov's regularization. An application of dynamic programming to train load identification is presented by Zhu et al. [7]. Among other possible approaches, Ronasi et al. [6] calculate the minimum of an objective function measuring the distance between experimental and analytical data. In the framework of an algebraic solution, Meli and Pugi [8] made hypotheses to simplify the load time-histories and adopted a multibody model for the railway vehicle. We develop here a load identification procedure within the framework of a Bayesian approach, also addressing problems of optimal choice of parameters and measurements [9, 10, 11]. Based on the results obtained from the investigation of the Fisher matrix, we were able to conclude that the load identification problem can be better formulated in two steps. The first step consists in a model updating performed on the grounds of the knowledge of the response to a known load. The second step is the actual load identification.

Experimental tests were used for validation: they consist of field measurements of the rail strain time-histories due to the transit of a two-axis locomotor at low speed.

2 DIRECT PROBLEM

The rail is represented as a plane beam with constant geometrical and mechanical properties resting on a linearly elastic foundation with viscous damping, and subjected to a Dirac delta load of amplitude P moving at constant speed v. On setting E the Young's modulus of the cross section, I its moment of inertia, the solution to this problem in terms of transverse displacement w as a function of time t and space z can be written in closed form:

$$w(z,t)=\begin{cases}\frac{P}{EI}\left(\frac{e^{(z-vt)k_3}}{(k_3-k_4)(k_3-k_1)(k_3-k_2)}+\frac{e^{(z-vt)k_2}}{(k_2-k_4)(k_2-k_1)(k_2-k_3)}\right) & z\leq 0\\ -\frac{P}{EI}\left(\frac{e^{(z-vt)k_4}}{(k_4-k_3)(k_4-k_2)(k_4-k_1)}+\frac{e^{(z-vt)k_1}}{(k_1-k_3)(k_1-k_2)(k_4-k_1)}\right) & z>0.\end{cases} \tag{1}$$

where k_1, k_2, k_3 and k_4 are wavenumbers. The time-history of the response at $z = 0$ for $P = 1$ N resulting from Equation (1) is represented in terms of curvatures $\chi(z,t) = \partial^2 w/\partial z^2$ in Figure 1, for different speeds and damping. Curvature is the observed quantity, since in the experimental tests we will measure the strain at the foot of the rail, which is tied to the curvature by the linear relation $\epsilon = h_G\chi$, valid under the hypothesis of small displacements, with h_G distance between the center of mass of the cross-section and the foot of the rail. Figure 1a shows that, for speeds of 28 km/h, approximately corresponding to 0.1 of the critical speed v_{cr}, the dependence of the maximum amplitude on damping is very limited. The critical speed $v_{cr} = (4kEI/(\rho A)^2)^{1/4}$, with k stiffness of the soil, ρ mass density, and A area of the cross-section, is the lowest speed at which a free wave can propagate in the rail [1]. When the speed increases, the response is no longer symmetric, and the maximum amplitude more and more depends on the train speed (Figure 1b). When the speed equals the critical velocity, a resonance occurs: the maximum response increases and strongly depends on damping (Figure 1c).

Figure 2a shows the analytical time-history of the strains due to a series of ten Dirac loads with P=78400 N, which is approximately the load insisting on one wheel of an unalden ETR324, obtained using Equation 1 for a travelling speed of 28 km/h. Figure 2 b reports, for comparison, the experimental time-history of an ETR324, travelling at around 30 km/h. The pattern of the time-history obtained from the model satisfactorily agrees with the time-history observed experimentally.

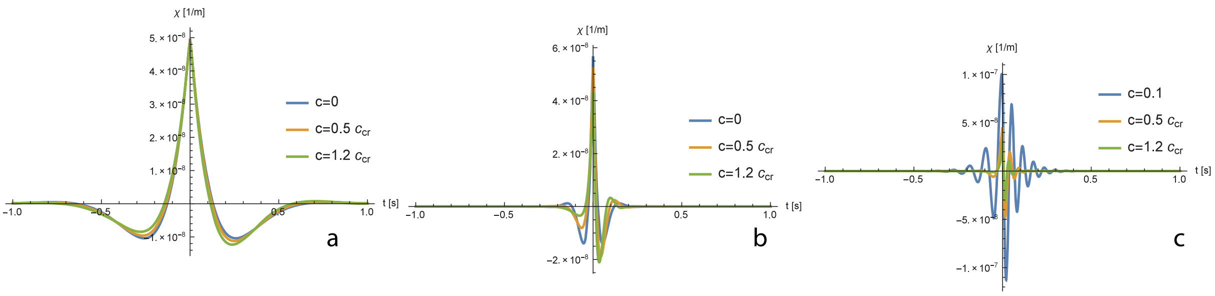
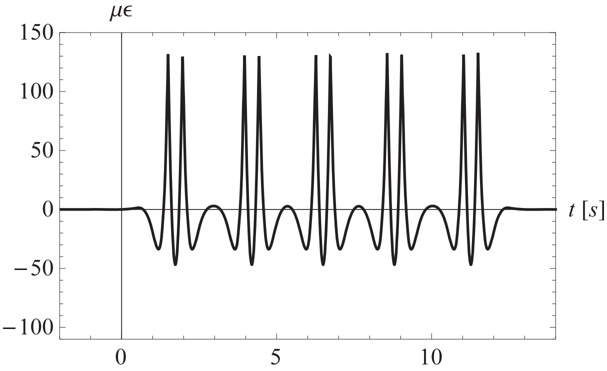


Figure 1: Time-histories of curvatures for different values of damping and speed: 28 km/h=$0.1v_{cr}$ (a), 140 km/h=$0.5v_{cr}$ (b), 280 km/h=v_{cr} (c). $c_{cr} = 2(k\rho A)^{1/2}$

3 INVERSE PROBLEM

The goal of the solution of the inverse problem consists in the identification of the amplitude of the Dirac deltas representing train loads. It must be considered that the model contains several parameters, whose values may be more or less uncertain, and that have an influence on the result of the load identification. Some parameters are mechanical and geometrical characteristics of the model, that is ρA, EI, k, c, while others concern the load: P and v. We will be dealing with recordings made at low speed, therefore we will assume $c = 0$, and, also, that v is known, since it is easily measurable. The vector of unknown parameters will be $\mathbf{x} = \{\rho A, EI, k, P\}^T$.

Let us call $\boldsymbol{\epsilon}(\mathbf{x})$ the vector of observed quantities as a function of the vector of parameters $\mathbf{x}$, and $\mathbf{z}$ the vector of corresponding measured quantities, that is, the time-discretized experimental



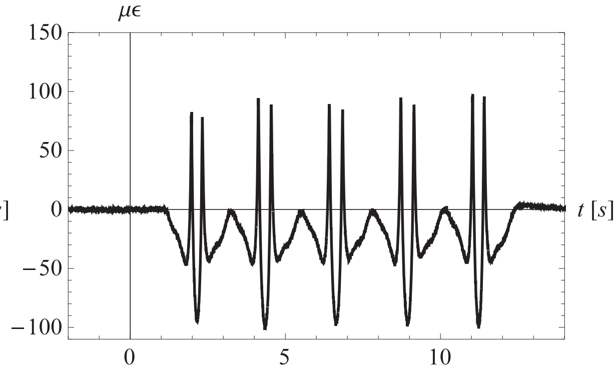


Figure 2: Analytical (a) and experimental (b) microstrain time-histories of an ETR324.

response at a given location. The vector of observed quantities is an m-element vector $\boldsymbol{\epsilon}(\mathbf{x}) = \{\epsilon_1(\mathbf{x}), ...\epsilon_m(\mathbf{x})\}$, whose i-th element represents the strain at a given abscissa and at the i-th time instant.

Within a Bayesian approach, parameters are random variables, and measured quantities satisfy the relationship

$$\mathbf{z} = \boldsymbol{\epsilon}(\mathbf{x}) + \mathbf{n} \tag{2}$$

where $\mathbf{n}$ is a vector of stochastic noise, independent of $\mathbf{x}$. Assuming a multivariate normal distrubution for $\mathbf{x}$ and $\mathbf{n}$, and being $\mathbf{S_x}$ and $\mathbf{S_n}$ the covariance matrices of the initial estimate of the parameters and of the noise, respectively, the maximum of the probability $p(\mathbf{x}|\mathbf{z})$ is attained for the optimal $\hat{\mathrm{x}}$ of $\mathbf{x}$ which minimizes the objective function:

$$l(\mathbf{x}) = \frac{1}{2}[\mathbf{z} - \boldsymbol{\epsilon}(\mathbf{x})]^T \mathbf{S_n}^{-1} [\mathbf{z} - \boldsymbol{\epsilon}(\mathbf{x})] + \frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \mathbf{S_x}^{-1} (\mathbf{x} - \mathbf{x}_0). \tag{3}$$

Assuming that the initial estimate $\mathbf{x}_0$ is not too far from $\hat{\mathrm{x}}$, a linearization of the relationship between observed quantities and parameters provides a recursive formula whose iterative application enables to reach the minimum of the objective function:

$$\hat{\mathbf{x}} = \mathbf{x}_0 + (\mathbf{H}^T \mathbf{S_n}^{-1} \mathbf{H} + \mathbf{S_x}^{-1})^{-1} \mathbf{H}^T \mathbf{S_n}^{-1} (\mathbf{z} - \boldsymbol{\epsilon}(\mathbf{x}_0)), \tag{4}$$

where $\mathbf{H}$ is the sensitivity matrix whose components are $H_{ij} = \partial \epsilon_i / \partial x_j$, and $\mathbf{S} = (\mathbf{H}^T \mathbf{S_n}^{-1} \mathbf{H} + \mathbf{S_x}^{-1})^{-1}$ is the a posteriori covariance matrix. Within this linearized framewok, the Hessian matrix associated to $l(\mathbf{x})$ is $\mathbf{H}_e = \mathbf{H}^T \mathbf{S_n}^{-1} \mathbf{H} + \mathbf{S_x}^{-1}$, whose first term is the Fisher or information matrix $\mathbf{A} = \mathbf{H}^T \mathbf{S_n}^{-1} \mathbf{H}$.

Figure 3a reports the time-history of the change of the curvature for a 10% variation of each of the three model parameters EI, ρA and k. These time-histories are, in practice, the columns of the sensitivity matrix each multiplied by the variation of the related parameter, and divided by h_G. Figure 3a shows that the parameters which play the most important role are EI and k, while ρA has a scarce influence. Identifiability is the actual possibility to determine a single set of optimal parameters $\hat{\mathrm{x}}$ such that the objective function is at a minimum. Some information about it is provided by $\mathbf{H}$, whose rank indicates the maximum number of identifiable parameters: in our case, the rank of $\mathbf{H}$ is 3, independently of the number of time samples, which are as many as the rows of the sensitivity matrix.

More precise information on the choice of parameters and measurements is provided by the Fisher matrix, which must be invertible for the solution (4) to be calculated. The list of its eigenvalues can put the parameters, or more often, their linear combination, in order of importance. This is displayed by the shape of the eigenvectors, which enables to point out possible coupling between parameters. Observing the eigenvectors of the Fisher matrix, which are reported in Figure 3b, it can finally be concluded that the parameters EI and k are coupled, while ρA is an independent parameter with scarce influence on the response, in fact, it is associated with the smallest eigenvalue and the related eigenvector has only the first component different from zero. In such situation, it seems to be appropriate to obtain the solution of the inverse problem in two steps: first the optimal parameters EI and k are sought using the response to a known load, then the updated model is used to determine the amplitude P, the real unknown of the problem.

The Fisher matrix also provides information on the choice of measurements by means of its Fisher inflow, which is the value of the terms on the principal diagonal as a function of the number of measurements. In fact, on assuming that $\mathbf{S_n}$ is a diagonal matrix, that is the noise is uncorrelated, the terms on the principal diagonal can be written as: $A_{ii} = \sum_k (\sigma_k^4)^{-1}(\partial\epsilon_k/\partial x_i)^2$. Since the summation occurs over the time instants, in the limit of the time instant between two samples tending to zero, it is obtained $A_{ij} = (1/T)\int_0^T (\sigma^4)^{-1}(\partial\epsilon/\partial x_i)^2 dt$, where T is the duration of the phenomenon. The terms on the principal diagonal of the Fisher matrix provide then an approximation of this integral, and tend to stabilize when the sampling frequency is sufficient, indicating that further samples do not provide further information. This is shown in Figure 4 a and b, where also the trace of $\mathbf{A}$ is reported, showing that a number of 40 measurements will provide a stable estimate of the covariance matrix, for our purposes, with a variation smaller than 5% for an increase of the number of measurements.

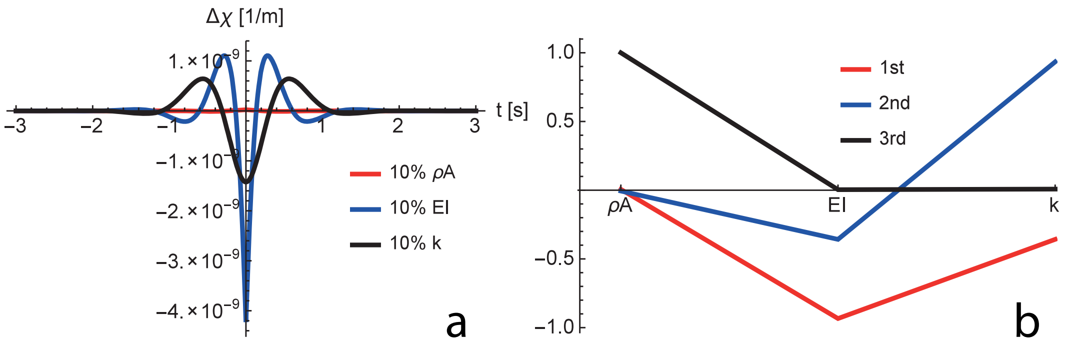


Figure 3: Time-histories of the change of curvature for a 10% variation of mechanical parameters (a) and eigenvalues of the Fisher matrix (b).

As proposed, we proceed to the identification of EI and k using 40 samples of a pseudo-experimental time-history generated by a known load. In the absence of noise and starting from initial parameters with a 30% error, we obtain convergence to the exact values within five iterative applications of Equation 4, according to the Newton method. The objective function is smooth and has a unique minimum, as it is shown in Figure 5. Using the identified parameters, if we proceed to the identification of the load, we obtain the exact value with a single application of Equation (4), in fact, since the dependence of the response on P is linear, Equation (4)

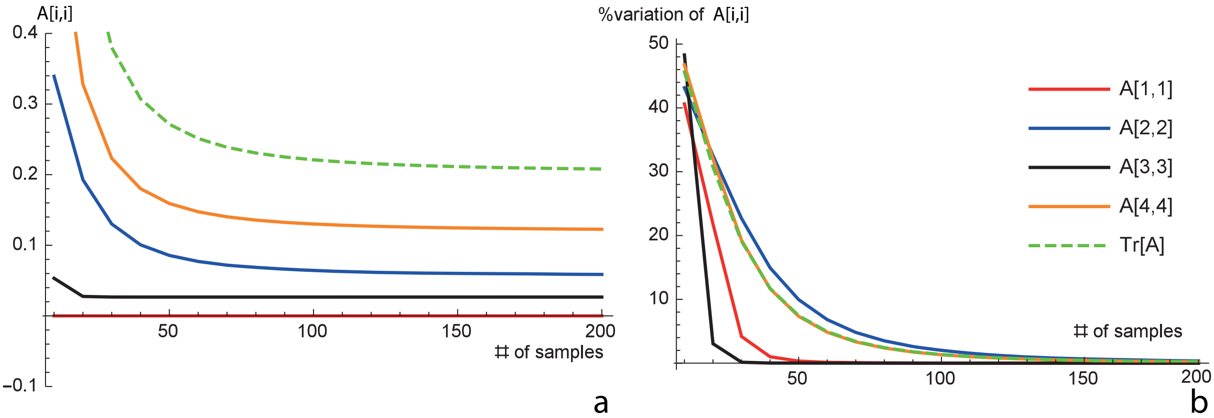


Figure 4: Fisher inflow and trace of the Fisher matrix (a), and % variation of their values (b) as a function of the number of samples.

does not constitute an approximation when the unknown parameter is P. It is instead an exact relationship, which does not requires iterations.

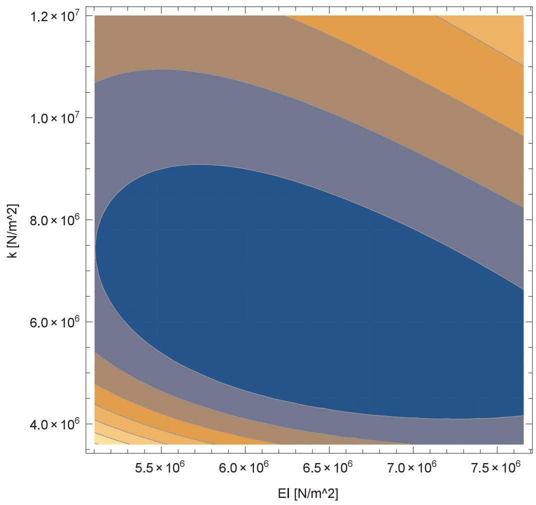


Figure 5: Contourplot of the objective function for pseudo-experimental data in the absence of error.

4 EXPERIMENTAL RESULTS

The procedure proposed is applied to experimental strains recorded at the foot of a rail on a stretch of line run by a locomotor moving at low constant speed. The weight of the locomotor is known and equal to 154.22 kN, so that two time-histories are firstly used to identify the soil stiffness. The resulting objective function is reported in Figure 6a, showing a distinct minimum. Then, a total of 10 locomotor transits with different constant speeds in the range 5-25 km/h were investigated. The updated model was used to identify the loads considered, at this stage, as unknowns, and the resulting identified values are reported in Table 1. These loads are in good agreement with the expected ones, with a mean error of 3.82%. The dispersion of errors, varying from -10.3 to 3.6%, can be ascribed to the simple interpretative model and to the the fact that experimental measurements were performed on one rail only, disregarding possible unbalance of the load. Figure 6b shows the comparison between the experimental time-histories of strains and those obtained from the updated model and the identified loads.

transit	1	2	3	4	5
weight	154.68	144.29	138.28	155.35	154.44
error [%]	0.3	-6.4	-10.3	0.7	0.14
transit	6	7	8	9	10
weight	155.46	144.63	157.71	142.73	159.73
error [%]	0.8	-6.2	2.3	-7.5	3.6

Table 1: Identified weights [kN] in the different transits and related error.

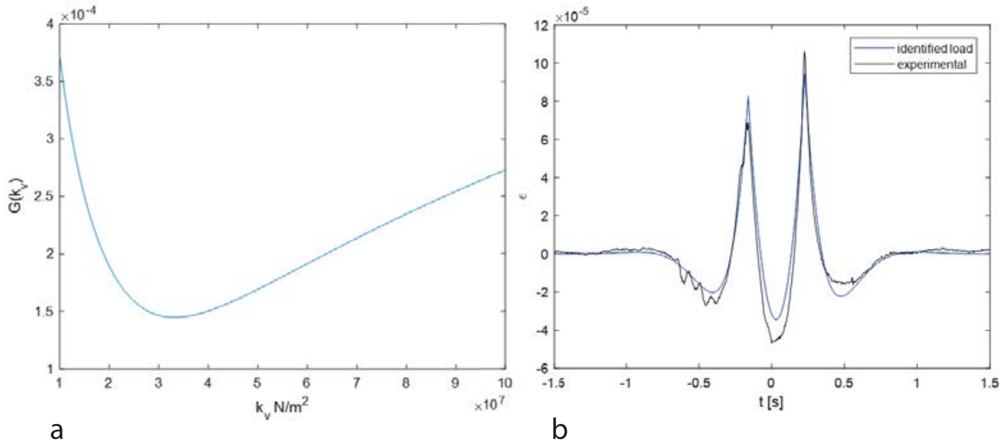


Figure 6: Objective function (a) and comparison between experimental and numerical time-histories of strains (b).

5 CONCLUSIONS

- We applied an approach for the identification of travelling loads of freight trains based on the minimization of the difference between the experimental time-history of strains at the base of the rail and their analytical counterpart. The model describing the response is a one-dimensional Euler-Bernoulli beam resting on a linearly elastic soil. The load is modelled as a Dirac delta load travelling at constant speed, whose amplitude is unknown and is the final goal of the identification procedure.

- As a result of a sensitivity analysis, among the model parameters, the soil and the beam stiffness resulted to be the most relevant, while damping was proved to be not significant for an appropriate description of the response in range of speeds away from the critical value. The procedure of identification is then performed in two steps, first updating the model using the response to a known load, then identifying the intensity of travelling loads.

- Experimental field tests were performed, in which the strains at the foot of the rail due to the transit of a locomotor at low speed were recorded. Making reference to the known loads, the model was updated by identifying the optimal value of the soil stiffness and, afterwards, used as interpretative model to identify the weight of the locomotor, now assumed as unknown. The loads were then identified with satisfactory accuracy, with a mean error smaller than 4%.

6 ACKNOWLEDGEMENTS

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