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Love in extrema ratio*

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The tidal deformability of a self-gravitating object leaves an imprint on the gravitational-wave signal of an inspiral which is paramount to measure the internal structure of the binary components. We unveil here a surprisingly unnoticed effect: in the extreme mass-ratio limit the tidal Love number of the central object (i.e. the quadrupole moment induced by the tidal field of its companion) affects the gravitational waveform at the leading order in the mass ratio. This effect acts as a magnifying glass for the tidal deformability of supermassive objects but was so far neglected, probably because the tidal Love numbers of a black hole (the most natural candidate for a compact supermassive object) are identically zero. We argue that extreme mass-ratio inspirals detectable by the future laser interferometric space antenna (LISA) mission might place constraints on the tidal Love numbers of the central object which are roughly eight orders of magnitude more stringent than current ones on neutron stars, potentially probing all models of black hole mimickers proposed so far.

"*Love all*, *trust a few*, *do wrong to none*."
William Shakespeare, All's Well That Ends Well

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Gravitational-wave (GW) measurements of the tidal deformability of neutron stars[1] — through the so-called tidal Love numbers (TLNs)[2] — provide one of the most accurate tools to date to probe the microphysics of the neutron-star interior.[3,4] It has been recently realized that tidal effects in coalescing binaries can also be used

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to distinguish black holes (BHs) from other ultracompact objects.[5–7] A remarkable result in classical General Relativity is that — owing to the one-way nature of the event horizon — the TLNs of a BH are identically zero,[8–13] whereas those of an ultracompact horizonless object are small but finite.[6,14,15] Besides posing an intriguing problem of "naturalness" in Einstein's theory,[15] this precise cancelation provides also an opportunity to test the BH paradigm: measuring a nonvanishing TLN with measurements errors small enough to exclude the null case would provide a smoking gun for the existence of new species of ultracompact massive objects.

The impact of the tidal deformability on the GW signal from a binary coalescence has been so far studied mostly in the case of comparable masses. This choice is well motivated for neutron star binaries, but it might be too restrictive in the context of tests of the nature of dark compact objects, especially because future GW observations are expected to unveil binaries with mass ratios departing significantly from unity. With this motivation in mind, here we explore the following question: *how much does the tidal deformability affect an extreme mass-ratio inspiral (EMRI)*[16] *when the massive central object has nonvanishing TLN*?

Let us consider a nonspinning compact binary, with masses m_i ($i = 1, 2$), total mass $m = m_1 + m_2$, and mass ratio $q = m_1/m_2 \geq 1$. At leading post-Newtonian order, the correction to the *instantaneous* GW phase due to the tidal deformability of the binary components reads[1,2] (we use $G = c = 1$ units)

$$\phi_{\rm tidal}(f) = -\frac{117}{8}\frac{(1+q)^2}{q}\frac{\Lambda}{m^5}v^5, \tag{1}$$

where $v = (\pi m f)^{1/3}$ is the orbital velocity, f is the GW frequency,

$$\Lambda = \frac{1}{26}\left(\left(1+\frac{12}{q}\right)\lambda_1 + (1+12q)\lambda_2\right) \tag{2}$$

is the weighted tidal deformability, whereas $\lambda_i = \frac{2}{3}m_i^5 k_i$ and k_i are the tidal deformability and the (dimensionless) TLN of the ith object, respectively. These can be defined in terms of the quadrupole moment $Q_{ab}^{(i)}$ of the ith object induced by the tidal field $G_{ab}^{(j)}$ produced by its companion, namely

$$Q_{ab}^{(i)} = \lambda_i G_{ab}^{(j)} \sim \lambda_i \frac{m_j}{r^3}, \quad i \neq j, \tag{3}$$

where $r \sim m v^{-2}$ is the orbital distance. For a typical neutron star, $k_i \approx 1000$ and $\Lambda/m_i^5 \approx 600$, the exact values depend on the equation-of-state. As a rule of thumb, the more compact an object, the smaller its TLN, so much so that a BH has $k_{\rm BH} = 0$.

It is enlightening to expand Eq. (1) in the extreme mass-ratio limit. Let us first consider the standard case in which the central object is a BH, so that $k_1 = 0$. In such case the first nonvanishing contribution is

$$\phi_{\rm tidal}(f) \sim -\frac{9}{2}k_2 v^5 \frac{1}{q^3} + \cdots, \qquad q \gg 1 \ \ (k_1 = 0), \tag{4}$$

which is proportional to the TLN of the *small* companion, and is suppressed by a q^{-3} factor. Therefore, for a typical EMRI with $q \approx 10^6$, the above term is negligibly small.

On the other hand, when $k_1 \neq 0$, the tidal phase *grows linearly* in q,

$$\phi_{\rm tidal}(f) \sim -\frac{3}{8}k_1 v^5 q + \cdots, \quad q \gg 1, \tag{5}$$

and is proportional to the TLN of the central object. It is remarkable that in this case the tidal phase enters at the leading order in the mass ratio just like the ordinary radiation-reaction term,

$$\phi_N(f) \sim \frac{3}{128} v^{-5} q + \cdots, \quad q \gg 1, \tag{6}$$

although the latter dominates at large binary separation, owing to the different scaling with the orbital velocity. The tidal phase contribution in Eq. (5) has the same scaling with q as the spin-induced quadrupolar deformations[17] and, as long as $k_1 \gtrsim 1/q$, it is even *larger* that the first-order correction due to the conservative part of the self-force (i.e. the self-interaction of a test-particle with its own gravitational field[18]), the latter being suppressed by a factor $1/q$ relative to Eq. (6).

The above intriguing result case be explained as follows. The GW phase can be obtained by solving for

$$\frac{d^2\phi(f)}{df^2} = \frac{2\pi}{\dot{E}}\frac{dE}{df}, \tag{7}$$

where E is the binding energy of the binary and $\dot{E}$ is the energy flux emitted in GWs. The TLNs enter both in conservative piece, $E(f)$, and in the dissipative piece, $\dot{E}$. To the leading order in post-Newtonian theory[19]

$$E(f) = -\frac{m_1}{2(1+q)} v^2 \left(1 - \epsilon_c \frac{6q(k_1 q^3 + k_2)}{(1+q)^5} v^{10}\right), \tag{8}$$

$$\dot{E}(f) = -\frac{32}{5}\frac{q^2}{(1+q)^4} v^{10}\left(1 + \epsilon_d \frac{4(q^4(3+q)k_1 + (1+3q)k_2)}{(1+q)^5} v^{10}\right), \tag{9}$$

where ϵ_c and ϵ_d are just book-keeping parameters for the correction to the conservative and dissipative term, respectively. Plugging this into Eq. (7) and solving at the leading order in the corrections, one finds

$$\phi(f) = \phi_N(f)(1 - 16\epsilon_d k_1 v^{10}), \quad q \gg 1, \tag{10}$$

whereas the correction coming from the conservative term is subleading. It is straightforward to check that the above equation yields Eq. (5). Thus, the enhancement of the tidal effect in the waveform is due to the contribution of the TLNs to the energy flux. When $k_1 \neq 0$, this term is not suppressed by any power of $1/q$ relative to the leading-order term, as evident from Eq. (9). In turn, this result can be obtained straightforwardly by using Eq. (3) and the quadrupole formula, $\dot{E} \sim \partial_t^3 Q_{ij}^{\rm tot} \partial_t^3 Q_{ij}^{\rm tot}$, where $Q_{ij}^{\rm tot}$ is the total quadrupole of the binary.

Let us now discuss the phenomenological implications of this enhancement. We consider an EMRI up to the innermost stable circular orbit (ISCO) of the central object. The total GW phase accumulated between $f_{\rm min}$ and $f_{\rm max} \sim f_{\rm ISCO} = \frac{1}{6\pi\sqrt{6}m_1} \gg f_{\rm min}$ due to the tidal deformability is simply

$$\phi_{\rm tidal}^{\rm tot} = -\frac{k_1}{96\sqrt{6}}q \approx -0.004k_1q. \qquad (11)$$

If for instance $q = 10^7$, by requiring a detectability threshold $\phi_{\rm tidal}^{\rm tot} > 1\,{\rm rad}$, we find that the effect might be measurable even for TLNs as small as $k_1 \approx 2 \times 10^{-5}$.

This bound is quite impressive at least for two reasons. First of all, it suggests that the future LISA mission,[20] which is expected to detect few to thousands EMRIs per year,[16,21] could set constraints on the TLN of the central object which are approximately *eight orders of magnitude smaller* than LIGO's current measurements on the tidal deformability of a neutron star.[3,4] Furthermore, in the most extreme models of horizonless compact objects[6] — some of which predicting quantum corrections at the horizon scale — the TLNs are of the order $k_1 \approx 10^{-3}$, well above the bound estimated in Eq. (11). Finally, it is easy to show that the relative measurement errors on k_1 scale as $q^{-1/2}$ in the high signal-to-noise ratio limit. Thus, EMRIs detectable by LISA might provide the ultimate tests for exotic compact objects (see Ref. 22 for a review).

Besides the aforementioned bounds on BH mimickers, the enhancement of the tidal phase in an EMRI might be relevant if primordial BHs with masses $m_2 \approx 10^{-4}\,M_\odot$ exist in nature. These objects might form an EMRI around a neutron star and pass through LIGO's band in less than a year before plunge. In such case the tidal phase would provide an unparalleled way to constrain the neutron-star equation-of-state through a measurement of the TLN at the level given by Eq. (11). Unfortunately, the event rates for neutron-star capture of primordial BHs seem extremely small in this mass range.[23]

Our derivation is based on a low-velocity expansion of the field equations, and the post-Newtonian series converges poorly in the large mass-ratio limit, at least in its dissipative sector.[24] Therefore, Eq. (10) cannot be used for a rigorous parameter estimation, but it should nonetheless provide the correct order of magnitude of the effect of the tidal deformability of the central object. We conclude that it would be very important to incorporate the tidal deformability terms consistently in an extreme mass-ratio expansion of Einstein's field equation for binary systems beyond post-Newtonian theory.

All in all, Love might be at play even in extreme encounters.

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