



ICMI STUDY 25

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EDITORS: HILDA BORKO & DESPINA POTARI

Teachers of Mathematics Working and Learning in Collaborative Groups



FEBRUARY 3-7, 2020

LISBON, PORTUGAL

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Teachers of Mathematics Working and Learning in Collaborative Groups

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Study Themes

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Theme A | Theoretical Perspectives on Studying Mathematics Teacher Collaboration

Theme Co-Chairs: João Pedro da Ponte, Portugal
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Theme B | Contexts, Forms and Outcomes of Mathematics Teacher Collaboration

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Theme C | Roles, Identities and Interactions of Various Participants in Mathematics Teacher Collaboration

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Theme D | Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration

Theme Co-Chairs: Ornella Robutti, Italy
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