OMA experimental identification of the damping properties of a sloshing system

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Abstract

In the paper, an experimental method based on the Operational Modal Analysis (OMA) approach will be developed for the estimate of the damping properties of a sloshing system. The considered system will mimic a wing-like structure carrying fluid simulating the sloshing typically occurring in a flying wing. The experimental investigation will take advantage of the environmental testing facility available at the Department of Mechanical and Aerospace Engineering of the University of Rome “La Sapienza”. The sensitivity of the damping estimates to the type, direction, and level of excitation as well as to the different filling levels will be studied and compared with similar results already achieved by the Department of Aerospace Engineering of the University of Bristol. The presented results, will serve the SLOshing Wing Dynamics (SLOWD) European funded project for the accuracy assessment.

1 Introduction

The understanding of sloshing dynamics has gained specific interest from aircraft manufacturers in their attempt to reduce design loads of flexible wing structures carrying liquid (fuel). For, it is essential the development of modeling capabilities that can describe the resulting wing loads from flying in atmospheric gusts, turbulence and landing impacts [1]. This objective could be achieved if a proper characterization of the damping properties of the fluid-structure system is carried out. From the experimental investigation point of view, the sloshing mechanism depends on how the dynamic energy is transferred and dissipated between the mechanical and the fluid systems. Several tests have been already carried out to estimate the damping properties of a fluid-structure system excited by harmonic loadings. However, such a periodic energy exchange between the systems is not accurately describing the damping mechanism occurring in a complex dynamic system as a flexible wing structure in real operating conditions. As an alternative to such harmonic, or forced, experiments other authors estimated the damping properties of a sloshing system from the system free responses [2], [3]. Still, existing decay experimental data are limited and they refer to response levels far lower than those corresponding to those characterizing an actual flying wing exhibiting a violent slosh in which the dissipating mechanism is driven by the rapid generation of vorticity, and therefore associated to high rate of dissipation of mechanical energy via viscous forces.

In this paper, an experimental identification of the damping properties of a wing-like structure carrying fluid simulating the sloshing behavior is presented. Specifically, the developed experimental methods, as well as the presented results, will serve the SLOshing Wing Dynamics (SLOWD) European funded project for the accuracy assessment.

An experimental method based on the Hilbert Transform Technique (HTM) Operational Modal Analysis (OMA) approach will be developed and the damping characterization of the sloshing system will be compared to those already available in the literature. The experimental investigation will take advantage of the environmental testing facility available at the Department of Mechanical and Aerospace Engineering of the University of Rome “La Sapienza”. First, the preliminary design of a cantilever beam with a tank located at its tip, based on the Froude scaling, is presented, then the analytical predicted dynamic properties will be compared to the corresponding ones gained from the experimental campaign. The sensitivity of the damping estimates to the type, direction, and level of excitation as well as to the different filling levels will be studied and compared with similar results already achieved by the Department of Aerospace Engineering of the University of Bristol.

2 Recalls on the Hilbert Transform Technique (HTM)

In this paper, the OMA method called Hilbert Transform Method will be used. More details on this approach can be found in [4]-[11]. However, it is worth noting that such an estimating method is included in the MATLAB-based OMA code called “NIMA” (Natural Input Modal Analysis) developed at the Department of Mechanical and Aerospace Engineering of the University of Rome “La Sapienza”.

2.1 Hilbert Transform Method

Starting from the output response, the spectral density function matrix can be built. The output spectral density function matrix is composed by spectral density functions defined between the *i-th* and the *j-th* output responses, at the *k-th* spectral line. The polar representation of a driving point Frequency Response Function (FRF) is:

$$H_{ii}(\omega) = |H_{ii}|\, e^{-j\phi_{ii}\omega} \tag{1}$$

or, by introducing the natural logarithm:

$$|H_{ii}| = G_{ii} - j\phi_{ii}(\omega) \tag{2}$$

where G_{ii} is the gain function. Considering that the real part of the FRF is an even function while the imaginary part is an odd function, the gain and the phase are respectively an even and an odd functions and, as a result, the left-hand side of Eq. 2, can be expressed as the sum of a pair of Hilbert transform functions:

$$\phi_{ii} = -\hat{G_{ii}}(\omega) \tag{3}$$

The gain function is also related to the spectral density function as:

$$G_{yy}(\omega_k) = H(\omega_k) G_{ff}(\omega_k) H^H(\omega_k) \tag{4}$$

where the input spectral density matrix, defined between the N_i inputs is assumed to be derived from a white noise excitation. This implies that $G_{ff}(\omega_k)$ is frequency independent, that is $G_{ff}(\omega_k) = G_{ff}$ and is a diagonal matrix when the input excitation is uncorrelated in the space domain. By applying the natural logarithm and using Hilbert transform, Eq. 4 becomes:

$$\mathcal{H}[\ln(G_{y_i y_i})] = 2\mathcal{H}[\ln|H_{ii}(\omega)|] \tag{5}$$

in which the input spectral density contribution, $G_{f_i f_i}$ is null, since the Hilbert transform of a constant is zero. Combining the previous Eqs. 3 and 5, it is possible to write

$$\phi_{ii}(\omega) = -\frac{1}{2}\mathcal{H}[\ln(G_{y_i y_i}(\omega))] \tag{6}$$

Therefore, the FRF in the i-th driving point is available by evaluating the modula and phase from the PSD function $G_{y_i y_i}$. It is possible to demonstrate that the off-diagonal terms of the FRF are derivable from the

Table 1: Mode shapes and natural frequencies of the bare beam by numerical analysis

Mode #	Mode Type	Frequency [Hz]
1	I B	19.5324
2	II B	122.197
3	I L	190.168
4	I T	238.622
5	III B	342.134

comparison between the commonly used estimators H_1 and H_2:

$$H_{ij}(\omega) = \frac{G_{y_i y_j}(\omega)}{\sqrt{G_{f_i f_i}} H^*_{ii}(\omega)} \tag{7}$$

Obviously the estimated functions are unbiased depending on the unknown input forces.

Once the FRFs are obtained it is possible to estimate the modal parameters of the system using the least square approximation, considering the expression of the FRF in pole-residue terms. In the frequency range of definition of the FRF the number of modes are unknown, therefore a stabilization diagram occur to estimate it by an iterative process ([13]-[16]).

3 Preliminary design of the wing-like structure

The structure is represented by a cantilever beam made of aluminium alloy clamped at one end that is, in turn, connected to the shaker system excitation system through an interface. On the other beam tip, a tank is located so that it can vibrate according to the controlled seismic motion.

The beam dimensions are designed so that its dynamic behavior is similar to a reference structure, that is both the actual system and the reference structure have the same Froude number to have the same effect of the gravitational forces acting on the fluid inside the tank. Specifically, the Froude number is defined as

$$Fr = \frac{U}{\sqrt{gL}} \tag{8}$$

being U the velocity, L the length, and g the gravitational acceleration. Thus, it is possible to obtain the span of the beam test article once the length of the wing of the aircraft considered as reference is known. Furthermore, by defining s as the ratio between the length of the wing of the aircraft and the beam span, the similarity between the natural frequencies is achieved if they are in the ratio equal to $\frac{1}{\sqrt{s}}$. Note that in this similarity analysis, the effects of the presence of the tank is neglected. According to the reference dynamics properties of the wing considered within the SLOWD project, and the test maximum dimensions, the span of the cantilever beam results of 0.65 m, whereas the width and the thickness are chosen equal to 0.1 m and 0.01 m respectively for practical reasons.

3.1 Finite Element Modeling: bare beam

In order to characterize the dynamic behavior of the bare beam, the finite element model of the previous cantilever beam, without the tank, is made by plate elements (CQUAD4). The clamping device has been modeled using two "L" bracket, on both side, with a length of 0.24 m, width 0.075 m, heigth 0.07 m and thickness 0.015 m and two vertical plates, one above and one below the beam, both with same dimensions of length 0.24 m, width 0.1 m and thickness 0.03 m. The total number of elements is 2277, to guarantee the numerical convergency up the fifth natural frequency. The corresponding modal parameters, as computed by the so called SOL 103 rigid solution sequence of the MSC.NASTRAN software, are reported in Tab. 1 where B, L, and T denote bending, lag, and torsional mode shape respectively.

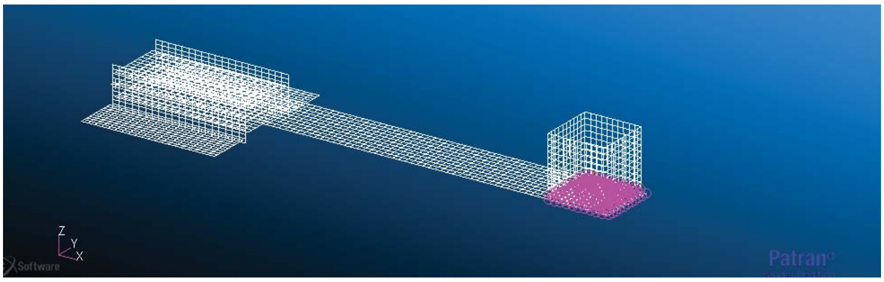

Figure 1: Configuration of the beam with the complete tank for m=0.578kg.

Table 2: Height of the filling 0.04 m (0.486 Kg).

Mode #	Mode Type	Frequency [Hz]
1	I B	13.5858
2	II B	107.84
3	I L-I T	119.548
4	I L-I T	193.548

3.2 Finite Element Modeling: beam with tank

The tank is located at the beam tip and is composed by the tank walls, made in plexiglass, and the liquid. In this analysis, the tank walls are represented by plate finite elements (CQUAD4) of thickness equal to 2 mm, whereas the liquid is assumed non-sloshing and represented by a lumped mass (CONM2), located at the liquid (water) center of gravity, connected to the bottom surface by rigid elements (RBE2), see Fig. 1 for the reference case of the liquid mass equal to 0.578 Kg.

In order to establish the range of variation of the natural frequencies corresponding to the different filling levels, three filling heights, 0.04 m, 0.05 m, and 0.06 m, with respect to the bottom of the tank, have been considered. The corresponding natural frequencies are reported in Tabs. 2, 3, and Tab. 4, whereas the mode shapes are depicted in Fig. 2. Note that, due to the considered configurations, mode shapes are not dependent on the different filling level, thus no reference has been made for such a plot to the height of the fluid.

As one can see, a slight dynamic contribution from the tank walls is observed. The effect of the different filling levels on the natural frequencies corresponding to the cantilever beam with and without the tank walls with four filling height of 0.04 m, 0.05 m, 0.06 m, and 0.07 m is reported in Fig. 3. The effect of the increase of the filling level is more and more important as the mode number is increased. Indeed, a strong contribution to the structural dynamics of the tank walls is reported, and this contribution seems to be greater at higher frequencies.

3.3 Dynamic response simulation

Finally, a dynamic response simulation is carried out in order to analytical evaluate the structural dynamic response during the seismic excitation provided by environmental testing facility. The considered simulation refers to the structure with the tank filled with water at the height of 0.05 m, and numerically excited at its root with a vertical random excitation, constant in the frequency range of $[0 - 250]$ Hz, with an overall

Table 3: Height of the filling 0.05 m (0.578 Kg).

Mode #	Mode Type	Frequency [Hz]
1	I B	13.0781
2	II B	106.397
3	I L-I T	111.822
4	I L-I T	194.444

Table 4: Height of the filling 0.06 m (0.670 Kg).

Mode #	Mode Type	Frequency [Hz]
1	I B	12.6171
2	I L-I T	104.238
3	II B	104.762
4	I L-I T	194.985

Table 5: Bare beam: experimental modal parameters.

Mode #	Mode Type	f_n [Hz]	ζ_n [%]
1	I B	17.187	1.01
2	II B	106.39	2.49
4	I T	222.13	0.25
5	III T	307.15	1.45

amplitude of 1g RMS, and using a damping ratio of 7%. The dynamic response analysis, carried out with the MSC.NASTRAN SOL 111 solution sequence, is synthesized in Fig. 4, where the transfer function between the vertical acceleration of the tip node with respect to the base acceleration is depicted. From such analysis, an amplification factor of 16.64 corresponding to the first natural frequency is foreseen, whereas a lower amplification factor of 2.638 is evaluated for the second natural frequency.

4 Experimental investigation

The actual sloshing system is manufactured by considering the outcomes of preliminary analysis reported in the previous Section.The cantilever beam-tank system, is constraint at the vibration shaker through a head expander and a structural interface. This last item is composed of one bottom plate and two top plates to properly constraint the beam-tank system, see Fig. 5.

The SIEMENS TEST.Lab software provided the drive signal to the Dong-Ling hardware in a close-loop control seismic excitation. The test setup considered one 1-D control accelerometer, located at the beam root, and three response accelerometers, uniformly placed along the beam span, to measure the system vertical accelerations.

4.1 Bare beam

In the first part of the experiments the considered beam is without the tank, to provide useful information for assessing the numerical model.

4.1.1 Bare beam: experimental modal analysis

The cantilever beam is excited with a modal hammer with a steel tip, whereas three axial roving accelerometers, have been used to record the vertical acceleration in correspondence of 11 measuring locations evenly distributed over the beam span. In Fig. 6 the roving configuration corresponding to accelerometers located at 0.650 m, 0.433 m, and at 0.216 m from the beam root is shown. Note that an uneven excitation, depicted with a red arrow, has been provided by the hammer to properly excite both the bending and the torsional modes. Using the SIEMENS Polymax technique, 4 modes are estimated from the stabilization diagram built on the H_2-estimated average frequency response function, Fig. 7, 5. By comparing the numerical and experimental natural frequencies of the corresponding modes it is possible to preliminary validate the numerical model of the beam since an average error of order of magnitude of 10% is achieved. It is expected to increase the correlation by refining the modeling of the clamping device.

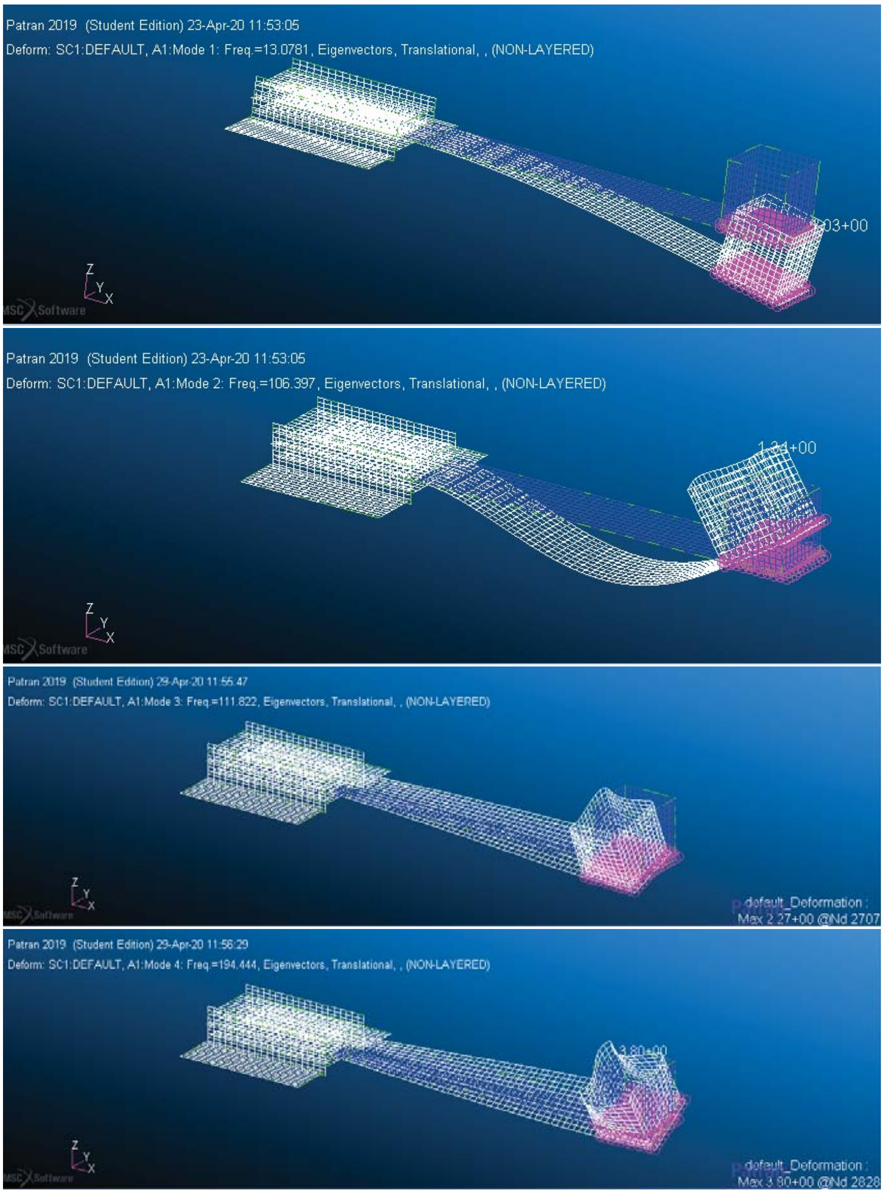

Figure 2: First four mode of the cantilever beam in the configuration with the complete tank for m=0.578kg.

4.1.2 Bare beam: experimental dynamic response

The bare beam is then seismically excited with the same random excitation of the numerical analysis, with the hardware and software described at the beginning of this Section. First, a non-linearity check has been performed. From the analysis of the transfer functions corresponding to different excitation levels, from 4g RMS to 6.8g RMS, no evident non-linearities have been identified in the frequency range of interest, see Fig. 8. Then the accuracy of the numerical dynamic response model has been assessed by comparing the transfer functions with the experimental ones. By comparing the numerically evaluated dynamic response of Fig. 4 with the experimental one reported in Fig. 9, a good correlation has been obtained in correspondence of the first natural frequency, for which comparable amplification factors are obtained. On the contrary, the amplification factors corresponding to second natural frequency exhibit an evident discrepancy, about one order of magnitude. This difference could be probably reduced by setting a more accurate value of the damping coefficient associated to the second mode shape and by introducing a more accurate fluid model.

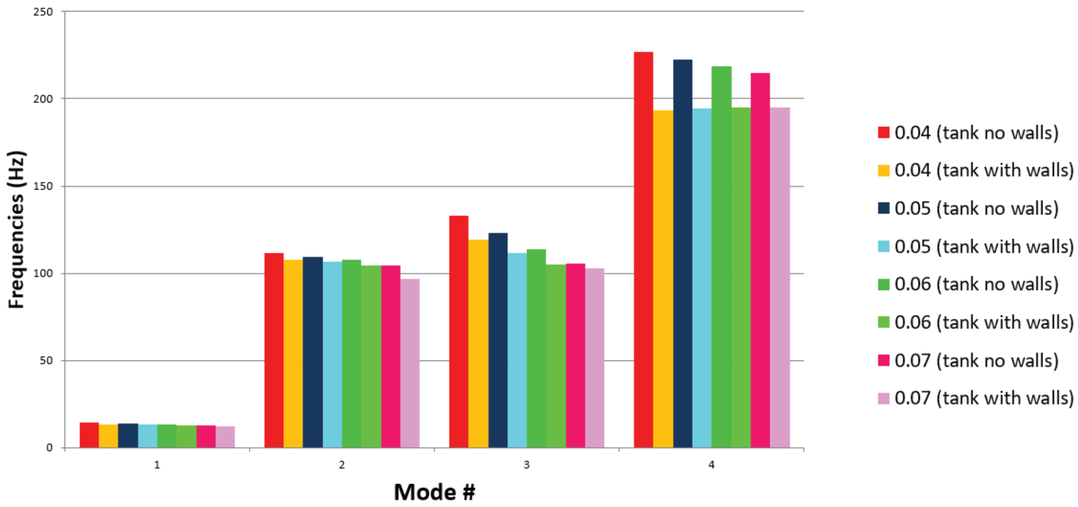

Figure 3: Comparison between the configurations of the beam for tank with and without wall.

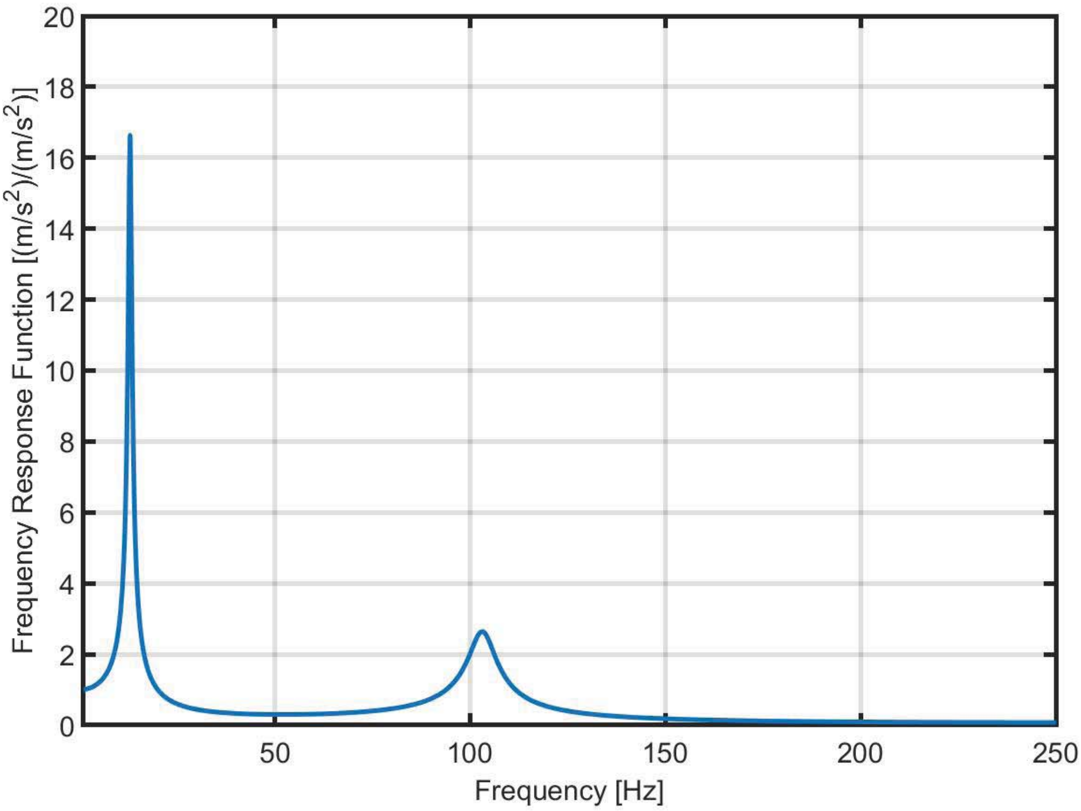

Figure 4: Dynamic response of the cantilever beam subjected to a random excitation of 1g in the range of 0Hz-250Hz.

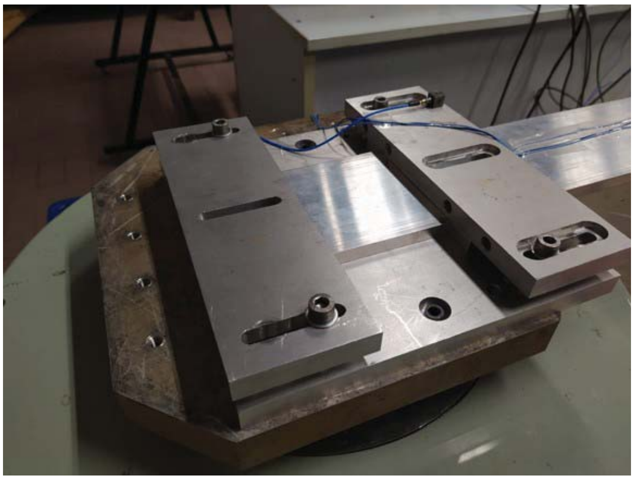

Figure 5: Detailed view of the cantilevering device.

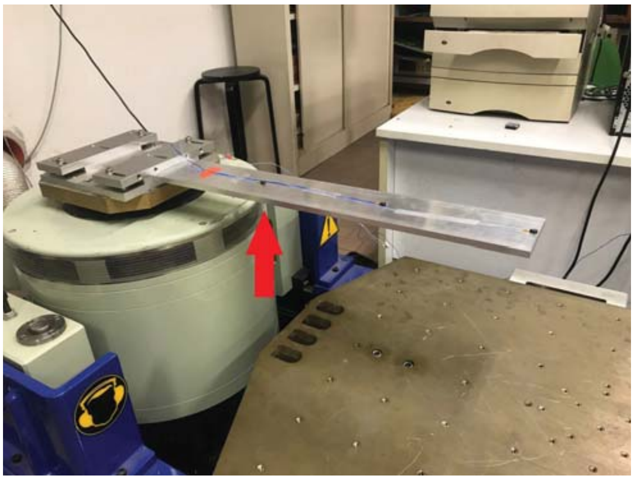

Figure 6: Bare beam: EMA test setup; red arrow = input.

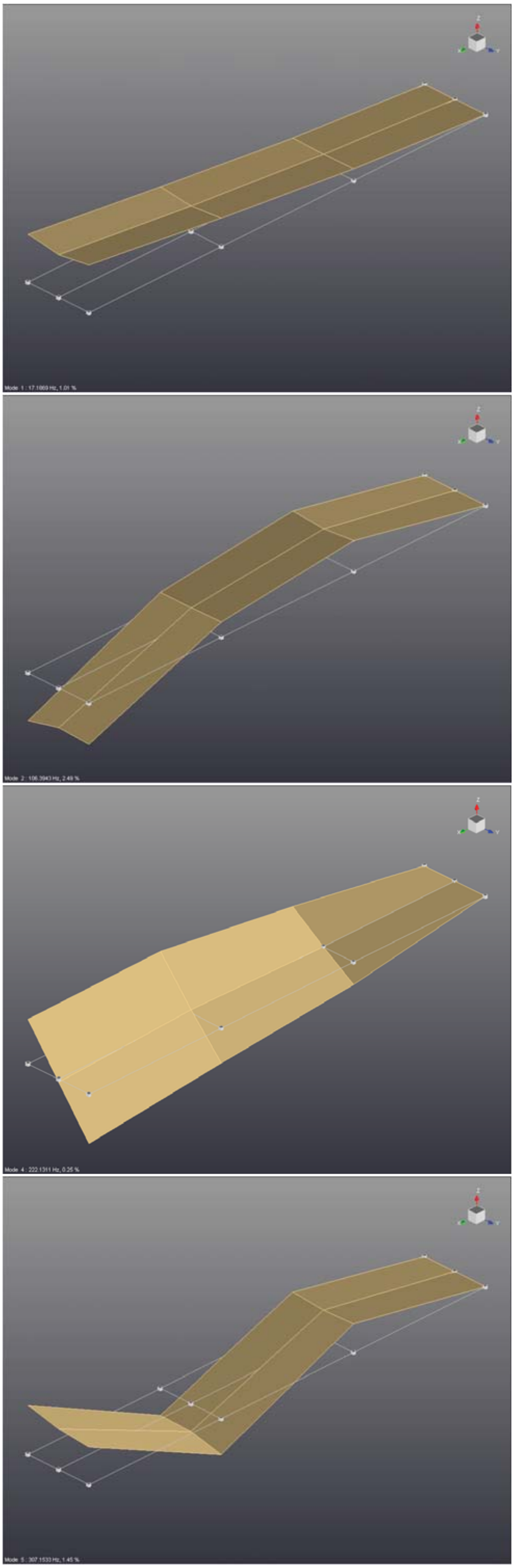

Figure 7: Natural modes of the bare beam excited with the modal hammer

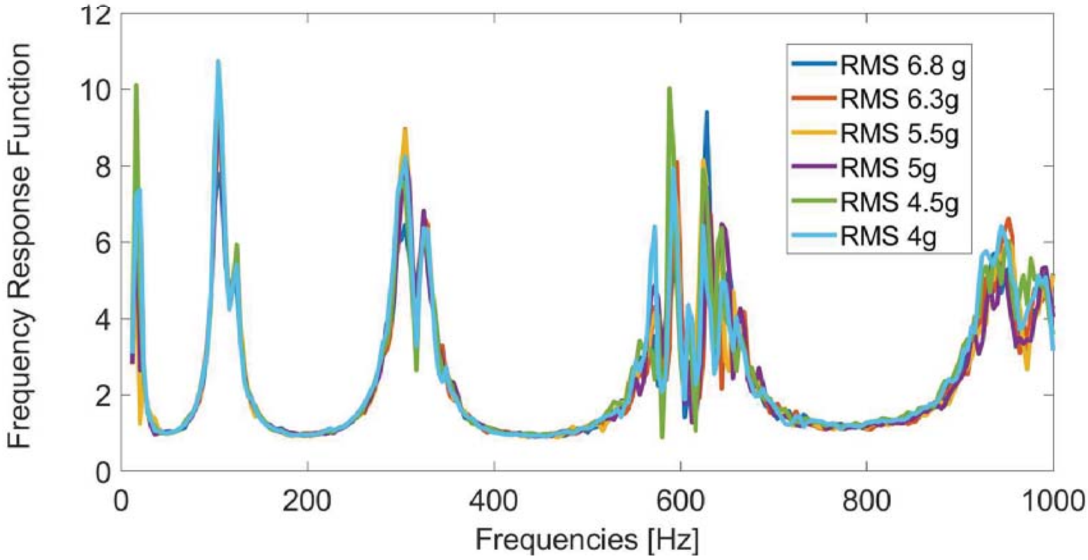

Figure 8: Experimental bare beam: linearity check.

Table 6: Experimental beam-tank system: estimate of the natural frequencies and damping ratios with empty tank.

Mode #	f_n [Hz]	ζ_n [%]
1	13.98	1.5137
2	93.65	1.9076
3	138.50	3.3921
4	258.97	2.6709

4.2 Beam with tank

A cubic tank of 10cm edge length is attached to the beam tip and an accelerometer, located at one corner of the top surface, is used to measure the vertical acceleration of the beam-tank system also including the effects of the liquid motion contained in the tank, see Fig. 10. The modal parameters of the beam-tank system are first evaluated in the case of empty tank. Such a modal parameters, reported in Tab. 6, are estimated using the HTM OMA technique from the output power spectral densities corresponding to a seismic excitation constant in frequency with an amplitude equal to 1g. Although the mode shapes are those of the previous experimental investigation the effect of the weight of the tank is evident in the lowering of the natural frequencies. Also, an overall increase of the damping ratios is reported. The beam-tank system is the considered with the tank filled with water. Three heights of the water filling level are considered as those of the numerical simulations, that is $h = 0.04$ m, 0.05 m and 0.06, whereas nine amplitudes of the base excitation, constant in the frequency range $[8 - 1000]$ Hz where used to excite the system, that is the RMS amplitudes of 0.35g, 0.50g, 0.75g, 1.00g, 1.35g, 1.50g, 1.75g, 2.00g, and 3.00g are considered. As a general comment, it is noticed that up to the excitation level of RMS$=$ 0.5g the fluid motion is characterized by small amplitudes of vibrations and no practical interaction with the tank walls is reported, see 11. From the excitation level of 0.75g the water

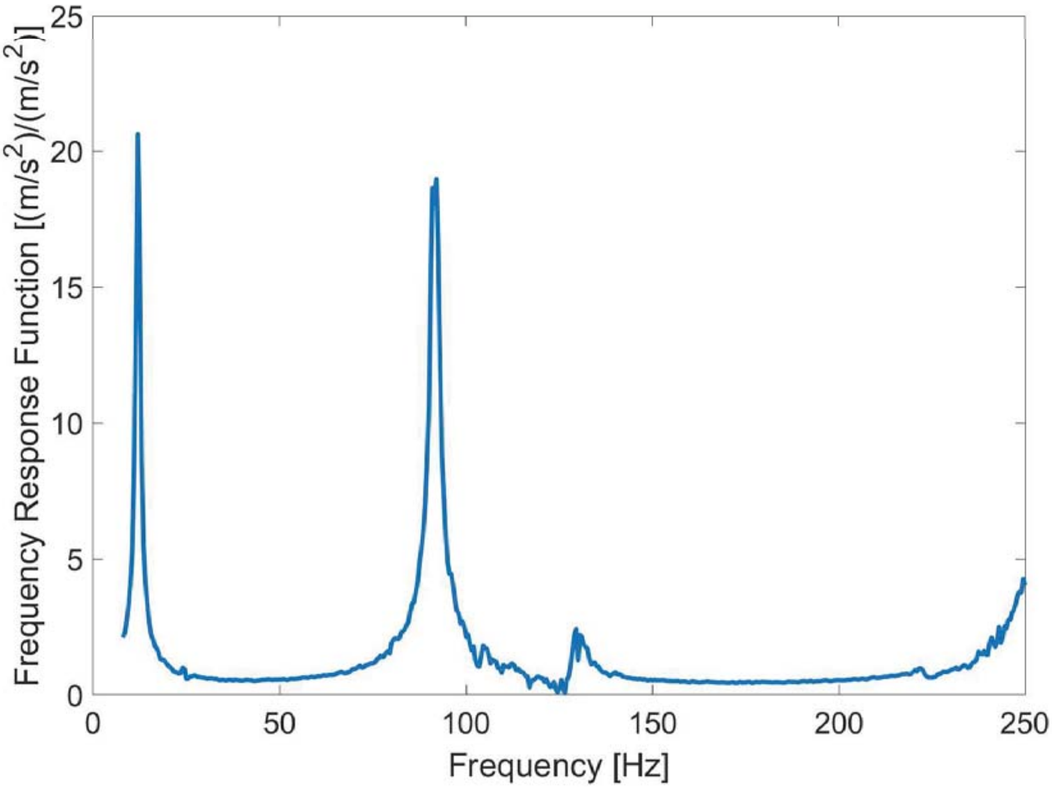

Figure 9: Experimental beam-tank system: dynamic response with filling height $= 0.05$ m, 1g random excitation in the frequency range of [0-250] Hz.

Table 7: Experimental beam-tank system: estimated natural frequencies for h=0.04 m for 0.75g.

Mode #	Frequency [Hz]
1	12.48
2	91.04
3	132.55
4	256

motion start to interact with the upper horizontal surface of the tank, and from eismic excitation of 1g the liquid sloshing effect is fully developed, see Fig. 12. The estimate of the natural frequencies corresponding to considered excitation and filling levels shows a very little sensitivity to the excitation level, as reported in Tabs. 7, and 9, results regarding the filling level $h = 0.04$m are onlt reported for brevity.

On the contrary, the damping ratios depend on the excitation level, for a given height of the the filling water, as expected. Indeed, an overall increase of the damping ratios with the excitation level is observed for the identified four modes, see figures from Fig. 13 to Fig. 16. Furthermore, the identified zigzagged shapes the previous trends, is showing the different effects on the damping mechanism of the different sloshing regimes for a given mode shape. Finally, it is also noticed a very large value of the damping ratio for the third mode of the beam-tank system when excited at 1.5g RMS with the liquid filling level $h = 0.06$ m. This could be the result of a strong second-third mode coupling that specific operating condition is highlighting.

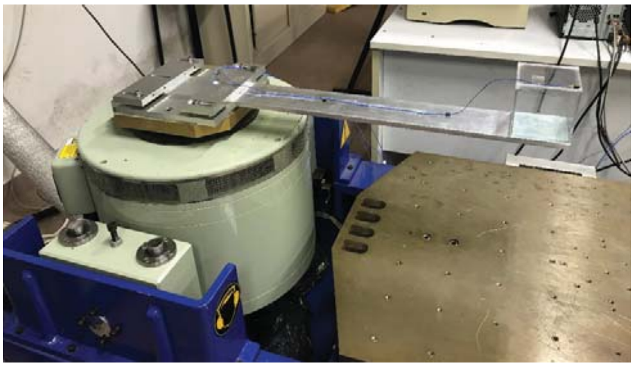

Figure 10: Experimental beam-tank system: test setup

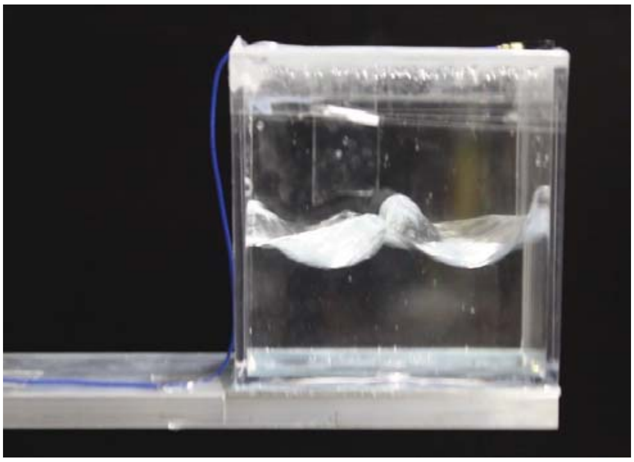

Figure 11: Experimental beam-tank system: video frame of the sloshing of the water inside the tank for h=0.04 m and a RMS=0.5g.

Concluding remarks

An experimental procedure has been developed to characterize the damping properties of a wing-like structure with a sloshing liquid contained in tank. Such a procedure estimated the modal parameters of the operating system by the developed Hilbert Transform Method (HTM) at different liquid filling levels inside the tank and different excitation levels. For, the closed-loop control environmental testing device, available at the Structural Dynamics Laboratory of the Department of Mechanical and Aerospace Engineering of the University of Rome “La Sapienza” has been used to guarantee the proper excitation levels. The experimental investigation shows a sensitivity of the damping ratios of the considered wing-like structure to the sloshing regimes. Specifically, an overall increase of the damping ratios is reported as the excitation level is increased in correspondence of the identified mode shapes. Also possible effects of the sloshing regimes to the damping ratios is observed. The presented results, contribute to the reference database to the SLOshing Wing Dynamics (SLOWD) European funded project for the accuracy assessment.

Acknowledgements

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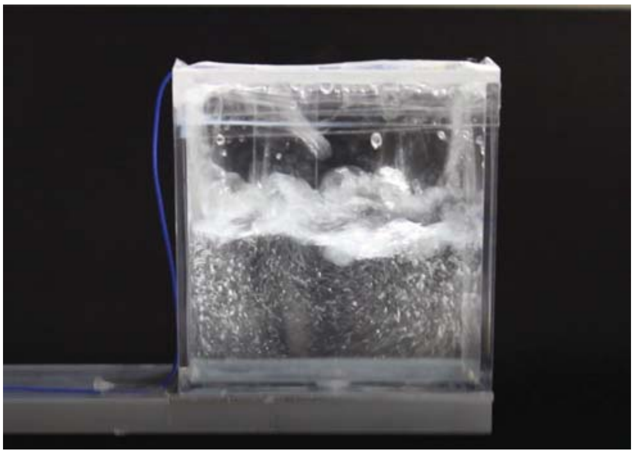

Figure 12: Experimental beam-tank system: video frame of the sloshing of the water inside the tank for h=0.05 m and a RMS=1.5g.

Table 8: Experimental beam-tank system: estimated natural frequencies for h=0.04 m for 1.35g.

Mode #	Frequency [Hz]
1	12.50
2	91.02
3	133.47
4	255.41

Table 9: Experimental beam-tank system: estimated natural frequencies for H=0.04 m for 1.75 g

Mode #	Frequency [Hz]
1	12.42
2	90.73
3	135.62
4	252.19

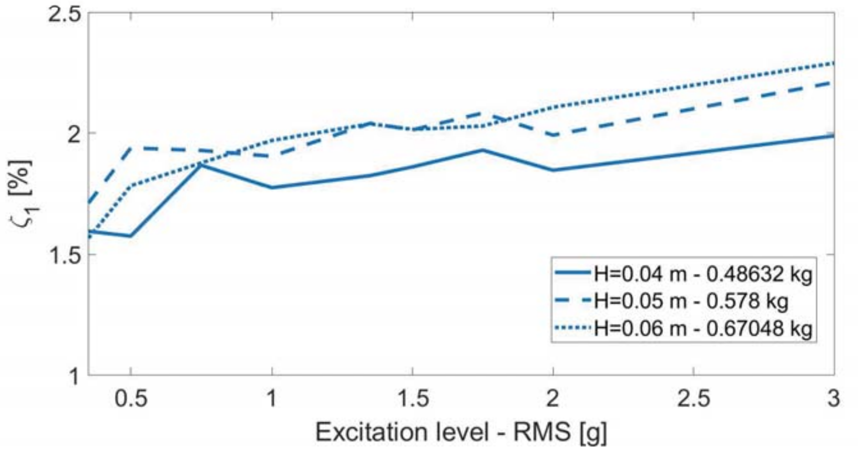

Figure 13: Experimental beam-tank system: sensitivity of the damping ratio associated to the mode #1 to the filling and excitation levels.

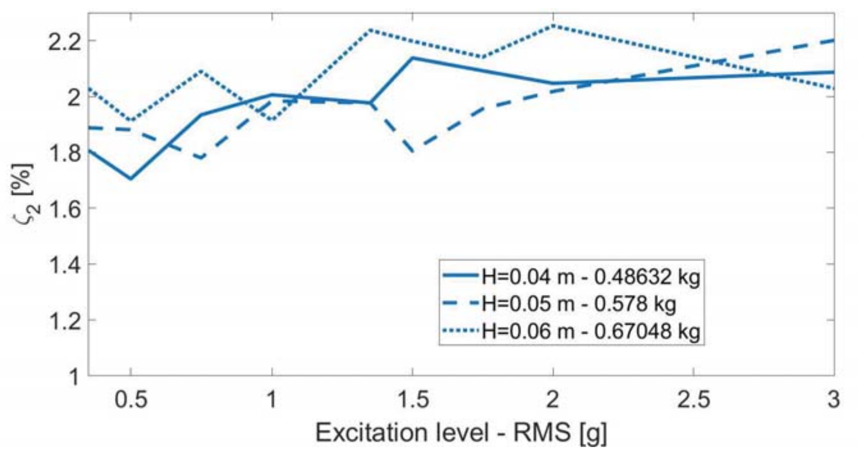

Figure 14: Experimental beam-tank system: sensitivity of the damping ratio associated to the mode #2 to the filling and excitation levels.

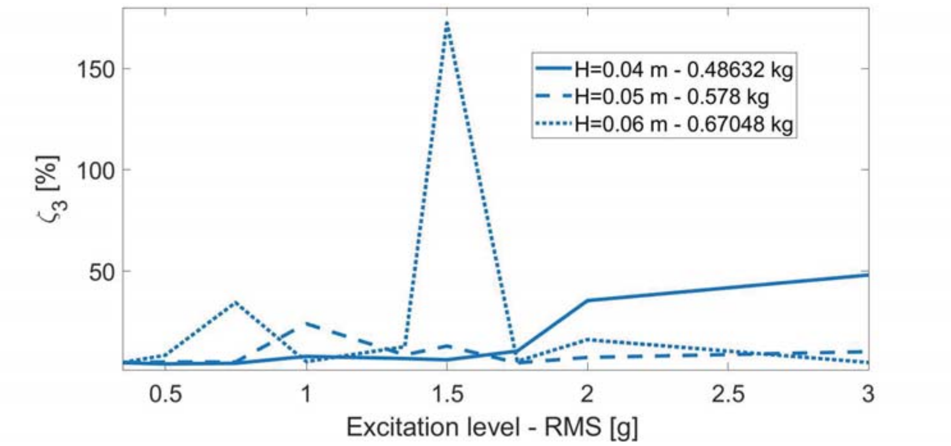

Figure 15: Experimental beam-tank system: sensitivity of the damping ratio associated to the mode #3 to the filling and excitation levels.

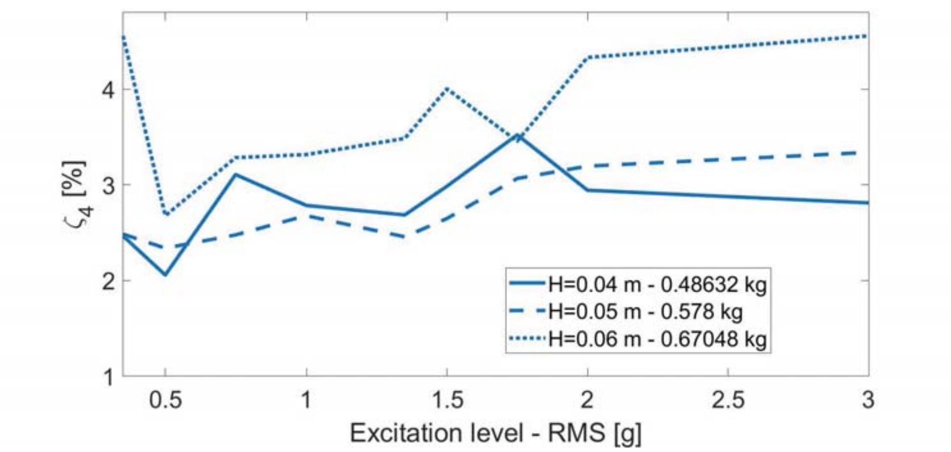

Figure 16: Experimental beam-tank system: sensitivity of the damping ratio associated to the mode #4 to the filling and excitation levels.

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