

LONGITUDINAL MODE-COUPPLING INSTABILITY: GALACLIC VLASOV SOLVER VS. MACROPARTICLE TRACKING SIMULATIONS

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Abstract

Following the same approach as for the recently developed GALACTIC Vlasov solver in the transverse plane and taking into account the potential-well distortion, a new Vlasov solver, called GALACLIC, was developed for the longitudinal plane. In parallel, a new mode analysis was implemented for the post-processing of the results obtained through macroparticle tracking simulations. The results of the several benchmarks performed between the two methods are presented.

INTRODUCTION

GALACLIC (for Garnier-LAclare Coherent Longitudinal Instabilities Code) is a new Vlasov solver, which is discussed (and compared to GALACTIC, for Garnier-LAclare Coherent Transverse Instabilities Code) in Ref. [1]. The purpose of this paper is to compare the results from GALACLIC to the ones obtained from the macroparticle tracking simulation code SBSC [2] (as well as BLoND [3] and MuSIC [4]) for the two cases of a Constant Inductive (CI) and Broad-Band Resonator (BBR) impedances above transition, assuming a “Parabolic Line Density” (PLD) longitudinal distribution [5].

GALACLIC

In the case of a PLD longitudinal distribution [5], the effect of the Potential-Well Distortion (PWD), which is slightly different from the case of the “Parabolic Amplitude Density” (PAD) longitudinal distribution discussed in Ref. [1], is given by (with Q_s and Q_{s0} the intensity-dependent and low-intensity synchrotron tunes respectively and Q the coherent synchrotron tune)

$$\frac{Q}{Q_{s0}} = \frac{Q}{Q_s} \times F_{PWD} \quad \text{with} \quad F_{PWD} = \frac{Q_s}{Q_{s0}} = \frac{1}{\sqrt{1 - \frac{3}{4}x}} \quad (1)$$

where x is a normalised parameter proportional to the bunch intensity given by

$$x = \frac{\text{Im} \left[\frac{Z_l(p)}{p} \right]_{p=0} 4 I_b}{\pi^2 B^3 \hat{V}_T h \cos \phi_s} \quad (2)$$

Here, the simplified case of a constant shape of the longitudinal distribution was assumed and $Z_l(p)/p$ is the longitudinal impedance (at the bunch spectrum line p), $I_b = N_b e f_0$ the bunch current (with e the elementary charge, N_b the number of charges and f_0 the revolution frequency), $B = f_0 \tau_b$ the bunching factor with τ_b the full (4σ) bunch length, \hat{V}_T the total (effective) peak volt-

age, h the harmonic number and ϕ_s the RF phase of the synchronous particle ($\cos \phi_s > 0$ below transition and $\cos \phi_s < 0$ above). It is important to note that B , \hat{V}_T and ϕ_s depend on the bunch intensity due to the PWD. The cases of CI and BBR impedances, above transition and taking into account PWD, are depicted on Figs. 1 and 2 respectively, considering the same numerical values as the ones used for the SBSC simulations discussed in the next section.

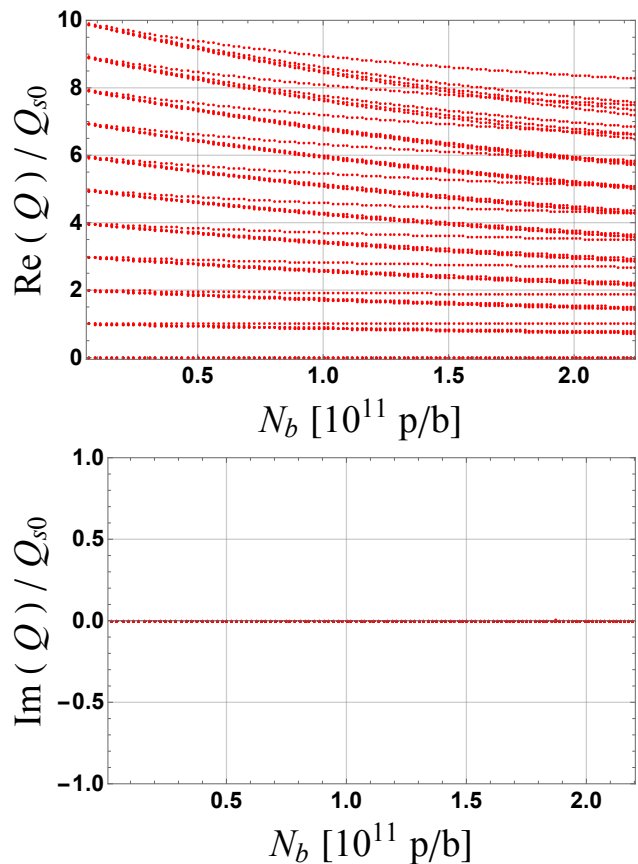


Figure 1: Normalised (to the low-intensity synchrotron tune) mode-frequency shifts from GALACLIC in the case of a CI impedance, above transition, taking into account the PWD and for a PLD longitudinal distribution [5], with the parameters mentioned below: (upper) real part and (lower) imaginary part.

SBSC

The SBSC code is a macroparticle tracking code (Single-Bunch Simulation Code) [2] for the longitudinal plane. The beam and machine parameters used for the benchmarks of this paper are the following (close to the CERN SPS case): the relativistic mass factor is $\gamma = 27.73$, the relativistic mass factor at transition is $\gamma_{tr} = 22.77$, the

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machine circumference is $C = 6911$ m, the peak RF voltage is $V_{RF} = 6$ MV, the harmonic number is $h = 462$ (instead of 4620 used in the CERN SPS, to be in a linear RF system and not mix other possible effects from the nonlinear RF system), the full bunch length (4σ) is $\tau_b = 2.7$ ns and the low-intensity synchrotron tune is $Q_{s0} = 3.26 \times 10^{-3}$. As concerns the impedance, a BBR model is considered, with a quality factor of 1, a resonance frequency f_r such that $f_r \tau_b = 2.7$: $f_r = 1$ GHz, and $\text{Im}[Z_l(p)/p] = 8.67 \Omega$ at low frequency. The case of a CI impedance corresponds to the case where the resonance frequency f_r is equal to infinity.

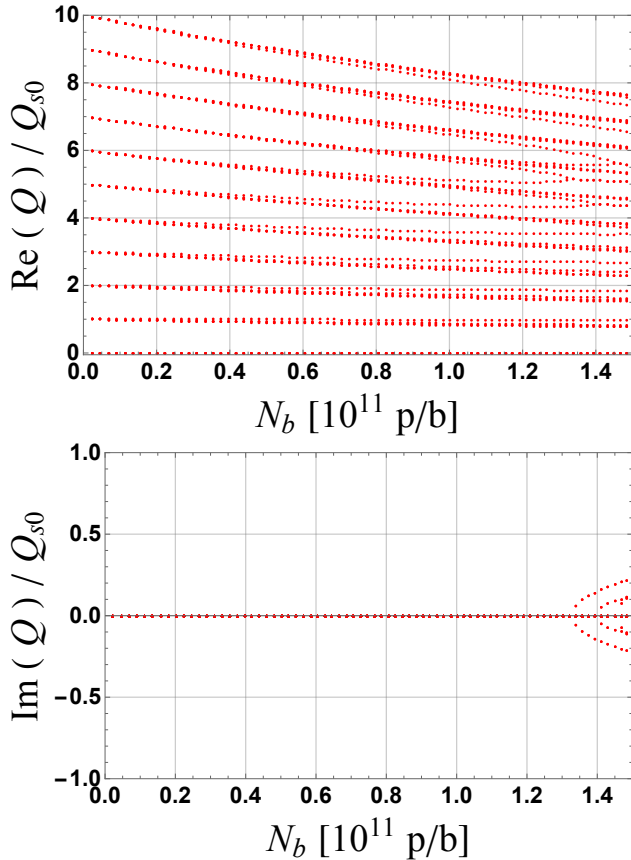


Figure 2: Normalised (to the low-intensity synchrotron tune) mode-frequency shifts from GALACLIC in the case of a BBR impedance (with a quality factor of 1 and a resonance frequency f_r such that $f_r \tau_b = 2.7$), above transition, taking into account the PWD and for a PLD longitudinal distribution [5], with the parameters mentioned above: (upper) real part and (lower) imaginary part.

The initial stationary distribution, taking into account collective effects for protons, has been obtained with BLonD [3] and a good agreement has been reached between SBSC and BLonD (and MuSIC), as can be seen from Fig. 3 revealing clearly the intensity threshold of the longitudinal “microwave instability” at $\sim 1.2 \times 10^{11}$ p/b for the case of the BBR impedance. In the case of CI impedance, no instability is observed as predicted from

GALACLIC (see Fig. 1). It is worth noting also from Fig. 3 that a perfect agreement has been obtained between GALACLIC and the macroparticle simulation codes for the bunch lengthening due to the PWD (see red point).

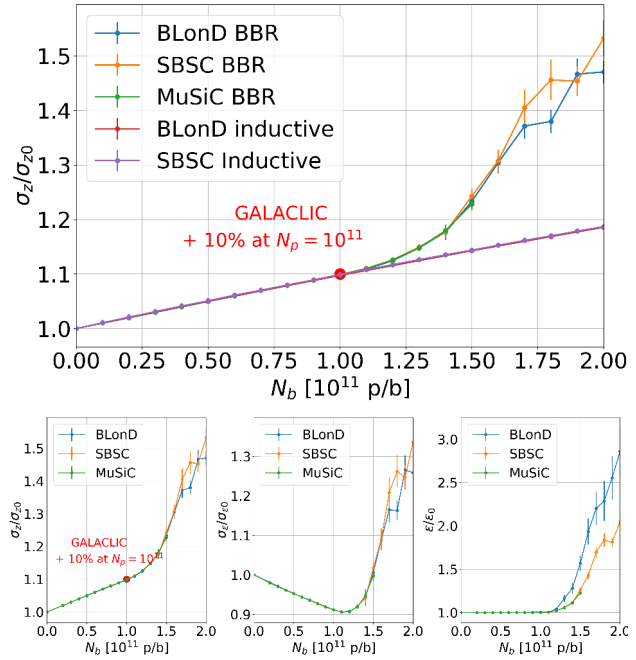


Figure 3: Simulation results from BLonD, SBSC and MuSIC codes with the parameters mentioned above: (upper) evolution of the normalised rms bunch length vs. bunch intensity for the cases of BBR and CI impedances; (lower) evolution of the normalised rms bunch length, energy spread and longitudinal emittance vs. bunch intensity for the case of the BBR impedance.

To analyse this instability in more detail, a new mode analysis was implemented for the post-processing of the results obtained through macroparticle tracking simulations, considering the momenta of the longitudinal distribution defined by

$$\bar{z} = \int_{-\infty}^{+\infty} z \lambda(z) dz \quad \text{and} \quad \sigma_k = \int_{-\infty}^{+\infty} (z - \bar{z})^k \lambda(z) dz. \quad (3)$$

The FFT of $\sqrt[k]{\sigma_k}$ is normalized to its maximum and for each bunch intensity the quantities are summed. The result is depicted in Fig. 4 as well as the intensity threshold deduced from Fig. 3. Some mode-coupling could be guessed but this is not easy to say from Fig. 4 alone.

GALACLIC VS. SBSC

Superimposing the plots from GALACLIC and SBSC, as shown in Fig. 5, a good agreement is obtained for both cases of a CI impedance and the BBR model, even if for the latter some slight shift is observed for the higher-order modes. This would need to be investigated in more detail in the future but it should be reminded that the simplest

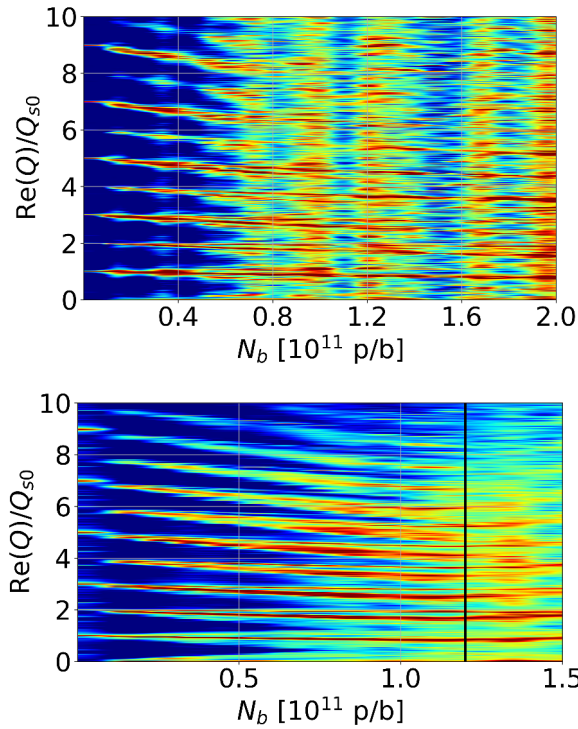


Figure 4: New mode analysis from SBSC, as described in Eq. (3), for the case of CI impedance (upper) and BBR impedance (lower) discussed before.

model of PWD was used here, which assumes that the shape of the longitudinal distribution does not change and that only the bunch length changes with the bunch intensity. The model could be refined in the future to take into account the variation of the bunch profile with intensity. However, the agreement seems already sufficiently good to state that the longitudinal “microwave instability” observed in Fig. 3 is a Longitudinal Mode-Coupling Instability (LMCI) of high-order modes (6 and 7).

SIMPLE FORMULA

The instability threshold deduced from GALACLIC (see Fig. 2 lower) corresponds to $x_{th} \approx -0.75$ [1] and the bunch intensity threshold is given by

$$N_b^{th} = \frac{|x_{th}| \pi^2}{4} \frac{B_0^3 V_{RF} h}{e f_0 \left| \frac{Z_L(\omega)}{p} \right|_{p=0}} \frac{B_0}{B}, \quad (4)$$

with

$$\left(\frac{B_0}{B} \right)_{PLD} = \left(1 - \frac{3}{4} x_{th} \right)^{-1/4}, \quad (5)$$

which leads to a numerical value of $\sim 1.2 \times 10^{11}$ p/b, i.e. very close to the one found from SBSC (see Fig. 3). It is worth noting that Eq. (4) is very similar to the simple formula obtained in Ref. [6], which was based on the mode-coupling between the two modes overlapping the peak of the real part of the impedance, taking into account also the PWD with a PLD longitudinal distribution: the

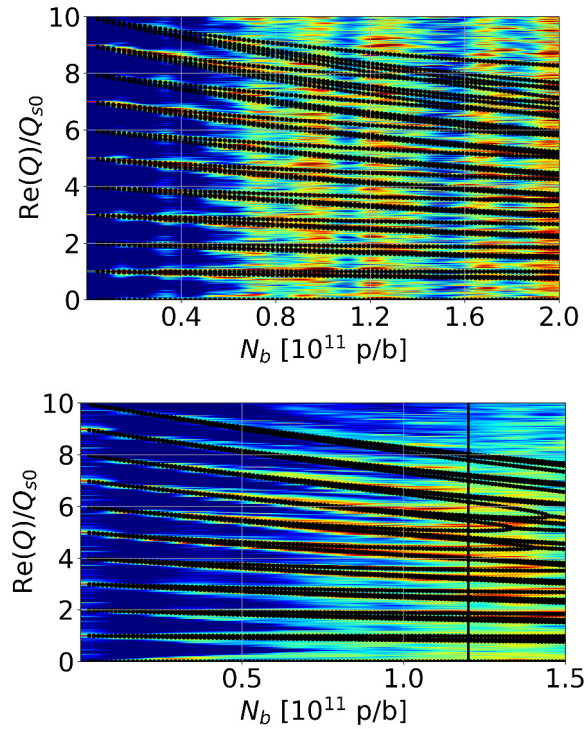


Figure 5: Real part of the normalised mode-frequency shifts: comparison between GALACLIC (black lines) and SBSC for the cases of CI (upper) and BBR (lower) impedances.

same formula was obtained, with $|x_{th}| \approx 0.75$ replaced by 1. This formula corresponds to the Keil-Schnell-Boussard criterion (i.e. the Keil-Schnell criterion for coasting beams applied with peak values for bunched beams as proposed by Boussard) [5,7].

CONCLUSION

A good agreement has been reached between the new GALACLIC Vlasov solver and the SBSC longitudinal macroparticle tracking code (as well as BLoND and MUSIC) for the two cases of CI and BBR impedances above transition. For the BBR impedance model, the longitudinal “microwave instability” observed in Fig. 3 has been undoubtedly explained by a Longitudinal Mode-Coupling Instability (LMCI), whose intensity threshold is very close to the Keil-Schnell-Boussard criterion.

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