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From internationalization to autarky: Mathematics in Rome between the two world wars

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Abstract. *The history of mathematics in Rome between the two world wars is characterized by a complete reversal of the enlightened international vision promoted by Guido Castelnuovo, Vito Volterra, Tullio Levi-Civita and Federigo Enriques, in favor of a short-sighted autarkic one, mainly personified by Francesco Severi.*

It is a story full of contradictions, in which internal trends to the community of Italian mathematicians intertwine inextricably with powerful social and political changes. This reversal of attitude compromises the international success of Italian mathematics and its capability to keep up with the big transformations which are changing the face of mathematics: topology, abstract algebra and abstract functional analysis, just to recall some of the fields where these changes are more radical.

1. Introduction

This work has not the goal to provide a complete overview on the development of mathematics in Rome between the two world wars but only to discuss some features of the *schools of mathematics* which are active in Rome in this period, in order to throw some light about the reasons for their difficult rise and sudden fall.

Since 1870 a program for making Rome the most important center for mathematics in Italy has been conceived by Luigi Cremona as part of the program of Quintino Sella for the *Rome of scientists*, the third Rome after the first Rome of emperors and the second Rome of Popes. This program, has been pursued struggling against many difficulties by Cremona, Castelnuovo, Volterra, Levi-Civita, Severi and Mauro Picone according to different angles. We claim that in 1920s the internationally recognized role of the mathematical schools in Rome testifies a substantial attainment of the goals of the program of Sella and Cremona while in 1930s the autarkic attitude epitomized by Severi destroys at the fundamentals the international character of the schools making them implode and drying the possible sources for their renewal.

We support our claims with ample reference to original documents: scientific journal articles, letters, newspaper articles and documents from archives. We put many efforts in providing english translation of material previously available only

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in Italian and that, in our opinion, is worth of international consideration. This justifies the abundance of citations and their amplitude. We hope that the effort of rendering the complexity of the original prose of many documents has not been unsuccessful. We think however that is worth to allow also easy access to all references in their original form. They can be downloaded at [58].

The paper is divided in three parts covering the following periods: 1870-1918; 1919-1931; 1931-1945. In each period, we will briefly review the characteristics of the main mathematical schools active in Rome in that period: protagonists, main research topics, international connections and difficulties to be faced.

Three appendices complete the work. The first one, with the list of international students visiting Rome between the two world war, is not complete but we think it provides a useful starting point for further research.

We acknowledge our debt to many authors, whose work we have tried to cite punctually. Their list is too long to be recalled here.

2. Mathematics in Rome between 1870 and 1918

On September 20th 1870, Italian troops enter in Rome putting an end to the secular power of Popes. On October 2nd, a plebiscite establishes the annexation of Rome and of the State of the Church to the Reign of Italy. The Italian Government, in particular the Minister of Finances Quintino Sella (1827–1884) and the Minister of Education Cesare Correnti (1815–1888), strongly believes that Rome should become a scientific center of excellence in order to oppose the secular cosmopolitanism of modern science to the ecumenicity of the Catholic Church.

A crucial battle of this war is that for the renewal of the educational system. According to Quintino Sella roman people should immediately feel that the Italians are able to provide a much more efficient and modern educational system than the old one and without any delay. Schools and Universities should open within a month of the annexation with new programs, new teachers and new infrastructures. The man in charge of this ambitious project is Francesco Brioschi (1824–1897), the leader of Italian mathematicians, the stronger and most influential Italian scientific community in those years.[1]

The first Italian Education Law, named after the Minister of Education, Count Gabrio Casati (1798–1873) was enacted in November 1859, entered into force in 1860 within the Reign of Sardinia and was progressively extended to all annexed territories. One of the innovations of the Casati Bill is the establishment of the Faculties of Science. For the Faculty of Rome, the Government strives to convince some of the most renowned scientists, and in particular some of the most famous mathematicians, to move to Rome. Giuseppe Battaglini (1826–1894), coming

[1]It was Brioschi that suggested to Enrico Betti (1823–1892) from Pisa and Angelo Genocchi (1817–1889) from Torino, a “mythical” journey to visit the most important mathematical centers of Europe (Paris, Berlin and Göttingen). He made it in 1859, with Betti and his brilliant pupil Felice Casorati (1835–1890). The (somewhat fictionalized) meaning of that journey is told by Volterra in his plenary lecture delivered at the International Congress of Mathematicians, held in Rome in 1908.

from the University of Naples and famous for his work in invariant theory and non-euclidean geometry is the first to accept, soon followed by Luigi Cremona (1830–1903) from the University of Bologna, two-time winner of the Steiner Prize for his outstanding work on projective algebraic geometry, and Eugenio Beltrami (1835–1900) from the University of Bologna, internationally famous for his fine works in mathematical physics and for his discoveries of the intrinsic and embedded models of hyperbolic geometry. The three participated actively in the resurgence movement (Risorgimento) and Cremona did not hesitate to take up arms in 1848 for defending Venezia during the first Italian war for independence.

From left to right: Luigi Cremona; Eugenio Beltrami; Giuseppe Battaglini.

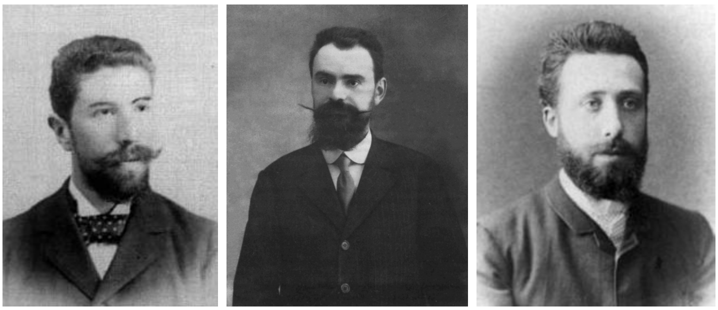
At the time of annexation, Rome is still a very provincial town, poorly connected with the rest of Italy and showing inadequate sanitation condition in many of its districts. Also its scientific milieu is depressing, in spite of all efforts put by Quintino Sella to revitalize it. Beltrami chooses to leave Rome after just one year, in spite of the vibrating opposition of Cremona, which clearly realizes how Beltrami's choice brings unexpected and formidable difficulties to the project to establish a prestigious school of Mathematics in Rome. Beltrami will come back to the capital of Italy in the last decade of the nineteenth century. Cremona, as director of the school of engineering, gets more and more involved in administrative tasks and in political affairs. Alongside, his creativity vein gets more and more exhausted. Moreover, his field of research, synthetic projective geometry is becoming rapidly obsolete and does no more occupy a central role in mathematical research in those years. Nevertheless Cremona's efforts to boost a high level school of projective geometry in Rome never fails. Many of his students give important contributions and prepare the way for the future flourishing of the Italian school of algebraic geometry. Among them, we recall: Ettore Caporali (1855–1886), Eugenio Bertini (1846–1933), Riccardo de Paolis (1854–1892), Giuseppe Veronese (1854–1917) and Giovan Battista Guccia (1855–1914).

Quintino Sella's project to make Rome a scientific center of excellence finds it hard to realize completely in the two decades following annexation, at least for mathematics, whose principal centers, in Italy, remain Pisa and Torino. The scientific vitality of both these centers is again closely connected to convinced choices of international opening: Betti's choice to send his best pupil, Ulisse Dini

(1845–1918), to study in Paris and, probably, Enrico D'Ovidio's[2] choice to encourage his young and brilliant student Corrado Segre (1863–1924) to follow Klein's[3] researches and begin an intense epistolary correspondence with him.

The development of mathematics in Rome gets new impetus in 1891 with the call of young (25 yo) Guido Castelnuovo (1865–1952) on the chair of Analytic and Projective Geometry. He is the one who strongly pursue Sella's dream ([43]). The next year, Federigo Enriques (1871–1946), comes to the capital of Italy on a one year grant and begins a fruitful collaboration with Castelnuovo. They quickly discover how to apply Segre's methods of hyperspatial projective geometry, already used for studying algebraic curves by Segre himself and by Castelnuovo, to the much more difficult and interesting case of algebraic surfaces.

Castelnuovo and Enriques get high international recognition. They further increase Italian mathematics in international acknowledgment, which was already high thanks to the works of Dini from Pisa, Salvatore Pincherle (1853–1936) from Bologna, Corrado Segre from Torino, and Vito Volterra (1860–1940) from Pisa until 1892 and later from Torino until 1900, when he finally moves to the University of Rome thanks to Castelnuovo's commitment, against the will of the majority of his colleagues, who would prefer to call Ulisse Dini on the chair of Mathematical Physics left vacant by Beltrami's death. Castelnuovo's vision of preferring a younger mathematician (15 years younger than Dini) at the top of his scientific career, produces the desired results, growing the prestige of mathematics in Rome. The association between Castelnuovo and Volterra gets its enshrining in the organization in Rome of the fourth International Congress of Mathematicians in 1908.



From left to right: Guido Castelnuovo; Federigo Enriques; Vito Volterra.

The city of Rome is acquiring in these years a scientific and cultural role of international importance. The seed sown by Sella in the immediate aftermath of annexation in order to foster that role does not bear its immediate fruits as expected. Only when Italy recovers from economic difficulties which plague the aftermath of its reunification and begins to take advantage of the "first economic miracle" of late nineteenth century the idea to make Rome an internationally

[2] Enrico D'Ovidio (1843–1933).
[3] Felix Klein (1849–1925).

renowned center for scientific culture accelerates sharply, attaining its climax with Nathan's mayoralty.[4]

From 1894 to 1914, for example, at least 27 international congresses on manifold subjects takes place in Rome. The organization of these congresses reflects a Secular and modernist culture which considers Science at the service of societies.

In his opening address to the International Congress of Mathematicians of 1908, Nathan connects explicitly his views with Sella's utopia of the "the Rome of Scientists" (after that of the Emperors and that of the Popes):

> *[with resurgence (Risorgimento) the people from Rome] turned their gaze to a new star rising in the firmament, whose serene light disclosed new ways, and the consciousness of a people, enlightened by Science, pointed at a third mission to the eternal city, between the nations. Gentlemen, on behalf of Science, of that new, intense and constant enlightenment, you are gathered here coming from all over the world.* [36], p. 26.

In order to consolidate the role acquired with the organization of 1908 Congress, Castelnuovo and Volterra devise a careful politics of Calls for Chairs in order to reinforce mathematical research in the principal fields. The two most brilliant young mathematicians are contacted: the mathematical-physicist Tullio Levi-Civita (1873–1941) from Padua and the analyst Eugenio Elia Levi (1883–1917) from Genoa. The first does not yet feel ready to abandon his family and university milieu, where he supported among his colleagues the call of his friend Francesco Severi (1879–1961), while the second meets his fate prematurely on October 28th 1917 at Subida (Cormons) during the first world war, after having shown his strong interest in moving to Rome, as we can read in his letter to Volterra, dated June 17th 1914, kept in Volterra's archives at the National Academy of Lincei.

> *My friend Vacca writes me that, apparently, the Faculty of Rome wants to have a professor of Analysis, [and suggested] whenever I wished to be called there, to write you, making me understand with it that you would be ready to support me. No attestation of esteem could be more complimentary for me than this one that comes from you; and I thank you heartily for it. If confirmed, because of the importance of the University where I would be called to teach, I could not appreciate more any other place - if not, possibly, my Turin.* [29].

Levi was a rising star in Italian Analysis and his tragic death is just one example among the many upheavals originated by first world war, a true watershed in the history of Europe, and of Italy in particular. All those changes, however, do not prevent that the politics of specific renewal of the academic staff of the Faculty of Science of the University of Rome continue to be inspired by the same principles that guided Castelnuovo and Volterra before the war.

[4]Ernesto Nathan (1845–1921) is major of Rome from November 1907 to December 1913.

Between summer and fall of 1918, in a climate of arduous but determined struggle to recover from the ruins of war, Levi-Civita's call to Rome finally goes through. The Paduan mathematician arrives in Rome at the peak of his scientific career in order to ensure to the Faculty the same momentum given by the call of Volterra 20 years before.

From left to right: Tullio Levi-Civita; Levi-Civita with Vito Volterra; Levi-Civita with Sommerfeld.

Castelnuovo, Levi-Civita and Volterra share the same desire to open the Faculty to international connections and open minded confrontation with the most dynamic realities in the world of mathematics and to call to Rome the most brilliant Italian mathematicians. They do not share however the same attitude towards German scientists. Volterra's choice is to support French ostracism against German speaking scientists and to exclude them from all scientific international authorities devised just after the war in order to coordinate scientific research and facilitate collaboration between the allied.

To Volterra's uncompromising attitude, Levi-Civita opposes a policy of reconciliation testified by his choice to support the organization of the International Congresses of Mechanics, open to the contribution of german scientists [3].

Another conflict between the two occurs about the choice for the tenures of Algebra and Analysis, left vacant since the death of Alberto Tonelli (1849–1920). Levi-Civita's proposal to call Enriques for Algebra and Severi for Analysis is vigorously opposed by Volterra who claims that the tenure of Analysis should be given to an analyst whose research is focused on the most recent and difficult problems and proposes Leonida Tonelli (1885–1946), the founder of the direct methods in the Calculus of Variations. At the end of a fierce academic discussion, Severi is called on the chair of Algebra and Giuseppe Bagnera (1865–1927) from Palermo, who expresses an unexpected desire to move to Rome, on that of Analysis. Enriques gets a provisional transfer from Bologna on the chair of "Matematiche Complementari" for the newly created degree course in Mathematics and Physics, designed to prepare high school teachers. He gets his definitive transfer to Rome two years later. Volterra tries again to call Tonelli in 1927, but again he finds the opposition of his colleagues. This time there are two chairs of mathematics which need to be covered: Analysis, left vacant by Bagnera at his death, and Descriptive Geometry, left vacant by Giulio Pittarelli (1852–1934) at his retirement. However the balance of power within the Faculty has changed and mathematicians does not receive the lion's share anymore. Therefore, only one chair in mathematics can be covered and the choice of the majority of the Faculty goes to the chair of Descriptive Geometry, a fundamental subject for the curriculum of engineers, for which is called Enrico Bompiani (1889–1975), a former student of Castelnuovo.

The decision not to call Tonelli blocks the development of Analysis in Rome until 1932, when Mauro Picone (1885–1977) is called.

From left to right: Francesco Severi; Giuseppe Bagnera; Leonida Tonelli.

3. Mathematics in Rome between 1919 and 1931

The commitment to make Rome one of the main international centers of mathematics grasps its ripest fruits between 1919 and 1931, and only for a little more than a decade. At the end of it, the autarkic delirium imposed by fascism to the entire nation heavily contributes to the sharp decline of the Italian school of mathematics that with great efforts and engagement established itself as one of the most vital in the international arena.

The American mathematician of Italian origin, Giancarlo Rota (1932–1999) remembers that Solomon Lefschetz (1884–1972) used to refer to Rome of the twenties as to the "Princeton of his time" [44], p. 17, because of the inspiring and cosmopolitan cultural atmosphere which attracts foreign students and visitors for collaborating with Volterra, Levi-Civita, Castelnuovo, Enriques e Severi. In appendix A we give a partial list of foreign mathematicians who spent a period of study at the University of Rome between the two world wars. It is an incomplete list, consisting of about fifty names, many of which appear in the list of biographies of famous mathematicians of *MacTutor History of mathematics archive* [37], because of the importance of their research. Evidence of the international role of the roman mathematical school during the twenties comes also from the records of the Rockefeller foundation [50] about the fellowships awarded for Rome. The Dutch mathematician Dirk Jan Struik (1894–2000), one of Rockefeller fellows in those years, remembers his experience with the following words:

> *In Rome there were other Rockefeller fellows, and we got fondly acquainted with Mandelbrojt and Zariski. Mandelbrojt was a pupil and a*

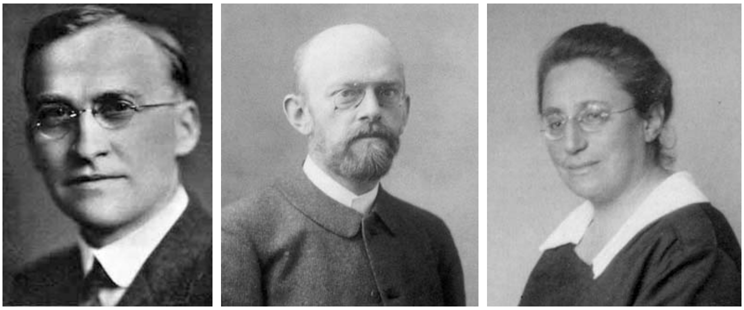
> *collaborator of Hadamard in Paris and was interested in analysis; Oscar Zariski was studying with algebraic geometers – both would become famous in their respective research fields, Mandelbrojt at the Sorbonne, Zariski at Harvard.* [51], p. 7.

A Rockefeller fellowship for Rome was a prestigious and coveted prize for a mathematician. Dirk Struik's brother Anton, teases him by writing

> *congratulations for your success in using Rockefeller's money to allow one of the enemies of their power to complete his education in a very aristocratic way, in Italy, in Rome, the "prix de Rome" for mathematicians! (...)* [51], p. 6.

The documents of Rockefeller Foundation offer valuable testimony about mathematics between the two world wars and testify the importance of the school of Rome. In a memorandum written by Georg David Birkhoff (1884–1944) for the International Education Board in 1926 we read

> *The conditions of Europe, from the point of view of mathematical research was very much better before the war than immediately after. There was a general lowering of morale, the reasons of which are obvious. At the present time, however, it may be said that the effects of the War have largely passed by, so that within ten more years a complete recovery may be looked for. As before the war the principal European mathematical centers are Paris, Gottingen and Rome. There are a number of lesser centers such as Zurich, Munich, Hamburg, Berlin, Amsterdam, Copenaghen, Stockholm, Oxford and Cambridge. (...) The numerical strength of the mathematical group and the power of tradition at Paris, Gottingen and Rome far transcends those at the other centers named. It seems to me, too, that Paris and Göttingen are of decidedly more importance than Rome. (...) The greatest mathematician of Europe is Hilbert at Gottingen, but he is nearly at the end of his career. Since the War, Hardy of Oxford has perhaps done the most spectacular work. In range and power Hadamard of Paris seems nearest to Hilbert. The principal leaders of European mathematics are: Volterra and Levi-Civita in Italy; Picard, Hadamard, Lebesgue and Borel in France; Hilbert, Landau, Heeke, Caratheodory in Germany; Brouwer in Holland; Weyl in Switzerland; H.Bohr in Denmark, and Hardy and Whittaker in Great Britain.* Birkhoff G. D., Report per l'International Education Board, September 1926, reproduced in [50], pages 265-271.



From left to right: George Birkhoff; David Hilbert; Emmy Amalie Noether.

Inaugurating a tradition of numerical evaluation of research, which continues nowadays with distorting - and sometimes pernicious - effects that are before the eyes of all who want to open them, Birkhoff assigns an international ranking to mathematical research giving the following scores: Germany 37, USA 26, France and Italy 22, England 14. Birkoff's ranking was based on assigning a score of one, two or three to the most representative mathematicians of each nation. Applying the same criterion to the most important centers of mathematics in Europe, Birkoff gets the following scores: Paris 20, Rome 12, Göttingen 11 [50] p. 51. From these premises we can easily deduce the scores that Birkhoff assigns to the mathematicians in Rome: Volterra 3, Levi-Civita 3, Severi 2, Enriques 2, Castelnuovo 2. The judgment of Birkhoff is very flattering towards Italian mathematics and in particular towards the mathematicians in Rome. However he subordinates the school of algebraic geometry of Castelnuovo, Enriques and Severi to that of mathematical physics, or rather, according to the terminology of the time, of "applied mathematics" of Volterra and Levi-Civita. This is also due to an evident bias of the American mathematician in favor of the fields which he controls better (among which is certainly not algebraic geometry) and to the doubts that are now insinuating themselves at an international level on the solidity of the foundations of the impressive building erected by Italian algebraic geometers. This judgement, in our opinion, has produced relevant consequences for the Italian school of algebraic geometry. In fact, the flow of international students in Rome supported by the Rockefeller scholarships is very unbalanced in favor of the researches of Volterra and Levi-Civita, clearly favored by the councilor Birkhoff. The visits of students interested in algebraic geometry are few and of less scientific importance. These, like Bartel Leendert Van der Waerden (1903–1996) and André Weil (1906–1998), are preferably addressed to Göttingen, without favoring the opening of an exchange channel between the German and the Italian reality even when, as in the case of Van der Waerden, the fellows themselves explicitly ask to complete their training in Rome. This lack of confidence in the possibility of a proper training in Italy for Rockefeller scholars interested in geometry and algebra may have influenced, and in our opinion has influenced, the perception by Roman algebraic geometers, Severi in particular, of a lack international esteem for the importance of the work done in Italy in the field of algebraic geometry, considered by many mathematicians a sort of esoteric practice for initiates. Against this negative judgement, according to Severi, it is necessary that Italian geometers defend themselves vigorously.

> *Most mathematicians who do not know Italian algebraic geometry and those who have managed to deal with its results being interested in certain collateral questions, consider our methods as something mysterious which can not to be handled safely apart from a small number of insiders; and they prefer to create or perfect other methods; they even sometimes believe that they should establish results that we have known for a long time with precision.* [45], p. 212.

To restore the prestige of the Italian school of algebraic geometry, it seems to Severi that it is not necessary to seek an international confrontation but that an autarkic scientific policy could and should be pursued, completely aligned with what the regime will follow with determination in the 1930s. Another point of possible influence of the politics of Rockefeller Foundation on the development of mathematics in Italy, in our opinion, can be seen in the choice by the International Board for Education to finance two important projects for mathematics in Europe; the Institute Henri Poincaré of Paris and the mathematics institute of Göttingen, while a similar initiative is not envisaged in favor of Rome, which could well aspire to host an international research center of applied mathematics or algebraic geometry. It is no coincidence that two such institutions will be set up in the 1930s: the National Institute of High Mathematics (Istituto Nazionale di Alta Matematica), headed by Severi; the National Institute for Calculus Applications (Istituto Nazionale per le Applicazioni del Calcolo), headed by Picone. Both the Institutes are characterized by a sharp nationalistic footprint, very different from the one we could have expected by institutes under International Board patronage.

In the rest of this section we will briefly review the characteristics of the main mathematical schools active in Rome in the 1920s: the protagonists, the research themes, the international relations and the difficulties they face in order to stay ahead.

3.1. The school of algebraic geometry

When Oskar Zariski (1899–1986) moves to Rome in 1921, the University of the Capital is considered the most important center of algebraic geometry in the world.

> *I had the great fortune of finding there on the faculty three great mathematicians, whose very names now symbolize classical algebraic geometry: G. Castelnuovo, F. Enriques, and F. Severi. Since even within the classical framework of algebraic geometry the algebraic background was clearly in evidence, it was inevitable that I should be attracted to that field.* [39] p.13.

Zariski does not hide his admiration for the stimulating climate he finds in Rome, which he does not hesitate to call a “geometric paradise”. Unfortunately, the relationships between the great masters of Italian algebraic geometry, in particular those between Severi and Enriques, are no longer cordial and this does not help to establish a climate of collaboration, capable of catalyzing the energies of the young mathematicians who intend to study in Rome in order to face and overcome the difficulties that risk to destroy the "paradise" glimpsed by Zariski. Despite the special appeal of the geometric approach to algebraic geometry, the crisis of the foundations is in fact evident.

> *Speaking of "geometric intuition", they pushed their way into the gray area between "proof" and "rigorous proof", on what would turn out to be an exciting but perilous journey. They used "whatever tools were at hand, whether algebro-geometric, transcendental or topological, coupled with a geometrical imagination that gave the subject a beauty to match that of the Italian scene".* [39] p. 13.

The crisis of the Italian way to algebraic geometry is experienced differently by its various protagonists. Castelnuovo is perhaps the first to fully realize the crisis. According to Zariski, as reported in [39] p. 25, Castelnuovo thought that "*the methods of the Italian School have reached a dead end and become inadequate for further progress in the field of algebraic geometry*" and encourages Zariski to study the work of Solomon Lefschetz, which introduces topological methods in the study of algebraic geometry. Zariski approaches these new methods with interest and manages to make use of them in important researches suggested by his Italian masters, but finding elsewhere the tools he seeks to give a rigorous foundation to Italian algebraic geometry, namely in the commutative algebra of Emmy Noether (1882–1935). Zariski regrets not having met them during his stay in Rome, where he could have used them to address the problems posed by his Italian masters: "*It was a pity that my Italian teachers never told me there was such a tremendous development of the algebra which is connected with algebraic geometry. I only discovered this much later, when I came to the United States*" [39] p. 26.

From left to right: Oskar Zariski; Solomon Lefschetz; Bartel Leendert Van der Waerden.

However, the "regret" does not concern only Zariski, but the entire sector of Italian algebraic geometry. The choice of not considering the language of commutative algebra developed by David Hilbert (1862–1943), by Emmy Noether, by

Hermann Weyl (1885–1955) and by the whole German school of Algebra means choosing to renounce to the instrument necessary to describe and control the intricate geometric situations that they are facing and which they cannot manage to solve with exclusively geometric methods. Significant in this regard is a letter dated 1934 by Fabio Conforto (1909–1954), perhaps the most brilliant of the Italian algebraic geometers of the new generation, to Enrico Bompiani, his research director, for his six-month stay in Germany (Göttingen and Berlin). The letter provides an important testimony, in perfect and probably independent harmony with what is said by Severi in the passage quoted above, of the incapacity or bad will of Italian geometers, to look out of their own garden, with the effect of letting others, (Zariski, Weil and van der Waerden), discover the right ways to progress. Conforto writes

> *I now come to the second order of studies, which is the most cultivated in Germany. I mean the order of studies, which I learned about through the lessons of Weyl and Miss Noether. I affirm that this kind of studies is the most cultivated in Germany not only on the basis of the examination of the scientific mileau of Göttingen, but also of all other university cities visited by me, as well as on the basis of the examination of the German scientific literature, most of which is dedicated to this address. It therefore seems reasonable to me to state that the main current of scientific studies in Germany is in this address, which is called "Modern Algebra". Trying to characterize this address in a few words, I will only say that it is an exposition of Algebra, considerably more abstract than the expositions used in earlier times in Germany and still used in other countries, an exposition which is based on the concept of field, defined by certain postulates, regardless of the nature of the elements making up the field itself. I believe that historically we can trace the origin of this address in the memory published by Mr. Steinitz in the year 1909 in the volume 137 of the newspaper of Crelle with the title: "Algebraische Theorie der Koerper". From this time on, the number of scholars has always increased, to the current situation, in which the subject represents the largest scientific topic and is practically taught in all universities. (Artin in Hamburg, Schur in Berlin, Van der Waerden in Leipzig, Miss Noether in Göttingen etc. etc.).*
>
> *I have been starting the study of "Modern Algebra" since the first months of my stay in Germany. This requires taking on a vocabulary and a system of notably complex and intricate designations, made even more difficult by the deliberately omission of every heuristic element in which German mathematicians usually present their writings.*
>
> *However, I did a fair amount of practice amidst this vast amount of definitions and concepts, the distinction of which is often very thin. As I gained the knowledge of terminology, however, it became increasingly clear to me that the new discipline could only have value from a methodical point of view. Its intrinsic content instead was simply*

> *found to be the content of what has always gone under the name of Higher Algebra. To take the case of Galois theory as an example, the results to be found in the expositions of "Modern Algebra" do not differ except for the form from those found in our treatise by Bianchi. What can at least certainly be stated is that the results added by German mathematicians are absolutely disproportionate to the difficulty created with the introduction of such a complicated phrasing. Furthermore, according to this mentality more critical and methodic than constructive, Algebra is cultivated in Germany only as a goal in itself and is indeed an indication that any new results that could suggest new research in other fields (I mean for example the relationship between the Galois theory and Picard-Vessiot's theory for linear differential equations) is completely left aside. However, if "Modern Algebra" does not represent an address of great originality, there are real and important problems in another field, which also, to tell the truth, is very much cultivated by German mathematicians. I intend to talk about the theory of numbers, which is a topic generally not cultivated in Italy, while in all of Germany and in Göttingen in particular, after Gauss and Riemann, it represents a tradition. This field, however, is in many parts necessarily not algebraic and makes use of transcendent means so as to be closely connected with the Theory of functions. Another part of the theory of numbers (and with this in particular the theory of algebraic numbers) has been deprived, in the exposition that is now given of it in Germany, of every transcendent element and has approached "Modern Algebra".*
>
> *With this I believe I have reported enough about the scientific mileau of Göttingen. If from an objective point of view it must be said that what is taught in Göttingen is not of great originality and that the truly higher courses are somewhat rare due to the very large number of pupils, from a subjective point of view I must however say that my staying there was very useful to me, since I was able to approach so many fields, of which one does not have the opportunity to hear about very frequently in Italy (for example the theory of finite groups in relation to Algebra), what contributed very much to expand my culture and to encourage the development of my ideas.* [16]

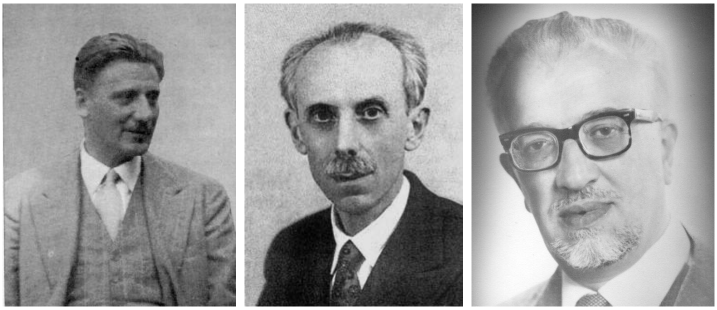
Although we have no information as to whether and how Conforto discussed his ideas on the value of modern algebra with his masters (Castelnuovo, Enriques and Severi), in fact his conclusion - "we can do without the new formal languages" - reflects the line pursued by all Italian geometers, though with occasional timid criticism. Italian algebraic geometry acquired a wide international reputation at the end of the nineteenth thanks to the results obtained by Castelnuovo and Enriques about the classification of algebraic surfaces.

The account made by Castelnuovo in his well-known report to the International Congress of Mathematicians of Bologna in 1928 highlights the need to build solid foundations. "*In the development of sciences the periods of intense work, where* *the eye is turned to the future and of the past it mainly interests what is good for daily research, alternate with periods of revision and adjustment*" [11] p. 191. According to Castlenuovo, algebraic geometry is in the 1920s in the second phase of its development, and it is necessary "*to look at the path traveled in the last fifty years (...) to take impetus and enthusiasm in view of future investigations*" [11] p. 191. At the end of the conference, Castelnuovo mentions the difficulties in dealing with algebraic varieties of higher dimension, and concludes by exhorting "*not to renounce to geometrical intuition, the only aid that has allowed so far to orientate oneself in this intricate territory*" [11] p. 201 in order not to "*extinguish the small flame that can guide us in the dark forest*" [11] p. 201.

Severi (and his school) are the sole to venture into this "dark forest", armed only with that tenuous flame of geometric intuition, elaborating the geometric theory of equivalence systems. We will see, however, in the next section, that the results of his adventure are not at all exciting and what Severi glimpses in his explorations, he is no longer able to communicate to the other mathematicians.

3.2. The school of Federigo Enriques

The prevailing scientific interest of Federigo Enriques concerns the classification of algebraic surfaces, on which it repeatedly returns in the period between the two wars to refine it, but Enriques's work cannot be limited to algebraic geometry and cannot be understood without reference to all its multiform interests. This is why we dedicate a separate paragraph to Enriques, although many themes are linked to those considered in the previous paragraph.



From left to right: Fabio Conforto; Oscar Chisini; Luigi Campedelli.

His philosophical, historical and pedagogical interests are inextricably intertwined with his purely mathematical ones, giving a unique character to his research and to the style of his writings. In his three great works, *the Geometric Theory of Functions and Algebraic Equations* [20], written in collaboration with Oscar Chisini (1889–1967), *Rational Surfaces* [21], written in collaboration with Fabio Conforto and *Algebraic Surfaces* [22], which resumes and extends a first 1934 edition, written in collaboration with Luigi Campedelli (1903–1978) and published posthumously by his last students, Alfredo Franchetta (1916–2011) and Giuseppe Pompilj (1913–1968), an ambitious cultural project is pursued which is characterized by "*a strongly historicist approach to the exposition of the disciplines treated.*

This is a very dynamic vision of the history of knowledge, which is not seen only as "literary erudition", meaning that it bears a mere "completion of chronological and bibliographic information"" [8] p. 244. For Enriques:

> *A dynamic view of science naturally leads to the terrain of history. The rigid distinction that is usually made between science and the history of science is based on the concept of history as pure literary erudition; understood in this way, history brings to the theory an extrinsic complement of chronological and bibliographical information. But the historical understanding of knowledge has a very different meaning, which seeks to discover in the possession [of historical understanding] the acquisition [of knowledge] and take advantage of that [possession of historical understanding] to clarify the development of ideas, and conceives this [knowledge] as extending beyond any provisionally reached term. Such a history becomes an integral part of science, and it has its role in the exposition of doctrines, although it would be useful to prune it - as much as possible - from too cumbersome wealth of quotations, which obscures the synthetic vision of progress in its broad lines. The reference to the past is not separated from the interest of the present, which only draws on the vision of a larger reality, and enlivens it by recreating the discovery.* [20] p. XI-XII.

This style of exposure involves a reworking of the categories of rigor and generality, which are declined according to a deep and personal critical vision, strongly original and divergent from the mainstream of contemporary mathematics. Enriques does not accept the model

> *Of a rational science logically ordered as a deductive theory, which must appear in all its parts, closed and perfect, and which, descending from the most general concepts to particular applications, rejects by itself the uncertain and changing suggestions of the concrete, all that is reminiscent of the dark history of research or discover new difficulties, breaking the harmony of the system.* [20] p. IX.

The outcome of this approach, in the concrete of his mathematical research, leads him to follow very different paths with respect to those followed by Severi. If the mathematician from Arezzo tries to shore up the building erected by Italian geometers by mixing any kind of material in his peculiar "geometric cement" which can help to "go always further", Enriques refrains from invading new fields (for example the one of algebraic varieties of dimension larger than two), and continues to refine, without any external contamination, its algebraic-geometric vision, by considering new classes of surface and new examples of curves without showing any interest in alternative methods.

> *He conceived the algebraic method as existing by himself, independently and external to us, regulated by a supreme law, which is the*

> *law of continuity, reflecting the analyticity of the entities under consideration. In trying to understand this world the main point is not so much to set an ideal of logical perfection; and even less is a matter of proceeding axiomatically, starting from postulates which are, in some way, of our choice. This could be done, Enriques used to say, in other parts of mathematics, such as, for example, in the theory of functions of real variable, where the entities to be studied are in some way determined by ourselves, and it is therefore possible, with some opportune limitations, to exclude certain objects or to let certain others enter our considerations. The algebraic world, on the other hand, exists by itself and the exclusion from it of certain entities, for example the exceptional ones, is impossible, because it would contrast the law of continuity. The exceptions must indeed be accepted and explained in the light of continuity itself. Therefore, understanding the algebraic world is not so much a question of correct deduction, but mainly and above all a question of "seeing".* [17].

It is therefore a vision of mathematics quite different from that which is commonly accepted. It is undoubtedly endowed with a great charm but is also, since "out of the chorus", quite difficult to share. This explains the relatively small number of Enriques students. Alongside the "strictly mathematical" ones, such as the already mentioned Chisini, Campedelli, Franchetta and Pompilj, one should also consider students of his history of mathematics school, such as Giorgio De Santillana (1902–1974) and Attilio Frajese (1902–1986).

The style of Enriques embodies the essence of the Italian road to algebraic geometry in its purest essence. However, while Enriques accepts the limits which this purely geometrical view puts on its applications and does not try to extend it to the study of higher dimensional varieties, Severi, who becomes the paladin of the school, stubbornly persists in trying to break those limits without changing its foundations.

3.3. The school of Volterra

As we have already had occasion to recall, Volterra is the best known Italian scientist abroad. In his scientific work we find the same broad scope that characterizes that of Levi-Civita and the same attention to applications to physical sciences, which place the two Italians, in the eyes of the international community, as the most prestigious "applied mathematicians" in Europe. In Volterra, research inspired by applications also extends to biology and economics. In his famous lecture of 1901, pronounced on the occasion of his call to the University of Rome, Volterra sketches a large programmatic plan for the application of mathematics to the problems raised by other disciplines, which he pursues with consistent commitment throughout his life.

> *The mathematician finds himself in possession of an admirable and precious instrument, created by the efforts accumulated over long peri-*

> *ods of centuries by the most acute minds and sublime minds that ever lived. He has, so to speak, the key that can open the door to many dark mysteries of the Universe, and a means to summarize in a few symbols a synthesis that embraces and connects vast and disparate results of different sciences. (...) It is around those sciences in which mathematics has only recently attempted to introduce its methods, biological and social sciences, that curiosity is more intense, since there is a strong desire to test if the classical methods, which have given such great results in the mechanical-physical sciences, are likely to be transported with equal success in the new and unexplored fields that open up before them.* [55]

Volterra is one of the fathers of functional analysis. In the introduction to his plenary conference, presented at the International Congress of Mathematicians of 1928 in Bologna, he clearly highlights the peculiarity of his approach to functional analysis, primarily aimed at applications.

> *The researches I started about 45 years ago about what I then called functions of lines, later called functionals, were based on the* principle of transition *from discontinuous to continuous (suggested by the analogous principle which is the basis of Integral Calculus) and, starting from the procedures of the Calculus of Variations, aimed at an extension of it. Indeed it can be better said, at a double extension of it, both because I gave the greatest possible generality to the way of making a quantity depend upon all values of a function in a given interval, (dependence which in the Calculus of Variations is limited to the quadrature process), and because I did not place any limitation on the nature of the problems in which the newly introduced elements appeared, problems which in the Calculus of Variations are restricted, instead, to those of maximum and minimum. The derivation of a function of line was the first concept I established, whence it arose that of differential of a function of line: if, however, I followed this path, it is certain that today it is better to follow another path, as Hadamard, Fréchet, Paul Lévy and others have observed. They have taken up the question again, started from the concept of differential and derived from it the concept of derivative.*
>
> *However, these fundamental concepts needed further clarification through a delicate and subtle analysis. In his beautiful work on functional analysis, Paul Lévy masterfully fulfilled this task by reproducing and extending what had been done before his work by Hadamard and by many others on this topic. I had not been able to complete this critical study, as my attention was immediately attracted in other directions: in fact the application of the new theoretical principles to newly arising problems, the possibilities of solving old unresolved problems, aroused immediately my curiosity and interest; so it was natural the tendency*

> *not to deepen immediately these more abstract parts of the research, postponing their study to a later moment.* [56] p.215.

The school of Volterra is set in the wake of the strong Italian tradition of applied mathematics (today we would say mathematical physics) that dates back to Betti and Beltrami. We use [6] to illustrate the roots of Volterra's research in the tradition of post-Risorgimento Italian mathematics, the extension of its school and its strong international vocation:[5]

> *To the problems of applied mathematics contributed actively in Italy Betti, who with his famous reciprocity theorem gave impulse to the wide and deep studies on elasticity; Beltrami, Dini, Bianchi, Arzela, Lauricella, Cerruti, Somigliana, Marcolango, Tedone, Burgatti, Almansi, Boggio; and particularly Volterra, who extended, in papers now classical, the method of Riemann to elastic vibrating bodies. Also to Volterra is due the creation of the theory of distortions of elastic multiply-connected bodies (a theory whose consequences were clearly verified by the experiments of Trabacchi and Corbino). Further in this group of works are to be mentioned the researches of Volterra on internal cyclical motions, of Almansi on the equilibrium of sands and of Signorini on reenforced concrete.*
>
> *The functional calculus as a creative branch of analysis arises with the work of Volterra. From physical problems he drew the idea of considering functions depending not only on the values of one or more parameters, but on other functions, that is, in geometrical terminology not functions of one or more points, but functions of lines, surfaces and so on; hence the name of functions of lines replaced more recently by that of functionals.*
>
> *It is impossible to refer in detail to the work of Volterra in this field, of which excellent expositions exist by Volterra himself and his school (Peres, P. Levy, Fantappié). It will suffice to remember the integral and integro-differential equations which constitute important classes of functional equations, and to associate with the name of Volterra those of Fredholm, Hilbert, Picard, and to remember the more general studies of E. H. Moore and Frechet on general analysis; and the list is necessarily too incomplete to give an idea of the enormous influence of the work of Volterra on all contemporary mathematical production. These studies of such a general and abstract character found their most concrete application in the physical field, from which they were born, and in mathematical economics. Volterra himself gave the mathematical theory of hereditary phenomena in which the present state of a system depends not only on the present circumstances but on all its history.*

[5]The paper by Bompiani from which we take the excerpt will be used again in the next section. This paper is taken from a lecture delivered at the Fourteenth Summer Meeting of the Mathematical Associati America at Providence, R. I., on Sept. 9, 1930.

> *Recently Volterra illustrated his theory of hereditary phenomena under considerations of energy: he showed that the work of external forces necessary to bring a system from a given state to a different state is always greater than the variation of a certain functional which depends exclusively on the present state of the system; and calculated the work dissipated by the external forces when a system returns to the initial condition*
>
> *To these considerations Volterra was led by his recent researches on mathematical biology, whose consequences have been verified by the exploration of the seas.*
>
> *To illustrate another field in which integral and integro-differential equations show ample possibilities for practical use, I shall recall that Evans., a pupil of Volterra, and Roos have initiated with these means the study of economical phenomena in the regime of monopoly, and that F. P. Cantelli has used them largely in questions of calculus of probabilities and mathematical statistics.*
>
> *Another branch of analysis is concerned with problems of the functional calculus: the calculus of variations whose first general treatment is due to Lagrange. Soon after his first essays Volterra had established the dependence of the calculus of variations on functional analysis. In fact, to the problems of maximum and minimum of a function correspond problems of maximum and minimum of a functional which constitute the very subject of the calculus of variations. The idea pointed out by Volterra was taken up again by Arzelà, who could not fully succeed because the functionals which presented themselves in the calculus of variations are not generally continuous. Tonelli, a pupil of Arzelà and Pincherle, was on the other hand completely successful. The Tonelli method is essentially based on the semicontinuity of such functionals, a concept analogous to that of semicontinuous functions given by Baire.* [6] pages 89-90.

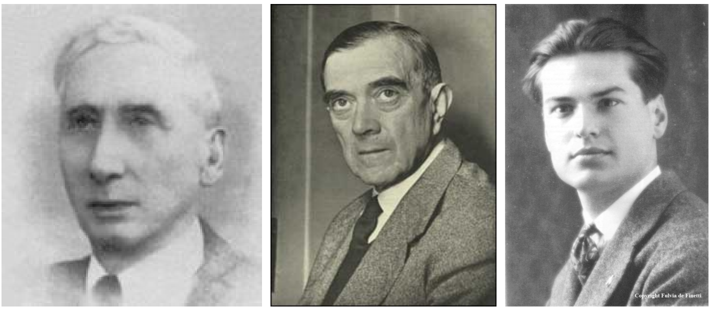
In the 1920s, students in mathematics who come to Rome to study with Volterra are fewer than those who come for Levi-Civita, but they are of great quality. We recall the names of André Weil and Szolem Mandelbrojt (1899–1983) among foreigners and that of Luigi Fantappié (1901–1956) among Italians, without forgetting those of well established mathematicians who continue to collaborate with Volterra, with whom they studied before the war, and under whose guidance they took the first steps in mathematical research, like for example Griffith Conrad Evans (1887–1973) and Joseph Pérès (1890–1962).

3.4. The school of Castelnuovo

Guido Castelnuovo, as we already said, is perhaps the first mathematician in the Italian school to realize the inadequacy of the purely geometric approach to Algebraic Geometry. At the beginning of the century he almost completely stops publishing research papers in this field but he continues to give his contribution to the training of mathematicians who come to Rome to perfect their preparation, as we are reminded, among others, by Zariski, who, in a letter to his wife Yole, writes the following words about the Bologna International Congress of 1928:

> *Today Castelnuovo gave a lecture; it was the best so far and it made a great impression on everyone, both for its content and for its elegant style. It was a real work of art. He did me the great honor of interrupting his lecture at a certain point ... in order to announce to the audience my upcoming communication to the Congress, in which, according to him, I have made an important step toward the solution of a fundamental problem which is still unresolved. ... Since there are hundreds of these brief reports given at the Congress, you can well understand how significant a sign of recognition this was.* [39] p. 45.

To evaluate the magisterium of Castelnuovo, in addition to the training of future geometers, we must also consider his activities for promoting new important fields of research: calculus of probabilities, statistics and its applications, mathematical economics and actuarial mathematics. His contributions in these fields is not only important on the scientific side, but also for his enlightened efforts to make these fields adequately cultivated in Rome, also because of their social importance. Stimulated by the organizational, scientific and educational work of Castelnuovo, important schools develop in Rome in: Calculus of Probabilities (Francesco Paolo Cantelli (1875–1966), Bruno de Finetti (1906–1985)); Statistics (Corrado Gini (1884 –1965)); mathematical economics (Luigi Amoroso (1886–1965)); actuarial mathematics (Cantelli and Paolo Medolaghi (1873–1950)). These schools quickly reach levels of absolute international prestige.



From left to right: Francesco Paolo Cantelli; Corrado Gini; Bruno de Finetti.

Here is an excerpt from the presentation of the School of Statistical and Actuarial Sciences in which Castelnuovo highlights some of the reasons that led him to broaden the horizons of his scientific commitment:

> *The developments of calculus of probabilities and its applications in recent decades has urged the need to create university chairs or groups of chairs even in countries where the tradition of such teachings was*

> *lacking. (...) In the school year 1914-15 I chose Calculus of Probability as the subject of my course in Higher Mathematics at the University of Rome. This teaching and the frequent conversations I had with my friend Prof. Cantelli at that time gave birth to my treatise, of which the first edition came out in 1919, and also led me to the conviction that such a course, to which students were very interested, should not be missing in our Faculty of Sciences. (...) It was easy, even with the support of the Hon. Gentile, (...) to set up a body of studies that could lead to a degree in Statistical and Actuarial Sciences [which] was established in 1927. It has a dual purpose:*
>
> 1. *a scientific purpose: to promote the study of calculus of probability and its applications to physical, biological and social sciences; to promote the applications of mathematics to social sciences in general (statistics, political economy);*
> 2. *a professional purpose: to provide the necessary preparation for statistical or actuarial offices in public or private administrations; to prepare actuaries (in the broadest sense of the word) which have a solid mathematical and economic culture.*
>
> *Among the courses developed in the last few years, or that will be held in the current year, we point out: Pure Economy (prof. Amoroso); Biometric Statistics (prof. Gini); Social Statistics (prof. Savorgnan); Techniques for social Insurances (Prof. Medolaghi, General Director of the Social Insurance Fund). (...) Outside of these university courses, the students of the School usually attend conferences of Insurance culture which, for a brilliant initiative of the National Insurance Institute, are held periodically at the Institute's headquarters, where current important problems are discussed about Statistics, Political Economy or Insurance.* [12] pages 109-110.

3.5. The school of Levi-Civita

Tullio Levi-Civita graduates in Padua in 1894 (at age 21!) with a thesis on the methods of tensor calculus developed by Gregorio Ricci Curbastro (1853–1925). The themes of his research are manifold and range from mechanics, where he provides essential contributions to the study of stability of motions, to the problem of three bodies and to the study of binary potentials; to differential geometry, where it enriches the tensorial methods of absolute differential calculus (of which he illustrates the importance for the formulation of general relativity) by introducing the notion of connection, that quickly becomes a fundamental tool for the study of Riemann varieties and, starting from the works of Weyl, also for the geometric formulation of gauge theories; hydrodynamics, where he indicates how to get the general integration of the irrotational plane motions endowed with a wake for each shape of the profile that causes them and develops the foundations of the general theory of channel waves.[6] His call to Rome in 1918 marks a turning point both for the Faculty of Science of the Capital of Italy and for Levi-Civita himself.

> *At the end of 1918 the Faculty of Science of Rome, aware of the high duties, that, in that tormented resumption of international scientific relations after the first great war, the traditions and the victory imposed on Italy, wanted to increase the prestige of its collegium of distinguished masters and called in its bosom Levi-Civita (...)*
>
> *Here in Rome, for another twenty years, he carried out his work as Maestro even more intensely and broadly, initiating and guiding a whole host of young mathematicians, who he guided along the ways he himself opened up, proposing to all, with inexhaustible imagination, new problems, lavishing (for all his students) with ample generosity and providing germs of ideas and directives. But his guidance was not compulsion, his advice was not imposition of particular methods or views. Understanding and respecting the inclinations and attitudes of each of his disciples, he supported them with assiduous and suggestive assistance, discreet and almost covert, to the point of arousing their first personal initiatives. Many of these young people were foreigners and, having returned to their countries, still carry the character of that clearly Italian speculative formation in their university teachings.* [1] p. 1141.

Beyond the themes already recalled, Levi-Civita adds new research interests in Rome, among which adiabatic invariants and the relativistic three bodies problem stand out prominently.

From left to right: Gheorghe Vranceanu; Dirk Struik; Marie-Louise Jacotin Dubreil.

In the 1920s, around Levi-Civita is produced an impressive amount of scientific research, fueled not only by his scientific stature but also by his great generosity toward students. The brief presentation made by Bompiani in [6] of his contributions in his Roman period, clearly highlights the international dimension of his research and the quality of his school, which does not limit itself to a mere reworking of the ideas of the master but is able to broaden them substantially:

[6] For a scientific and human profile of Levi-Civita, see [33].

I shall mention the researches of Levi-Civita and of his school (Bisconcini, Signorini, Armellini) on the problem of three bodies, which have prepared the ground for the result of Sundmann. To the classic mechanics belongs also the problem of motion of a body of variable mass, either by increase or by diminution. Levi-Civita has demonstrated, on the basis of two natural hypotheses, namely independence of effects and statistical isotropy, that the application of the principles of classical mechanics leads not to the equation of Lagrange (mass times acceleration equal to force) but to the theorem of quantity of motion (rate of change of the quantity of motion, or momentum, is equal to the force) which, as is known, remains still valid in the restricted relativity theory. The conclusions of Levi-Civita were applied by one of his pupils, Vranceanu, to the problem of two bodies of variable masses. This problem was already treated by Armellini with the use of higher analytical- methods: Levi-Civita has tackled again the problem using the hypothesis of adiabaticity (not necessary in the solution of Armellini) and has made important applications to astronomy and has succeeded in assigning the conditions of minimum energy also in the case of revolving bodies. This is a consequence of a theory developed by Levi-Civita of the adiabatic invariants of differential systems of Liouville, in which are framed the theorems of Gibbs, Hertz and Burgers. Geppert, under the guidance of Levi-Civita, has extended the theory of the adiabatic invariants to more general differential systems.

To applied mathematics belong the famous paper of Levi-Civita on waves, which has given a new impulse to the study of plane hydrodynamics (by a suitable use of analytic functions) and in which have amply participated Cisotti, Signorini, Colonnetti, Finzi, Pistolesi, Masotti and many others; the rigorous solution, given by Levi-Civita, of the problem of Airy on the progressive waves of permanent type in straight-channels and the extension of the theory to circular channels (as is necessary for the experimental verification) given by Geppert; the study of Masotti on the motions of a perfect liquid which take place in non-plane strata; and finally the extension of the theorem of Bernoulli to homogeneous viscous liquids which is due to Lelli. [6] pages 85-87.

4. Mathematics in Rome between 1931 and 1945

The stimulating cosmopolitan climate of Roman mathematics of the 1920s is rapidly deteriorating with the evolution of fascism. In 1931 the request for the oath of allegiance to the regime and the consequent removal from the University of Volterra, who refuses the oath,[7] establishes the onset of a very different climate from that recalled in the previous section.

[7] Only twelve professors refused the oath, see [4].

The architects of the oath: Francesco Severi; Benito Mussolini; Giovanni Gentile.

Here is a significant excerpt from the minutes of the meetings of the Faculty of Sciences, preserved in the Archives of the University of Rome.

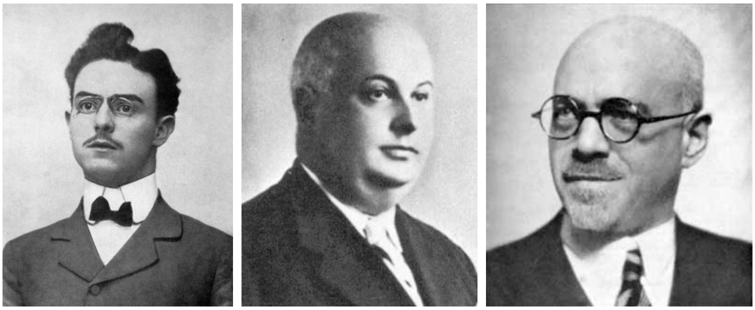
> *[The Dean communicates that the Min. of the National Education] wishes to be promptly informed of the conferences that are intended to be given by foreign professors in our universities since it is necessary to examine each time whether the act of homage to a foreigner is politically appropriate.* [2], vol 11, Meeting of November 25th, 1931.

The removal of Volterra is immediately ratified by the new dean of the Faculty, the chemist Nicola Parravano (1883–1938), who requests his replacement without any delay .

> *The Dean [Parravano] regretfully reports that a Ministerial statement, dated December 29th, communicates that prof. Vito Volterra was excused from the service from 1 January 1932, and invited the Faculty to propose arrangements for the teaching of mathematical physics.* [2], vol 11, Meeting of February 3rd, 1932.

The Faculty, with the favorable opinion of the mathematicians, decides to use the vacancy of a chair to call the analyst Mauro Picone from Napoli, who finds in Rome the necessary support to complete the realization of his project for a National Institute for Applications of Calculus, perhaps the first of this kind in the world.

Despite Picone's willingness to promote scientific exchanges with foreign mathematicians, both young and established, and a constant flow of students from countries siding with fascist Italy, the 1930s marks a continuous deterioration of scientific relations with democratic countries. The strengthening of contacts with Germany, Romania, Argentina, Brazil, Spain, Portugal and Japan is witnessed by the numerous cultural propaganda journeys they make in these countries mathematicians like Severi and Fantappié.



From left to right: Mauro Picone; Luigi Fantappié; Enrico Bompiani.

The personality who prevails in these years among mathematicians is that of Francesco Severi. The mathematician from Arezzo fights his tough battle to develop an original approach to overcome the difficulties that algebraic geometry is encountering, avoiding an in-depth comparison with the alternatives that are proposed and explored outside Italy, except from trying to bring them back in the wake of Italian tradition and, whenever possible, claiming priorities on the results obtained with these methods, and hiding their merits. Severi follows the same path that Fascist Italy decides to take after the economic sanctions imposed by the League of Nations for the war of aggression unleashed by the regime against Ethiopia in 1936, that is the road of autarky in the field of mathematics.

4.1. Mathematics and autarky

The first meeting of the Italian Mathematical Union (UMI) takes place in 1937 - a good 15 years after its foundation (1922) - the year after the 1936 International Congress of Mathematicians in Oslo, to which Italy does not participate "because the Norway was a country which applied sanctions", that is, a country that shares the sanctions imposed on Italy by the League of Nations because of the fascist colonial adventure in East Africa. Italian mathematicians are expected to participate to the International Congresses of 1940 (planned in the United States).[8] The International Congress of 1940 is not held however due to the outbreak of the Second World War. But if it had taken place, Guido Fubini (1879-1943), exile at Princeton, admonishes the Italian mathematicians that they would not be well received by the American mathematical community because of their behavior of servile acquiescence to the racial laws of Fascism.[9] Therefore Italian mathematicians meet at a Conference only between domestic walls, the first time in 1937 and the second time in 1940. And in this second congress they accept to exclude their "Jewish" colleagues, removed from schools and universities with the infamous decrees of the autumn of 1938.

[8] The only international mathematical congress held in Italy during fascism is the one organized by Bompiani and Severi on behalf of the UMI of 1942 with the mathematicians of the "fascistized" countries. The proceedings are published in 1945. The reports received for the Volta conference, strongly supported by Severi, and which does not take place due to the outbreak of the Second World War, were published in 1943, see [9].

[9] Letter of Guido Fubini to Mauro Picone, January 31st, 1940, see [24].

At the second National Congress of the UMI, of which Severi is the undisputed dominus, strong of its felucca of "academic of Italy", the Italian mathematicians listen and applaude the Minister of National Education Giuseppe Bottai (1895–1959), who pompously affirms the "Italian primacy"

> *in algebraic geometry, in Calculus of Variations, in projective-differential geometry; its leading position in the theory of functions, of differential equations, of algebras, of relativity, of thermoelastic transformations, in the studies of probability and actuarial calculus, in the history of mathematics.* [7], p. 5.

But do these "primacies" really exist? Despite the self-congratulatory climate of this congress, Italian mathematics does not enjoy good health in 1938. The ground lost to the more dynamic nations and the weakened contacts with the most original foreign mathematicians has carved a rift that is difficult to heal. The main protagonists of the research lines for which Bottai claimed "Italian excellence" are: Severi (algebraic geometry), Tonelli (calculus of variations), Bompiani (projective-differential geometry), Picone (differential equations), Gaetano Scorza (1876–1939) (theory of algebras), Antonio Signorini (1888–1963) (thermoelastic transformations) and de Finetti (probability). In 1938, all these protagonists, with the sole exception of Leonida Tonelli, who will be called a few months later, are teaching at the University of Rome, from which Levi-Civita and Enriques have just been thrown out as a result of the infamous fascist laws on the race.

Two of the three sectors for which Bottai affirms the primacy "over all other nations", algebraic geometry and the calculus of variations, share, in Italy, the difficulties of keeping up with the new international ferments. The Italian school appears rigid and plastered and the leaders, Severi above all but, in some ways also, Tonelli, "*the last illustrious representative of the Italian school of real analysis that strongly believes in its rich specificities, and has no time and eyes except its territories*" ([25], p. 166) are overshadowed by nationalist rhetoric and do not urge young people to critically confront themselves with the novelties coming from abroad. The situation, as we have already mentioned, appears more dynamic in Picone's school, thanks also to the maestro's greater openness to the novelties and to the scientific profile of his most brilliant pupil, Renato Caccioppoli (1904–1959). Caccioppoli knows and appreciates modern functional analysis and decisively takes the road of renewal that will allow Italian analysis to resume, after the war, his important and internationally recognized role in the fields of calculus of variations, geometric measure theory and partial differential equations.

From left to right: Renato Caccioppoli (student of Picone); Beniamino Segre (student of Severi); Enzo Martinelli (Student of Severi).

In the field of algebraic geometry, on the other hand, the suffocating presence of Severi does not grant his students an autonomy comparable to that granted by Picone to Caccioppoli and his best students. Zariski is the only young "Italian" geometer who manage to get out of the shoals in which the school is stuck, but he has to do it away from Italy. Next generation Italian algebraic geometry scholars will follow the same path as Zariski and go to the United States to learn the techniques necessary to face the problems that Italian algebraic geometry has posed but has not been able to solve.

In the thirties, the mathematicians who embraced fascism, Severi, Picone and Bompiani, contribute to build an official image of Italian mathematics in which Jewish mathematicians, who have given so much prestige to Rome and Italy, are less and less tolerated and finally openly hindered.

Volterra, while continuing to work thanks to his international contacts - publishes with Pérès the Théorie générale des fonctionnelles in 1936, not in Italy but in Paris - is removed from the University and the Accademia dei Lincei and increasingly becomes an alien body to Italian mathematics. His work on the applications of mathematics to biology and hereditary phenomena, to which he does not stop to devote himself with passion and competence, continues to be appreciated abroad but he no longer finds the opportunities to advance his research and foster his school in Italy. Volterra is forced to publish his work in English and French journals.

The regime tries also to put the gag on Levi-Civita. He is no longer called to be a member of the examining boards for professorships and his magisterium for training foreign students becomes less fruitful when the Rockefeller scholarship program is suspended.

It also seems that Severi intends to use the racial laws to settle accounts with his ancient Master Enriques. Severi, in order to make his triumph complete, not only replaces Enriques in the teaching of the course of history of mathematics, but also in the management of the Science History seminar, although he never showed a true interest in the History of Mathematics before. Severi does not even find words of private or public solidarity toward Levi-Civita, his great friend at Padua and tutelary deity of his academic career, who has fought so much for his call to Rome in 1921. The aryanisation process does not allow for derogations. This is the aberrant meaning of the infamous vote of the UMI scientific commission of

December 10th, 1938:

> *The Italian mathematical school, which has acquired vast resonance throughout the scientific world, is almost entirely the creation of scientists of the Italic race (aryan). (...) Even after the elimination of some followers of the Jewish race, it counts scientists who, by number and quality, are sufficient to maintain the tone of Italian mathematics, and it has masters who, with their intense work of scientific proselytism, assure the Nation elements which are worth of covering all the necessary professorships [left vacant at Universities].* [15]

The abject distortion of the contribution of Italian mathematicians of Jewish origin perpetrated by shameless scholars like Severi is, in our opinion, best illustrated by simply juxtaposing the following quotations about Levi-Civita contribution to differential geometry.

The first, by Severi

> *A completely different direction, which also originated in Italy and gave rise to vast international repercussions in geometry, mechanics, and modern physics, is the one prepared by the popular work of Luigi Bianchi and derived essentially from the monumental creation of the absolute differential calculation of Ricci Curbastro, whence, after all, derive general relativity and the ingenious notion of parallelism in Riemann's space due to Levi-Civita, a disciple of Ricci Curbastro, and immediately illuminated, in its intrinsic geometric meaning, by Severi.* [48], p.3.

The second by Élie Cartan

> *It was reserved to Levi-Civita to bring a last improvement [to differential geometry] through the discovery of the concept of parallel transport in 1917. By making the fundamental notions of absolute differential calculus more intuitive, he put a theory, hitherto purely analytical, under the domain of Geometry. This resulted in profound repercussions on the development of Geometry itself, to which the discovery of Levi-Civita [started] a new development, comparable in size to that [started] by Klein nearly half a century before.* [10], p. 234.

In the rest of this section we briefly review the principal characteristics of the main mathematical schools in Rome in the 1930s, similarly to what we have done in the previous section.

4.2. The school of Levi-Civita

The Paduan mathematician continues to receive numerous invitations from prestigious foreign universities, for example for spending an entire semester at Princeton in 1936, to work with Einstein, but his international activity is reduced and hardly tolerated by the regime, more interested in propaganda than in scientific relations.

His visits abroad often embarrass the local Italian consuls because of his unwelcome appreciation of foreign school systems or because of the absolute reluctance to make propaganda for the regime, as we read in an Information note of September 20th, 1937 from Talamo Atenolfi, Marchese di Castelnuovo, the Italian ambassador in Lima and published in [32] p. 133.

> *I would like to point out that although the aforementioned professor has carried out a commendable activity during his time in Lima, it would be more appropriate, for the purposes of our propaganda in this country, to come up with less strictly technical elements but more suitable for spreading our thinking in countries like this, with a low cultural level.* [52]

In the 1930s, Levi-Civita's research focuses mainly on the of n-bodies relativistic problem, which still remains one of the most interesting and stimulating line of research among those followed in Rome in those years. The famous French mathematical physicist André Lichnerowicz (1915 – 1998), after recalling that

> *we owe to Tullio Levi-Civita to have created or developed many of the principal instruments that have enabled either the birth or the intelligence and exploitation of this theory.* [31], p. 127.

continues with the following words

> *With his memoir titled "the relativistic problem of several bodies" published in 1937 in the American Journal of Mathematics, Levi-Civita pioneered and paved the way for a series of research of particular importance, due mainly to Einstein, Infeld, Hoffman on the one hand, and Fock on the other, which bore fruit well into the 1950s.* [31], p. 127.

The last years of Levi-Civita are tragically marred by the racial laws that destroy his research, his school, his professional activity, his international travels, in a word, his life. His physical death, which takes place in Rome on December 29, 1941 (of heartbreak, literally), follows shortly after his civil death to which the regime condemns him.

From left to right: Albert Einstein; Herman Weyl; Vladimir Aleksandrovich Fock.

In a letter to the Swedish physicist Carl Wilhelm Oseen (1879–1944) who, on behalf of the specific committee of the Swedish Academy of Sciences, invites him to propose names for the Nobel prize for physics of 1939, he replies with infinite sadness: " *Because of the anti-Semitic campaign that rages here, I no longer have contacts with the Italian academic world.*" [30].

4.3. The school of Severi and the National Institute of High Mathematics

Since the 1930s, Severi has been making his greatest effort to reaffirm the ability of Italian algebraic geometry to deal with its foundational problems and address further developments making use of its geometric methods and the language developed at the beginning of the century. This program begins in 1932, with two ponderous works appearing in the Helvetic Mathematical Commentaries and in the Memoirs of the Italian Academy, in which the concept of equivalence series is introduced [8]). However this effort, in which the mathematician from Arezzo tries to involve his disciples, appears completely inadequate. It is made possible and even favored by a political context that welcomes the biased distortions of assessments of what is happening in the world of mathematics for reasons of pure nationalistic propaganda.

At the first congress of the Italian Mathematical Union (Florence, 1-3 April 1937), Severi takes stock of the "autarkic program" for the renewal of algebraic geometry, begun in 1932, and outlines its ambitious perspectives with the following words:

> *In 1932 I began to consider simultaneous couples, triples, etc. of rational functions of the point of an algebraic variety; while previously one only single rational functions were studied.*
>
> *It is evident that the list of invariants of the varieties, with respect to birational transformations, is thus enlarged in a remarkable way. Why then did it take so long to consider such essential problems in a branch of geometry whose origins are linked to Riemann and therefore date back almost a century ago?*
>
> *In my work on the new theory I have indicated some reasons for this delay. I myself have reflected on these problems almost from the beginning of my scientific life and yet I have been able to achieve the goal only after thirty years of reflection.* [46] p. 58.

He adds moreover,

> *I hope that the above is enough to give a precise idea of the fundamental facts of the new theory, which raises many high and important problems and provides new tools to tackle even the oldest problems remained unsolved in algebraic geometry (e.g. the conditions of rationality of a variety). I hope our young mathematicians turn to these problems with a tenacious will and with faith to succeed; they need to*

> *do it to preserve Italy's primacy in a magnificent branch of geometry.* [46] p. 68.

The theory of intersection and of equivalence on an algebraic variety proposed by Severi serves as a stimulus to replace geometrical methods with algebraic ones, in order to make effective the often confused and contradictory intuitions of his proponent. Among the mathematicians who develops the algebraic language necessary to address these issues, we mention Zariski, Weil and Van der Waerden, who have all had deep contacts with Italian geometers but have not let themselves be seduced by their methods and have searched in commutative algebra the key for a rigorous foundation of the concepts and insights of the Italian school. A student of Zariski, Pierre Samuel (1921–2009), who in his doctoral thesis (1951) lays the foundations of modern Intersection Theory on a variety, states that the idea of many of his proofs develops "*following closely the geometric reasoning of F. Severi, but insists on some points of more algebraic nature left aside by this Author*" [8], p. 215. The recognition of the importance of the work of Severi by Samuel does not correspond to a recognition by Severi of the validity of the algebraic methods, as is evident from the famous controversy at the International Congress of Mathematicians held in Amsterdam in 1954, during which Samuel, Weil and Van der Waerden strongly criticize the conference given by Severi.[10]

An echo of the contents of that controversy emerges from Severi's letter to André Weil, which we reproduce in full in the appendix, as an important testimony of the stubborn and in some way heroic, but desperate, battle that Severi insists on fighting and that Weil does not even seem to be able to understand or take seriously. The letters were published in [5] and we think it is useful to provide an English translation of them.

From the debate of the Congress of Amsterdam emerges, from Van der Waerden and Samuel, the proposal of a modification of Severi definition of intersection of varieties, which is finally published in [54]. Despite the criticism of Severi's statements, the point of view of the Dutch mathematician resumes, after almost forty years, that of the Italian mathematician with full recognition of his intuitions: "*It seems to me that Severi's theory is a highly important generalization of the classical theory of linear systems of cycles*" [54], p. 256.

We conclude, with Ciliberto and Sallent [14], that Severi's contribution to algebraic geometry, after World War II, is above all a contribution of prophetic insights but it does not provide any definitive result. This does not affect the importance of the work of this complex character, but strongly limits, in our opinion, the judgment on the school that Severi created in defense of outdated and unproductive points of view, theorizing his ability to "do by himself" as Benito

[10] At the risk of appearing schematic we believe that, reduced to the bone, the genesis of the controversy between Severi and his critics is all in the way of conceiving the mathematical objects under consideration. The geometric vision is always, in some way, constructive. For example, a complicated singularity of an algebraic surface must always be seen as a degeneration of a family of simpler surfaces. The algebraic vision instead introduces the operations necessary to construct an "automatic calculation" that produces the desired results without having to check their geometric meaning step by step.

Mussolini (1883–1945) theorized for fascist Italy.

The creation of the National Institute of High Mathematics deserves a separate discussion, parallel to the one we will reserve in the next section to the National Institute for the Applications of Picone. The creation of the two institutions certainly represents important achievements for Italy. Both have accompanied and helped the difficult recovery of Italian mathematics after the war, but, especially the second, gives a late and inadequate response to the growing isolation of Italian mathematics, caused also by the blind ambitions of its founder himself, who presents the two initiatives with the following words:

> *I repeatedly mentioned the National Institute for the Applications of the Calculation of the Research Council, directed by Picone, and the Institute for High Mathematics, which I presided over. They are original institutions of the Regime, the first of which makes excellent services to the Army, to industries, to scientific laboratories and establishes continuous and fruitful relations between pure mathematics and applications; while the latter, while cooperating with the former in the solutions of high-science problems claimed by applications, has the primary purpose of promoting research in the new branches of mathematics, of maintaining contact with science and with foreign scientists, of preserving high our mathematical prestige in the world.*
>
> *I believe that Italian science can be satisfied with what has been achieved so far in the two Institutes, that foreigners look at with growing interest and that they begin to imitate.* [48], pp. 34-35.

For Severi, the Institute for High Mathematics is the place for the revival of Italian mathematics.[11]

[11] Beside his commitment in research, his duties as a member of Academia d'Italia and his role in the building of Istituto Nazionale di Alta Matematica, Severi, as we said already, is the promoter of two important International Congresses, modeled after the International Congresses of Mathematicians from which Italy was excluded in 1936: the Volta Congress of 1939 and the International Congress of 1942. The first does not take place because of the outbreak of World War II, but Severi collects and publishes the text of most of the scheduled conferences in [49]. See also [9]. For the latter, actually a Congress of Mathematicians from Fascists and few Neutral Countries, the Proceedings [28] are edited in 1945. The principles that inspire Severi in the organization of both these Congresses, are the same which we have already considered and discussed talking about the birth of the Institute of High Mathematics. Severi understands the need to open Italian Mathematics to the new trends coming from abroad, but is firmly convinced that these novelties can easily be mastered and implanted in organisms, especially that of Algebraic Geometry, whose general structure is well established and does not need substantial modifications or revisions.

Inauguration of the Institute of High Mathematics: Severi with Mussolini and Bottai.

We report a significant excerpt from the general conference of the mathematician from Arezzo at the second congress of the UMI, in which he clarifies his project.

> *The aims of the Institute for High Mathematics are clearly indicated in its constitutive law: development of the teaching of (higher) mathematics; coordination of the national mathematical movement with the foreign one; organization of an updated bibliography of worldwide mathematical literature; dissemination of the most important directions of national thought; link between high mathematics research and the sciences that benefit from mathematical tools; overview of the progress of our science in history and in the general philosophical movement; collaboration with the National Institute for Applications of Calculus of the National Research Council in all matters concerning technical problems, most directly related to experimental sciences and technical and autarkic applications.* [47], p. 27.

4.4. The school of Picone and the National Institute for Applications of Calculus

Mauro Picone moves to Rome after the expulsion of Volterra because of his refusal to take the oath of allegiance to the regime. Levi-Civita in the discussion in the Faculty concerning the transfer of Picone strives to obtain a sign of recognition of the Faculty in respect of Volterra for his thirty years of balanced and enlightened activity profused without reserve as a member of the Faculty in behalf of Science. The dean, the aforementioned fascist chemist Nicola Parravano, totally ignores the request. Levi-Civita asks for a pause for reflection for suggesting to the Faculty at least three possible candidates, worthy of the Faculty itself. His proposal is to accompany the name of Picone with those of Leonida Tonelli and Guido Fubini. Also this request is disregarded and again, for the third time, the Faculty blocks the road to Tonelli and votes compact for the call of Picone.[12]

[12] Political reasons contributed probably to this choice. Tonelli sign "Manifesto Croce" and was not considered a fascist. Also the choice to make him the director of Scuola Normale superiore di Pisa, sponsored by Gentile, was contrasted for this reason.

Picone graduated from the Normal school in Pisa in 1907 with Dini, in the same period in which Eugenio Elia Levi himself studied in Pisa. Picone always considers Levi as his second master. During the war, after a period dedicated to research in abstract analysis, the Sicilian mathematician, organizes the work of a small computer center dedicated to improving shooting tables, and develops his great interest in applications. At the resumption of civil life, after a first call to Catania and a brief return to Pisa on the chair that had been of his teacher Dini, he arrives in Naples where he recreates a small computer center and is lucky enough to meet a young mathematician of absolute value, immediately recognized by him: Renato Caccioppoli. This is the start of a thriving school of analysis that produce dozens of excellent researchers. Picone will always show a great flair for recognizing mathematical talent and a great ability to value inclinations properly. Picone's students will always receive generous and up-to-date indications from him, about the most fertile research to undertake and the better opportunities for their career development. One of Picone's characteristics is his openness to the most recent developments in analysis, and his interest in abstract functional analysis, as cultivated by Polish mathematicians, with whom he is in constant and close contact.

On the other side of the coin, the obscurities of his texts and his lectures has led someone to wonder

> *how a professor, not endowed with exceptional pedagogic qualities, could soon become the most illustrious master of Italian mathematics, from whose forge at least three quarters of the professors of Analysis of Italian universities have directly or indirectly come out and not – mind you well – for backroom intrigues, but for authentic merits. (...) [An] explanation can paradoxically be found in the abstruse difficulty of his university teaching - while his teaching proved to be indigestible to mediocre students - he spurred the best (as W. Gröbner, who also got this experience, sharply pointed out) not to be satisfied with his notes but to carry out personal studies in the library, noting sometimes that things that Picone had been able to make very complicated and difficult (or, as he said, "high") in other texts were instead clear and easy* [53]

Among the merits of Picone, the main one is undoubtedly the creation of the National Institute for Application of Calculus (INAC).

Calculators at the Institute for Application of Calculus.

In Picone's own words, the reasons that led him to the foundation of the Institute and its judgment on the importance of the theoretical work developed within the institute are:

> *The need to arrive at quantitative determinations in the tasks assigned by the High Technique to the Institute for the Applications of Calculus, has, from hand to hand, led the mathematicians of this Institute to attempt new methods of investigating problems, some even classics, related to the integration of linear equations to the partial derivatives of mathematical physics, which those tasks pose and impose. These problems often lack any possibility of reference to the classic conditions of existence and uniqueness of the solution, so that new purely existential researches are also needed to support the reasoning and eventually correct the intuition of the Physicist in the construction of the mathematical scheme of the phenomenon under investigation. It is therefore easy to foresee that, very often, the mathematicians of the Institute will have to content themselves with coming to those analytical clarifications of the proposed problems which, made public, will allow them to enter the domain of pure mathematics research.* [40], p. 213.

4.5. The school of projective differential geometry

In these years, a new discipline in geometric research emerges, which becomes soon very popular in Italy. It is projective differential geometry, which allows the combined use of techniques of differential geometry, initially the non tensorial techniques of Bianchi and later those of the absolute differential calculus of Ricci and Levi-Civita, and of projective and algebraic geometry. It is a synthesis of researches originated or that have had a great development in Italy and therefore in Italy finds fertile development ground. Foundational questions are less important for projective differential geometry than for algebraic geometry, but its contents and perspectives appear more limited. This is a research topic in some ways artificial, as Fichera observes in [23] that will not assume a central importance in the development of mathematics and that will almost disappear in the 1960s. Only recently mathematicians are beginning to look at projective differential geometry from a more advanced point of view. [38]

Projective geometry is initially developed, in Italy, by Corrado Segre and Guido Fubini and is cultivated, among others, by Alessandro Terracini (1889-1969) and Enrico Bompiani.

Mussolini's visit at the library of the Mathematical Institute with Severi and Bompiani.

To summarize the character of these studies we use the words written by Severi in [48] in which he draws up an ideological balance of the discipline in the twenty years between the wars. They testify vividly the conditioning that the regime exercised on scientists. We cannot believe that Severi is not aware of the ridicule he is spreading above himself with this acquiescent reconstruction of the history of Italian mathematics, shamefully purified by the contributions of the great mathematicians of Jewish origin. Is this the price that was necessary to pay to maintain the role and power that fascism had granted him? Or is it an obligation he could not escape? In both cases it cannot be from these people that post-war Italian science can begin its renewal.

> *In differential geometry, the two decades [1920s and 1930s] began while the strong influence of Luigi Bianchi, one of the great Italian mathematicians of the generations immediately preceding mine, continued (and I remember among them, besides Cremona, Betti, Brioschi, Dini, Beltrami) Bertini, Peano, Veronese, Cesàro, Arzelà, Ricci Curbastro, that the regime is preparing to honor with complete editions of their works, curated by Unione Matematica Italiana. But at the same time new addresses were developed or deepened which dominated since them our differential geometry throughout the twenty years period [of Fascism]. Projective-differential geometry, after the first isolated and old contributions (of Brioschi, Halphen and others, including Del Pezzo and Berzolari), can be said to have been born almost completely within an Italian framework (by Jewish mathematicians, like C . Segre, Fubini, Terracini and arian mathematicians, like Bompiani, Enea Bortolotti - passed away prematurely for science - Calapso, Sannia).* [48], p. 30.

The "arian" character of Italian mathematics of the 1920s and 1930s is shamelessly repeated in every sentence of the article and probably on every public occasion to which "his excellence Severi" intervened, in Italy or abroad, as the caustic blurb of the Weekly political and satirical anti-newspaper "il Cantachiaro" reminds us.

> *If we return to the argument Severi, already treated by us and by others, it is not for political reasons. It is rather because Francesco Sev-*

eri has the sad merit of having succeeded in being the first in bringing delinquency into mathematics.

Around 1920, Severi was a young man who, equipped with some good cards to play, foreshadowed good achievements. He himself expected them, [the good achievements], roaring, to smooth the way, according to masonic, socialist and anticlerical fashion. Gentile goodsoul, who favored dynamism, made him at once Rector of the University of Rome.

Let's turn the page. Later, here is our man, propagandist of the regime in orbace like a hierarch. But we do not accuse him of this. It has already been pointed out by others that Severi has represented the most scandalous case of the international "vague Venus" in scientific world. His travels in Japan and other places at the sound of fascist trumpets have been recalled. His academic speeches about the "sleepless helmsman" and other bagatelles have already been quoted. All this, in the end, is nothing but a light sin, for which absolution can also be given. We, on the other hand, do not want that the main accusation against F. Severi be obscured, which is that of extortion and robbery in the field of thought.

It was a thing well known to everybody: it was not conceivable to approach mathematics, without getting stuck in this grim and sinful big guy, twisted in the soul as in the look, which demanded the toll of a flattery or genuflection with the look of a member of a military squad.

He had created an institute for High Mathematics, to preside over it; and from there it held the High Mathematics, which thrived so well in Italy without an Institute.

He introduced small variations in the theories of others and he appropriated them; so that all the Higher geometry in Italy became a list of Severi's theorems. And, like the dragon Fafnir, he hatched his evil acquired heritage in the journals and in the academic meetings, ready to destroy with fire-blows those who did not prostrate themselves before him.

As a "sleepless helmsman", he assiduously supervised the middle school, threatening the poor teacher who adopted a text that was not his own, denouncing him for incompetence and lack of fascist sensitivity.

But one thing still disturbed his sleep. It was known by all that he came from a great mathematical school, honored in the world, whose masters, not all dead yet, looked at him in silence.

But, thanks to the supreme wisdom of the regime, the racial laws arrived and the monuments of the work of these distinguished men were demolished.

Thus, in the end, all Italian mathematics could be summed up in the sole name of Severi.

In 1942 a great prize was instituted in Berlin named after Copernicus (for the occasion made German); and Severi neither doubted nor

> *blushed, seeing himself being awarded the first prize, while only the second was awarded to the great Heisenberg.*
>
> *He showed no doubts, we have said, as he had no hesitation when he confiscated the heritage of his masters; indeed, peculiar author, being also shameless censor of his own works, he surpassed himself when he wrote the motivation of his extorted first prize.*
>
> *Here it is, this motivation, in a style never used in academic annals, not even in the case of Galileo:*
>
> *"...the great Algebrist of our times, the head of the Italian school of algebraic geometry, the master who has dominion over mathematical methods, the founder of one of the best-known mathematical schools among those of all civilized nations, the brilliant organizer, the man who, above its own specialty, has greatly enriched the spiritual life of our times in the natural and philosophical sciences ...".*
>
> *Every comment is superfluous. And noe, the big man, grim and sinful, is still living and perhaps wants to march to the rescue.* [19]

5. Conclusions

It results from our analysis an evident moment of fracture in the development of Mathematics, which coincides with the oath imposed by fascism on university professors, strongly suggested by Severi to cancel his adhesion to the manifest Croce, see [26]. Before the oath, Mathematics in Rome appears to be well established and respected internationally, capable of profiting from the numerous opportunities for exchange between the best students and researchers who come to study with Volterra, Levi-Civita, Castelnuovo and Enriques. After 1931 the music changes completely, when the role of Rome in Italian Mathematics becomes more representative than stimulus to research and an institutional arrogance is revealed that suffocates the confrontation, precisely in the moment of the greatest need for renewal. This is evident in the person of Severi, while it is more nuanced in the other protagonist of the Mathematics in Rome of that period, Mauro Picone. His appearance of devoted supporter of the regime hides clearly contradictory behaviors. Picone is described as a character who "*attacks the horse where his master wants*" as Caccioppoli reproaches him, but who, protected from the acquiescent role he knows well how to interpret, does not let himself be overshadowed by the autarkic delirium around him, maintaining a greater balance, and a greater humility, in the evaluation of the condition of his research and of the international relations that are necessary for it.[13]

[13] In this paper, not much is said about Enrico Bompiani, another major figure of mathematics in Rome from 1927 to 1959. He was vice - president of the Italian Mathematical Union from 1938 to 1943; president of the committee for mathematics of the Italian Research Council; an excellent director of the institute of mathematics from 1939 to 1959. Even if his contributes to the field of projective differential geometry are substantial, he was not the leader of a mathematical school of importance comparable to those headed by the other mathematicians considered in this paper. Therefore he plays a less important role in our story. For further notices about Bompiani, see

Severi, on the contrary decides with a conscious calculation of "becoming" fascist in the conviction of being able to use the regime to satisfy his pathological desire for affirmation and finding himself in perfect harmony with the head of fascism, which he praises with embarrassing words, quickly forgotten from the academic world. Someone, unfortunately only ex post, and from the columns of a satirical newspaper, reminds him

> *it did not seem an outrage to the intelligence to write words like: "... the sleepless helmsman, superb and faithful synthesis of the feelings and of the aspirations of the genius of the race, leads the prow of Italy in the storm". Nothing was missing from your always smoking thurible of expressions that today, in reading them, thicken in our throats. "The synthetic power of your genius" and the "trusting confidence that you will be able to dominate, as always, the events" and "Your imperial vision" and "our superiority and our right to dominate" are all grains of incense burned by you, together with your dignity as a man of science, on the altar of that uneducated, vulgar, plethoric dictator.* [18]

Mussolini, pickaxer of Italian people and gravedigger of Italian mathematics.

[13], especially the last paragraphs for the long period he spent in Rome.

A. Appendix

Foreign mathematicians who spent a period of study in Rome between the two wars (an asterisk marks those appearing in [37]).

LAST NAME	FIRST NAME	NATION	ADVISOR	DATE
Bachiller *	Tomàs Rodrìguez	SPA	Severi?	1939
Behman	Heinrich	GER	Enriques	1926-27
Brauner	Karl	AUT	Levi-Civita	1928-29
Brelot	Marcel	FRA	Volterra	1929-30
Burniat	Pol	BEL	Enriques	
Busemann	Herbert	GER	Levi-Civita	1930-31
Davies *	Evan Tom	RSA	Levi-Civita	
Deuring *	Max	GER	Severi	1928-29
Dresden	Arnold	USA	Calcolo delle variazioni	1935-36
Dubreil *	Paul	FRA	Enriques	1929-31
Du Val *	Patrick	GBR	Enriques	1930-32
Evans *	Griffith	USA	Volterra	1910-12
Fenchel	Werner	GER	Levi-Civita	1930-31
Féraud	Lucien	FRA	Levi-Civita	1928-29
Geppert	Harald	GER	Levi-Civita	1928-29
Geppert	Maria Pia	GER	Cantelli, Castelnuovo, Gini	1932-33
Haimovici	Mendel	ROM	Levi-Civita	1933
Hlavaty	Vaclav	TCH	Levi-Civita	1927-28
Jacotin Dubreil *	Marie-Louise	FRA	Levi-Civita	1930-31
Kahler *	Erich	GER	Levi-Civita, Castelnuovo	1930-31
Lane	Ernest Preston	USA	Proj. diff. geom.	1926-27
Lauriers des	Guérard	FRA	Levi-Civita	1925-26
Levy	Harry	USA	Levi-Civita	1929-31
Lewy *	Hans	GER	Levi-Civita, Enriques ?	1929-31
Logdson	Mayme	USA	Enriques	1925-26
Mandelbrojt *	Sz.	POL	Volterra	1924-26
Mazet	Robert	FRA	Levi-Civita	1926-27
Mc Connel	Albert	IRL	Levi-Civita	1927-28
Mihoc *	Gheorghe	ROM	Castelnuovo	1928-30
Moisil *	Grigoire	ROM	Volterra	1931-32
Moufang *	Ruth	GER	Enriques ?	1931-32
Onicescu	Octav	ROM	Levi-Civita	1919-20
Peierls *	Rudolf Ernst	GER	Fermi	1932
Pérès *	Joseph	FRA	Volterra	1914-15
Pic *	Gheorge	ROM	Levi Civita?	1930-32
Reidemeister *	Kurt	GER		1933
Roth *	Leonard	GBR	Severi et al.	1930-31
Sebastiao e Silva *	José	POR	Enriques (?) Severi Fantappié	1942
Stouffer	Ellis Bagley	USA	Proj. diff. geom.	1926-27
Struik *	Dirk	HOL	Levi-Civita	1926-27
Thomsen	Gerhard	HOL	Levi-Civita	1926-27
Thullen	Peter	GER	Severi	1931-32
Uhlenbeck *	George Eugene	HOL	Fermi	1923-25
Vranceanu *	Gheorghe	ROM	Levi-Civita	1923-24
Weil *	André	FRA	Borsa Sorbonne	1925-26
Weinstein *	Alexander	URSS	Levi-Civita	1926-27
Wintner *	Aurel	GER	Levi-Civita	1929-30
Zariski *	Oskar	URSS	Castelnuovo	1921-26

B. Courses of mathematics taught at the University of Rome 1921-45.

We have left the title of the courses in Italian. The symbol (i) indicates that the course was given ad interim.

FIRST AND SECOND YEAR

Analisi Algebrica 1921-27 Francesco Severi; 1927-36 Ugo Amaldi (i)

Analisi Infinitesimale 1921-22 Francesco Severi (i); 1922-26 Giuseppe Bagnera; 1926-27 Bagnera (dead on may 12-th); 1927-36 Francesco Severi

Analisi matematica (algebrica e infinitesimale) 1936-39 Francesco Severi – Ugo Amaldi (i); 1939-42 Leonida Tonelli – Ugo Amaldi (i); 1942-45 Tullio Viola (i) – Ugo Amaldi

Geometria analitica e Proiettiva 1921-35 Guido Castelnuovo.

Geometria descrittiva con disegno e applicazioni 1921-27 Giulio Pittarelli; 1927-35 Enrico Bompiani

Geometria analitica con elementi di proiettiva e geometria descrittiva con disegno 1935-1939 Gaetano Scorza – Enrico Bompiani; 1939-45 Fabio Conforto – Enrico Bompiani

Meccanica Razionale 1921-23 Tullio Levi-Civita (suppl. Almansi); 1923-38 Tullio Levi-Civita; 1938-45 Antonio Signorini.

THIRD AND FOURTH YEAR – FUNDAMENTAL

Analisi Superiore 1921-23 Tullio Levi-Civita; 1923-26 Bagnera (i); 1926-27 Bagnera (dead on may 12-th); 1927-32 Bompiani (i) 1933-42 Mauro Picone; 1942-44 Mauro Picone (i); 1944-45 Mauro Picone

Fisica Matematica 1921-31 Vito Volterra; 1935-39 Luigi Sobrero (i); 1939-43 Carlo Cattaneo (i); 1944-45 Carlo Cattaneo (i).

Geometria superiore 1921-23 Castelnuovo (i); 1923-38 Enriques; 1938-43 Severi; 1943-44 Severi (Enriques from June); 1944-45 Enriques

THIRD AND FOURTH YEAR –OPTIONAL

Astronomia 1921-22 Alfonso di Legge; 1922-45 Armellini

Calcoli numerici e grafici 1938-39 Mauro Picone (i); 1943-45 Aldo Ghizzetti

Calcolo delle probabilità e statistica 1926-27 Francesco Cantelli (i); 1927-28 Guido Castelnuovo (i); Calcolo delle probabilità 1928-34 Guido Castelnuovo (i); 1937-38 Castelnuovo (corso libero); 1941-45 Giuseppe Ottaviani (i)

Geodesia 1921-22 Umberto Crudeli (i); 1924-26 Armellini (i); 1943-45 Armellini (i)

Geometria algebrica 1939-41 Enzo Martinelli (i); 1941-45 Guido Zappa (i)

Geometria Differenziale 1932-45 Enrico Bompiani (i)

Matematica attuariale 1921- 34 Francesco Cantelli (i); 1938-39 Francesco Cantelli (i); 1941-42 Francesco Cantelli (i)

Tecniche per l'assicurazione libera sulla vita umana 1941-42 Francesco Cantelli (i)

Matematica attuariale e tecnica delle assicurazioni 1942-43 Maria Castellani (i); 1943-45 Cantelli (i)

Matematiche Complementari 1921-23 Federigo Enriques (comando); 1923-24 Alfredo Perna (i); 1924-26 Castelnuovo (i) - Esercitazioni di Matematiche complementari - Perna (i); 1926-28 Alfredo Perna (i); 1928-29 Enriques (i); 1929-31 Perna (i); 1931-32 Francesco Severi (i), Alfredo Perna (i); 1932-45 Alfredo Perna (i).

Matematiche superiori 1923-24 Armellini – Bagnera – Castelnuovo (probably on three different subjects: Meccanica, Analisi, Geometria); 1924-26 Ugo Amaldi (i) (with title Introduzione alle matematiche superiori); 1932-39 Francesco Severi (i); 1939-45 Fantappié (i).

Meccanica statistica 1936-39 Giovanni Lampariello (i)

Meccanica superiore 1921-22 Vito Volterra (i); 1924-26 Volterra (i); 1926-27 Luigi Fantappié (i); 1927-29 Vito Volterra (i); 1929-45 Giulio Krall (i)

Metodologia matematica 1925-26 Alfredo Perna (i).

Statistica matematica 1921-26 Francesco Cantelli (i); 1928–34 Francesco Cantelli (i)

Storia delle Matematiche 1923-29 Giovanni Vacca (i) (professor of "Storia e geografia dell'Asia" at the Faculty of Humanities); 1936-38 Enriques (i); 1938-39 Fabio Conforto (i); 1939-45 Attilio Frajese (i).

Teoria dei numeri 1936-39 Scorza (i); 1939-43 Conforto (i); 1944-45 Conforto (i)

Teoria delle funzioni 1935-37 Carlo Miranda (i) 1937-38 Lamberto Cesari (i) 1938-45 Tullio Viola (i)

Topologia 1941-45 Enzo Martinelli (i).

C. Letter exchange between Severi and Weil

The letters were published in [5] and we think it is useful to provide here an English translation of them.

C.0.0.1. Severi to Weil [Archive number: HS 652: 8402] Roma, 29 Marzo 1956

> *Still embittered by the form, which seemed unusual to me, with which you intervened on the occasion of the Congress of Amsterdam at my second conference there, I am feeling embittered all the more, recalling the cordial relations I had had with you about 30 years ago, when you came to visit me in Italy and we talked several times with you and in particular in my country house, since then you were a newcomer in Mathematics, where you gave so many remarkable proofs of your value (I still remember the Parisian dissertation of arithmetic-algebraic-geometric, [for an] academic title at which I was not an alien and whom you had the goodness to send me with affectionate dedication), I found with regret that my name is avoided, as if it were on a black list, in your brief memory on the equivalence criteria in algebraic geometry, published in volume 128 of the Mathematische Annalen (Weil, 1954). You write that the criteria of linear equivalence and algebraic equivalence are due for the most part to Italian geometers, and refer to Zariski for this, which from the strictly logical point of view (I would say bourbakist) can justify the form of your quotation, because in fact Zariski does not quote others than me for these criteria and this quotation is completely true. The reality is that none of the Italian geometers of the period, whom you call classic, has dealt with either linear equivalence criteria or equivalence relations (in particular algebraic equivalence). Those who have dealt with this question have dealt*

it exclusively following my suggestions. It is therefore not right to make such approximate quotations, especially by professors who, teaching our science in American universities, from which it spreads in the world, together with many beautiful things, also a kind of cultural imperialism, end up to be the oracles and creators not only of what they create in one go, but also of what they transform or generalize. I do not mean by this that I do not give great value to the abstract research of a remarkable part of contemporary mathematics, and in particular to everything that brings algebraic geometry closer to abstract algebra, number theory and other questions of analysis, but the historical precedents of the theories have great value, also for the purposes of the subsequent cultural production, and you teach it to me, because you, a person of vast interests and vast culture (I remember how much in your youth you have been interested for example in Sanskrit and our talks about it), know how important it is to return to the ideas expressed in the ancient form, because there is always something new, even in imperfections. I only deplore the attitude of many very young mathematicians, who believe that modern mathematics has been born today and have an Olympic disdain for what has been produced in the past, ignoring, because they are made to ignore, that "classic" production contains the generating ideas of much of what is done today.

You can say that, if you did not talk about me in the M.A., I got a profit, because I had not to be direct subject to criticism "ad personam": "malheureusement les ouvrages qui pretendent traiter cette matière sont loin de founir à cet egard ce qu'on serait en droit d'en attendre".[14]

Well, I prefer to take the critics personally and respond when they are well specified. You will find an essay of the precision, with which I like to respond to criticism, about the untimely interrogations addressed to me in Amsterdam, in a Note whose drafts I have already corrected for the volume in honor of Lefschetz. It refers to the value of the symbol of intersection of virtual varieties (or products of such varieties when their set is considered as a ring).

Your questions of Amsterdam 1954 were, in my opinion, untimely, because the notion that you was asking for your personal understanding, I had to presuppose to be known by each of my listeners, because of the nature of the lectures I was supposed to deliver, not directed at those preliminaries (not even I had the duty to prepare listener by listener); and they were a bit cruel to a man who, at least for his age, had the right to expect a different treatment and yet, despite not being allowed to answer in his own language, he tried to stay in the saddle, and did not lose his temper!

With regard to the theory of algebraic or rational equivalence, which my lecture assumed, I hope that also the Notes of the Academy of Lincei

[14] Unfortunately, the works which claim to treat this subject are far from providing in this respect what one has the right to expect.

of the 1955 entitled "Complementi alla teoria dell'equivalenza su varietà algebriche"[15] *may clarify your understanding. I send you these Notes together with the extracts of some of my most recent works, of which I still have a copy. I am now correcting the drafts of the Memory of the Mathematics Annals, about the theorem of irregularities of varieties, in relation to differential forms of the first and second kind, referred to in the Notes of the C.R. and of the Lincei, which you will find in the envelope.*

I repeat that it does not correspond to historical accuracy to attribute in a generic way the equivalence criteria to Italian geometers, because Castelnuovo-Enriques have never dealt with such questions. My criteria of linear equivalence are part, is true, of the domain of geometry that they have magnificently illustrated with their work, but they had never dealt with them.

The general notions about continuous systems of varieties or about the concepts of equivalence, have never caught the attention of my colleagues and teachers older than me! There is only one Note of the Academy of Lincei by Enriques about a "series of equivalence" which he called series of Severi on a surface.

Since I finally took the pen to spend some time with you (and if I wasn't always overwhelmed by the work, I would have done it much earlier), I would like to empty the sack. I must remark that the name you adopted of Albanese variety for what I call the second variety of Picard in a volume to be published soon, is not justified at all. I draw your attention to this fact, because many American mathematicians living in America (Japanese and even Italian), following the example perhaps given first by Osuka and confirmed by you, call Albanese variety the one which is relative to the periods of simple integrals of the first kind and is related to the first and oldest variety of Picard (Castelnuovo). Now there is nothing in the work of my ancient disciple Albanese that can justify such a nomenclature. Poor Albanese quoted my work regularly. The second variety of Picard was considered by me for the first time since 1913 in a Note of the Proceedings of the Istituto Veneto entitled "Un teorema di inversione per gli integrali semplici di prima specie appartenenti ad una superficie algebrica".[16] *In this regard there are some important works and others simply interesting, but in any case well written, of my young disciples Andreotti (professor at the University of Turin, who was at Princeton for one year), Rosati and Benedicty, disciples who are still here with me. Andreotti in particular has found that each of the two varieties of Picard can be considered as an involution on the other, thus falling again into that sort of relation of duality between the two varieties, which you had discovered yourself. Both Andreotti and Rosati gave answer to the problem of knowing when*

[15] Complements to the theory of equivalences on algebraic varieties.

[16] An inversion theorem for the simple integrals of the first kind belonging to an algebraic surface.

the two varieties coincide.

Dear Weil, you may not believe it, but we arrive, at the point that, having called the attention of an Italian geometer who now lives in America, about this fact, I was told: "You are right, but I must continue to call it Albanese variety, because in America they all call it that way"*. Here is a small episode of that cultural imperialism of which I spoke above.*

I know that this will give you a reason for practicing your sarcastic attitudes. I cannot blame you, because I also make sometimes use, albeit with benevolence, of the weapon of irony. I wouldn't be from Arezzo if I wasn't like that.

I send this letter and the package of which I told you, in Paris, because in the Mathematische Annalen is given your address in Paris. It may be that at this time you are in Chicago instead. In this case letter and envelope will remain with your relatives in Paris. And perhaps your relatives will think about forwarding you the letter, giving you the package on your return.

One last thing. Since I have had repeated proofs that the manifestations of low personal benevolence of some mathematical circles towards me have originated from legends that run on my account, in relation with fascism and such kind of things; while I do not care to give explanations for what concerns my behavior as an Italian among Italians (who alone can judge it, and have recently judged it, because with a law approved by the Italian Parliament and issued by the President of the Republic I have been appointed President for life of the National Institute for High Mathematics), I worry about debunking legends that deeply touch my humanity as a man among men. I am talking about rumors that have spread on top of my alleged anti-Semitism. I believe that you could usefully have knowledge of a letter (of which I enclose a copy), which my disciple Beniamino Segre have recently had the opportunity to write to a non-Italian colleague recently. (I must keep silence about the name of my colleague, who I do not have the right to make intervene, even indirectly. I should have asked Segre for consent too, but he is so fond of me that his consent is not doubtful).

I hope that you will consider this long reading with the same spirit of friendship with which I wrote it. Cordial greetings.

C.0.0.2. Weil's reply to Severi [archive number: HS 652: 8403a] Paris, le 9 Avril 1956

Dear and illustrious Master,

Coming back from a short trip, I find your letter of March 29th. I am sorry for having involuntarily displeased you, but I do not see what *you reproach me for. In my memory of Math. Ann. it was not at all part of my purpose to make the story of the subject - this story being, in my opinion, very well done in the volume of Zariski to which I refer the reader.*

For quite accidental reasons, the name "Albanese Variety" is now in use. The name "Picard Variety" has been introduced for quite accidental reasons too, it seems to me; similarly for "Fuchsian functions", "Taylor formula" etc. All those who are interested in the history of mathematics are well aware that these denominations must never be given importance. The important thing is to understand each other. It seems extremely probable that we will continue, for some time at least, to speak of "Picard variety" and "Albanese variety", and I see, for the moment, much more advantages than drawbacks to retain this terminology until further notice.

If I thought that I had behaved in a manner unfavorable to a master whom I admire and respect at the Congress of Amsterdam, I would have apologized to you since a long time; and I am willingly do so now, since your letter indicates that I have offended you, although I did not think I exceeded the tone permitted in a scientific discussion.

Finally, I assure you that it has never occurred to me to attribute to you anti-Semitic sentiments, and that it was quite useless to send me a copy of Segre's letter on this subject. I know you are far too clever, anyway, to assume that you have ever been able to take seriously the absurd racial theories that Hitler's successes have imposed for some time, on Germany first and on a great deal of Europe later. Happily, all this happily belongs to the past.

Believe me, I beg you, that I am always most grateful for the kind welcome I received from you in Rome and Arezzo, at the time of my first steps in mathematics. As for saying what I owe you scientifically, it suffices, I think, to say that I am an algebraic geometer. Everyone knows enough that, without your work, algebraic geometry would not be what it is.

Please I pray you, to always believe in my gratitude and my deep admiration.

PS Will you be very surprised if I tell you that the words "the works which pretend to treat this matter are far from providing what one would have the right to expect" were addressed to my own Foundations, and that is what all my friends understood right away? My friends know that, while it does not always displease me to make my irony over others, I am able of exercising it in my own regard either. That this could serve at least to reassure you that I wrote the sentence in question without thinking of Italian geometers.

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