Beamwidth Evaluation of Finite-Length 1-D Bidirectional Leaky-Wave Antennas

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***Abstract*—In this work we analyze the beamwidth properties of one-dimensional (1-D) bidirectional leaky-wave antennas (LWAs) when the antenna is pointing close to broadside. Existing formulas neglect the effect of the aperture truncation, which affects the exact value of the beamwidth for practical realizations of 1-D LWAs. Using a rigorous theoretical framework, we find a transcendental beamwidth equation, whose numerical solution furnishes the exact value of the beamwidth for finite-length 1-D bidirectional LWAs. Numerical results confirm that the aperture truncation considerably affects the beamwidth properties, especially for limited efficiencies.**

I. Introduction

One-dimensional (1-D) bidirectional leaky-wave antennas (LWAs) are a class of LWAs [1] for which radiation occurs from two identical leaky waves traveling along both the $\pm z$-axis. As opposed to 1-D *unidirectional* LWAs, the *bidirectional* ones can easily generate a broadside beam as a superposition of two beams pointing close to broadside, but in opposite directions [see Fig. 1(a)]. This is one of the main reasons for which 1-D bidirectional LWAs have attracted a lot of interest in the antenna community [1].

As is well known [1], the main radiating features (e.g., pointing angle, half-power beamwidth, etc.) can be determined from the knowledge of the phase and attenuation constants, β and α, respectively, of the relevant complex wavenumber of the dominant leaky wave $k_z = \beta - j\alpha$.

In the past, formulas for the radiating features of both unidirectional and bidirectional 1-D LWAs [1], [2], were derived assuming an *infinite* aperture. Only very recently, the effect of the aperture truncation has been accounted for in 1-D *unidirectional* LWAs pointing from broadside to ordinary endfire ($\beta = k_0$) [3], and beyond ordinary endfire ($\beta > k_0$) [4] (where $k_0 = 2\pi/\lambda$ is the free-space wavenumber, λ being the operating wavelength). In these works, approximate analytical formulas have been proposed to rigorously take into account the finiteness of the antenna structure.

Conversely, for 1-D *bidirectional* LWAs or 2-D LWAs the current formulas always assume an *infinite* aperture [2]. Here, we discuss the effects of the aperture truncation in 1-D *bidirectional* LWAs with *finite* apertures, for the relevant case of broadside radiation. Starting from a well-established theoretical framework, we derive an exact beamwidth equation whose numerical solution furnishes the exact value of the beamwidth. These results are then compared with the formulas for infinite apertures to show that, in some cases, the latter do not provide reliable results in terms of accuracy.

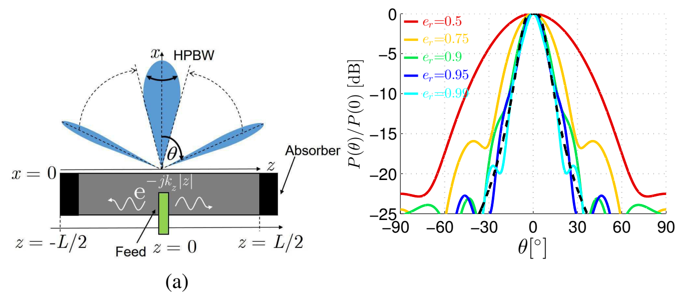


Fig. 1. (a) A sketch of a 1-D bidirectional LWA of length L. Depending on the operating frequency, the antenna can scan either two beams pointing off-broadside, or a single beam pointing at broadside. (b) Normalized pattern $\bar{P}(\theta)$ vs. θ for $e_r = 0.5$, 0.75, 0.90, 0.95, 0.99 (in solid colored lines). Results are compared with the infinite aperture case (in black dashed line).

In Section II, we briefly review the radiating properties of 1-D bidirectional LWAs. In Section III, we derive a beamwidth equation for the finite-length case and compare the results with the infinite-length case. Conclusions are drawn in Section IV.

II. Radiation Patterns of 1-D Bidirectional LWAs

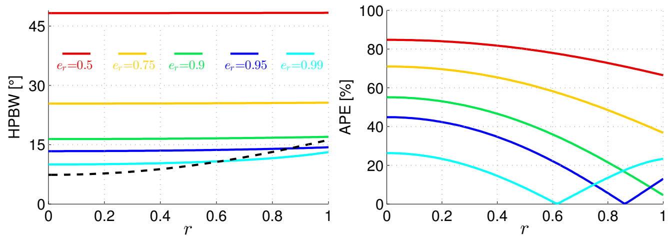
In this work we consider 1-D bidirectional LWAs as sketched in Fig. 1(a). In order to apply the following results to any 1-D bidirectional LWA, we do not discuss a specific LWA design. The analysis is then based on the space factor (SF) only, ignoring the element pattern and hence aperture polarization effects. Also, we assume that radiation from the structure is accurately described by a single dominant bidirectional leaky mode, so that the aperture-field distribution takes the form $e^{-jk_z|z|}$.

Under this assumption, the space factor of a bidirectional LWA of length L is [1]

$$\mathrm{SF} = \int_{-L/2}^{L/2} e^{-jk_z|z|} e^{jk_0 z \sin\theta} dz, \tag{1}$$

where θ is the angle from zenith (thus, broadside is at $\theta = 0°$).

When $L < \infty$, the formula for the space factor is provided in [1], after analytical evaluation of Eq. (1). Since we are



(a) (b)

Fig. 2. (a) HPBW vs. $r = \beta/\alpha$ for $e_r = 0.5, 0.75, 0.90, 0.95, 0.99$ (in solid colored lines). Results are compared with the infinite aperture case (in black dashed line). (b) APE vs. r for the same efficiency values as in (a).

interested in evaluating the half-power beamwdith (HPBW) of LWAs radiating at broadside, it is useful to define a normalized power distribution as $\bar{P}(\theta) = P(\theta)/P(0)$, where $P(\theta) = |\mathrm{SF}|^2$ and $P(0)$ is the power radiated at broadside. Starting from Eq. (1), an expression of $\bar{P}(\theta)$ is found as

$$\bar{P}(\theta) = \frac{|p|^2}{|p^2 - t^2|^2} \frac{\left|p - e^{-jp}(p\cos t + jt\sin t)\right|^2 e^a}{4|\sin(p/2)|^2}, \qquad (2)$$

where $p = k_z L/2$, $a = \alpha L/2$, and $t = l \sin\theta$, with $l = k_0 L/2$. When $L \to \infty$ the normalized power distribution is given by

$$\bar{P}_\infty(\theta) = |k_z|^2/|k_z^2 - k_0 \sin^2\theta|^2. \qquad (3)$$

From this formula, it has been derived in [2] that for $\beta \leq \alpha$ the antenna points at broadside, and for $\beta > \alpha$ the beam is split and has maxima on either side of broadside. The condition $\beta/\alpha = 1$ is thus known as the *splitting condition*.

At this point, it is worth comparing the patterns [see Fig. 1(b)] of an infinite-length 1-D bidirectional LWA (dashed black lines) with those of finite-length (solid colored lines) for different efficiency values, at the splitting condition. Since the radiation efficiency is related to $\hat{\alpha} = \alpha/k_0$ and L through

$$e_r = 1 - \exp(-2a) = 1 - \exp(-2\hat{\alpha}\pi L/\lambda), \qquad (4)$$

different radiation efficiencies correspond to different values of L for a fixed $\hat{\alpha}$. For $\hat{\alpha} = 0.1$ [the case shown in Fig. 1(b)], the considered radiation efficiencies $e_r = 0.5, 0.75, 0.90, 0.95, 0.99$ would approximately correspond to lengths $L/\lambda = 1, 2, 4, 5, 7$.

As is shown, the beamwidth considerably enlarges as the efficiency is reduced down to 0.5. Conversely, as the efficiency approaches the unity value, the pattern of the finite-case converges to that of the infinite case. However, even when the antenna is designed to radiate 0.9 of its power (the most common choice in LWA design), there is an appreciable difference in the HPBW, thus motivating the need for a more accurate formula.

III. Beamwidth Equation

In this Section we evaluate the HPBW of 1-D bidirectional LWAs radiating at broadside. Therefore, we will limit our analysis to ranges $0 < r \leq 1$, where $r = \beta/\alpha$. It is important to stress here that a finite-length 1-D bidirectional LWA may also radiate at broadside for $r > 1$ [5]. However, results will be confined here to $0 < r < 1$ for brevity. More results, including the case $r > 1$, will be presented at the conference.

The *one-sided* half-power beamwidth is defined as the angle $\theta_{3\mathrm{dB}}$ such that $\bar{P}(\theta_{3\mathrm{dB}}) = 1/2$. As shown in [2], for an infinite-length 1-D bidirectional LWA pointing at broadside ($r \leq 1$), HPBW $= 2\theta_{3\mathrm{dB}}$ is given by the following analytical formula

$$\mathrm{HPBW}_\infty = 2\sqrt{\hat{\beta}^2 - \hat{\alpha}^2 + \sqrt{2(\hat{\beta}^2 - \hat{\alpha}^2)^2 + 4\hat{\beta}^2\hat{\alpha}^2}}. \qquad (5)$$

Unfortunately, for finite-length 1-D bidirectional LWAs no analytical solutions are found, and one has to numerically solve for the roots of the following transcendental equation [obtained by setting to 1/2 the right-hand side of Eq. (2)]

$$\frac{|p|^2}{|p^2 - t_\mathrm{h}^2|^2} \frac{\left|p - e^{-jp}(p\cos t_\mathrm{h} + jt_\mathrm{h}\sin t_\mathrm{h})\right|^2 e^a}{4|\sin(p/2)|^2} = \frac{1}{2}, \qquad (6)$$

where $t_\mathrm{h} = l\sin(\mathrm{HPBW}/2)$. Note that t_h, which is a normalized beamwidth, only depends on r and e_r. CAD formulas for t_h have been obtained, and will be presented at the conference. The HPBW then follows directly from the definition of t_h.

In Fig. 2(a) results are shown for $\hat{\alpha} = 0.1$, solving Eq. (6) in the range $0 < r < 1$ for $e_r = 0.5, 0.75, 0.90, 0.95, 0.99$ (solid colored lines), and comparing the curves with that of the infinite case (dashed black line). As is shown, the HPBW is rather stable with respet to r, except for very high efficiencies. However, the beamwidth is considerably underestimated by the formulas for the infinite case as the efficiency reduces down to 0.9 and lower.

To better show the disagreement between the results for the infinite and finite cases, the absolute percent error (APE), calculated as $\mathrm{APE} = 100 \cdot |\mathrm{HPBW} - \mathrm{HPBW}_\infty|/|\mathrm{HPBW}|$, is shown in Fig. 2(b). Remarkably, the error is appreciably large, even for efficiencies greater than 0.9.

IV. Conclusion

In this work we have discussed the beamwidth properties of finite-length 1-D bidirectional LWAs and compared them with those of infinite-length 1-D bidirectional LWAs. This has confirmed the need for a more accurate tool for the beamwidth estimation for this class of LWAs. Numerical results are shown for some relevant cases, corroborating the utility of this investigation.

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