Strategic interaction and NFNE: The case of CO_2 Concentration

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Abstract (English)

Abstract

After the first formulation of optimal control problems in the middle of the 20th century, model predictive control (MPC) and nonlinear model predictive control (NMPC) gradually became popular methods in both science and industry. Considering the advantages of NMPC as a practical method to solve dynamic decision problems numerically, this method could attract more attention in economics too. Later, in order to model strategic interactions between different policymakers, NMPC was extended to the NMPC feedback Nash equilibrium (NFNE).

With focusing on these novel and practical techniques, the aims of this thesis are considered as follows. First, taking into account the privileges of mentioned techniques (NMPC and NFNE) such as repetitive solution of an optimal control problem in a receding horizon fashion and considering the time horizon, we use them in an environmental topic and assess the effects of different regimes on the climate change. Second, referring to the time horizon as a significant factor in these methods, we evaluate the effect of “Different time horizons” on the results and finally, we extend the current NFNE method in order to have more accurate predictions, specifically where we face more than one state variable in our optimization problems.

In this thesis, we consider a significant climate issue as global warming. Since using non-renewable energy and as its result CO_2 concentration leads to the negative externalities and affects individual welfare, for evaluating the interactions between different policymakers, we use a canonical growth model augmented with damages in the household's welfare function. We assess the CO_2 concentration level when players operate under different regimes, in this thesis by different regimes, we refer to the cooperative and non-cooperative policies and we consider the period from 2019 to 2100. We start considering one common state variable as CO_2 concentration and one control variable as using non-renewable energy. Our result shows a big difference in the CO_2 concentration level in the cooperative and non-cooperative situation. Although, with cooperation between independent policymakers we can reach a lower level of externality but, still it is not the best emission pathway. However, if policymakers find it difficult (e.g., for political reasons) to accept international binding agreements, and prefer to rely on their preferences for consuming non-renewal recourses instead of considering the global warming, the negative externalities and damaging effects may be quite severe.

Moreover, along the line of Sims's idea that agents often make decisions under information constraint, we interpret the finite horizon as a measure of inattention or myopia. When we apply different time horizons for introducing the policymakers' myopia, we observe that interestingly, less myopic policymakers anticipate much less CO_2 concentration above the pre-industrial level. However, results state that also if we find a way to remove policy uncertainty or constraints and have a more precise prediction (i.e., longer time horizon), still the result will not be satisfying by the year 2100.

Then, in our extended form of NFNE, we consider the transition from non-renewable to renewable energy as an important way to combat global warming. This transition can also be considered as an additional instrument in cooperative situations. Hence, we suppose two state variables. Along with CO_2 concentration which is the common state variable, we take into account the capital stock to produce renewable energy. Also, we have two control variables as the extraction rate of fossil fuels and consumption. But, this extension in our model requires an extension in the method. So, we extend the current NFNE method and build two separate loops as two different games for optimization problems. These loops should be solved independently but simultaneously to find those fixed points that players have no incentive to change that at each point of time. Results show that despite having the renewable resources, since there is not a suitable cooperation between countries/policymakers, we cannot expect to have a transition from non-renewable to renewable resources. But interestingly, if policymakers accept a high degree of cooperation, we will reach really good results in CO_2 concentration and eventually temperature.

Contents

Introduction

Nonlinear model predictive control (NMPC) is an approach to solve dynamic decision models and it is applied to optimal feedback control of nonlinear systems. This method has some significant advantages. For example, it avoids gridding the state space, and considers finite horizon optimal trajectories in order to find the infinite horizon optimal trajectory. Also, comparing NMPC solution with other differential games such as open-loop and feedback Nash equilibrium, we can suppose that NMPC is a method using both open-loop and feedback information structures of the initial state value at the same time.

Furthermore, Di Bartolomeo et al. (2017) introduced an extension of NMPC called the NMPC feedback Nash Equilibrium (NFNE) which focuses on the length of forecasting horizon. NFNE aims at modeling the strategic interactions between different policymakers. In this method, the procedure works in a loop consist of repeating the maximization/minimization of the optimal control problem to find an optimal strategy that players have no incentive to change it, at each point of time.

So, considering the efficiency of these methods in economics, at first, we have a comprehensive review of the mentioned methods and their application in economics. Also, strategies, and equilibrium concepts both in the NMPC and NFNE will be discussed. In the next step, we use these novel methods in a climate change issue. In other words, using the mentioned techniques and considering different regimes, we aim at providing a more precise prediction.

Global warming, as a controversial issue, is a result of raising in CO_2 concentration.[1] Due to the fact that CO_2 emission has a high correlation with the consumption of fossil fuels and this consumption increase rapidly, a large number of studies try to predict the level of CO_2 concentration in the future and focus on mitigation policies in order to combat global warming. But on one hand, most of the methods used before, have a deficiency related to the "Time Horizon" and this deficiency can be more prominent when we notice that there is a big difference of the forecasting horizon between economics and climate issues. On the other hand, previous studies did not pay enough attention to this point that there are different policymakers who decide about these mitigations policies, not the United Nations and consequently it is about policymakers' decisions to emit under cooperation or in a non-cooperative situation.

Considering global warming as a global public good which affects all countries, a significant question is how much CO_2 concentration will be expected by emitting under different regimes. Also, referring to the time horizon issue in NMPC and NFNE, on one hand, policymakers need, less information when making decisions because of using receding horizon solutions and on the other hand, we can assess the effects of policymakers' short-termism on the predicted CO_2 concentration level and the global mean temperature. It is worth noting that shorter lengths in policymaker's time horizons, can be associated with political economy or information constraints.[2]

Finally, we extend our research in two directions. At first, we extend NFNE technique to apply for the optimization problem with more than one dynamic, and then from the economic point of view, we take into account the ability to shift from polluting to non-

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[1] IPCC (2018).

[2] See Sims (2005, 2006).

polluting energy in order to have a more realistic view about our predictions. In the context of the transition from non-renewable to renewable resources, some studies have been done and renewable resources such as wind, water, solar, or nuclear energy are considered as a promising way to combat global warming. However, still there is an important question that if this transition will work efficiently, or it cannot bring enough incentives to encourage policymakers/countries to shift to these non-polluting energies. Hence, in this contribution, we extend the NFNE method and add another state variable to the previous economic framework called capital stock which is used to build the non-polluting energy resources.

In the end, it should be noted that models are coded and run in MATLAB in order to solve the optimization problems and simulates the responses of both economic and environmental variables under different regimes.

Outline and contribution

So, the remainder of this thesis is organized as follows:

Chapter 1 presents the Nonlinear model predictive control theory, optimization problem, its advantages, and its application in economics.

In Chapter 2, we define the NFNE method and explore its difference with other differential games.

Chapter 3 using explained techniques, investigates the level of CO_2 concentration under different regimes by the year 2100, and strategic interactions. Also, we assess the effect of different forecasting horizons on the results.

In chapter 4, we have an extension of the previous method and framework. We consider two sources of energy as renewable and non-renewable and investigate the possibility of transition between them.

Finally, Chapter 5 summarizes the major results as well as interesting topics that one could further investigate on.

Chapter 1

Nonlinear model predictive control theory

Nonlinear model predictive control (NMPC) is generally considered as an optimization-based method to solve dynamic decision models. This approach relies on iterative and finite-horizon optimization. It starts with implementing the initial value to the system and then repeatedly solves the original problem on a defined horizon by implementing the first element of the solution to the system.

In the middle of the 20th century, NMPC had developed from the theory of optimal control, and the first paper which was formulated the central idea of model predictive control was published by Propoi (1963). In the late 1970s, after a gap due to the inefficiency of computer hardware and software, MPC for linear systems and gradually, NMPC which is applied to optimal feedback stabilization problems,[3] became popular in control engineering and the industrial practice. However, after establishing successfully in engineering, they were extended in economic applications too. The NMPC algorithm which we use today was published for the first time by Chen and Shaw (1982) and then after the basic principles of NMPC had been clarified, more advanced topics such as efficiency of numerical algorithms and robustness of stability had been done.

In this chapter, we introduce the concept of NMPC and then we describe the topic of time as discrete and continuous in this method. At the end, we explain the privileges of this method which make it different from other approaches.

1.1 Fundamentals of Nonlinear model predictive control

The aim of this technique is to predict future system behavior. Assume that we are given a vector of state variable $x(t)$ which has been influenced by a control input $u(t)$. We consider T as the NMPC policy horizon also, the current state is sampled at time (t) and the control problem will be optimized between t and $t + T$.

In order to predict future system behavior, we start with the most recent measurement of the state variable as the initial value $x(0)$. we define a dynamical system to construct a prediction trajectory, $x^{*}\ (t)$ as

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[3] See for example Rawlings and Mayne (2009) and Grüne and Pannek (2011)

$$x(t+1, x_0) = \varphi\big(x(t, x_0), u(t)\big) \qquad t \in \mathbb{N} \tag{1.1}$$

$$x(0) = x_0$$

Equation (1.1) describes how the state variable of the dynamical system develops under the influence of the control variable. The main idea of NMPC is that the control can be regularly adjusted and keep constant for a small finite interval. Hence, we assume that control variables are set at the beginning of each period and will be kept constant until its end.

Considering the above dynamic, we define the infinite horizon discounted optimal control problem by:

$$V(x_0) = min \sum_{t=0}^{\infty} \beta^t \; g\big(x(t), u(t)\big) \qquad x(0) = x_0 \tag{1.2}$$

$$\text{s.t.} \quad x(t+1, x_0) = \varphi\big(x(t, x_0), u(t)\big) \qquad t \in \mathbb{N}$$

The outcome of the optimal control problem is a vector of T controls, u_t^*. We define the NMPC feedback as $\mu(x(t))$, and set $\mu\big(x(t)\big) \coloneqq u_t^*(1)$, i.e., the first element of this sequence is the optimal NMPC at time t. NMPC does not want to involve an optimization over the entire planning horizon, but it involves the repetitive solution of an optimal control problem in a receding horizon fashion. Hence, in the next step, we use NMPC feedback, $u_t^*(1)$, in the next sampling period. It should be noted that in the new step, the state variable is sampled between $t+1$ and $t+T+1$, i.e., the prediction horizon keeps being shifted forward and because of this NMPC is called receding horizon control. We will use the notation of prediction trajectory as $x^{*}(t)$ for a trajectory resulting from the control sequence u_t^{*} and with the initial value $x(0)$.

Based on these definitions we can formulate NMPC solution algorithm.

Algorithm 1.1 (NMPC Solution Algorithm). Given time horizon T; the initial condition of the dynamic system: $x(0) = x_0$. Consider a dynamic problem, e.g., (1.1) - (1.2). At each sampling time $t \in \mathbb{N}$:

(1) Solve the following optimal control problem:

$min \sum_{t=0}^{T} \beta^{t} \; g\big(x(t), u(t)\big)$

s.t. $x(t+1) = \varphi\big(x(t), u(t)\big)$ and $x(0) = x_0$

and receive an optimal control sequence, u_t^{*}.

(2) Define the NMPC-feedback value $\mu\big(x(t)\big) \coloneqq u_t^{*}(1)$ and apply it to the system, i.e., $x(t+1) \coloneqq \varphi\Big(x(t), \mu\big(x(t)\big)\Big)$

(3) By repeating the above procedure, the prediction trajectory $x^{*}(t)$ will be obtained.

This procedure leads to the infinite closed-loop trajectory $x^{*}(t),\ t = 1, 2, 3, \ldots$ along with its control sequence, $u^{*}(t)$, which consists of all the first elements of the optimization sequences. In this method, we use open-loop finite horizon optimal trajectories, in order to find an infinite closed-loop prediction trajectory.

Above procedure is sketched in Figure. 1.1.

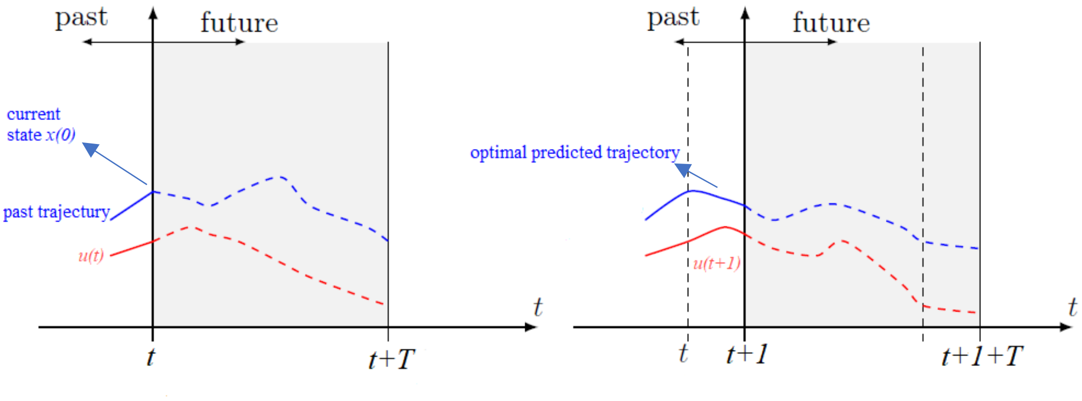

Figure 1.1 Illustration of the NMPC

Example 1.1[4] Consider a simple growth model as

$$f\big(x(t),u(t)\big) = \ln(Ax^{\alpha} - u)$$

s.t $x(t+1) = u(t)$

Where Ax^{α} is a production function and x is the capital stock.

Solution will be reached by:

$$V(x) = B + C\ln x, \quad \text{with } C\frac{\alpha}{1-\alpha\beta} \quad \text{and } B = \frac{\ln((1-\alpha\beta)A)+\frac{\beta\alpha}{1-\beta\alpha}\ln(\alpha\beta A)}{1-\beta}$$

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[4] Grüne et al. (2015).

If initial value $x(0)$ and policy horizon T, are set equal to 5 and 3 respectively and $\alpha = 0.34$, $\beta = 0.95$ and $A = 5$, then the optimal solution is:

$$u^*(t) = (2.59, 2.07, 1.92, 1.87, 1.85)\ .$$

In figure 1.2, we see the first open-loop optimal trajectories (dashed) as A-C which is a vector of 3 controls. Then, by picking up the first element of this vector and applying it to the system, point B has been reached. B is the value of the state variable for the next step $x(1)$.

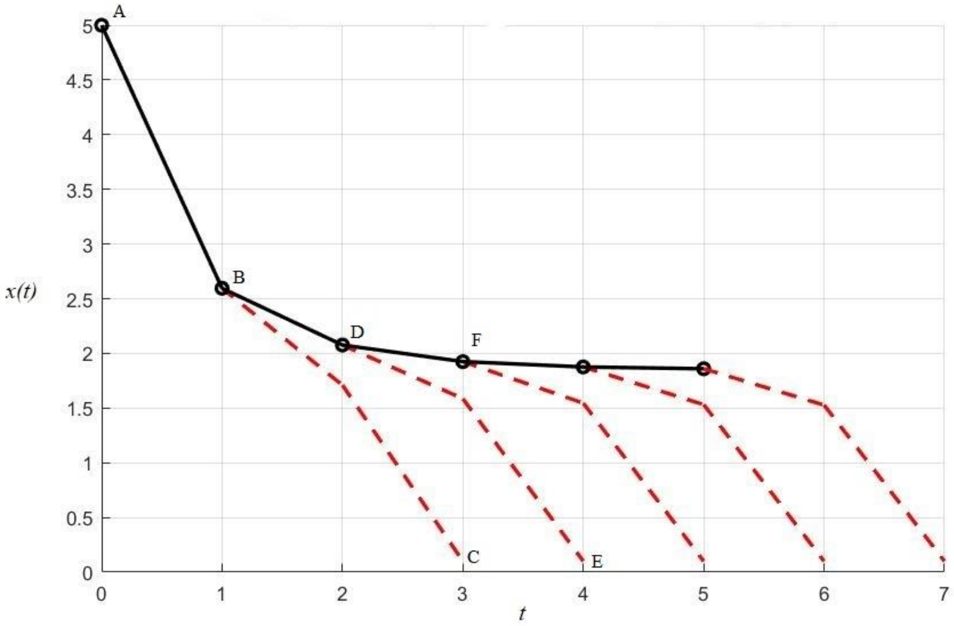

Figure 1.2 Open-loop optimal trajectories (dashed) and Closed-loop trajectory (solid) for $T = 3$ and $x_0 = 5$

1.2 Continuous and Discrete Time Models

Basically, NMPC is applying in discrete-time problems. However, we usually face with continuous-time models in form of differential equations. Hence, the continuous-time problems need to be discretized in time in order to apply in this method.[5]

In the following, we define how we can convert any continuous-time model into discrete-time model. But at first, we describe the timing assumption that we are going to use in the current thesis.

1.2.1 Timing assumptions

We can consider a continuous-time models in general, or a particular time assumption. In this thesis, we are going to use a mixed-time-structure model.

Assume that we have the infinite horizon discounted optimal control problems in continuous time where $t \in \mathbb{R}_0^+$

$$V(x_0) := min \int_0^\infty e^{-\rho t} \, g\big(x(t), u(t)\big) dt \tag{1.3}$$

ρ is the discount rate [6] and

$$x(t+1) = \varphi\big(x(t), u(t)\big), \qquad x(0) = x_0 \tag{1.4}$$

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[5] Grüne et al. (2015).

[6] In the literature the most common discount function is the exponential discount function.

We consider a specific time structure as a mixed-time-structure model where state variables evolve in a continuous-time, i.e., $t \in [0, \infty)$, whereas controls are constant in the interval Δ, which occurs between $\tau - 1$ and τ .

This happens because policymakers set their controls in a discrete fashion (policy instruments are set at $\tau \in \mathbb{N}$). In other words, the state variable $x(t)$ is evolved in continuous time while the control ones $u(t)$ can be regularly adjusted and kept constant for Δ.[7]

So, the assumption needs to implement as a receding horizon solution, but it is without loss of generality since Δ can be arbitrarily small. Here, it is worth noting that for simplicity, we assume control variables are set at the beginning of each period. We assume $\Delta = 1$ and for any $x \in \mathbb{R}_0^+$, we indicate $ceil(x)$ and $floor(x)$ by $\lceil x \rceil$ and $\lfloor x \rfloor$, respectively. Then we define $\tau = \lceil t \rceil$.

Fig. 1.3 can clarify the above notion.[8] For instance, we can see that $\tau = 1$ for any $t \in [0,1)$ or $\tau = 6$ for any $t \in [5,6)$ and because of our small supposed interval $(\Delta = 1)$ it can be interpreted as continuous-time while we should notice that control variables used in each period have been set at the beginning of that period.

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[7] Di Bartolomeo et al. (2017).

[8] Grüne and Pannek (2011).

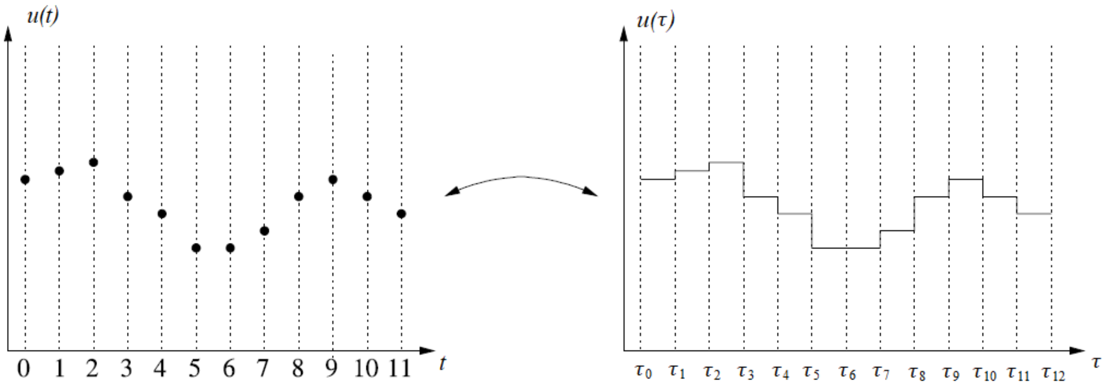

Fig 1.3 The sequence at $u(t)$ on the left corresponds to the constant control functions with $u(\tau) = u(t)$ for almost all at $t \in [\tau - 1, \tau)$

Considering the above explanation, NMPC in continuous-time or mixed-time-structure can be defined as follows.

Definition 1.1 (NMPC Solution, mixed-time-structure)

Given the initial condition of the dynamic system: $x(0) = x_0$,defining the vector of policies between t and $t + T$ as $u^t \in \mathbb{R}^T$ and $u(\tau) = u(t)$ for almost all at $t \in [\tau - 1, \tau)$:

NMPC feedback solution is a sequence $\{u^*(t)\}_1^\infty$ such that each element is $u^*(t) = u_t^*(1)$, where policymaker aims to maximize/minimize

$$L^t(u^t) = \sum_{\tau=t}^{t+T} \rho^{\tau-1} \int_{\tau-1}^{\tau} g\big(x(t), u^t(\tau)\big) dt,$$

s.t. $\dot{x} = f\big(x(t), u(\tau)\big)$ and $t \in \mathbb{R}_0^+, \tau \in \mathbb{N}$

Hence, during the time, economy evolves according to the following differential equation

$$\dot{x}(t) = f(x(t), u^{*}(t)) \tag{1.5}$$

And the policymaker aims to optimize the following inter–temporal loss/benefit:

$$V(x_0) = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \int_{\tau-1}^{\tau} g\big(x(t), u(\tau)\big)dt \tag{1.6}$$

Where ρ is the discount factor.[9]

So, given a dynamical system i.e. (1.5) and according to the above definitions, we can use the basic NMPC algorithm (1.1) in our mixed-time-structure model.

To solve the optimal control problem in continuous-time numerically, we need to convert it to the optimal control problem in discrete-time. For this purpose, it needs to be discretized in time in order to apply in the NMPC method. Hence, we use the first step of semi–Lagrangian discretization technique which is in time.

In Appendix A, it is explained that how we are able to solve the optimal control problem in the NMPC algorithm numerically, by illustrating the discretization technique.

1.3 Advantages of NMPC

The first important advantage of NMPC is considering as the policymakers' time horizons. The time horizon is strongly related to the two different arguments. The first refers to the different time perspectives for different policymakers because of policy

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[99] Di Bartolomeo et al. (2017).

uncertainty or policy constrains. The second argument refers to the limited capabilities of policymakers in order to forecast the effect of their policies and/or the policies of other policymakers. This limitation is the direct result of making decisions under limited information. Shorter horizons can be interpreted as the measure of short-termism.[10] This bounded rationality also was discussed by Simon (1957) and Forte (2012). Forte believes that the public choice approach is fundamentally micro-economic and is based on limited rationally.

Sims (2005, 2006) showed that agents make decisions under limited information which might be

a) the result of not available information.

b) imprecise answers to available information.

According to the Sims, with increasing the agents' information and information processing capacity this limitation can be solved and better approximation for infinite trajectory will be possible.

We can deal with these constraints, referring to the main principles of NMPC. As we explained, we compute finite horizon optimal trajectories in order to find the infinite horizon optimal trajectory so, in the NMPC compared with the other infinite horizon models, agents need, less requirement of information when making decisions. In other words, we make a decision for the control of the next step by looking at the problem on a shorter time horizon.

Another significant advantage of NMPC is that this method is one of the most advanced control approaches for multi-dimensional systems. In this approach, state and

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[10] Di Bartolomeo et al. (2018).

control constraints allow us to consider more complex dynamics due to the less possibility for solutions to stick in dimensionality trap. NMPC only computes one optimal trajectory at a time, therefore, it avoids to grid the state space. For this reason, the computational demand grows much more moderately with the space dimension.[11]

It is worth noting that the principles of NMPC can also create a deficiency for this method. As we explained in the NMPC procedure, we start from the current state $x(0)$ and only the first decision step is implemented, however the given control sequence will be $u(0),\dots,u(T-1)$. So, considering the policy horizon T, in the closed-loop solution we are always $T-1$ periods away from the final decision. It refers to the point that, we never can see the effects which appear at the end of the policy horizon. According to Grüne, et al. (2015) for this problem, in the shorter decision horizon we can use the salvage value which can be determined in a reasonable way.

This deficiency implies that the optimization horizon T plays a significant role in this method because there is a tradeoff between good approximation of T and numerical accuracy. This tradeoff happens because on one hand, we can expect a good approximation of the infinite horizon optimal trajectories when the optimization horizon is sufficiently large, but on the other hand, large horizons increase the decision horizon and it may lead to the dimensionality and numerical problems. So, the length of the time horizon should be considered in NMPC precisely.[12]

Considering explained advantages, NMPC can be considered as a practical option to solve dynamic decision problems numerically, in order to find global solutions. This approach can be useful in different economic areas such as economic growth and

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[11] Grüne et al. (2015).

[12] See for example Grüne et al. (2015).

ecosystem management, when we face a more complicated dynamic while using commonly numerical techniques for solving them are difficult. Moreover, the possibility of using NMPC to solve nonlinear dynamic decision problems, both continuous and discrete-time models and considering regime changes makes this technique attractive in economics.

1.4 Alternative interpretations

Short-termism which is the result of uncertainties of policymakers, can be interpreted as "policymakers' inattention" or "myopia". Bounded rationality which was formalized in NMPC by Grüne et al. (2015) and Di Bartolomeo et al. (2017) refers to this notion. Since NMPC optimization involves the repetitive solution at each sampling of time, instead of the entire planning horizon, in the context of bounded rationality, a shorter horizon can be interpreted as measuring stronger inattention.[13]

In this context, some notable researches have been done. For instance, Di Bartolomeo et al. (2017) focused on pollution regulation policies and inattention. They found that inattention basically affects the transition dynamics and it leads not only to quicker, but also more costly, transitions. Moreover, their results show that inattention may accelerate climate change by under-evaluating the environmental cost.

Also, referring to this fact that policymakers generally are stuck in time, Di Bartolomeo et al. (2018) used inattention as "policy myopia". They tried to distinguish

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[13] Di Bartolomeo et al. (2017).

the effects of policymakers' time horizons on debt stabilization. According to their results policy myopia induces policymakers to be more aggressive in stabilizing the debt at the beginning, but it is less effective in reducing excessive public debts in the long run.

In this thesis, we consider different policy horizons in order to investigate the effect of policymakers' myopia on climate change.

Chapter 2

NMPC Feedback Nash Equilibrium (NFNE)

In this chapter, we want to define a novel concept as the NMPC feedback Nash Equilibrium (NFNE). NFNE is the extended form of NMPC and it is modeling the strategic interactions between different policymakers. This method proposes the results with feedback structure in an infinite horizon control problem. It is worth noting that NFNE as well as NMPC is related to the bounded rationality and inattention. We start with the definition and notion of NFNE and then describe the interaction between policymakers. Furthermore, we make a comparison between NFNE and other differential games.

2.1 Preliminary Definitions

Di Bartolomeo et al. (2017), with focusing on the length of the forecasting horizon, introduced NFNE. This method which applies the NMPC technique to differential strategic games, is the combination of the NMPC method in economics, proposed by Grüne et al. (2015), and moving horizon LQ–control in dynamic games, used by Van den Broek (2002). In order to solve problems involving multiple interacting agents, they formalized an equilibrium concept as the NMPC Feedback Nash Equilibrium and developed a routine to compute it. Their work can be used in different areas such as environmental economics, industrial organizations, decision and management science, marketing, and quantitative methods.

NMPC is a method which let policymakers to predict the effects of their actions and/or their opponent's actions on a finite receding horizon. Moreover, the length of the forecasting horizon can be interpreted in two ways: first, as a specific aspect of bounded rationality, second, with regard to different policymakers that may have different time perspectives.

In the NFNE method, the policy equilibrium is obtained by applying nonlinear model predictive control techniques to differential strategic games. It means that policymakers' problems involve the repetitive solution of an optimal control problem at each instant of time. But since we consider strategic interactions – differently from the usual NMPC solution – players interact in each instant of time in an infinite time setting and they try to predict the dynamics and the opponents' moves during a given time horizon. Each player works along a receding horizon strategy and the same as the NMPC, when a vector of control variables is calculated, only the first element will be used. Each of the optimal control problems must result in an open-loop Nash equilibrium. So, Di Bartolomeo et al. (2017) referred to this kind of equilibrium as Non-linear model predictive control Feedback Nash Equilibrium (NFNE).

2.2 Problem formulation

In this section, we describe the basic principles of the NFNE. As we mentioned before, for a given time horizon, NFNE allows one to predict the dynamics of decision variables of the other players in each instant of time. For the sake of simplicity, we suppose the case of two players.

Hence, the economy evolves according to the decisions of two policymakers $i \in \{1,2\}$ based on the following differential equation:

$$\dot{x}(t) = g(x(t), u_1(t), u_2(t)) \tag{2.1}$$

Equation (2.1) describes how the state x of a dynamical system evolves in time t under the influence of both the control u_1 and u_2.

And, both players want to minimize(maximize) the loss(benefit) such as:

$$V_i(x_0) = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \int_{\tau-1}^{\tau} g_i\big(x(t), u_i(\tau), u_{-i}(\tau)\big)dt \tag{2.2}$$

Where ρ is the discount rate.

Same as NMPC, we assume that policymakers' optimal problems involve the repetitive solution of a receding horizon fashion, instead of over the entire planning horizon.

Since we assume that their problem is a Nash equilibrium in each instant of time, so we should find an optimal strategy in which players have no incentive to change their policies.

Using the definition of NMPC we can define the NFNE as follow:

Definition 2.1 (NFNE) Given the initial condition of the dynamic system: $x(0) = x_0$, and the control of the other player, defining the vector of policies between t and $t + T$ as $u^t \in \mathbb{R}^T$ and $u(\tau) = u(t)$ for almost all at $t \in [\tau - 1, \tau)$ and two policymakers as $i \in \{1,2\}$:

NFNE feedback solution is a sequence of two elements $u_1^*(t), u_2^*(t)\,\}_1^\infty$ where $u_i^*(t) = u_i{}^{\mathrm{t}}\,(1)$ and $i \in \{1,2\}$ i.e., the first element of each sequence is the optimal NFNE at time t and Policymaker aim to maximize/minimize

$$L_i^t(u^t) = \sum_{\tau=t}^{t+T} \rho^{\tau-1} \int_{\tau-1}^{\tau} g_i(x(t), u_i^t(\tau), u_{-i}^t(\tau))dt \qquad\qquad i \in \{1,2\}$$

$$\text{s.t.} \qquad \dot{x} = f(x(t), u_i(\tau), u_{-i}(\tau)\,\}_1^\infty) \qquad\qquad t \in \mathbb{R}_0^+, \tau \in \mathbb{N}$$

Hence, during the time, economy evolves according to the following differential equation

$$\dot{x}(t) = g\big(x(t), u_1^*(t), u_2^*(t)\big) \tag{2.3}$$

and policymakers aim to optimize the following problem:

$$V_i(x_0) = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \int_{\tau-1}^{\tau} g_i\big(x(t), u_i(\tau), u_{-i}(\tau)\big)dt \qquad\qquad i \in \{1,2\}$$

Where ρ is the discount factor.

NFNE includes the strategies that players have no incentive to change them at each period of time τ and it is explained by the optimal strategies $u_1^*(\tau)$ and $u_2^*(\tau)$, as:

$$NFNE := \{u_1^*(\tau), u_2^*(\tau)\}, \forall_\tau \in \mathbb{N} \tag{2.4}$$

Hence, given a dynamical system i.e., (2.3) and according to the above definitions and the algorithm1.1, we can find the prediction trajectory and optimal strategies, $u_1^*(\tau)$ and $u_2^*(\tau)$, via algorithm 2.1 as follows.

Algorithm 2.1 (NFNE). Considering the first period $\tau = 1$, given time horizon T, the initial condition of the dynamic system: $x(0) = x_0$ and an initial guess for the policy of player 2, optimal strategies of two countries will be found as follows:

(1) Solve optimal control problem of first player between 1 and T:

$$min\, V_i(x_0)$$

$$\text{s.t.} \quad \dot{x}(t) = g(x(t), u_1(t), u_2(t))$$

The outcome is a vector of T controls, $\tilde{u}^1$.

(2) Then, using the result vector as the guess for the policy of the first player, i.e., $u_1 = \tilde{u}^1$ (obtained from step (1)) and $x(0) = x_0$, then we should solve the optimal control problem for player 2 between 1 and T,

$$min\, V_{-i}(x_0)$$

$$\text{s.t.} \quad \dot{x}(t) = g\big(x(t), u_1(t), u_2(t)\big)$$

Again, the outcome is a vector of T controls, $\tilde{u}^2$.

(3) Repeat step 1 and 2 using $\tilde{u}^2$ for the first player and new $\tilde{u}^1$ for the second player, which are resulted by the optimization process, until a fixed point is found, i.e. vectors $u_1^o(1)$ and $u_2^o(1)$.

(4) At this point, we take the first elements of vectors $u_1^o(1)$ and $u_2^o(1)$, which are the optimal NFNE at time $\tau = 1$, i.e., $u_1^*(1)$ and $u_2^*(1)$. And then by applying the optimal NFNE solution in usual dynamic, $\dot{x}(t) = \; g\big(x(t), u_1(t), u_2(t)\big)$ we obtain $x^*(1)$.

(5) Then, having $x^*(1)$ and using $u_2^o(1)$ as the guess vector for the policy of player 2, repeat the same procedure as just described in order to find e.g. $u_1^*(2)$ and $u_2^*(2)\ldots.$

(6) The optimal policy vectors $u_i^*(t), i \in \{1,2\}$, are found by repeating the procedure just described.

Above algorithm is generalized to two players, hence dynamic for $x(t)$ is implied by NFNE as:

$\dot{x} = \; f(x(t), u_1^*(\tau), u_2^*(\tau))$ for in $t \in \mathbb{R}_0^+, \tau \in \mathbb{N}$

Based on the above explanation we can say that the NFNE technique can be used in the games as a novel technique where we want to predict the dynamics of other players.

2.3 NFNE vs other differential games

In this section, we want to describe the advantages of NFNE as a prediction method and investigate the differences between NFNE and other differential games.

Differential games can be used widely in economic problem analysis and in dynamic games. It should be mention that in dynamic games, information that is available for the player is a significant issue because it leads to the different strategies adopted by players and different game situations.

Hence, in order to describe the game, at first, we need to identify the available information at each time t. For instance, while open-loop solution depends on time and initial state of the system, feedback controls depend on time and current state.[14] In the former game, players are just aware of the initial state, the game structure and they determine their actions for the entire planning horizon before the process starts. Whereas, in the latter one, players are allowed to observe at every point in time the current state of the process and determine their actions based on this observation.[15]

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[14] Ngendakuriyo, (2010).

[15] Basar and Olsder (1982).

Comparing NFNE with these two differential games (i.e., Open-loop and Feedback Nash Equilibrium) NFNE solution also depends on time but, if we consider information structures, NFNE is a method using both open-loop and feedback information structures at the same time. In other words, as we explained before NFNE is a receding horizon strategy which means that at each instant of time the horizon is moving toward the future, using the first control signal of the sequences calculated at previous steps. Hence, at the beginning of each period, the initial state variable is fixed over the time interval but once we apply the new control variable resulted from the optimization, a new measurement of initial state variable appears and it will be used for the next period. So, each player plays a non-cooperative strategy in each iteration and uses the most valid information at each instant of time to predict the future.

Moreover, in NFNE – as an extended form of NMPC – we do not need to the linearization techniques to solve nonlinear dynamic problems globally. Also, in this technique, by adding state and control constraints and by having a sufficiently long time horizon, we avoid difficulties of nonlinear games such as multiple equilibria.

In this thesis, we aim at using NMPC and NFNE in the environmental context under different regimes i.e., cooperative and non-cooperative. Considering the climate issue, we need to be concerned with a long time horizon of centuries while, in the context of economics, in the best case, we have the prediction only for a few decades which means that policymakers in these two fields have different time perspective. So, using a receding horizon method which also does not have the difficulties of other nonlinear games, can provide better predictions in the environmental issues. However, to reach our purpose, we should extend the current NFNE method which before has been applied only for the models with one state variable.[16]

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[16] See for example Di Bartolomeo et al. (2017, 2018).

Chapter 3

Strategic Interactions and CO_2 Concentration

Considering the privileges of NMPC and NFNE, in this chapter we use the explained techniques in an environmental topic. We start with investigating the CO_2 concentration level, under different policy regimes. In this thesis, by different regimes, we refer to the cooperative and non-cooperative polices. For this purpose, we use a canonical growth model which is augmented with damages in the welfare function. Compared with other assessment models, this approach ties economic activity with externalities and feedback effects. Our primary goal is not to evaluate different abatement policies, but we want to study the interaction between different policymakers under different strategies. Then we investigate the effect of policymakers' myopia on the results and assess how different time horizons can change the result.

3.1 An overview of global warming

Nowadays, global warming is considered as one of the most controversial issues and it has recently attracted considerable scientific and policy attention. Considering the importance of global warming, at first, we need to figure out the reason of this climate change. According to a large number of studies, the main reason has been recognized as raising in greenhouse gases (GHGs) concentration, and specifically Carbon dioxide (CO_2).

Before the industrial revolution, it was a balance between inflows of GHGs and outflows of carbon absorbed by ocean and plants but increasing the use of fossil fuels including coal, natural gas, and oil are recognized as the main human activities which changed that balance and led to the CO_2 emission by more than 3% per year on average in the 2000s.[17] These human activities have made approximately 1.0°C of global warming above pre-industrial level and if we continue to emit at the same current rate it is 'likely[18] with high confidence' to reach 1.5°C between 2030 and 2052.[19]

According to the United States Environmental Protection Agency (EPA), in 2017, CO_2 emissions from fossil fuel combustion, were the largest source of GHGs emissions – with about 76% of total Global Warming Potential (GWP)[20] – and it was

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[17] Garnaut (2011).

[18] 'Likely' refers to the level of confidence: '66–100%'.

[19] IPCC (2018)

[20] Global Warming Potential: The cumulative radiative forcing effects of a gas over a specified time horizon resulting from the emission of a unit mass of gas relative to a reference gas. The GWP-weighted emissions of direct greenhouse gases in the U.S. Inventory are presented in terms of equivalent emissions of carbon dioxide (CO2).

accounted for all the US. It should be mentioned that although emissions from other sources, e.g., industrial processes, agriculture, waste, land use, land-use change and forestry are also significant, they are not included in these estimations due to some reasons such as lack of data availability, higher level of uncertainty in quantification methods, and smaller contribution to total emissions.

Hence, in this thesis, we concentrate on the cumulative global emissions of CO_2 as a significant part of GHGs and the energy sector and specifically in this chapter, we investigate the level of CO_2 concentration and global mean temperature, under different policy regimes.

3.2 Importance of CO_2 concentration

Externalities and negative effects of CO_2 concentration not only affect the present but also the future. So, in order to limit global warming, we need to limit the total cumulative global emissions of CO_2 since the pre-industrial period. Scientific evidence shows that before the Industrial Revolution, the global average amount of CO_2 was about 280 parts per million (ppm), however the average CO_2 concentration in December 2018 has been recorded around 408 ppm.[21] According to the IPCC (2018), emissions scenarios that limit the concentration level, up to 450 ppm are likely to achieve

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[21] Dr. Pieter Tans, NOAA/ESRL (www.esrl.noaa.gov/gmd/ccgg/trends/) and Dr. Ralph Keeling, Scripps Institution of Oceanography (scrippsco2.ucsd.edu/).

2°C above pre-industrial temperatures while, scenarios that reach CO_2 concentration of 650 ppm will lead to 3°C by 2100 with the same level of confidence.

Hence, different actions have been proposed by researchers in order to limit climate change. These efforts generally elaborate on subjects such as mitigation policies e.g., carbon tax and cap and trade**.** However, it should be noted that climate policies usually imply tradeoffs at the domestic level and externalities at the international one, which makes their implementation difficult because on one hand, policymakers need to do an immediate action to control the climate change and on the other hand, any attempt to reduce CO_2 emissions (as the main factor) such as reduction in the use of fossil fuels, transition from fossil fuels to renewable energy and new technologies would be economically costly[22] and may lead to changes in major economic issues such as production, consumption, and investment. These tradeoffs refer to a fact that governments prefer to set the plans for national emissions according to their national interests, i.e., under non-cooperative situations, and because of these preferences still world community is struggling to achieve an effective international agreement in order to reduce CO_2 emission. We can refer to some of the most significant meetings and commitments about the climate change as Kyoto (1997), Copenhagen (2009), Doha (2012), Warsaw (2013) and Paris (2015). However, theses international negotiations generally face difficulties and these difficulties show that global warming is considered as a case of global public good (GPG).

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[22] Nordhaus (2008).

3.3 Global warming as a global public good

Global warming is considered as a polar case of Global Public Good (GPG). An important point about global public goods such as climate change issues is that the effects of them are not limited to a special nation, country, or group. GPGs not only affect all parts of the world but also, in contrast with other economic activities, are difficult to deal with in an efficient mechanism.

This problem happens due to the characters of GPGs. The main feathers of public goods such as being non-excludable and non-rivalrous, bring the climate change issue in this category. Also, along with those properties, "stock externalities" can reveal that why global warming is count as a polar case. Stock externalities refer to the point that, the impacts of global public goods depend on a stock of a variable that is accumulated over time.[23] Since these accumulations usually happen slowly and have irreversible consequences, sometimes we see the symptoms when it is too late for doing any remedy. Also, it should be mentioned that CO_2 concentration as the main reason of global warming, has an atmospheric residence time with a half-life in the order of a century, which can make this topic bolder.

It is worth noting that the case of global warming is even more complicated compare with some other GPGs because:

a) The number of policymakers involved in the climate change issue is quite high, and this makes it more difficult to reach an efficient agreement.
b) Good results of effective policies are not obvious to most people.

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[23] See Nordhaus (2005).

c) Estimating and balancing costs and benefits cannot be measured easily and requires global concerns constantly.

Furthermore, decisions are taken by decentralized governments who deal with global warming, and it is possible that some more difficulties in making decisions may occur i.e., the so-called Westphalian dilemma. This dilemma would manifest within a global scale beyond national boundaries and refer to the international law that no governance can enforce a coordination among nations with the same legal force found in a sovereign nation.[24]

So, global warming as a GPG, means that climate change is a global challenge and dealing with this problem requires commitments of different countries/policymakers with their different preferences, instead of considering one globally aggregated approach. This implies that in this situation, international relations play an important role to handle the global warming issue. Generally, we face different local policymakers making different decisions that affect the final outcomes hence, it is important to find out how different policy regimes may lead to different results.

Generally, there are some different strategies that can be applied when we face the GPGs. The first approach which is more realistic but does not seem so efficient is "Non-cooperative policies". This concept can be clarified by the prisoner's dilemma with two possible strategies {Pollute, Abate} for each policymaker which {Pollute, Pollute} is a Nash equilibrium,[25] and consequently make policymakers act non-cooperatively.

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[24] See Nordhaus (2006, 2007).

[25] Wood (2010).

Another approach which is also the main purpose of all international negotiations is "Cooperative policies" which can be considered as aspirational or persuasive agreements (e.g., the FCCC)[26]. Although this approach can be considered as the most efficient way to combat global warming, according to the previous experiences, it seems unrealistic and requires a high level of cooperation to agree on a globally efficient way to reduce CO_2 emissions.

3.4 Economic assessment models

Generally, in order to evaluate the CO_2 concentration and assess the effect of economic activities on climate change, integrated assessment models (IAMs) have been used. These models mainly evaluate the effect of mitigation policies on climate systems, including resources, emissions, and consequence of CO_2/GHGs emission. In other words, they are integrating the economic activity with the climate system. The Representative Concentration Pathways (RCPs) which describe the 21st century pathways of CO_2 concentrations, have been developed using a range of approaches, from simple idealized experiments to different IAMs.[27]

Earlier IAMs such as DICE (Nordhaus, 1992, 1994), CETA (Peck and Teisberg, 1991) and MERGE (Manne and Richels, 1995) are just focusing on a "globally aggregated approach" but Nordhaus and Yang (1996) for the first time presented an

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[26] Nordhaus (2007).

[27] IPCC (2014)

integrated assessment model which considered ‘different nations’ as environmental policymakers. This model which was named RICE or Regional Integrated model of Climate and the Economy, in the structural equations is the same as DICE model. The difference is that in RICE model also they consider production, consumption, emissions, and damages for 10 different regions also, in this model they observe ‘Market’, ‘Cooperative’ and ‘Non-cooperative’ policies. Nordhaus and Yang (1996) believe that previous models ignore the fact that policy decisions that are taken to reduce CO_2 emissions are taken primarily at the national level and it is single nations, not the United Nations, that determine energy and environmental policy. After introducing the RICE model, some studies started to consider the effect of different policymakers and their different attitudes toward optimal emission.[28]

Also, another important point which these models mainly did not look at that seriously, is the effect of “Time Horizon”. Considering the climate issue, we need to be concerned with a long time horizon of centuries while, in the context of economics, in the best case, we have the prediction only for a few decades.[29] However, in this case, very few researches have been done such as Wong et al. (2015) which concentrated on the impact of changing the time horizon specification on the mean social cost of carbon dioxide.

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[28] See for example FUND (Tol, 1999), DART (Deke et al., 2001) and WIAGEM (Kemfert, 2001).

[29] Deke et al. (2001).

3.5 A two-country GPG-dynamic-provision game

In this thesis, taking into account the negative externalities from the non-renewable energy, we aim at modeling different policies in a simple stylized strategic context. In other words, we see how different policies and different policymaker's time horizons affect the CO_2 concentrations as the result of their domestic and international trade-offs.

We consider a simple two-country GPG-provision game. Specifically, in a simple differential game, two policymakers face domestic and international trade-offs. The former captures the cost of emission regulation, while the latter formalizes the GPG nature of the problem. Each of policymakers needs to regulate emissions. At the domestic level, policymakers balance the costs and benefits of reducing them. However, domestic decisions will not be optimal at the international level since the policymakers do not internalize the external effects of their choices. Considering two policymakers, both prefer that the other would take care of reducing global warming at its expense. Therefore, they are trapped in a suboptimal equilibrium until they do not coordinate their actions.

We borrow from the Greiner et al. (2014) the definition of economic growth and damages in the household's welfare function to build a two-country GPG-provision game and explore the externality of CO_2 concentrations. We assume that the optimal solution takes into account the negative externality from the non-renewable energy. Compare with other IAMs, this direct disutility approach ties economic activity with their externalities and feedback effects and better captures impacts of climate change such as health and ecological loss.[30]

However, in this thesis in contrast to the paper by Greiner et al. (2014), we focus on the dynamic interaction between two different policymakers (two countries/ groups or coalition of countries) instead of considering one economy which is populated by a continuum of homogeneous agents. i.e., we discuss the interaction between different policymakers. Moreover, in this model instead of mapping emissions to temperature changes and finally reduction in productivity, we investigate the effects of different regimes, different energy resources (renewable and non-renewable), and even time horizon on our predictions.

Our model has two optimization problems i.e., one for each country. In this chapter, each policymaker has its own control variable as the use of non-renewable resources which is set at the beginning of each period and will be kept constant until its end, and a common state variable as CO_2 concentration as the result of using non-renewable energy. Our setup reflects the fact that global warming as a GPG has the same effect on everyone.

Also, it should be noted that we consider discounted optimal control problems in order to value the benefits and cost of limiting future climate change. The discount rate is a significant notion that affects the outcome of a benefit-cost analysis or damage valuation study. The importance of discount rate in environmental economics is related to the fact that CO_2 has a very long residence time in the atmosphere and we need to value the impacts of today's emissions into the future climate change.

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[30] Semmler et al. (2018).

If we tend to put less weight for the future and consequently less investment to combat global warming, we will have a high discount rate. In other words, when future outcomes are discounted for the economy at a higher rate, the cost of non-renewable resource extraction in present falls and it leads to the higher extraction rate.[31] In contrast, considering climate change as an important issue that requires an immediate reaction, we will have a low discount rate i.e., in cost–benefit analyses more importance is given to future generations' wellbeing.

In order to report the numerical solutions, we employ Nonlinear Model Predictive Control (NMPC) in the cooperative situation and NMPC Feedback Nash Equilibrium (NFNE) in the non-cooperative situation. In the game, we find an optimal strategy in which both players have no incentive to change that, at each point of time and we observe how non-renewable energy sources move on. Then, we analyze the situation under the cooperative regime and make a comparison between these two situations.

Also, we refer to shorter lengths in policymaker's time horizons as political short-termism or myopia.[32] In a dynamic global public good game (GPG), short-termism is formalized by using the NMPC technique and consequently, policy equilibrium is obtained by applying the NMPC technique to differential strategic games. Policymakers' problems involve the repetitive solution of an optimal control problem in a receding horizon fashion but, as we consider strategic interactions, each of these optimal control problems must result in an open-loop Nash equilibrium. In this system, we have the control constraint which determines an upper bound for using fossil fuels and our

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[31] Semmler et al. (2018).

[32] Short-termism which is associated with political economy or information constraints was explained in the first chapter.

solution is a collection of policies extracted from a set of open-loop Nash equilibria, which refers to the NFNE. Using this method, we can provide a more accurate prediction and find the possible differences with other predictions which may be observed.

3.5.1 Global mean temperature

Climate change is characterized by changes in the global mean surface temperature. Moreover, while the temperature increase, it is assumed to increase the economic impacts of climate change. However, the impacts of climate change are the most uncertain part of any model.[33]

In this section, we provide simple mathematical formulas that show the relationship between CO_2 concentration and the global temperature change. Although our model differs from other models (such as DICE) that map emissions to temperature changes, but for making a comparison between our results and previous studies we convert the level of CO_2 concentrations to the possible temperature above the pre-industrial level.

For this purpose, we need to start with the definition of "Radiative Forcing". Radiative forcing is a measure which calculates the influence of GHGs concentration on changing the balance of incoming and outgoing energy in the earth-atmosphere system.[34] However, since 1750 the largest contribution to total radiative forcing is caused by

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[33] Nordhaus and Yang (1996).

[34] Greiner and Semmler (2005)

the increase in the CO_2 concentration.[35] Also, in the context of environmental economics and policy analyses CO_2 -equivalent concentration (CO_2-eq) generally is considered as a measure of radiative forcing because it gives the same radiative forcing as the actual mix of greenhouse gases.[36]

Radiative forcing is defined by:

$$\Delta F = 5.35 \ln\left(\frac{g}{g_0}\right) \tag{3.1}$$

Where g_0 is the pre-industrial level of CO_2 concentration equal to 280 ppm and g refers to the CO_2 -eq concentration. Unit of radiative forcing is W m^{-2} (Watts per square meter).

Moreover, considering λ as the climate sensitivity, it defines the response of the climate system to a given radiative forcing. Hence, the change in the Earth's average surface temperature ΔT_S is calculated by:

$$\Delta T_S = \lambda \Delta F \tag{3.2}$$

Where λ has the unit of degree Centigrade per W m^{-2}. Also, it is common to give λ in units of degrees Centigrade per CO_2 doubling (usually from 280 to 560 ppm in experiments with Global Climate Models (GCMs).

Then, the conversion between λ (degC per W m^{-2}) and λ^{dblCO2} (degC per CO_2 doubling) is $\lambda^{dblCO2} = 3.71\lambda$ and 3.71 W m^{-2} is the radiative forcing for CO_2 doubling from Equation (3.1).

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[35] IPCC (2013)

[36] See for example Stern (2006) and Garnaut (2008).

Hence,

$$\Delta T_s = \frac{\lambda^{dblCO2}}{ln(2)} ln\left(\frac{g}{g_0}\right) \tag{3.3}$$

According to the IPCC (2007a), the current best estimate is $\lambda^{dblCO2} = 3\ degC$ per CO_2 doubling. However, taking into consideration that the climate sensitivity is one of the most uncertain parameters in the climate change, the uncertainty range is large i.e., range 2 to 4.5 degC.[37]

3.6 The economic framework: CO_2 Concentration

We assume that two countries (or two coalitions of countries) face a domestic tradeoff between boost economic activity and limiting the use of fossil fuels that leads to the CO_2 concentration in the atmosphere and climate change. Hence, we face with the CO_2 stabilization dynamic. Moreover, each country has its own decision variable determined by its policymakers.

The use of non-renewable energy in each country ($x_1(t)$ or $x_2(t)$), leads to an increase of CO_2 concentration $g(t)$. So, the CO_2 concentration evolves according to

$$\dot{g}(t) = -\mu \cdot g(t) + \beta(x_1(t) + x_2(t)) \tag{3.4}$$

where $\mu \in (0,1)$ is the inverse of the atmospheric lifetime of CO_2 and $\beta \in (0,1)$ gives that part of CO_2 that remains in the atmosphere.

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[37] Raupach et al. (2011). Also, for further details we refer to Equation for Global Warming, Robert Ellis (2013).

Equation (3.4) clearly implies a tradeoff between CO_2 concentration and domestic consumption of fossil fuels for each country. Moreover, it highlights the negative externality associated with domestic productions. Any increase in the consumption of non-renewable energy, increases CO_2 concentration in both countries.

3.6.1 The social planners' problem

In our game**,** considering the externality, policymakers aim to maximize net social benefits. As to the utility function U, we use a generalization of the one, presented in Byrne (1997) and used by Greiner et al. (2014):

$$U = \frac{x^{1-\sigma}(g-g_0)^{-\xi(1-\sigma)}-1}{1-\sigma} \tag{3.5}$$

g_0 is the pre-industrial level of CO_2 concentration which is given in ppm. Moreover, $1/\sigma > 0$ is the parameter that is used to state the inter-temporal elasticity of substitution of using fossil fuels between two points in time. Also, $\xi > 0$ shows the (dis)utility of the CO_2 concentration exceeding the pre-industrial level, i.e., it expresses the effect of disutility (or the disaster effects) on our well-being.

Equation (3.5) shows that when the intertemporal elasticity of substitution is larger than one, if CO_2 rises, the marginal utility of consumption will decline and when the intertemporal elasticity of substitution is smaller than one, an increase in using fossil fuels will reduce the negative effect of pollution at the margin.[38]

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[38] Greiner et al. (2014)

To see the effect of CO_2 on the marginal utility, we calculate the cross derivative of the utility function:

$$\frac{\partial^2 U}{\partial X \partial M} = -\xi(1-\sigma)X^{-\sigma}(g-g_0)^{-\xi(1-\sigma)-1} > (<) 0 \leftrightarrow 1/\sigma < (>) 1 \qquad (3.6)$$

Equation (3.6) suggests that for $1/\sigma > 1$ consumption of fossil fuels and a clean environment are complementary, i.e., the marginal utility of using fossil fuels increases with a decline in the level of CO_2 emissions. But, for $1/\sigma < 1$, use of fossil fuels and CO_2 emissions are considered as substitutes because in this case, the marginal (dis)utility of additional pollution declines with a rising level of consumption.[39] However, it should be mentioned that $\sigma = 1$ makes the utility function logarithmic in using fossil fuels and externality of CO_2 concentration.

In this thesis, we will use the simplified preference i.e., $\sigma = 1$ for two reasons. First, for having a logarithmic form which implies that damages are a convex function of CO_2 concentration exceeding the pre-industrial level g_0. Second, because a more simplified welfare function in the NMPC works much faster for additively separable preferences than for the multiplicative form of equation (3.5).[40]

Hence, utility function can be written as:

$$U_i\big(x_i(t), g(t)\big) = \ln\big(x_i(t)\big) - \gamma \ln(g(t) - g_0) \qquad i \in \{1,2\} \qquad (3.8)$$

$\gamma > 0$ denotes the (dis)utility of CO_2 concentration and this function implies that damages are a function of CO_2 exceeding the pre-industrial level g_0.

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[39] Greiner et al. (2014)

[40] Greiner et al. (2014)

As we explained before, we focus on the dynamic interaction between two different policymakers. This can be interpreted as two continuums of homogeneous countries or as a coalition of countries.

3.6.2 Policy equilibrium

For the numerical solution, we employ two procedures. For observing the effect of coordination, we use the NMPC and for the strategic interaction the policy equilibrium is obtained by applying NFNE and solution is a collection of policies extracted from a set of open-loop Nash equilibria.

Policymakers try to choose a level of emissions in order to maximize net social benefits along with considering the CO_2 concentration. Thus, policymakers face the following optimization problem:

$$max \int_0^\infty e^{-\rho t} U_i dt, \qquad i \in \{1,2\} \tag{3.9}$$

$$\text{Subjected to:} \quad \dot{g}(t) = -\mu \cdot g(t) + \beta(x_1(t) + x_2(t))$$

$$x(0) = x_0, \;\; t \in \mathbb{R}_0^+, \;\; \tau \in \mathbb{N}$$

3.7 Cooperative solution

The international community is trying to achieve global cooperation to reduce CO_2 emissions. In a cooperative situation, players communicate and agree in order to co-operate to achieve their targets.

Designing a game can be considered as a mechanism to achieve cooperation. [41] To have a cooperative behavior, we need to accept a coalition. Hence, cooperative game theory, as a mechanism, try to investigate the situations in which policymakers would form these coalitions. The cooperative solution is obtained from maximizing the Nash product of two countries i.e., two policymakers inter a Nash bargaining process to determine the rate of emission.[42]

In other words, international coordination is formalized as an alternative regime which its outcomes can be obtained by the Pareto optimal control solution when two policy-makers jointly maximize the Nash product:

$$max(U_1)^{\omega}(U_2)^{1-\omega} \tag{3.10}$$

U_1 and U_2 refers to the emissions benefit functions minus emissions damage functions i.e., utilities augmented with damages in welfare.

Also, ω and $1-\omega$ measure policymakers' relative bargaining powers. Bargaining powers is used to specify the share of collective gains and according to those shares, policymakers evaluate the incentives to join an agreement.[43] However, it should be noted that since our gains from cooperation can only be reached after we obtain an

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[41] Wood (2010)

[42] Acocella and Di Bartolomeo (2019)

[43] Yu et al. (2017)

agreement, any delay to achieve an agreement is costly because if we can have an agreement sooner, we will face less climate damages.[44] Thus, The discount factor can be used as an indicator of the negotiators' bargaining power because it reflects a player's willingness to wait.[45]

In this thesis, for the sake of simplicity, we assume equal bargaining powers between the two players. Also, both policymakers enjoy the same forecasting horizon.

3.8 Results

This chapter presents the numerical analysis derived from dynamic decision problems. As explained before, we evaluate the expected CO_2 concentration and the global mean temperature under two different regimes. The procedure starts by considering the non-cooperative situation and then we make a comparison with the level of CO_2 concentration which is obtained in the cooperative regime. Solutions rely on the implementation of Algorithm1.1 and 2.1.

3.8.1 Calibration

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[44] See for example Muthoo (1999), Rubinstein (1982) and Courtois and Tazdait (2014).

[45] Yu et al. (2017)

For the numerical solutions we calibrate our model as Greiner et al. (2014). Hence, the inverse of the atmospheric lifetime of CO_2, μ , and the part of CO_2 that remains in the atmosphere β are set as $\mu = 0.1$, $\beta = 0.49$, the unit of both parameters are in percentage and the discount factor is considered as $\rho = 0.03$. According to the IPCC, the pre-industrial level of CO_2 concentration is considered around 280 ppm that we normalized it to one (i.e. $g_0 = 1$). Moreover, in the utility function, there is a parameter as γ which refers to the disutility of CO_2 concentration exceeding the pre-industrial level. This parameter shows that how much disutility (or the disaster effects) will affect our well-being, i.e., γ expresses the weight of a disaster and it will have an impact on the control variables.[46] Taking into account that we face different countries who will be affected by this damage differently, we can suppose that γ would be different e.g., in developed and developing countries. A larger γ (when $g > g_0$) means a bigger impact on the welfare of households in developing countries, because they have fewer means of adaptation (building sea walls or other protective measures). So, considering the interaction between non-symmetric policymakers/countries, γ for the group with a bigger impact on the welfare is set as 2.50 and for the group with the lower impact on welfare is considered equal to 2.35 i.e., interaction between developing and developed countries respectively. Finally, in order to compute the result, we assume the value of forecasting horizon length as $T = 3$ but later we compare the effect of different time horizons.

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[46] Semmler et al. (2018).

3.8.2 CO_2 Concentration example

At first, we describe the application of our NFNE algorithm to a period between 1959 to 2019. This optimization is important to verify the accuracy of numerical procedures in our research. results of the first optimization problem illustrated in figure 3.1 and because similar results with evidence have been observed during the same period, hence the accuracy of the respective method and calibration can be approved.

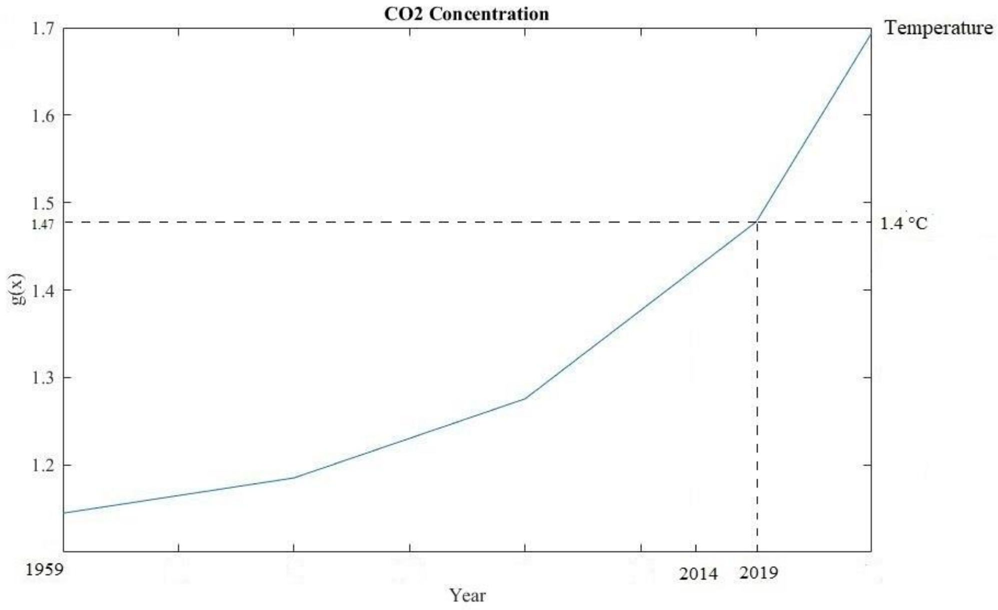

Fig.3.1 CO_2 concentration and global mean temperature 1959-2019 by NFNE method

Figure 3.1 shows the normalized level of CO_2 concentration (left scale) during 1959-2019 and the global mean temperature (right scale). The initial value of CO_2 concentration, i.e., $g(0)$, is considered as 315.97 ppm (or 1.128 as the normalized form) of CO_2 Concentration in 1959. Numerical analysis ends in 2019 with CO_2 concentration equal to 411.44 ppm (or 1.47, the normalized form).[47] According to the Berkeley Earth, the level of CO_2 concentration compared to 1951-1980 averages, in 2019 has increased 1.32 ± 0.04 °C. The curve shows good compatibility with observed data during these years.

Hence, these results support our model and demonstrate the NFNE as an efficient way of solving dynamic decision problems numerically.

3.8.3 Non-cooperative VS Cooperative regime

In this section, we observe the prediction of CO_2 concentration level under cooperative and non-cooperative regimes for 2019-2100. Non-cooperative Nash equilibrium will be obtained by solving the optimization problem (3.9) for two policymakers/countries and the result can be compared to the cooperative solution which is obtained from maximizing the Nash product of two players, i.e., equation (3.10).

Figure 3.2 reports the CO_2 concentration and the global mean temperature under non-cooperative and cooperative regimes during the next 80 years. The numerical analysis starts from 2019, assuming $g(0) = 1.47$ (411.44 ppm) and considering time horizon as $T = 3$.

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[47] Dr. Pieter Tans, NOAA/ESRL (www.esrl.noaa.gov/gmd/ccgg/trends/) and Dr. Ralph Keeling, Scripps Institution of Oceanography (scrippsco2.ucsd.edu/).

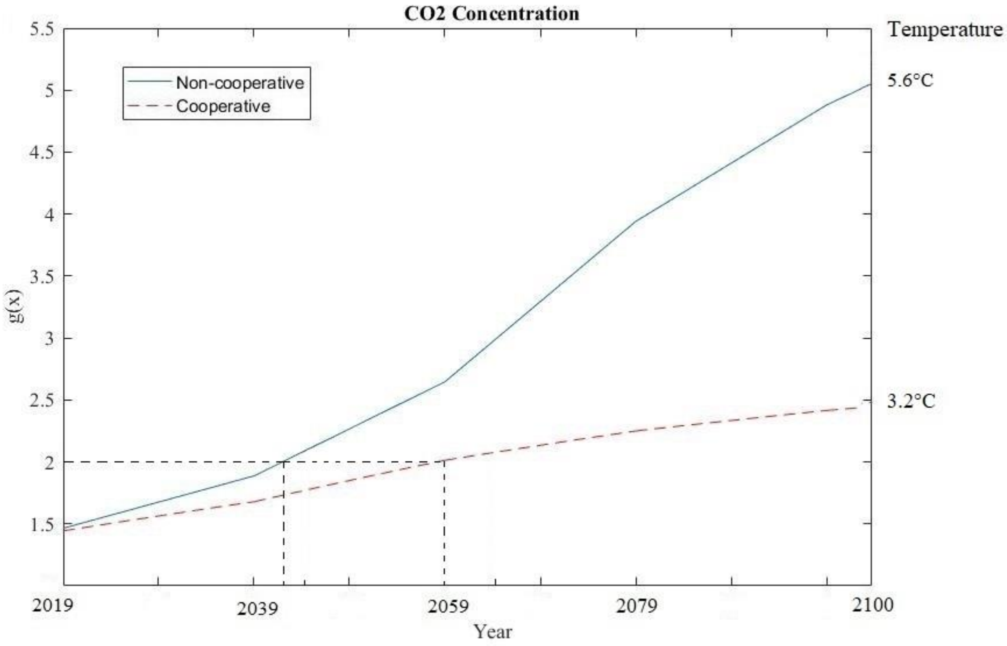

Fig.3.2 CO_2 concentration and global mean temperature under different regimes 2019-2100

As we expected, the non-cooperative strategy leads to a higher level of CO_2 concentration compared with the cooperative policy. In the beginning, there is not a big difference in CO_2 concentration for two policies, but after 2039 we can observe a notable increase in its level under non-cooperative situation which eventually will arrive at 1456 ppm (5.2 as a normalized form). this level of CO_2 concentration shows the temperature around 5.6°C above the pre-industrial level by 2100. While, under the cooperative situation, CO_2 concentration reaches 700 ppm (2.5 as a normalized form) which leads to 3.2°C above pre-industrial temperatures.[48] According to the IPCC (2014), baseline scenarios – i.e., those without additional mitigation – in The Representative Concentration Pathways (RCPs) fall into the >1000 ppm and are anticipated

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[48] It should be mentioned that the surface temperature increase is including the CO_2 concentration with water vapor feedback.

to increase the global mean temperature to the range from 2.5°C to 7.8°C above pre-industrial level with high confidence when including climate uncertainty.

Comparing our results with the RICE model, CO_2 concentration in our model shows the damages in a concave function while the RICE model implies that damages are a convex function of CO_2 concentration in both cooperative and non-cooperative situations. Moreover, in the RICE model, by 2100, we observe the level of CO_2 concentration around 754 ppm and 731 ppm for non-cooperative and cooperative regimes respectively and consequently the estimated temperatures were around 3°C and 2.8°C. While our results predict higher levels in both situation; around 1456 ppm for non-cooperative policy and 700 ppm in the cooperative situation.

We can explain these differences by the methods that we have used i.e., while the RICE model predicts for the entire time from beginning, we use a receding horizon model which compared to the infinite horizon models, agents need less information because at each instant of time the horizon is moving toward the future and players use the first control signal of the sequence calculated at previous steps for an especial time horizon. However, under cooperative regimes, this difference also can be interpreted by climate policy. In the RICE model *"carbon tax"* was considered as the climate policy under cooperative regimes which can reduce the level of CO_2 concentration more, while in our model under the cooperative situation policymakers just reduced the use of fossil fuels to find the Pareto optimal cooperative solution also, in NFNE, along with trying to maximize net social benefits, control constraint prescribes an upper bound for using fossil fuels and, the state constraint places a cap on the total level of CO_2 in the atmosphere in each period.[49] In other words, our model implies

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[49] Semmler et al. (2018).

that under supposed conditions, clear climate policies such as carbon tax work more efficiently compared with an international climate agreement which is self-enforcing and it aims to decrease the use of fossil fuels without any explicit climate policy.

But it should be noted that still there are other climate policies such as using new technologies or substituting non-renewable energy with renewable energy which should be assessed and there is the possibility that they work more efficiently than carbon tax that we will discuss in the next chapter completely.

Also, it is worth noting that considering the date of doubling of CO_2 concentration relative to preindustrial level, if the benchmark is taken to be 565 ppm (2.01 normalized form),[50] in the RICE model the doubling date is 2070 for cooperative, and 2065 for the non-cooperative regimes. While, the date of the doubling of CO_2 concentration in our model are around 2064 for the cooperative and 2044 for the non-cooperative policies. This means that without any actions to reduce emissions, CO_2 concentrations are likely to pass double pre-industrial level more rapidly than those dates which have been anticipated before. Moreover, these results show that although an efficient co-operation will lead to a much lower temperature by the year 2100 but it warns us that we may face damages of getting the double level of CO_2 concentration earlier than that time, we already are prepared for.

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[50] Nordhaus and Yang (1996).

3.9 Policymakers’ myopia

Considering the concept of NFNE, the length of the policymaker’s horizon is an exogenous parameter that plays an important role in achieving an accurate prediction. In this section, we investigate the effects of policymakers’ short-termism on CO_2 concentration.

As we explained in chapter 1, by short-termism, we refer to the policymaker’s time horizons and the shorter lengths in time horizons may be associated with political economy or information constraints. In other words, policymakers tend to have short-time horizon decisions as the result of making decisions under limited information or limited information processing.[51]

We formally deal with this issue assuming that policymakers’ problems do not involve the optimization over the entire planning horizon, but it just involves repetitive solutions of dynamic decision problems at each instant of time in a receding horizon fashion. In other words, by focusing on changes in policymakers rather than changes in the policymakers’ ideology (ideological turnover), we can measure policy instability by the policymakers’ time preferences.[52]

When we work with a finite horizon model, this difficulty of seeing far enough, can be interpreted as inattention or myopia, i.e., shorter horizons measure greater myopia. The main idea is that policymakers need to weigh the short-run cost of information rising with longer horizon, against the long-run benefits.

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[51] See for example Sims, 2005, 2006

[52] Di Bartolomeo et al. (2018).

Our results are displayed in Figure 3.3. Two different values of the forecasting horizon are considered as the higher and lower myopia i.e., T=3 and T=4 respectively.

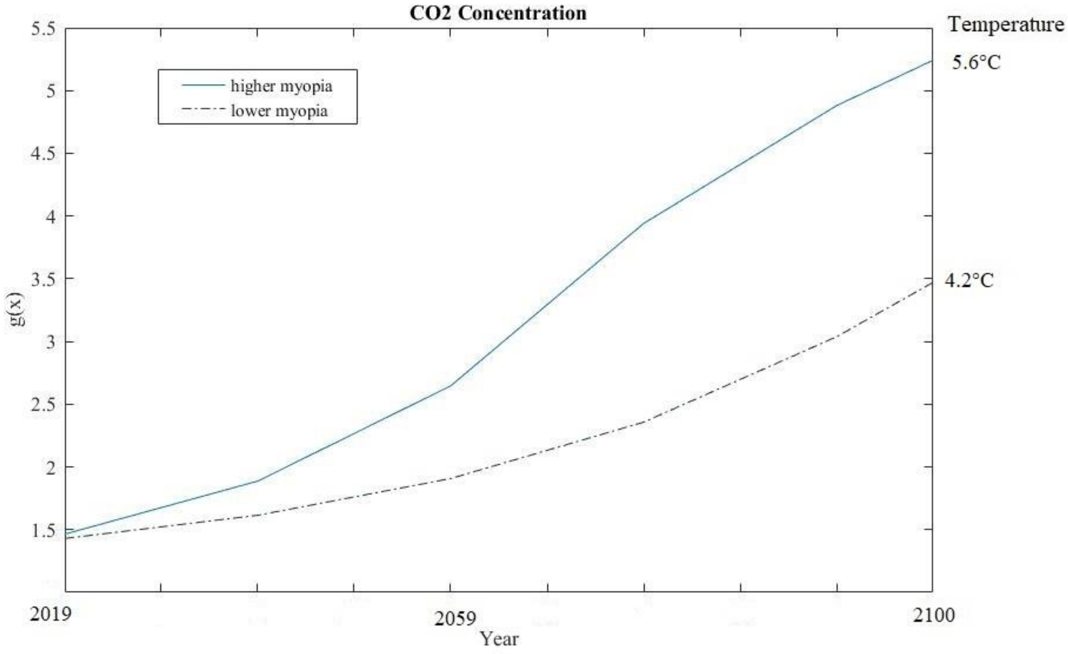

Fig.3.3 CO_2 concentration and policy myopia (T=3 and T=4 respectively).

Figure 3.3 shows that myopic policymakers will observe a higher level of CO_2 concentration compared to the less myopic policymakers. Assessing the temperature in the next 80 years, we see that myopic policymakers predict 5.6°C above preindustrial level while, less myopic policymakers, anticipate much less CO_2 concentration which leads to the temperature around 4.2°C above the pre-industrial level.

Moreover, while higher myopic policy shows that damage is a concave function of CO_2 concentration, lower myopia shows a convex function that is nearer to the RICE model in shape. Also, our result implies that this tendency to short-termism in our model, may lead to the over-evaluating the level of CO_2 Concentration compared with less myopic policymaker.

As Sims (2005, 2006) explained, with increasing the agents' information and information processing capacity we can have a more accurate approximation for infinite trajectory.

3.10 Concluding remarks

In this chapter considering the strategic interaction between two different policymakers, we have studied the level of CO_2 concentration and eventually global mean temperature in cooperative and non-cooperative situations. We described our model by considering the use of non-renewable energy as the control variable and CO_2 concentration as the state variable. Then, we explained the optimization problem under different regimes. The numerical part of the chapter shows that CO_2 concentration will be significantly lower if players operate under a cooperative policy. It seems that with cooperation between independent policymakers we can reach much less CO_2 concentration. However, still it does not lead to the best emission pathway. Moreover, if policymakers find it difficult (e.g., for political reasons) to conclude international binding agreements, and prefer to rely on their preferences for consuming non-renewal recourses instead of considering the global warming, the negative externalities and damaging effects may be quite severe.

Also, along the line of Sims's idea that agents often make decisions under information constraint, we interpret the finite horizon as a measure of inattention or myopia. This difference in the time horizons, leads to different results. Hence, we examine optimization problems characterized by two different values of the decision horizon length (respectively 3 and 4). Results show a significant difference between higher myopic and lower myopic policymakers. Interestingly lower myopic policy does not lead to the previous harsh result. It implies that making decisions under myopia, which can be the result of limited information and/or limited information processing, leads to a higher prediction of environmental damage, by over-evaluating the level of CO_2 Concentration compared with less myopic policymakers. However, in the absence of any cooperation, if we continue to emit at the same rate, even less myopic policymakers anticipate a severe result by 2100. However, we should note that these results are very sensitive with respect to our assumptions and calibration.

Chapter 4

Extension of NFNE method: Transition from non-renewable to renewable energy

The purpose of this chapter is to build an extended form of NFNE with two dynamics, in order to deal with an important environmental economics issue which is substituting non-renewable by the renewable energy sector.

Although, this substitution is considered as a significant way to reduce the CO_2 emission and as an additional instrument of cooperative policy, still there is some ambiguity about that. Hence, in this chapter, considering the interaction between different policymakers, we specifically investigate the effect of different regimes on transition from non-renewable to renewable resources.

For this purpose, we need to move on from the usual NFNE with just one common dynamic to NFNE with two dynamics. This means that we need a change in the form. The NFNE method which has been applied before, can be used in the models with only one state variable. Hence, this extension in the number of state variables requires building two separate loops for each country i.e., one loop for repetitive solution for common state variable (CO_2 concentration) and the other one for the second state variable (capital stock for renewable resource). These loops are going to be solved independently but simultaneously to find those fixed points that players have no incentive to change that at each point of time.

4.1 Related literature

The feasibility of limiting CO_2 concentration has attracted considerable scientific and policy attention. Different actions have been proposed to limit climate change. Efforts generally elaborate on subjects such as using new “green” technologies, and mitigation policies (e.g., carbon tax and tradable emission permits).

A large number of studies are focused on mitigation policies and their welfare effects. However, in this context, there is an important research direction that its primary goal is to focus on the transition from non-renewable to renewable energy.[53] In the long-run, substituting renewable energy for non-renewable energy, not only decrease negative externality but also, increase the individual welfare. Although, this transition of

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[53] See for example see Edenhofer et al. (2006), Heinzel and Winkler (2011), Jacobson and Delucchi (2011, 2011a).

energy sectors can be considered as a promising way to reduce CO_2 emissions and combat global warming, but still there are some doubts that if it works efficiently or not, especially while we notice that countries usually prefer others to take care of reducing the global warming at their own expenses, i.e., non-cooperative situation.

Renewable resources can be classified from different natural resources (e.g., wind, water, solar) to man-made capital stock (in order to produce renewable energy). For the first time, Krautkraemer (1985) presented a growth model where fossil fuels can be used to produce output that can be consumed or invested for the capital stock (in order to make renewable energy). In this model, capital stock can be either an imperfect or a perfect substitute for the non-renewable resource. In more recent contributions, Hoel and Kverndokk (1996) and Van der Ploeg and Withagen (2011, 2012) presented models that have the same structure belong to the general class of models analyzed by Krautkraemer (1998). Their models show that non-renewable resources can be perfectly substituted by renewable resources at a given cost, with both variables being control variables.

In this context, Greiner et al. (2014) combined two previous approaches of Krautkraemer (1985), Hoel and Kverndokk (1996) and Ploeg and Withagan (2011, 2012) and worked on a canonical growth model augmented with damages in the welfare function with two energy sectors as non-renewable and renewable resources. They tried to investigate the time of transition from polluting to non-polluting resources and find out whether fossil fuels will be extracted completely or left partially. In their model energy can be produced either from a non-renewable energy source, such as fossil fuels, or from a renewable source that requires investment in capital stock. While, renewable energy is considered as a non-polluting energy and does not contribute to global warming, using fossil fuels leads to the negative externality and consequently generate damages to welfare. According to their results a country – apart from its initial deposits of fossil fuels –will start to build a capital stock for the production of renewable energy before the non-renewable resource is completely exhausted. However, results state that higher initial value of fossil fuels would prolong this starting point. Moreover, they found that in the case of high capital stock or low initial value of non-renewables, more of fossil fuels are left unextracted and consequently we will face the lower level of CO_2 concentration.

4.2 Dynamic-provision game

In this chapter, we aim to investigate the transition from polluting to non-polluting resources when policymakers operate under different policies. Hence, we borrow from Greiner et al. (2014) the definition of economic growth and damages in the household's welfare function and two energy resources, i.e., non-renewable and renewable energy while for producing the latter, capital stock must be built up.

In this thesis in contrast to the paper by Greiner et al. (2014), our model has two optimization problems and we focus on dynamic interactions between two policymakers (two countries or coalition of countries) instead of an economy and we investigate the effect of different regimes on the level of CO_2 concentration and capital stock. Each maximization problem has two control variables, consumption and extraction rate of fossil fuels,[54] and two state variables as CO_2 concentration and capital stock where the former is the common dynamic for both countries and the latter is considered as the second state variable for each country separately. Our setup reflects the fact that global warming as a GPG has the same effect on everyone.

To achieve this purpose, we need to extend the previous NFNE method. For the numerical solutions, we employ NMPC in the cooperative situation and NFNE in the non-cooperative situation. However, according to the regular definition of NFNE we cannot solve a model with more than one state variable. This means that we should build a new loop involved in the procedure of finding equilibrium strategies. With adding another dynamic to the model, we should build two separate loops for each country which these two procedures should work independently but simultaneously. This procedure is used to find optimal control problems and fixed points that players have no incentive to change that at each point of time. This extension is a big step in solving more complicated NMPC games.

So, our results can be summarized in two directions. First, we extend the current technique for using it in our model. Then, we investigate the effects of different regimes on the level of CO_2 concentration and the transition of resources.

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[54] Remember that according to the NMPC, control variables are set at the beginning of each period and will be kept constant until the end of each period.

4.3 Structure of NFNE with two state variables

According to the NFNE definition, we solve a procedure consist of two players to find an optimal strategy in which players have no incentive to change that at each point time. As we explained in chapter 2, this procedure works in a loop consist of repeating the maximization/minimization of the optimal control problem.

In order to add another dynamic, we need to extend our previous NFNE method and increase the loops involved in the procedure of finding equilibrium strategies. In our model, we face two separate loops for each country i.e., the first loop for common state variable, CO_2 concentration, and the second one for the capital stock.

In this case, the first state variable evolves according to the decisions of two policy-makers, i.e., $i \in \{1,2\}$, that each of them faces with two control variables, i.e., $j \in \{1,2\}$:

$$\dot{x}(t) = g\left(x(t), u_{i,j}(t), u_{-i,j}(t)\right) \tag{4.1}$$

We suppose x as the state variable and $u_{i,j}(t)$ and $u_{-i,j}(t)$ are the first control variables for the first and second players respectively. Equation (4.1) is considered for each policymaker separately however since we consider it as CO_2 concentration in our model and it is a common state variable, it can be considered the same for both players.

Then the second dynamic for each player evolves according to:

$$\dot{y_{\imath}}(t) = f\left(y_{i,}(t), u_{i,-j}(t), u_{-i,-j}(t)\right) \tag{4.2}$$

$$\dot{y_{-\imath}}(t) = f\left(y_{-i}(t), u_{i,-j}(t), u_{-i,-j}(t)\right) \tag{4.3}$$

Where $\dot{y}_i(t)$ and $\dot{y}_{-i}(t)$ refer to the second state variables of each country. Also, $u_{i,-j}(t)$ and $u_{-i,-j}(t)$ are considered as the second control variable of the first and second players respectively.

As we explained before, both players want to maximize welfare.[55] Thus, for each one we have the following optimization problem:

$$V_i(x_0, y_0) = \sum_{\tau=1}^{\infty} \rho^{\tau-1} \int_{\tau-1}^{\tau} g_i\left(x(t), y(t), u_{i,j}(\tau)\right) dt \quad i \in \{1,2\}, j \in \{1,2\}$$

Where ρ is the discount factor.

Given their prediction horizon $[\tau - 1, \tau + T]$, players find fixed points where they have no incentive to change strategies at each period of time τ. NFNE is explained by the optimal strategies $u_1^*(\tau)$ and $u_2^*(\tau)$, hence:

$$NFNE := \left\{u_{1,1}^*(\tau), u_{2,1}^*(\tau), u_{1,2}^*(\tau), u_{2,2}^*(\tau)\right\} \quad , \forall_\tau \in \mathbb{N} \tag{4.4}$$

Given dynamical systems i.e., (4.1), (4.2), (4.3) and according to above definitions, by extending the algorithm 2.1, we can build the prediction trajectory and optimal strategies, via algorithm 4.1 as following.

Algorithm 4.1 (Extended NFNE). Given time horizon T; the initial condition of the dynamic system: $x(0) = x_0$, $y_1(0) = y_{1,0}$, $y_2(0) = y_{2,0}$; and initial guess for the policy of player 2: $u_{2,1}^{g0} \in \mathbb{R}^T$ and $u_{2,2}^{g0} \in \mathbb{R}^T$, the NFNE is found by the following steps:

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[55] Note on t vs. τ and discretization, see section 1.2.1 Timing assumptions.

At time $\tau = 1$,

(1) Use vectors $u_{2,1}^{g0}$=$u_{2,1}^{g}$ and $u_{2,2}^{g0} = u_{2,2}^{g}$ as guess for player 2 policy and $x(0) = x_0$ and $y_1(0) = y_{1,0}$, and find the optimal policies ($u_{1,1}^{o} \in \mathbb{R}^T$ and $u_{1,2}^{o} \in \mathbb{R}^T$) solving the optimal control problem of first player between 1 and T subjected to two dynamic constraints:

$$max\, V_1\left(x_0, y_{1,0}\right)$$

$$\text{s.t.} \qquad \dot{x}(t) = g\left(x(t), u_{1,1}(t), u_{2,1}^{g}(t)\right)$$

$$\dot{y}_1(t) = f_1\left(y_1(t), u_{1,2}(t), u_{2,2}^{g}(t)\right)$$

The outcomes are two vectors $u_{1,1}^{o}$ and $u_{1,2}^{o}$ of T controls.

(2) Then we use – as guesses for player 1 policies – the $u_{1,1}^{g} = u_{1,1}^{o}$ and $u_{1,2}^{g} = u_{1,2}^{o}$ obtained from step (1) and $x(0) = x_0$ and $y_2(0) = y_{2,0}$, to find the optimal policies for player 2 (i.e., $u_{2,1}^{o} \in \mathbb{R}^T$ and $u_{2,2}^{o} \in \mathbb{R}^T$) solving the optimal control problem of second player between 1 and T, i.e.,

$$max\, V_2\left(x_0, y_{2,0}\right)$$

$$\text{s.t.} \qquad \dot{x}(t) = g\left(x(t), u_{1,1}^{o}(t), u_{2,1}(t)\right)$$

$$\dot{y}_2(t) = f_2\left(y_2(t), u_{1,2}^{o}(t), u_{2,2}(t)\right)$$

Again, the outcomes are two vectors $u_{2,1}^{o}$ and $u_{2,2}^{o}$ of T controls.

(3) Then, repeat step (1) by using $u^{o}_{2,1}(t)$ and $u^{o}_{2,2}(t)$ as guesses for player 2's policies, and step (2) – by using the results we achieve from its previous step – until a fixed point is found, i.e., $u^{f}_{1,1}$, $u^{f}_{1,2}$, $u^{f}_{2,1}$, and $u^{f}_{2,2}$.

(4) Then we take the first elements of fixed vectors, i.e., $u^{f}_{1,1}(1), u^{f}_{1,2}(1), u^{f}_{2,1}(1)$, and $u^{f}_{2,2}(1)$, which are the optimal NFNE at time $\tau = 1$: $u^{*}_{1,1}(1)$, $u^{*}_{1,2}(1)$, $u^{*}_{2,1}(1)$ and $u^{*}_{2,2}(1)$, and then by applying theses mentioned optimal NFNE solutions in usual dynamics, i.e., $\dot{x}(t) = g\left(x(t), u_{1,1}(t), u_{2,1}(t)\right)$, $\dot{y_1}(t) = f_1\big(y_1(t), u_{1,2}(t), u_{2,2}(t)\big)$ and $\dot{y_2}(t) = f_2\big(y_2(t), u_{1,2}(t), u_{2,2}(t)\big)$, we obtain $x^{*}(1)$, $y^{*}_{1}(1)$, and $y^{*}_{2}(1)$.

At time $\tau > 1$,

(5) Use vectors $u^{g}_{2,1}$=$u^{f}_{2,1}$ and $u^{g}_{2,2} = u^{f}_{2,2}$ as guess for player 2 policy $x(\tau - 1) = x^{*}(\tau - 1)$ and $y_1(\tau - 1)$=$y^{*}_{1}(\tau - 1)$ as initial conditions, and find the optimal policies ($u^{o}_{1,1} \in \mathbb{R}^{T}$ and $u^{o}_{1,2} \in \mathbb{R}^{T}$) solving the optimal control problem of first player between τ and $\tau + T$ subjected to two dynamic constraints:

$$max\, V_1\big(x^{*}(\tau - 1), y^{*}_{1}(\tau - 1)\big)$$

$$\text{s.t.} \qquad \dot{x}(t) = g\left(x(t), u_{1,1}(t), u^{f}_{2,1}(t)\right)$$

$$\dot{y_1}(t) = f_1\big(y_1(t), u_{1,2}(t), u^{f}_{2,2}(t)\big)$$

The outcomes are two vectors $u^{o}_{1,1}$ and $u^{o}_{1,2}$ of T controls.

(6) Then we use, as guesses for player 1 policies, the $u^g_{1,1} = u^o_{1,1}$ and $u^g_{1,2} = u^o_{1,2}$ obtained from step (5) and $x(\tau-1) = x^*(\tau-1)$ and $y_2(\tau-1)$=$y_2^*(\tau-1)$ as initial conditions, to find the optimal policies for player 2 (i.e., $u^o_{2,1} \in \mathbb{R}^T$ and $u^o_{2,2} \in \mathbb{R}^T$) solving the optimal control problem of second player between τ and $\tau + T$, i.e.,

$$\max V_2\big(x^*(\tau-1), y_2^*(\tau-1)\big)$$

$$\text{s.t.} \qquad \dot{x}(t) = g\left(x(t), u^o_{1,1}(t), u_{2,1}(t)\right)$$

$$\dot{y}_2(t) = f_2\big(y_2(t), u^o_{1,2}(t), u_{2,2}(t)\big)$$

Again, the outcomes are two vectors $u^o_{2,1}$ and $u^o_{2,2}$ of T controls.

(7) Then, repeat step (5) – by using $u^o_{2,1}(t)$ and $u^o_{2,2}(t)$ as guesses for player 2's policies – and step (6) until a fixed point is found.

(8) Next, we take the first elements of vectors, i.e., $u^f_{1,1}(1)$, $u^f_{1,2}(1)$, $u^f_{2,1}(1)$, and $u^f_{2,2}(1)$, which are the optimal NFNE at time τ: $u^*_{1,1}(\tau)$, $u^*_{1,2}(\tau)$, $u^*_{2,1}(\tau)$ and $u^*_{2,2}(\tau)$, and then by applying theses mentioned optimal NFNE solutions in usual dynamics, i.e., $\dot{x}(t) = g\left(x(t), u_{1,1}(t), u_{2,1}(t)\right)$, $\dot{y}_1(t) = f_1\big(y_1(t), u_{1,2}(t), u_{2,2}(t)\big)$ and $\dot{y}_2(t) = f_2\big(y_2(t), u_{1,2}(t), u_{2,2}(t)\big)$ we obtain $x^*(\tau)$, $y_1^*(\tau)$, and $y_2^*(\tau)$ for $\tau > 1$.

(9) Repeated (5)-(8) to find the NFNE vectors:

$$NFNE := \{u^*_{1,1}(\tau), u^*_{2,1}(\tau), u^*_{1,2}(\tau), u^*_{2,2}(\tau)\}, \forall \tau \in \mathbb{N}$$

And dynamics for $x(t)$ and $y(t)$ for i & $j \in \{1,2\}$, $t \in \mathbb{R}_0^+$, $\tau \in \mathbb{N}$ can be explained by NFNE as:

$$\dot{x} = f\big(x(t), u^*_{i,j}(\tau), u^*_{-i,j}(\tau)\big) \tag{4.5}$$

$$\dot{y_i} = f_i\big(y_i(t), u^*_{i,-j}(\tau), u^*_{-i,-j}(\tau)\big) \tag{4.6}$$

$$\dot{y_{-i}} = f_{-i}\big(y_{-i}(t), u^*_{i,-j}(\tau), u^*_{-i,-j}(\tau)\big) \tag{4.7}$$

Applying the above algorithm to our model, will generate the global equilibrium dynamics of CO_2 concentration and capital stocks in the two countries.

4.4 The economic framework

We consider a stylized model for GPG provision with two sectors of energy as non-renewable and renewable resources. As we mentioned before, in this chapter we have two dynamics as CO_2 concentration which affecting the welfare of households and the capital stock which must be built up to produce renewable energy. Moreover, each country has its own decision variables as consumption and the extraction rate of fossil fuels at time t, which are determined by their policymakers.

The use of fossil fuels in each country ($x_1(t)$ or $x_2(t)$), leads to an increase of CO_2 concentration ($g(t)$) and it evolves according to:

$$\dot{g}(t) = -\mu \cdot g(t) + \beta\big(x_1(t) + x_2(t)\big) \tag{4.8}$$

where $\mu \in (0,1)$ is the inverse of the atmospheric lifetime of CO_2 and $\beta \in (0,1)$ gives that part of CO_2 that remains in the atmosphere. Equation (4.8) implies a tradeoff between CO_2 concentration and domestic consumption of fossil fuels for each country and it proves the negative international externality associated with the domestic production.

Moreover, non-renewable energy can be used to make the capital stock for producing renewable energy. Generally, total energy output E is the combination of energy produced from a renewable energy sector E_r, and from a non-renewable energy sector as E_n. Hence, Y is the production of the final good as a concave function of energy:[56]

$$Y = A(A_r K + A_n u)^{\alpha} \ = AE^{\alpha} \tag{4.9}$$

Where A_r and A_n denote efficiency indices of renewable and non-renewable resources respectively. u is the amount of fossil fuels used at time t and K is a stock of capital using renewable sources to generate energy with $0 < \alpha \leq 1$ and $A > 0$.[57]

Hence, if $i \in \{1,2\}$, the second dynamic as the stock of renewable energy is K and for each country it evolves according to:

$$\dot{K}(t) = Y - \left(z_i(t) + \delta z_{-i}(t)\right) - \theta K \tag{4.10}$$

$z_1(t)$ and $z_2(t)$) refer to the consumption for the first and the second country, respectively. Also, θ is the decay rate of capital stock. Equation (4.10) characterize the accumulation of capital stock.

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[56] Greiner et al. (2014).

[57] It should be noted that in Greiner et al. (2014) energy is a homogeneous good so modeling the two types as perfect substitutes can be justified.

4.5 The social planners’ problem and policy equilibrium

While the aim of the policymakers is to maximize net social welfare, policymakers face a domestic tradeoff between boost economic activity and limiting the use of fossil fuels and climate change.

As to the utility function U, we use

$$U = \frac{z^{1-\sigma}(g-g_0)^{-\xi(1-\sigma)}-1}{1-\sigma} \tag{4.11}$$

Where ρ is the discount rate, g_0 is the pre-industrial level of CO_2 concentration and $\xi > 0$ shows the (dis)utility of the CO_2 concentration exceeding the pre-industrial level.

As we mentioned in chapter 3, $\sigma = 1$ makes the utility function logarithmic in using fossil fuels and pollution. Hence, take into account the negative externality of CO_2 and the marginal utility of consumption, utility function can be written as:

$$u_i\big(z_i(t), g(t)\big) = \ln\big(z_i(t)\big) - \gamma \ln(g(t) - g_0) \qquad i \in \{1,2\} \tag{4.12}$$

where the same as before g_0 is the pre-industrial level of CO_2 concentration and $\gamma > 0$ refers to the (dis)utility of the CO_2 concentration exceeding the pre-industrial level.

Since, policymakers try to choose a level of emissions to maximize net social benefits considering the externality of CO_2 concentration, where $i \in \{1,2\}$, policymakers face the following optimization problem:

$$max\ u_i\big(z_i(t), g(t)\big), \tag{4.13}$$

$$\text{Subjected to:}\quad \dot{g}(t) = -\mu \cdot g(t) + \beta(x_i(t) + x_{-i}(t))$$

$$\dot{K}(t) = Y - \big(z_i(t) + \delta z_{-i}(t)\big) - \theta K$$

In the following, we consider the cooperative situation to assess the effect of international cooperation and make a comparison between cooperative and non-cooperative situations.

We consider cooperative game theory, as a mechanism to investigate the situations in which policymakers would form a coalition. The cooperative solution is obtained from the maximizing of the Nash product of two countries. Policymakers inter a Nash bargaining process as

$$max(U_1)^{\omega}(U_2)^{1-\omega} \tag{4.14}$$

U_1 and U_2 refers to the utilities augmented with damages in welfare. ω and $1 - \omega$ measure policymakers' relative bargaining powers. It should be mentioned that we assume an equal bargaining powers between the two players and both policymakers enjoy the same forecasting horizon.

For the numerical solution, we apply NFNE and NMPC techniques under non-cooperative and cooperative regimes.

4.6 Results

In this section, we evaluate the expected CO_2 concentration and the stock of capital under different regimes. The non-cooperative Nash equilibrium will be obtained by solving the optimization problem (4.13) for two policymakers simultaneously. Then the result can be compared to the cooperative solution which is obtained from maximizing the Nash product of two countries, i.e., equation (4.14). Solutions rely on the implementation of Algorithm1.1 and 4.1.

4.6.1 Calibration

In the numerical analysis below, we calibrate our model as Greiner et al. (2014). μ as the inverse of the atmospheric lifetime of CO_2 is equal to 0.1 and the part of CO_2 that is not taken up by oceans (β) is set at 0.49 and the units of both parameters are in percentage. The discount rate is $\rho = 0.03$ and decay rate θ , is set as 0.05. Also, we take $A = 1$, $A_r = 1$ and a rather large $A_n = 1000$ which gives rise to reasonable steady-state results.

The pre-industrial level of CO_2 – which is around 280ppm – is normalized to one (i.e. g_0 = 1). The initial value of CO_2 concentration, i.e., $g(0)$, is considered 1.46 as the normalized form of CO_2 concentration at the beginning of 2019 which is equal to 408.52 ppm.[58] Moreover, the initial value of capital stock, i.e., $K_1(0) = 9.1$ and $K_2(0) = 7.7$ are considered as the normalized form of investment in renewable energy

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[58] Dr. Pieter Tans, NOAA/ESRL (www.esrl.noaa.gov/gmd/ccgg/trends/) and Dr. Ralph Keeling, Scripps Institution of Oceanography (scrippsco2.ucsd.edu/).

capacity in developing and developed countries at the beginning of 2019.[59] Finally, forecasting horizon length is $T = 3$.

4.6.2 Non-cooperative regime

Since in the previous chapter we numerically verified the accuracy of procedures between 1959 to 2019, in this chapter we present the numerical solution between 2019 to 2100 and investigate the level of CO_2 concentration and capital stock.

Figure 4.1 shows the result of emitting under non-cooperative situation and we can observe the changes in capital stock K, and negative externalities g , during 2019-2100. As we can see, the capital stock starts to deplete quickly up to 2039 an then it is monotonically decreasing up to 2059 where finally reaches zero. In the absence of any cooperation/coalition, i.e., a completely non-cooperative situation, if we suppose that there is sufficient deposit of non-renewable energy, capital stock – which is used for the production of renewable energy – will be exhausted completely by 2059. It shows that in this situation policymakers prefer to rely only on non-renewable energy which leads to a dramatic increase in externalities instead of investing in capital stock. In this situation, CO_2 concentration reaches 1484 ppm which shows the temperature around 5.6°C above the pre-industrial level by 2100.

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[59] Global Trends in Renewable Energy Investment (2019)

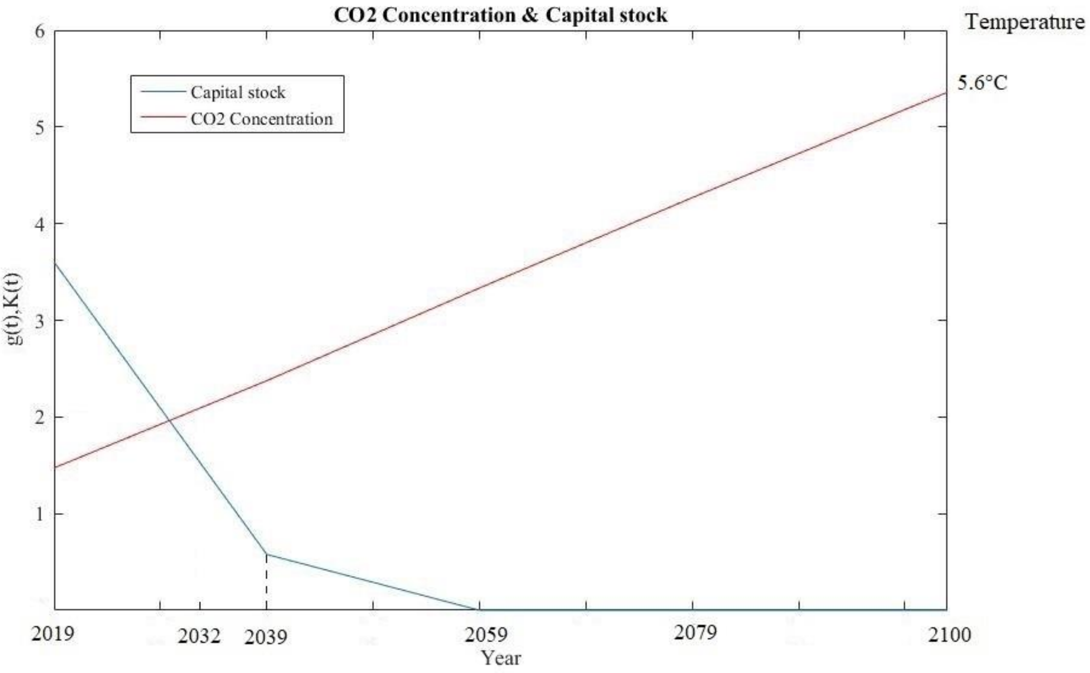

Fig.4.1 CO_2 concentration and stock of renewable energy (non-cooperative regime)

Comparing our results with other studies that considered this substitution, our results are in contrast with Van der Ploeg and Withagen (2012) where they show that until the government does not interfere, the suboptimal market solution uses too much of the fossil fuel source, but when the initial stock of the polluting resource is small, it is optimal to use only renewables resources.

Also, these results are in contrast to Greiner et al. (2014). According to their results, it is optimal to let the capital stock be exhausted completely and then, when fossil fuels tend to be depleted, the capital stock will be built up to a high level. But, our results do not show any increase in capital stock and no one tends to invest in the capital stock for renewable energy.

However, even if we suppose that this growth in capital stock will happen in the far future, for two important reasons we cannot expect that it would be useful:

1. As we can see, using non-renewable resources in a non-cooperative situation leads to a high level of negative externalities which is too much costly. So, in order to combat these negative externalities and damaging effects – which is included health impacts ecological loss and reduced productivity – a huge amount of capital stock will be required.

2. Even if we suppose that we can prepare that large amount of capital stock, we should consider the issue of "time'". According to our results, under the non-cooperative situation, CO_2 concentration increase rapidly so, if policymakers do not coordinate their actions sooner, we will pass the dangerous level of CO_2 concentration very fast.

It should be noted that also global data shows that governments should re-think and find a way to develop global investment in renewable energy more efficiently. In this context, we can refer to Bloomberg New Energy Finance (2019), which states that over the last years, global investment in renewable energy has been declined. for example, this investment in 2018 fell by 11.5%, and in the first half of 2019 compared to the same period in 2018 we can observe a 14% decline.

So, these results imply that despite of having two different energy resources, while there is no clear cooperation/coalition to reduce the emission, policymakers prefer to boost the economic activity instead of limiting the use of fossil fuels.

4.6.3 Cooperative regime

In the following, we want to assess the effect of cooperation on negative externalities and damaging effects. As we mentioned, the initial value of capital stock, in developing countries is set equal to $K_1(0) = 9.1$ and for developed countries as $K_2(0) = 7.7$ which are the normalized form of investment in renewable energy at the beginning of 2019 and interestingly this amount in developed countries is less than developing countries.

In figure 4.2, both capital stocks first raising up to 2039, and then they gradually decrease to the levels a little bit less than their initial points. This increase in the investment of the renewable energy sector can be justified with global environmental concerns and the immediate need to agree on an efficient way. In this situation, nations agree to reduce CO_2 emissions with a high degree of cooperation.

This increase in the capital stock of renewable energy, leads to a much lower level of CO_2 concentration during the time. In the beginning, we observe that CO_2 concentration increase slowly up to 2039 and then reaches its steady-state value, around 655 ppm as is the result of using renewable energy which has been built.

As we expected in the case of transition from non-renewable to renewable energy sector, under cooperation, we observe much lower level of CO_2 concentration i.e., 655 ppm compared with the results of chapter 3 i.e., 700 ppm, which rely only on reducing the use of fossil fuels.

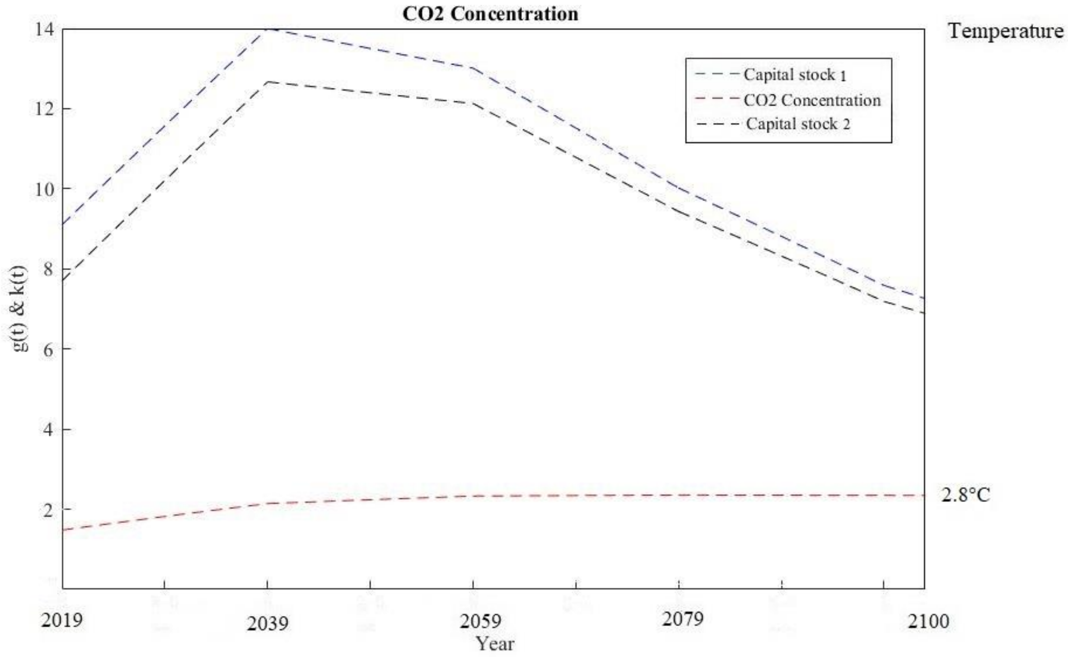

Fig.4.2 CO$_2$ concentration and stock of renewable energy (cooperative regime)

Comparing our results with the RICE model – as the first integrated assessment model which considered different policymakers and different regimes – in the RICE model, by 2100, we observe the level of CO$_2$ concentration around 754 ppm and 731 ppm for non-cooperative and cooperative regimes respectively, while our results predict these levels around 1484 ppm for non-cooperative policy and 655 ppm in the cooperative situation.

Although under non-cooperative regime the level of CO$_2$ concentration and consequently the temperature in our model is higher than RICE model, i.e., 5.6°C and 3°C respectively, but interestingly under cooperative regime where there is a high commitment of substituting non-renewable resources with renewable resources, our results show the level of CO$_2$ concentration and global mean temperature near to the expected level by RICE model, i.e., 655ppm compared to 731 ppm respectively.

Hence, results show that despite of having the renewable resources, since countries free-ride on the climate-change policies of other countries, they cut back their own efforts and we cannot expect to have a transition from non-renewable to renewable resources. But taking into account that cooperative regime, in the RICE model, “carbon tax” is considered as the climate policy, our results imply that if policymakers accept a high degree of cooperation in substituting non-renewable energy by renewable energy, we can expect that this climate policy leads to really good levels of CO_2 concentration and eventually temperature which are equal to the result when we consider the carbon tax as our climate policy.

Although, these results state that as we expected before, cooperation between countries/policymakers will lead to better conditions compare with the non-cooperative regime, but it should be mentioned that, this prediction requires a high degree of commitment.

4.7 Concluding remarks

In this paper, we extended the current NFNE method in order to investigate the efficiency of transition from non-renewable to renewable energy sector which is an additional instrument of cooperative policy. However, this extension in the economic framework requires an extension in the technique. So, we built two separate loops for each country i.e., one loop for CO_2 concentration and a new loop for capital stock. The second loop is supposed to work separately but simultaneously with the first loop.

This procedure is used to find optimal control problems and fixed points that players have no incentive to change that at each point of time.

Moreover, our economic model consists of two optimization problems each one includes two control variables as consumption and extraction rate of the non-renewable resources and we focused on the dynamic interaction between two policymakers instead of an economy to investigate the effect of different regimes on the level of CO_2 concentration and capital stock.

Results showed that despite having the renewable resources, since there is not an efficient cooperation between countries/policymakers, we cannot expect to have a suitable transition to non-polluting resources. But interestingly, if policymakers agree on a coalition and show a high degree of cooperation, we will reach really good results in CO_2 concentration and temperature by 2100 which also can be expected to be near the results when we consider carbon tax as our climate policy.

It is worth noting that still some other conditions such as level of development, different bargaining powers, and some uncertainties such as political situation and international relations, strongly affect the final results and they may change the result in different directions. In other words, these results are very sensitive concerning our assumptions and calibrations.

Chapter 5

Summary and Conclusions

This research has presented the use of Nonlinear model predictive control (NMPC) and NMPC Feedback Nash equilibrium (NFNE) for the prediction of CO_2 concentration level and temperature under different regimes by the year 2100. In this research, we focused on the effects of regimes and their results on global warming. With these techniques we can use a finite time horizon in order to forecast infinite horizon optimal trajectories. Also, there is the possibility to consider a game where players interact with each other and predict the dynamics of state variables and opponents' moves.

Furthermore, Considering the privileges of mentioned techniques and importance of global warming, we can enjoy the feature of receding horizon fashion and have more accurate predictions.

Although in the climate context, various international meetings and conferences have been held such as the meetings in Kyoto (1997), Copenhagen (2009) and Paris (2015) but still countries debate on the programs to control climate change and it seems that the global warming issue, requires more attention and collaborations of policymakers. As far as human activities, especially the burning of fossil fuels, play a significate role in global warming, forecasting the atmospheric CO_2 concentrations as the main reason for climate change is crucial in this field.

Hence, in the first and second chapters, we formally introduced the concept of NMPC and NFNE and described their advantages and differences with other approaches which make these techniques more prominent.

Then, in the third chapter, we used a canonical growth model augmented with externalities to household's welfare function to investigate the CO_2 concentration and consequently global mean temperature above pre-industrial level. We considered a common state variable as CO_2 concentration along with the control variable of using non-renewable resources. We analyzed how much would be the difference of CO_2 concentration levels if policymakers emit under different regimes, i.e., cooperation and non-cooperation. Expectedly, non-cooperative policies lead to a much higher amount of CO_2 concentration and eventually higher temperatures compare with the coordinated solution. However, we observed that even with an international cooperation but in the absence of effective incentives to use new technologies or to substitute non-renewable energy, we cannot expect to reach the desired CO_2 emission pathway by the end of our sample (the year 2100). Also, we investigated the effect of different time horizons which we called it policymakers' myopia. This result showed that optimization with the less myopic policymakers leads to a lower level of CO_2 concentration compared to the myopic ones. This means that in order to have a better picture of the future we should try to remove or decrease our deficiency in information and/or information processing. However, using NMPC and NFNE methods, we make decisions for the control of the next steps by looking at the problem on a shorter time horizon so, compared with the other infinite horizon models, agents need, less requirement of information when making decisions.

Finally, we consider the transition from polluting to the non-polluting energy sector as an instrument for the cooperative situation and an efficient way to combat global warming. We extended the state variables and set capital stock as a source of renewable energy for both countries/policymakers. But, this extension in the economic framework required an extension in the current NFNE method. Hence, we explained two separate loops as two different games for optimization problems. Results showed that despite having the renewable resources, if policymakers decide under non-cooperative policy, we cannot expect to have a transition between two resources. But interestingly, if policymakers emit under cooperation along with having non-polluting resources, we will reach really good results in CO_2 concentration and temperature by the year 2100.

Future Research

It should be mentioned that our results are very sensitive with respect to our assumptions and calibration which implies that any changes such as increasing the agents' information and information processing capacity in the future, my change the results. Another point is related to the time horizons. We assume the same time horizon for both policymakers however there is no obligation to have the same horizons for each player.

Another assumption is about the discount rate. As a controversial issue future research can concentrate on the impact of different discount rates and investigate the effect of different rates for different groups of countries.

Also, since bargaining powers by calculating the share of collective gains, can be considered as the incentives to join a climate agreement, the effect of unequal bargaining powers can be a topic for future research.

Appendix A

A.1 Discretization

In this section, we explain the basic discretization technique. Here, we want to describe how this technique presents the numerical solution in order to solve the optimal control problems in continuous-time using NMPC. For this purpose, we use the first step of semi–Lagrangian discretization technique which is considering the time.[60]

For using this approach, we consider optimal control problem in continuous time where $t \in \mathbb{R}_0^+$:

$$V(x_0) \coloneqq min \int_0^\infty \rho^{-\delta t} \, g\big(x(t), u(t)\big) dt$$

And

$$\frac{d}{dt} x(t) = f\big(x(t), u(t)\big), \quad x(0) = \ x_0 \in X \subseteq \mathbb{R}^n$$

the continuous-time optimal control problem is replaced by a first order discrete-time approximation given by:

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[60] See Grüne and Semmler (2004) for more details.

$V_h(x_0) \coloneqq \min J_h\,(x_0, u), \quad J_h(x, u) \coloneqq \sum_{t=0}^{\infty} \beta^k g_h\big(\tilde{x}(t), u(t)\big)$

Where $\beta = e^{-\delta h}$, $g_h(x, u) = h_g(x, u)$ and $\tilde{x}(t)$ is defined by the discrete dynamics

$\tilde{x}(0) = x_0, \qquad \tilde{x}(t+1) = \varphi\big(\tilde{x}(t), u(t)\big)$

$h > 0$, is the discretization time step and φ_h is a numerical approximation to the continuous-time solution at time h. It should be mentioned that in discrete-time optimal control problems, $h = 1$ and $\tilde{x}(t) = x(t)$.

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