



# Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education

**Editors:** Uffe Thomas Jankvist, Marja van den Heuvel-Panhuizen, Michiel Veldhuis  
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