Book of Short Papers
SIS 2020

Editors: Alessio Pollice, Nicola Salvati and Francesco Schirripa Spagnolo

Published by Pearson

www.pearson.com

ISBN 9788891910776

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Adding MIDAS terms to Linear ARCH models in a Quantile Regression framework

Regressione quantilica con l'aggiunta di termini MIDAS per modelli Linear ARCH

Vincenzo Candila, Lea Petrella

Abstract Recent financial crises have placed an increased accent on methods dealing with risk management. Despite some critiques, the Value-at-Risk (VaR) still plays today a leading role among the risk measures. For this reason, the financial econometrics literature has been involved in proposing as much as possible accurate VaR models. Recently, the quantile regression (QR) approach has been used to directly forecast the VaR measures. Within such a QR framework, we add a (MI(xed)-DA(ta) Sampling) term to the well known Linear ARCH (LARCH) model. The MIDAS term allows the inclusion of macroeconomic variables usually observed at low frequencies (monthly, quarterly, and so forth) in contexts where the dependent variable is generally observed at higher frequencies (mainly, daily). The resulting model, named Quantile LARCH-MIDAS (Q–LARCH–MIDAS), is the first model incorporating the MIDAS approach within the QR framework.
Abstract *Le recenti crisi finanziarie hanno portato un enorme interesse verso i metodi per la gestione del rischio. Nonostante alcune critiche, il Value-at-Risk (VaR) ha ancora oggi un ruolo primario tra le misure di rischio. Per questa ragione, la letteratura econometrica-finanziaria ha posto l'attenzione sui modelli per la stima del VaR. Recentemente, la regressione quantilica (QR) è stata usata per calcolare direttamente il VaR. In questo contesto di QR, un termine MIDAS (MI(xed)-DA(ta) Sampling) è aggiunto al noto modello Linear ARCH (LARCH). Il termine MIDAS permette l'inclusione di variabli macro, solitamente osservate a frequenza mensile o quadrimestrale, in contesti dove, di solito, la variabile dipendente è osservata a cadenza giornaliera. Il modello risultante, chiamato Quantile LARCH-MIDAS (Q–LARCH–MIDAS), è il primo modello che incorpora l'approccio MIDAS all'interno di un contesto di QR.*

Key words: Value-at-Risk, Quantile Regression, MIDAS term.

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1 Introduction

During the last decades, financial econometrics literature has paid particular attention to the methods for risk management. Among the different risk measures proposed in the literature, the Value-at-Risk (VaR) plays still today a leading role. Despite some criticisms (Artzner et al., 1999), according to the Basel frameworks, the VaR measures are fundamental in order to set aside risk capital adequately (Burchi and Martelli, 2016). The methodology used to obtain the VaR measures can be broadly divided into three main categories: parametric, non-parametric, and semi-parametric (Jorion, 1997). The parametric approach requires the estimation of the volatility of the asset under investigation as a primer step. Typically, the GARCH (Bollerslev, 1986) class of models is used. Secondly, the VaR measures are indirectly obtained by considering these volatility estimates and the quantile at a fixed level of the presumed distribution of the asset. Usually, the Normal distribution is taken into account. Contrary to the parametric approach, the non-parametric technique does not make any distribution assumption concerning the daily returns. The semi-parametric technique specifies the updating dynamics of the model but does not require any distributional assumptions. Contributions based on the quantile regression (QR) (Koenker and Bassett, 1978; Engle and Manganelli, 2004) framework belong to the class of semi-parametric methods for the estimation of the VaR measures. Recent works employing the QR methods are Laporta et al. (2018); Bernardi et al. (2015), among others. Within the QR context, this paper aims to investigate the profitability of including a MIDAS (MI(xed)-DA(ta) Sampling) (Ghysels et al., 2007) component in the well known Linear ARCH (LARCH) (Taylor, 1986) model. The MIDAS component allows to filter the information coming from variables observed at lower frequencies (say, monthly, quarterly) in contexts where the dependent variable is usually observed daily. In this respect, many contributions (Amendola et al. (2019) and Conrad and Loch (2015), among others) highlight that the macroeconomic variables, which are typically observed at a monthly or quarterly frequencies, are driving forces of (daily) assets' variability. To the best of our knowledge, this is the first time that the MIDAS approach is incorporated within the QR framework. Therefore, the advantage of using the proposed Quantile LARCH–MIDAS (Q–LARCH–MIDAS) model to estimate the VaR measures is that these latter values are directly obtained as conditional quantiles of the daily return process, which in turn may depend on some exogenous variables observed at lower frequencies.

The rest of the paper is organized as follows. Section 2 introduces the Q–LARCH–MIDAS model and Section 3 is devoted to the empirical application.

2 Q–LARCH–MIDAS model

Let r_i be a (log-) return of an asset observed at time i, with $i = 1, \cdots, N$. Usually, i represents a day, but sometimes it can refer to periods observed at lower frequencies. In the general heteroskedastic framework, it is assumed that:

$$r_i = \sigma_i z_i, \tag{1}$$

where σ_i denotes the standard deviation, conditionally to the information set $\mathscr{F}_{i-1}$, and z_i is an *iid* random variable with $E(z_i) = 0$ and $Var(z_i) = 1$.

The conditional (one-step-ahead) VaR for day i, at τ level, is defined as VaR_i, and represents the quantity such that

$$Pr(r_i < VaR_i|\mathscr{F}_{i-1}) = \tau. \tag{2}$$

Therefore, by definition, the VaR at time i is the $\tau-$th conditional quantile of the series r_i, given $\mathscr{F}_{t-1}$. For this reason, the VaR can be also expressed as $Q_{r_i}(\tau|\mathscr{F}_{i-1})$.

Within the parametric context, VaR_i can be (indirectly) obtained once got an estimate of the (one-step-ahead) conditional standard deviation, $\hat{\sigma}_i$ and once a distribution function $F(\cdot)$ for z_i is assumed. That is:

$$VaR_i = F(z_i;\tau)\hat{\sigma}_i, \tag{3}$$

where $F(z_i;\tau)$ denotes the quantile at τ% of z_i. Many options are available for the conditional standard deviation of r_i. For instance, it can be estimated by means of a specification belonging to the GARCH class of models. The density function of z_i is usually assumed Normal, as in the seminal work of Engle (1982). Instead of following a parametric approach to obtain the VaR measures, the goal of this paper is to directly estimate the VaR by means of the quantile regressions, in a semi-parametric context.

In the linear regression model, the relationship between a dependent variable y_i, at time i, and a set of covariates $\boldsymbol{x}_i$ is represented by the following equation:

$$y_i = \boldsymbol{x}_i^{'}\boldsymbol{\beta} + u_i, \tag{4}$$

where the vector $\boldsymbol{x}_i$ includes an intercept and $k-1$ covariates, while the zero mean *iid* error term u_i, with quantile function $Q_u(\tau)$, is left with an unspecified distribution. As demonstrated by Koenker and Bassett (1978), the τ-th quantile of y_i, conditional to $\boldsymbol{x}_i$, is:

$$Q_{y_i}(\tau|\boldsymbol{x}_i) = \boldsymbol{x}_i^{'}\boldsymbol{\beta}(\tau), \tag{5}$$

where, in line with Xiao et al. (2015), the $k\times 1$ vector $\boldsymbol{\beta}(\tau) = (\beta_1 + Q_u(\tau), \beta_2, \cdots, \beta_{k-1})^{'}$ is obtained minimizing the following loss function:

$$\hat{\boldsymbol{\beta}}(\tau) = \underset{\boldsymbol{\beta}\in\mathscr{R}^k}{\arg\min}\left[\sum_{i\in\left\{i:y_i\geq \boldsymbol{x}_i^{'}\boldsymbol{\beta}\right\}} \tau|y_i - \boldsymbol{x}_i^{'}\boldsymbol{\beta}| + \sum_{i\in\left\{i:y_i< \boldsymbol{x}_i^{'}\boldsymbol{\beta}\right\}} (1-\tau)|y_i - \boldsymbol{x}_i^{'}\boldsymbol{\beta}|\right]. \tag{6}$$

The asymptotic properties of the regression quantile estimator in (6) are discussed in Bassett and Koenker (1978).

The work of Koenker and Zhao (1996) is the first contribution where the VaR (or any other quantile of interest) is estimated within the QR framework. In particular, the authors consider the LARCH(q) model of Taylor (1986), defined by:

$$r_i = (\beta_0 + \beta_1 |r_{i-1}| + \cdots + \beta_q |r_{i-q}|) z_i, \quad \text{with } i = 1, \cdots, N, \tag{7}$$

where $z_i \overset{iid}{\sim} (0,1)$. The vector $\boldsymbol{\beta}(\tau) = (\beta_0(\tau), \beta_1(\tau), \cdots, \beta_q(\tau))^{'}$ is obtained minimizing the function in (6), replacing y_i with r_i and $\boldsymbol{x}_i = (1, |r_{i-1}|, \cdots, |r_{i-q}|)^{'}$. Koenker and Zhao (1996) illustrate the asymptotic properties of such estimator. Under this notation, the τ-th conditional quantile of r_i, that is the VaR at $\tau\%$, is:

$$\hat{Q}_{r_i}(\tau|\boldsymbol{x}_i) = \boldsymbol{x}_i^{'} \hat{\boldsymbol{\beta}}(\tau). \tag{8}$$

The innovation of this work is to enlarge the set of covariates of Eq. (7). More in detail, we add a MIDAS term, allowing the inclusion of variable(s) observed at different frequencies with respect to that of the dependent variable. More in detail, let t be the period of observation for the MIDAS variable. This period may be a week, a month, a quarter, and so forth. The LARCH(q) of Eq. (7) changes to the proposed Q–LARCH–MIDAS specification, that is:

$$r_{i,t} = (\beta_0 + \beta_1 |r_{i-1,t}| + \cdots + \beta_q |r_{i-q,t}| + \theta \sum_{j=1}^{K} \delta_k(\omega) |X_{t-j}|) z_{i,t}, \tag{9}$$

where $r_{i,t}$ is the log-returns of day i within the period t, the coefficient θ signals the impact of the weighted summation of the K realizations of the additional variable X_t, observed each period t. The variable X_t could be a macro-economic variable driving the log-returns of $r_{i,t}$ or a proxy of volatility at lower frequency (for instance: weekly or monthly aggregated realized volatility). The only condition that the Q–LARCH–MIDAS requires is the (weak) stationarity of X_t. Globally, there are N_t days for the period t and there are T different "lower frequency" periods. In total, there are N days, obtained from $N = \sum_{t=1}^{T} N_t$. In order to take benefit of the information coming from variable(s) observed at lower frequencies, the MIDAS component is a one-sided filter of the K lagged realizations of a given variable X_t, through the weighting function $\delta_k(\omega)$, calculated for $k = 1, \cdots, K$. As in the related literature, we use the Beta function, which is:

$$\delta_k(\omega) = \frac{(k/K)^{\omega_1 - 1}(1 - k/K)^{\omega_2 - 1}}{\sum_{j=1}^{K} (j/K)^{\omega_1 - 1}(1 - j/K)^{\omega_2 - 1}}. \tag{10}$$

Given that we are only interested in the cases where the most recent observations have a larger weight, we set $\omega_1 = 1$ and $\omega_2 \geq 1$. This will allow only for a monotonic decreasing system of weights.

The parameter space of the Q–LARCH–MIDAS model consists of the following vector $\Theta = \{\beta_0, \beta_1, \cdots, \beta_q, \theta, \omega_2\}$. The estimation of $\hat{\boldsymbol{\beta}}(\tau)$ is obtained by minimizing the loss function in (6), where as above y_i is replaced by $r_{i,t}$ and $\boldsymbol{x}_i = (1, |r_{i-1,t}|, \cdots, |r_{i-q,t}|, WS_{i-1,t})^{'}$, with $WS_{i-1,t} = \sum_{j=1}^{K} \delta_k(\omega) |X_{t-j}|$.

3 Empirical Analysis

The main application of this work focuses on the estimate of the VaR measures for the S&P 500 Index. Most of the data have been collected from the "realised library" of the Oxford-Man Institute. The returns of interest are the open-to-close daily log-returns, for the period 3 July 2000 – 12 November 2019 (4861 daily observations). The additional MIDAS component in the Q–LARCH–MIDAS model is the realized volatility, after a weekly aggregation (with $K = 4$ and $K = 8$). Moreover, we also use the Q–LARCH–MIDAS with a monthly MIDAS term: the (first difference of the) U.S. Industrial Production (IP), collected from the Federal Reserve Economic Data (FRED) archive, with $K = 6$ and $K = 12$ as number of lagged realizations. The competing models of the proposed Q–LARCH–MIDAS specification are: Q–LARCH, GARCH (G) and GARCH with Student's t-distribution (G–t), RiskMetrics (RM), standard GARCH–MIDAS (G–M) Engle et al. (2013), with IP as MIDAS component and $K = 12$, Symmetric Absolute Value (SAV), Asymmetric Slope (AS) and Indirect GARCH (IG) specifications for the CAViaR (Engle and Manganelli, 2004) model.

We evaluate the performance of the proposed model through the Model Confidence Set (MCS) (Hansen et al., 2011), employed with the VaR loss function proposed by González-Rivera et al. (2004). The full sample period has been further divided into three sub-periods: the first two correspond to the same periods used in Laurent et al. (2012) and the third period represents the Great Recession (according to the NBER dates). Overall, the proposed model behaves very well. In fact, for $\tau = 0.05$, the Q–LARCH–MIDAS with weekly MIDAS component model always enters the MCS. Moreover, looking at the full sample period, only the models based with the MIDAS component belong to the set of superior models.

Table 1 S&P 500: Model Confidence Set for 5% VaR measures

	Q–L–M	Q–L–M	Q–L–M	Q–L–M	Q–L	G	G–t	RM	G–M	SAV	AS	IG
X_t	RV	RV	IP	IP					IP			
Freq.	W	W	M	M					M			
K	4	8	6	12					12			
Period 1	0.1405	0.1405	0.1438	0.1432	0.1436	0.1406	0.1404	0.1406	0.1373	0.1414	0.1349	0.1386
Period 2	0.0786	0.0786	0.0796	0.0796	0.0807	0.0780	0.0781	0.0785	0.0774	0.0785	0.0794	0.0781
Period 3	0.2193	0.2199	0.2073	0.2088	0.2158	0.2351	0.2345	0.2320	0.2283	0.2410	0.2356	0.2325
Full Period	0.1109	0.1110	0.1109	0.1109	0.1118	0.1119	0.1121	0.1118	0.1100	0.1131	0.1116	0.1116

Notes: The table reports the averages for the loss function proposed by González-Rivera et al. (2004), under four different periods. Shades of gray denote inclusion in the MCS at significance level $\alpha = 0.25$. Models' labels are in the text. X_t indicates the MIDAS component, "Freq." its frequency (Weekly or Monthly) and K the number of lagged realizations of the MIDAS component included in the model. Period 1: July 2000 to March 2003 (681 obs.); Period 2: April 2003 to July 2007 (1088 obs.); Period 3: December 2007 to June 2009 (397 obs.); Full Period: July 2000 to November 2019 (4861 obs.).

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