SPACECRAFT ATTITUDE MOTION PLANNING ON SO(3) USING GRADIENT-BASED OPTIMIZATION: CASE STUDIES

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ABSTRACT

A method for spacecraft attitude motion planning presented in [Celani and Lucarelli, 2019] is applied to relevant case studies. The goal of the method is to design a control torque so that a spacecraft performs a desired rest-to-rest rotation while satisfying pointing constraints. The method possesses the following features. Attitude is represented on the group of three dimensional rotations SO(3) thus avoiding singularities and ambiguities that affect other attitude representations. Moreover, from a practical point of view, the control torque resulting from the method is continuously differentiable and vanishes at its endpoints. Thus, the resulting controls are easier to implement on real spacecraft than time-optimal control torques that often do not vanish at endpoints and can be discontinuous during the maneuver. A unique feature of the method is the use of basis functions parameterizing the angular rates. The case studies we consider are presented in [Spiller et al., 2016] in which a satellite for Earth observation in low Earth orbit must perform a roll rotation. The satellite is equipped with a star tracker that must avoid Sun and Moon directions with prescribed offset angles during the maneuver.

Keywords: attitude motion planning, pointing constraints, Slepian sequences

1 INTRODUCTION

Attitude motion planning is necessary in mission scenarios in which the spacecraft must perform rest-to-rest large angle maneuvers with the additional requirement that sensitive instruments must not point to bright celestial objects. These so-called ``keep-out cones" define constraints that must be satisfied along the instrument trajectory.

Many methods for performing such a task have been devised (see e.g. [1,2] and references therein). In this work the focus is on the method presented in [3] which possesses the following features. Attitude is represented on the group of three dimensional rotations SO(3) thus avoiding singularities and ambiguities that affect other attitude representations. Moreover, from a practical point of view, the control torque resulting from the method is continuously differentiable and vanishes at its endpoints. Thus, the resulting controls are easier to implement on real spacecraft than time-optimal control torques that often do not vanish at endpoints and can be discontinuous during the maneuver.

The method proposed in [3] consists of two steps. In the first, path-planning is performed to determine an appropriate time behavior for the angular rate so that the spacecraft is reoriented to the desired attitude while avoiding keep-out cones. Path-planning is performed first by expressing the angular rates as linear combination of some basis function. Thus, path-planning is formulated as an optimization problem in which the decision variables are the weights of the linear combination. In the second step, known as motion planning, the actual control torque is simply determined by the use of inverse attitude dynamics, and a time scaling is introduced to reduce the torque amplitude to within the allowed limits.

In a numerical example presented in paper [3] a number M of so-called Slepian sequences [4] were selected as basis functions. Slepian sequences are characterized by two parameters: length N and half-bandwidth W. In the example in [3], proceeding by trial-and-error it was set M=4, N=500 and $W=0.015$. It turned out that such a choice of M, N, and W led to a satisfactory solution to the attitude motion planning problem. In particular, the obtained solution seemed intuitively to have some "minimum-path property" in the space of the admissible manoeuvres. Such property corresponds intuitively to minimum-time manoeuvring within the set of admissible manoeuvres considered in the method.

The goal of the current paper is applying the method in [3] to some new case studies. In particular it is interesting to investigate if the choice of Slepian sequences as basis functions and the values of the parameters M, N, and W selected in the numerical example in [3] are effective also for those case studies. The new case studies considered here were originally formulated in [5].

2 CASE STUDIES

In the following case studies we consider a satellite for Earth observation in low Earth orbit which must perform a 60 deg roll rotation. The spacecraft attitude is represented by $\boldsymbol{R} \in SO(3)$. Setting the initial attitude as $\boldsymbol{R}_i = \boldsymbol{I}_{3\times 3}$ the desired final attitude is equal to

$$\boldsymbol{R_f} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) \\ 0 & \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) \end{bmatrix}$$

The satellite is equipped with a star tracker that must avoid Sun and Moon directions with prescribed offset angles during the maneuver. The pointing direction of the star tracker sensor is expressed in body coordinates by the following unit vector

$$\boldsymbol{r} = [0 - 0.62 - 0.79]^T$$

The spacecraft inertia matrix is given by

$$\boldsymbol{J} = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 4500 & 0 \\ 0 & 0 & 6000 \end{bmatrix} \text{ kg m}^2$$

The maximum torque on each body axis is equal to $T_{max} = 0.25$ N m. Two keep-out cones are considered corresponding to the Sun and the Moon direction. The corresponding half offset angles are equal to $\theta_1 = 40$ deg and $\theta_2 = 17$ deg respectively. Regarding the pointing directions to the Sun and to the Moon, the following two different case studies are investigated.

2.1 Case study 1

In this case study the pointing directions to the Sun and to the Moon resolved in an appropriate inertial frame are given by

$$\boldsymbol{w}_1 = [-0.58 \ -0.08 \ -0.81]$$

$$\boldsymbol{w}_2 = [0.40 - 0.13 - 0.90]$$

Thus, the pointing constraints can be expressed as follows

$$\boldsymbol{c}_i(t) = \boldsymbol{r}^T \boldsymbol{R}(t)^T \boldsymbol{w}_i - \cos\theta_i \leq 0 \quad i = 1,2 \tag{1}$$

The path planning step is solved by choosing as in [3] *M*=4 Slepian sequences as basis functions with parameters *N*=500 and *W*=0.015 (see Figure 1).

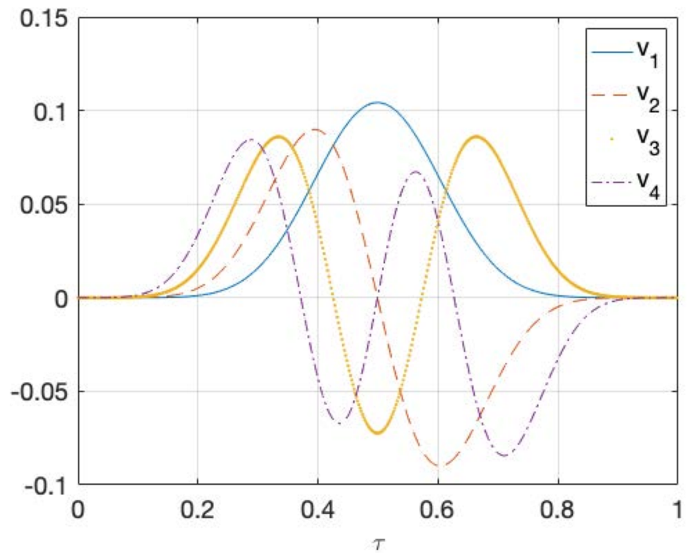

Figure 1: Samples of basis functions $v_k(\tau_\ell)$ $k = 1, \ldots, 4$ ℓ=1,…,500

The outcome of the path-planning step is given by the time samples of the angular-rate in normalized time $\omega^*(\tau_\ell)$ ℓ= 1, . . . , 500 which are represented in Figure 2. The motion planning step leads to maneuvering time t_f =478 sec and to the time behavior for the torque reported in Figure 3. Tests with different values of *N* and *W* lead to higher maneuvering times. Figure 3 shows that, from a practical point of view, $T(t)$ can be considered continuously differentiable and vanishing at its endpoints. Both properties are consequences of our choice for the samples $v_k(\tau_\ell)$ (see Figure 1). Both properties make the implementation of $T(t)$ on real spacecraft easier compared to time-optimal control torques that often do not vanish at endpoints and are sometimes discontinuous during the maneuver.

To validate the effectiveness of the obtained input torque, the attitude equations with initial conditions $\boldsymbol{R}(0) = \boldsymbol{R}_i = \boldsymbol{I}_{3\times 3}$, $\boldsymbol{\omega}(0) = \boldsymbol{0}$ are integrated numerically and the following results are obtained. The obtained final attitude $\boldsymbol{R}(t_f)$ satisfies the following

$$3 - \mathrm{tr}\left[\boldsymbol{R}_f^T \boldsymbol{R}(t_f)\right] = 1.66\ 10^{-9}$$

and $\|\boldsymbol{\omega}(t_f)\| = 7.12\ 10^{-8}$. Thus, the spacecraft reaches the desired final attitude $\boldsymbol{R}_f$ with zero angular velocity.

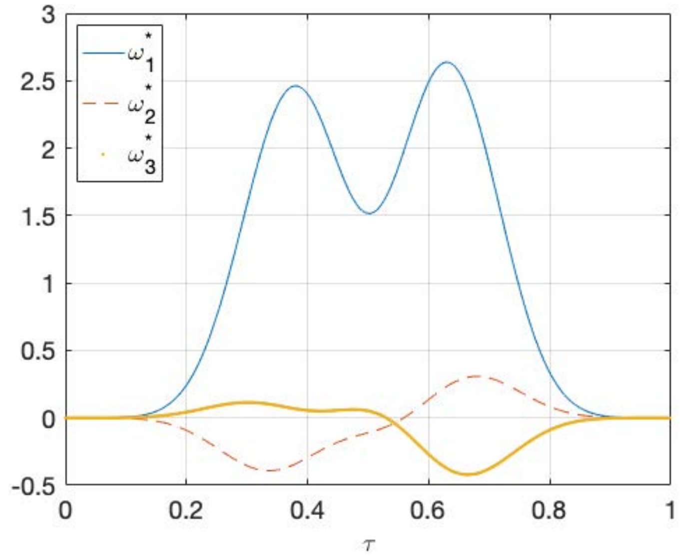

Figure 2: Samples $\omega^*(\tau_\ell)$ ℓ= 1, . . . , 500 (Case 1)

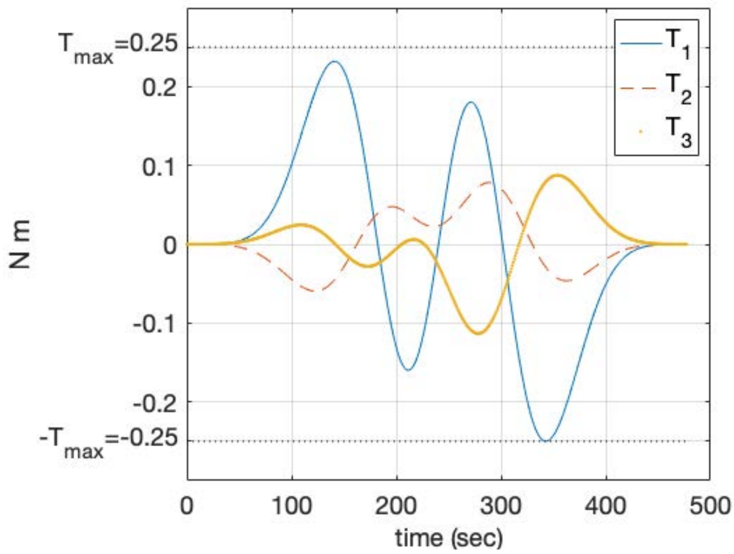

Figure 3: Torque $T(t)$ (Case 1)

The time behaviors of $\boldsymbol{c_i}(t)$ $\boldsymbol{i} = \mathbf{1}, \mathbf{2}$ from Equation (1) are shown in Figure 4 confirming that the two pointing constraints are fulfilled. The path of the sensitive direction, the two exclusion cones, the initial and the desired final sensitive directions are all displayed in Figure 5. By inspection the proposed method leads to a solution that apparently minimizes the length of the path.

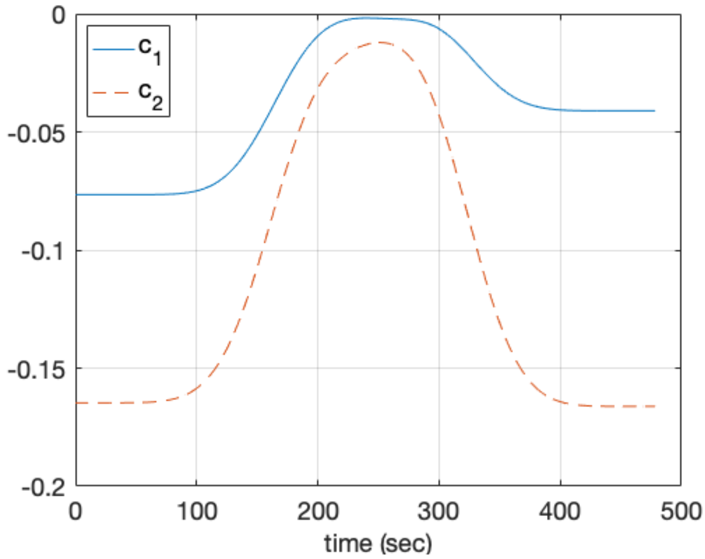

Figure 4: Pointing constraints (Case 1)

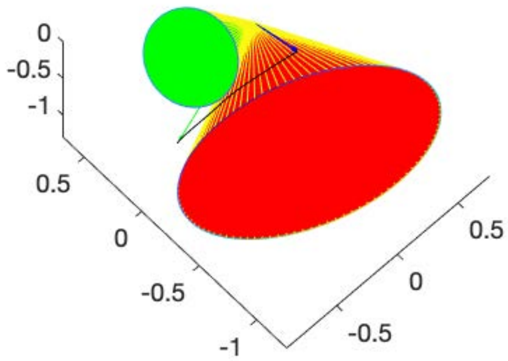

Figure 5: Path of sensitive direction (black curve), exclusion cones, initial sensitive direction (green arrow), desired final sensitive direction (blue arrow) (Case 1)

Time-optimal approaches to the same attitude motion planning problem are presented in [5] and achieve a maneuvering time of about 230 sec. The method proposed here does not explicitly minimize time and performs the maneuver in 478 sec which is substantially longer. However, our approach leads to a continuously differentiable control torque that vanishes at its endpoints (see Figure 3) making it is easier to implement on real spacecraft compared to the control torques in Figure 9 of [5]. In fact, the latter do not vanish at endpoints and some of them present discontinuities.

2.2 Case study 2

In this case study the pointing directions to the Sun and to the Moon resolved in the same inertial frame as Case 1, are given by

$$\boldsymbol{w}_1 = [-0.65\ 0.28\ -0.707]$$

$$\boldsymbol{w}_2 = [0.17 - 0.26 - 0.95]$$

The path planning step is solved by choosing as before *M*=4 Slepian sequences as basis functions with parameters *N*=500 and *W*=0.015 (see Figure 1). The outcome of the path-planning step is given by the time samples of the angular-rate in normalized time $\omega^*(\tau_\ell)$ ℓ= 1, . . . , 500 which are represented in Figure 6. The motion planning step leads to maneuvering time t_f =490 sec and to the time behavior for the torque reported in Figure 7. As in Case 1 the latter figure shows that, from a practical point of view, $T(t)$ can be considered continuously differentiable and vanishing at its endpoints. This makes the implementation of $T(t)$ on real spacecraft easier compared to time-optimal control torques that often do not vanish at endpoints and are sometimes discontinuous during the maneuver.
To validate the effectiveness of the obtained input torque, the attitude equations with initial conditions $\boldsymbol{R}(0) = \boldsymbol{R}_i = \boldsymbol{I}_{3\times 3}$, $\boldsymbol{\omega}(0) = \boldsymbol{0}$ are integrated numerically and the following results are obtained. The obtained final attitude $\boldsymbol{R}(t_f)$ satisfies the following

$$3 - \mathrm{tr}\left[\boldsymbol{R}_f^T \boldsymbol{R}(t_f)\right] = 3.60\ 10^{-9}$$

and $\|\boldsymbol{\omega}(t_f)\|$ = 8.59 10^{-9}. Thus, the spacecraft reaches the desired final attitude $\boldsymbol{R_f}$ with zero angular velocity.

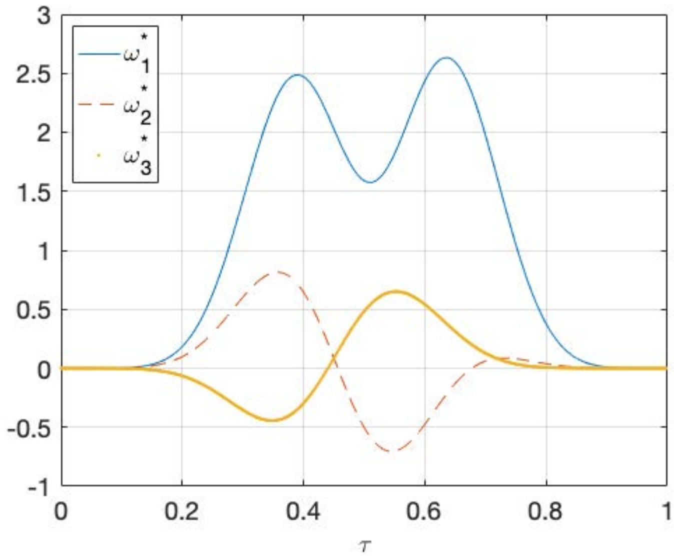

Figure 6: Samples $\omega^*(\tau_\ell)$ ℓ= 1, . . . , 500 (Case 2)

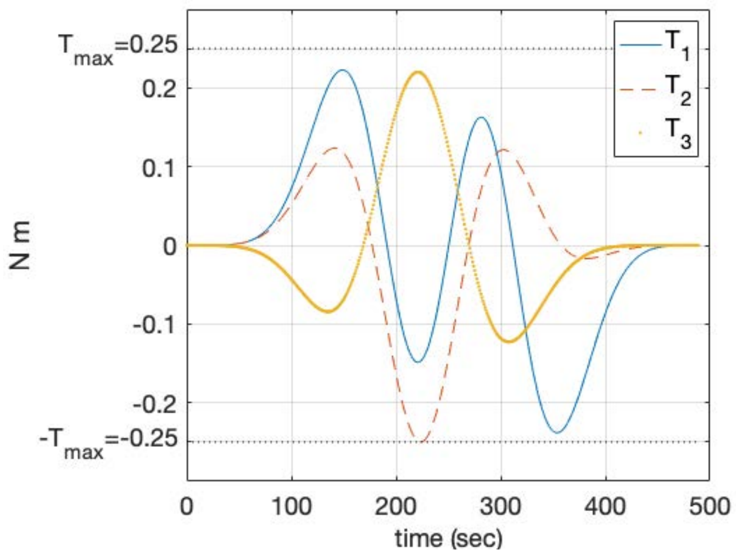

Figure 7: Torque $T(t)$ (Case 2)

The time behaviors of $\boldsymbol{c_i}(t)$ $\boldsymbol{i} = \mathbf{1}, \mathbf{2}$ from Equation (1) are shown in Figure 8 confirming that the two pointing constraints are fulfilled. The path of the sensitive direction, the two exclusion cones, the initial and the desired final sensitive directions are all displayed in Figure 9. By inspection the proposed method leads to a solution that apparently almost minimizes the length of the path.

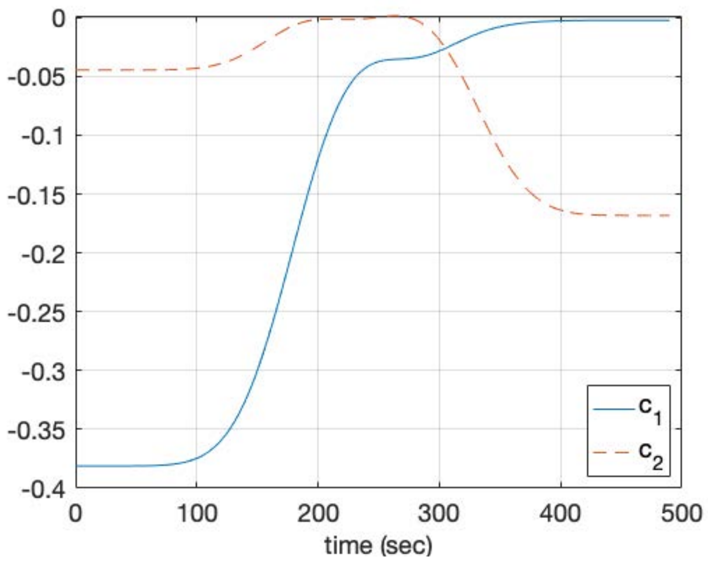

Figure 8: Pointing constraints (Case 2)

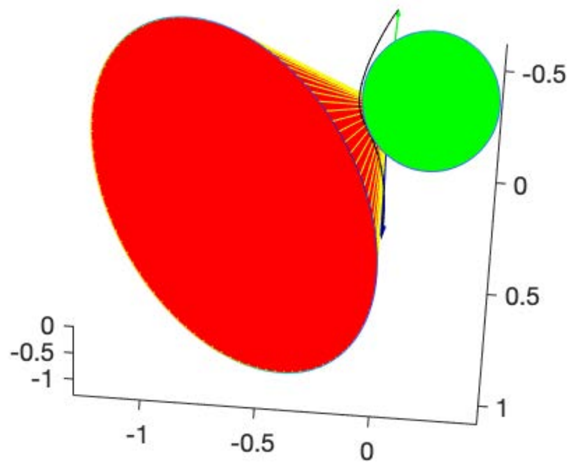

Figure 9: Path of sensitive direction (black curve), exclusion cones, initial sensitive direction (green arrow), desired final sensitive direction (blue arrow) (Case 2)

3 CONCLUDING REMARKS

In this paper a method for performing constrained spacecraft attitude motion planning has been applied to two case studies. The method obtains control torques that are continuously differentiable and vanish at their endpoints thus making the controls suitable for implementation on a real spacecraft. In both case studies, Slepian sequences have been selected as basis functions for the angular rate. The paths obtained by the method appear to minimize length in the space of admissible manoeuvres and support the choice of the Slepian sequences as an effective functional representation of the angular rates.

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