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Theoretical analysis of stone pavers in pedestrian areas

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Abstract

This paper compared two approaches used to analyze a modular pedestrian pavement made of hexagonal basalt pavers. In presence of occasional heavy traffic roads, the pavement should be verified using methods currently used for road pavements. Different loading conditions were examined varying the geometry of the blocks, and the magnitude of the vertical load. In all cases, the results obtained from the analytical theory of Westergaard were higher than those obtained from a finite element model (FEM). Therefore, a parametric study was performed in order to use the analytical method as an alternative to the costly FEM approach. The results of comparison gave a correction factor, valid for hexagonal pavers: it permits to analytically estimate with good approximation the stresses induced by heavy loads applied to natural stone blocks.

Keywords: Block Pavement; Commercial vehicle loads; Finite Element Model; Westergaard; Hexagonal Stone block; Paver; Pedestrian area

1. Introduction

Block or modular pavements are structures composed of pre-formed modular pavers made of brick or concrete that have been successfully used worldwide for low volume roads. In the last decades, the use of this type of structures has increased, especially for pedestrian zones, cycle paths, residential driveways, parking lots, industrial areas and historical city centers (National Highway Sector Schemes, 2013; Di Mascio, 2002).

The success of a stone pavement mainly depends on the correct determination of the expected applied loads. Indeed, in pedestrian priority or space sharing systems, emergency vehicles and other heavy vehicles (i.e. over 30

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kN) are exceptionally admitted for parking or circulation at speed lower than 40 km/h. Therefore, heavy loads could seriously affect the level of service during their service life (Corazza et al., 2016).

Pavers are generally made of concrete or rock (e.g. basalt, granite): mechanical properties of concrete are more constant than those of rocks because is not affected by the natural “geological” heterogeneity which characterizes natural stone (Sharp & Armstrong, 1984).

Materials used for bottom layers are the same currently used for asphalt and concrete pavements (Huang, 2004).

Some design methods used for modular pavements assume that the response of the structure is equivalent to that of flexible pavements, if pavements are composed of small blocks (e.g. 200 mm large x 100 mm wide x 65 mm thick, as reported by Cook & Knapton (1996)). Indeed, for larger blocks the mechanical response of the system starts becoming more similar to a rigid pavement than to a flexible pavement (Westergaard, 1926).

Empirical methods have been developed especially for modular concrete pavements, mainly because their structural response is easier to be estimated than natural stones.

Mechanistic design methods allow the calculation of the stress-strain conditions induced by a load. In the literature, four main approaches are (Hassani & Jamshidi, 2006): layered elastic analysis (Houben et al., 1984), finite element analysis (Di Mascio et al., 2019), distinct element analysis (Cundall, 1988).

Moreover, several catalogues of modular structures for low-volume roads are available, but they only refer to concrete block pavements (Roussel & Griselin, 1993; Abril, 1994).

Although in some cases it is appropriate to consider occasional heavy loads, the literature does not offer a practical method for designing stone block pavements, and the use of FEM is necessary, long, and costly. For this reason, the authors carried out a parametric study to evaluate a more efficient method to design stone modular pavements. The study focuses on a pedestrian pavement composed of hexagonal basalt pavers. The proposed innovative approach is based on the Westergaard’s analytical theory, commonly used for the analysis of concrete slab pavements. A comparison between the maximum tensile stress values obtained through the analytical approach and that obtained by mean a FEM approach was performed. Particularly, several loading magnitudes and block geometries were considered. The results show that the mechanical performances obtained through both approaches are comparable when a correction factor is applied to the analytical solutions. Therefore, the Westergaard’s theory can be used, as efficient alternative to costlier FEMs, in order to estimate the mechanical response of pavers.

2. Materials and methods

A common method to analyze a block pavement consists on using a FEM to represent modular blocks laid on the bottom layers of the examined pavement (Loprencipe et al., 2019). In the analysis of road pavements, wheel loads are modeled by means of a tire pressure uniformly distributed within a circular area on the top surface of the pavement (Zheng, 2012). In presence of widespread not interlocked blocks, it is possible to apply the load over a single block (Fig. 1). Therefore, the study of the isolated element does not model the lateral contacts of the elements; this condition complies with the hypotheses of the Westergaard’s theory (i.e. stress-free edges).

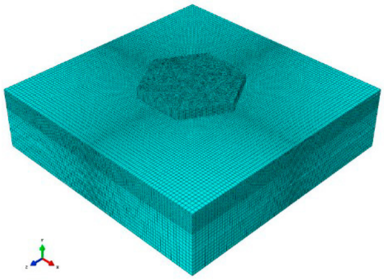

Fig. 1. Example of a finite element model of a block pavement with hexagonal pavers.

Different types of FEM analysis could be performed to calculate the maximum stresses developed on a block pavement: static, quasi-static and dynamic. The first ones are simpler and easier than the former, which requires the knowledge of additional mechanical parameters (e.g. material damping coefficients) and appropriate boundary conditions, to avoid disturbing effects such as wave reflections at the edge of the model (Zoccali et al., 2015). These aspects increase difficulty and computational cost of the analysis. Therefore, if the examined physical phenomenon allows it, it is preferable to use static or quasi-static analyses. In these cases, it is also possible to introduce an adequate Dynamic Amplification Factor (*DAF*) to account for the effects induced by dynamic application of traffic loads (Knapton, 2007).

Other important aspects related with FEM analyses are: the choice of the type of the finite elements, the dimension of the discretization (i.e. mesh size) to be used, and the boundary conditions of the model, since these factors are critical to ensure the convergence, stability and quality of the results.

In the present work, it was decided to perform static analyses, introducing an appropriate DAF. Existing works in the literature report that the average dynamic increment due to road traffic is near 30% (Bonin et al. 2007). Therefore, a DAF of 1.3 was applied to stress values obtained through a static FEM. This study compares the stresses from FEM and the Westergaard's induced by load traffic on stone modular pavements. The Westergaard theory is based on the theory of thin slabs (i.e. the slab thickness is smaller than its length and width dimensions), and it provides theoretical closed-form solutions for concrete slabs (Westergaard, 1926).

The starting configuration of the parametric study coincides with the case study previously analyzed by the authors (Zoccali *et al*., 2018); all the performed analyses considers the same layers under the blocks (Fig. 2).

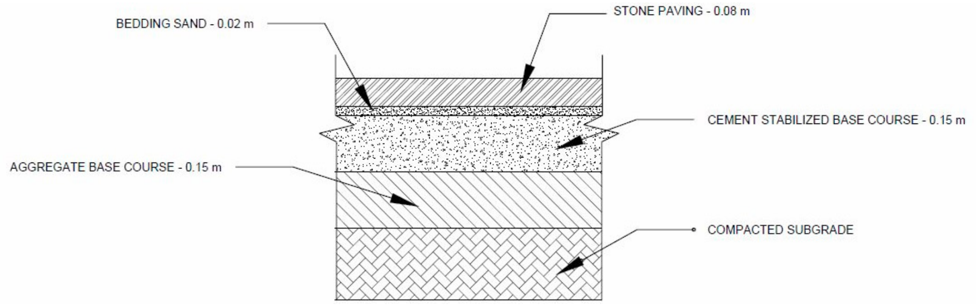

Fig. 2. Example of a finite element model of a block pavement with hexagonal pavers.

Table 1 lists the physical and mechanical properties of the starting configuration.

Table 1. Physical and mechanical properties of the starting configuration.

Material	Thickness (*cm*)	Young's modulus (*MPa*)	Poisson ratio	Reaction modulus (*MPa/m*)
Basalt	8	80,000	0.20	-
Bedding sand	2	150	0.35	-
Cement bound base course	15	500	0.20	-
Aggregate subbase course	15	300	0.35	-
Subgrade	Infinite	100	0.40	40

The considered basalt blocks are regularly shaped hexagonal pavers. Each side of the pavers is 42-cm long.

After a preliminary convergence analysis, several analyses were performed to compare the maximum tensile stress at the bottom of the blocks, obtained using the FEM approach and the Westergaard's theory. The role of three parameters was investigated: the block thickness (h_b), the applied wheel load (P), and the ratio between the radius of the circle inscribed in the hexagon and the radius of the load footprint (r_0).

Six values of h_b (3, 5, 8, 10, 12, and 15 cm), three values of P (50, 20, and 5 kN), and six values of r_0 (1.71, 2.05, 2.4, 3.08, 3.77, 4.45) were examined (Table 2).

Table 2. Examined wheel loads.

Notation of P	Wheel load (kN)	Tire pressure (MPa)	Footprint Area (m^2)	Radius (m)
P_1	50	0.75	0.067	0.146
P_2	20	0.75	0.033	0.103
P_3	5	0.25	0.020	0.080

3. Results

A single basalt block laid on the pavement layers was implemented in the FEM software Abaqus® because the pavers are not interconnected. The boundary conditions adopted in the model were: restrained horizontal displacements on the sides of the model, and fully constrained bottom layers. A preliminary convergence static analysis was carried out applying P_1 (Table 2) at the center of the block (Fig. 3).

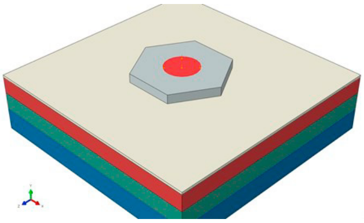

Fig. 3. Configuration of the convergence analyses: the circle represents the applied load.

Three configurations have been examined to choose the shape of finite elements and select their mesh size (Table 3) comparing the maximum tensile stress (σ_T) induced by P_1.

Table 3. Examined wheel loads.

Type of elements	Dimension of the elements (m)		σ_T (MPa)
	under the load	at the edge of the model	
*Hexaedral (hex C3D8)**	0.01	0.02	0.067
	0.02	0.05	0.033
*Tetrahedral (tet C3D4)**	0.02	0.05	0.020

The obtained differences are lower than 5% (Table 3), therefore, all three models gave results with good approximation. A static model meshed with hexahedral elements with a seed of 0.02 m under the load (Table 3) was used to reduce the computational cost of simulations.

The first comparison involved the results obtained through the Westergaard's theory and the FEM approach, having P_1 in different positions: center, vertex, and edge of the paver (respectively Fig. 3, Fig. 4a and Fig. 4b).

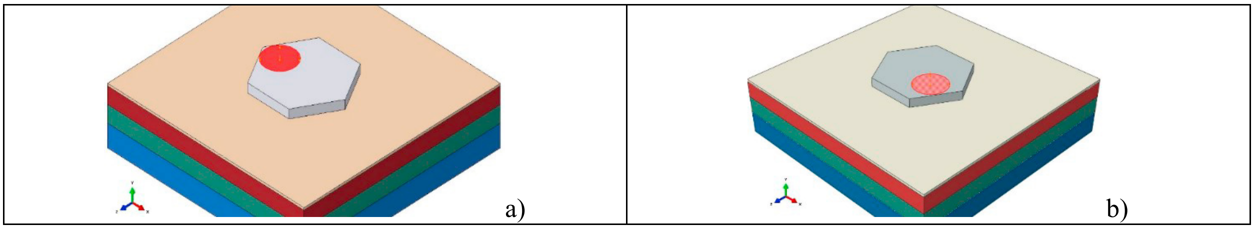

Fig. 4. Corner (a) and edge (b) loading of the basaltic block.

For each loading condition, the maximum tensile stresses at the bottom of the block (σ_T) were calculated using the Westergaard’s theory and the FEM, and compared according to Equation 1:

$$APD = \left| \frac{\sigma_{Westergaard} - \sigma_{FEM}}{\sigma_{FEM}} \right| \cdot 100 \qquad (1)$$

where $\sigma_{Westergaard}$ is the maximum tensile stress obtained using the Westergaard’s theory and σ_{FEM} is the maximum tensile stress obtained using the FEM.

For load applied at the block center (Fig. 3), the two methods gave values of σ_T that converge increasing the block thickness (Fig. 5a): for h_b more than 8 cm, *APD* is less than 50%. A similar trend was also obtained in Fig. 5b when the load is applied at the edge or at the vertex of the block, but for h_b = 8 cm *APD* is almost 100%; for 15 cm-thick pavers, $\sigma_{Westergaard}$ is about twice σ_{FEM}.

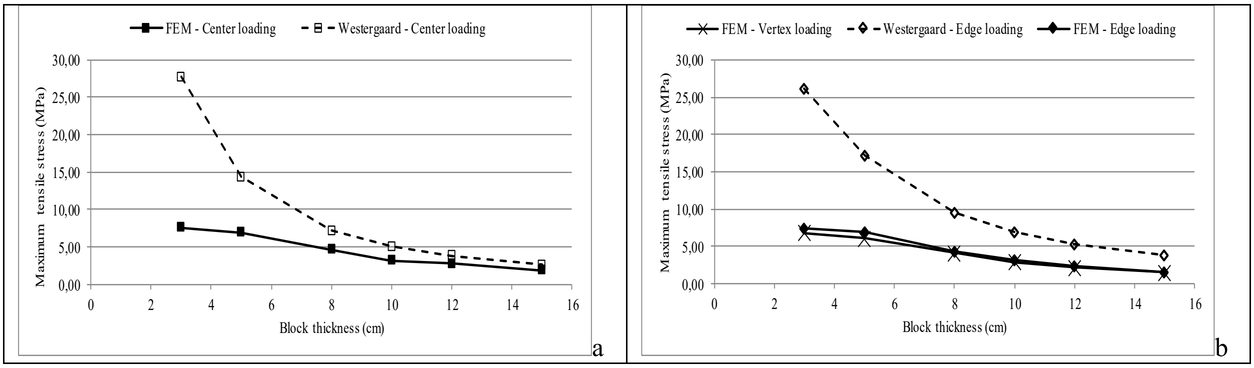

Fig. 5. Comparison of $\sigma_{Westergaard}$ and σ_{FEM} for P_1 applied a) at the center of the paver; b) at the edge and vertex of the block.

The shape of the examined block, which is far from the assumption of square slab considered in the Westergaard’s theory, could justify the curves in Fig. 5. However, the values of σ_{FEM} for the three different loading positions are very close each other (Fig. 6): the average differences are less than 12% for all the examined block thicknesses and the conditions with load applied at the center of the paver (Fig. 3) are the most critical ones (upper curve in Fig. 6).

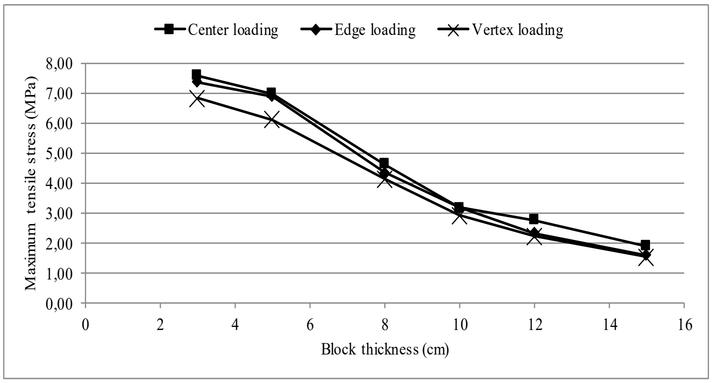

Fig. 6. σ_{FEM} for different loading positions of P_1.

Fig. 7 compares $\sigma_{Westergaard}$ and σ_{FEM} for P_2 and P_3 (Table 2). The two obtained couples of curves are closer than those obtained for P_1 (i.e. Fig. 7 versus Fig. 5a).

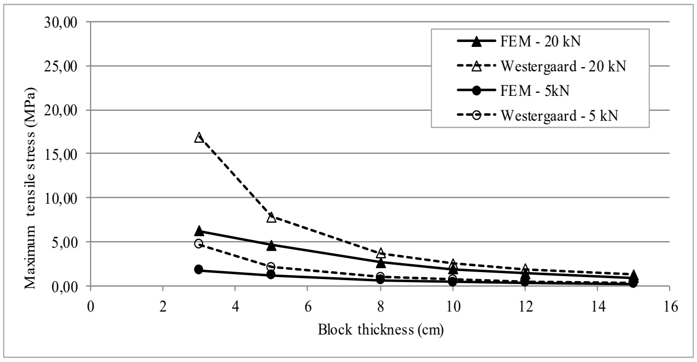

Fig. 7. σ_{FEM} for different loading positions of P_2 and P_3.

Fig. 8 shows σ_T for different plant dimension of hexagons: the analysis considered different values of r_0, having P_1 and 8 and 15 cm-thick blocks. According to the Westergaard's theory, the plant extension of the slab does not affect the calculation of σ_T (Fig. 8). *APDs* are less than 40% for h_b more than 8 cm, while for h_b lower than 8 cm, *APD* significantly increases. For 8 cm-thick blocks and r_0 not less than 3.08 (hexagon edge over than 0.52 m), *APD* is constant ($\approx$ 50%). When h_b is 15 cm, the trend is more linear: it suggests that lower *APDs* could be obtained for r_0 higher than the maximum value considered in this study and h_b greater than 8 cm. Increasing h_b and r_0 leads to a block pavement configuration closer to that assumed in the Westergaard's theory (Ioannides, 1999). It justifies the influence of h_b in the changes of σ_T for different values of r_0.

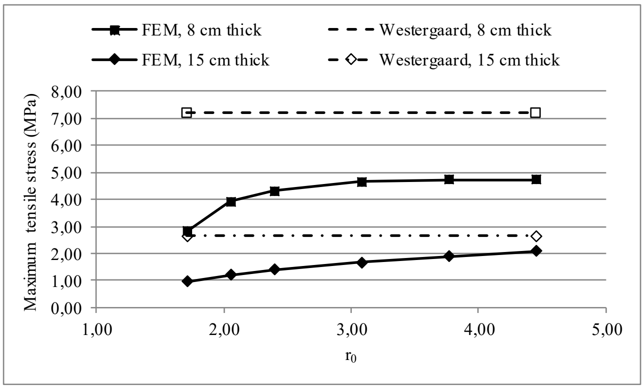

Fig. 8. Maximum tensile stress for different r_0 values, and 8 and 15 cm thick blocks.

For all the 30 examined cases, $\sigma_{Westergaard}$ are higher than σ_{FEM}: the analytical method overestimates the mechanical response of the hexagonal block pavement under study. Therefore, the authors assumed a correction coefficient k_c equal to 0.67 to correct the observed *APDs*: it is valid for the herein analysed hexagonal block shape, pavement structure, wheel loads, block thicknesses and horizontal shape to loading area ratios. For each case study, Table 4 lists σ_{FEM} and $\sigma_{Westergaard}$, its relative *APD*, $\sigma_{CORRECTED}$ calculated according to Equation 2 and its relative $APD_{CORRECTED}$ (*APDC*) calculated according to Equation 3.

$$\sigma_{CORRECTED} = \sigma_{Westergaard} \cdot k_C \tag{2}$$

$$APDC = \left| \frac{\sigma_{CORRECTED} - \sigma_{FEM}}{\sigma_{FEM}} \right| \cdot 100 \tag{3}$$

Table 4. Results of the convergence static analysis.

Wheel load (*kN*)	Hexagon edge size (*m*)	Block thickness (*m*)	σ_{FEM} (*MPa*)	$\sigma_{Westergaard}$ (*MPa*)	$\sigma_{CORRECTED}$ (*MPa*)	*APDC* (*%*)
50	0.42	0.03	7.58	27.73	18.58	145.12
		0.05	6.97	14.37	9.63	38.16
		0.08	4.64	7.19	4.82	3.88
		0.10	3.18	5.08	3.41	7.23
		0.12	2.76	3.80	2.55	7.61
		0.15	1.90	2.65	1.77	6.84
20	0.42	0.03	6.24	16.93	11.34	81.73
		0.05	4.61	7.85	5.26	14.10
		0.08	2.68	3.70	2.48	7.46
		0.10	1.91	2.56	1.71	10.47
		0.12	1.40	1.89	1.26	10.00
		0.15	0.92	1.29	0.87	5.43
5	0.42	0.03	1.73	4.69	3.14	81.50
		0.05	1.17	2.13	1.43	22.22
		0.08	0.66	0.99	0.66	0.00
		0.10	0.46	0.68	0.46	0.00
		0.12	0.33	0.50	0.34	3.03
		0.15	0.22	0.34	0.23	4.55
50	0.08	0.29	2.81	7.19	4.82	71.53
		0.35	3.93	7.19	4.82	22.65
		0.40	4.31	7.19	4.82	11.83
		0.52	4.66	7.19	4.82	3.43
		0.64	4.74	7.19	4.82	1.69
		0.75	4.73	7.19	4.82	1.90
50	0.15	0.29	0.95	2.65	1.77	86.32
		0.35	1.20	2.65	1.77	47.50
		0.40	1.40	2.65	1.77	26.43
		0.52	1.68	2.65	1.77	5.36
		0.64	1.89	2.65	1.77	6.35
		0.75	2.08	2.65	1.77	14.90

The obtained results show that for the starting plant paver, an acceptable approximation is obtained for the examined loads when the block thickness ranges between 8 cm and 15 cm. Nevertheless, similar parametric analysis could be conducted to determine the 'correction factors' for other geometrical block configurations that could allow designers to adjust the results offered by the Westergaard's theory to perform a valid preliminary design analysis.

4. Conclusions

Existing statistical data show that the use of pedestrian block pavements is increasing. These structures are often constructed in pedestrian priority or space sharing systems, where heavy and commercial vehicles are admitted under exceptional conditions (e.g. emergency or public order). The presence of these traffic loads should be considered as part of the design of the pavement to prevent its premature failure.

The paper examined a pedestrian block made of hexagonal basalt pavers loaded by occasional commercial traffic. The stresses induced by the traffic were calculated according to two approaches: an analytical Westergaard's theory and a finite element analysis performed using the software Abaqus®. Several configurations of loads and geometry of the model have been considered to compare the maximum tensile stress induced by the load. The results show the analytical method overestimates the mechanical response of the examined block pavement. Therefore, a correction factor equal to 0.67 has been proposed to evaluate the stress conditions induced by heavy loads. It reduces the maximum tensile stress provided by Westergaard's theory to obtain results coherent to those provided by the FEM analysis. This is an interesting result since the analytical theory provides an inexpensive procedure for the analysis of block pavements. Parametric analyses like that presented could be conducted for other paver geometries, until a list of correction factors useful to follow the Westergaard's approach rather than developing costly FEMs. On the other hand, the proposed comparative approach is a starting point to analyze larger complex of blocks, even interlocked.

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